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ROBUST CONTROLLER DESIGN WITH HARD INPUT CONSTRAINTS. TIME DOMAIN APPROACH

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Abstract. *A novel approach to robust controller design with hard input constraints is presented. The proposed robust controller design procedure is based on the robust stability condition developed using Affine or Parameter-Dependent Quadratic Stability approach. The obtained feasible design procedures are in the form of BMI or LMI. The obtained design results and their properties are illustrated in simulation examples.*
Keywords: Robust controller, Affine, Parameter-dependent Lyapunov function, Hard input constraints

1. **Introduction.** All control actuation devices are subject to magnitude and/or rate limits and this leads to degradation of the performance and even instability of closed loop control systems. Hard input constraints belong to the very important task in the controller design. A chronological bibliography on saturating actuators can be found in [2,7]. Necessary and sufficient conditions for controllability of linear systems subject to input/state constraints are given in [5]. Robust stabilization of uncertain systems subject to input constraints is considered in [6,14]. A piecewise linear control ensuring the input constraints is parameterized by algebraic Riccati equation. Youla parametrization of uncertain plant and stabilizing controllers which guarantee prescribed hard bounds on the control signal can be found in [9]. The use of a two stage IMC anti-windup design for stable plant and input constraints is described at [1]. The invariant set and an algorithm approach similar to Soft Variable Structure Control ensuring soft input constraints for the model predictive plant control systems are described by [15]. In the papers by [10,11] deal with the design of linear systems with saturating actuators, where the actuator limitations have to be incorporated into control design. The semiglobal approach includes stabilization, input additive disturbance rejection, and robust stabilization. In the book by [12] methods are proposed and algorithms are designed on the basis of the state space approach to overcome the effects of actuator saturation ensuring the global stability of closed loop systems. Book by [13] is devoted to these directions: first, anti-windup strategies, a second model predictive control with constraints and finally development of a stability and stabilization method for constrained systems. The paper [16] investigated the problem of decentralized robust stabilization of a class of uncertain nonlinear systems with input saturation. Robust constrained control for MIMO nonlinear systems can be found in [3].

The above short survey implies that in the field of robust controller design with input constraints there are many different approaches. The above observation motivated us to study the following research problem which has not been sufficiently solved yet. Design the robust controller with hard input constraints which will guarantee:

- stability and robustness properties of the closed loop system when the uncertain plant parameters $\Pi \in \Omega$ lie in the given polytopic (convex) uncertainty box;
- guaranteed cost (performance) for the closed loop system for all $\Pi \in \Omega$;
- Affine or Parameter-Dependent Quadratic Stability;
- the BMI or LMI design procedure.

In this paper, we provide, to the authors' best knowledge, an alternative novel approach to the robust controller design with hard input constraints. The proposed uncertainty model, introduced in Section 2, is used to formulate a robust control problem with hard input constraints. The main results are presented in Section 3. The design procedure is based on the new developed robust stability condition for the hard input constraints controller and numerical examples in Section 4 illustrate the effectiveness of the proposed approaches. Some conclusions are made in Section 5.

2. **Problem Formulation and Preliminaries.** This paper is concerned with the class of uncertain linear systems that can be described as

$$\begin{aligned} \dot{x} &= \left(A_0 + \sum_{i=1}^{p} A_i\theta_i\right)x + \left(B_0 + \sum_{i=1}^{p} B_i\theta_i\right)u \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^l$ are the state, control input and controlled output, respectively; matrices A_i, B_i, $i = 0, 1, \ldots, p$ are constants with appropriate dimensions; $\theta = [\theta_1, \ldots, \theta_p]$ is a vector of uncertain and possible time varying real parameters satisfying $\theta \in \langle \underline{\theta} \quad \overline{\theta} \rangle \in \Omega$, $\dot{\theta} \in \left\langle \underline{\dot{\theta}} \quad \overline{\dot{\theta}} \right\rangle \in \Omega_t$ which belong to the known boundaries.

There are two possibilities for θ parameter changes:

- parameter θ is unknown but time varying,
- parameter θ is unknown and constant.

For the first case the modified affine quadratic stability will be adapted to obtain the closed loop stability conditions with input constraints and for the second case the modified parameter dependent quadratic stability will be used.

For the class of uncertain systems (1) the following problem is studied in this paper.

Design a robust static output feedback controller with control algorithm

$$u = F(\theta_0)y = F(\theta_0)Cx \tag{2}$$

with hard constraints $|u_j| \le u_{Mj}$, $j = 1, 2, \ldots, m$ such that for the closed-loop system (3) the designed controller guarantees robust affine or parameter dependent quadratic stability and guaranteed cost with respect to the performance index (4)

$$\dot{x} = (A(\theta) + B(\theta)F(\theta_0)C)x = A_c x \tag{3}$$

where $F(\theta_0) = F\theta_0$, $\theta_0 = diag\{\theta_{0j}\}$, $j = 1, 2, \ldots, m$, $\left[\theta_0 \in \langle \underline{\theta_0} \quad \overline{\theta_0} \rangle, \dot{\theta}_0 \in \left\langle \underline{\dot{\theta}_0} \quad \overline{\dot{\theta}_0} \right\rangle\right] \in \Phi$ is known time varying parameter which serve for ensure the hard input constraints. To assess the performance quality, the following performance index is associated with system (1)

$$J = \int_0^\infty \left(x^T Q x + u^T R u + \dot{x}^T S \dot{x}\right) dt = \int_0^\infty J(t)dt \tag{4}$$

The respective notion of guaranteed cost is given in the next definition.

Definition 2.1. *Consider system (1) and controller (2). If there exist a control law u^* and a positive scalar J^* such that the respective closed-loop system (3) is stable and the value of the closed-loop cost function (4) satisfies $J \leq J^*$, then J^* is said to be the guaranteed cost and u^* is said to be the guaranteed cost control law for system (1).*

Recall the well known result from LQ theory which will be used below to prove one of the main results.

Lemma 2.1. [8] *Consider the system (1) with control algorithm (2). Control algorithm (2) is the guaranteed cost control law for the closed-loop system (3) if and only if there exists a Lyapunov function $V(\theta)$ such that for all $\theta \in \Omega$, $\dot{\theta} \in \Omega_t$ and θ_0, $\dot{\theta}_0$, the following condition holds*

$$B_e(\theta) = \left\{ \frac{dV(\theta)}{dt} + J(t) \right\} = -\varepsilon x^T x \quad \varepsilon \to 0 \quad \varepsilon \geq 0 \tag{5}$$

Uncertain system (1) with control algorithm (2) conforming to Lemma 2.1 is called robust stable with guaranteed cost. Note that for concrete structure of $V(\theta)$ "if and only if" may to be decreased to "if".

3. **Main Results.**

3.1. **Affine Quadratic Stability.**

This subsection formulates the theoretical approach to the robust static output feedback controller design for uncertain system (1) which ensures closed-loop system affine quadratic-stability and guaranteed cost (4), for all uncertain plant parameters $\Pi \in \Omega$ and hard input constraints.

Definition 3.1. [4] *The linear time varying system (3) is affine quadratically stable if there exist $p+2$ symmetric matrices $P, P_0, P_1, \ldots, P_p$ such that*

$$P(\theta) = P + P_0\theta_0 + \sum_{i=1}^{p} P_i\theta_i > 0 \tag{6}$$

$$\frac{dV(\theta)}{dt} = x^T \left(A_c^T P(\theta) + P(\theta)A_c + P_0\dot{\theta}_0 + \sum_{i=1}^{p} P_i\dot{\theta}_i \right) x \leq 0 \tag{7}$$

for all $\theta \in \Omega$, $\dot{\theta} \in \Omega_t$, $\left(\theta_0, \dot{\theta}_0\right) \in \Phi$.

Then for the closed loop system, Lyapunov function holds

$$V(\theta) = x^T P(\theta) x$$

Equation (7) can be rewritten as follows

$$\frac{dV(\theta)}{dt} = \begin{bmatrix} \dot{x}^T & x^T \end{bmatrix} \begin{bmatrix} 0 & P(\theta) \\ P(\theta) & P\left(\dot{\theta}\right) \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \end{bmatrix} \tag{8}$$

$P\left(\dot{\theta}\right) = P_0\dot{\theta}_0 + \sum_{i=1}^{p} P_i\dot{\theta}_i$.

To isolate two matrices $P(\theta)$ and A_c using auxiliary matrices $N_1, N_2 \in R^{n\times n}$ from (3) we obtain

$$(2N_1\dot{x} + 2N_2x)^T(\dot{x} - A_cx) = 0$$

or

$$\begin{bmatrix} \dot{x}^T & x^T \end{bmatrix} \begin{bmatrix} N_1^T + N_1 & -N_1^T A_c + N_2 \\ -A_c^T N_1 + N_2^T & -N_2^T A_c - A_c^T N_2 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \end{bmatrix} = 0 \tag{9}$$

Substitute control algorithm (2) to performance (4)

$$J(t) = \begin{bmatrix} \dot{x}^T & x^T \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & Q + C^T F^T R F C \theta_0^2 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \end{bmatrix} \tag{10}$$

Substituting (8), (9) and (10) to (5) one obtains

$$B_e = \begin{bmatrix} \dot{x}^T & x^T \end{bmatrix} W \begin{bmatrix} \dot{x}^T & x^T \end{bmatrix}^T \leq 0 \tag{11}$$

where

$$W = W_{01} + W_{02}\theta_0 + W_{03}\theta_0^2 + \sum_{i=1}^{p} (W_{i1} + W_{i2}\theta_0)\theta_i \tag{12}$$

$$W_{01} = \begin{bmatrix} N_1^T + N_1 + S & -N_1^T A_0 + N_2 + P \\ * & -N_2^T A_0 - A_0^T N_2 + Q + P\left(\dot{\theta}\right) \end{bmatrix}$$

$$W_{02} = \begin{bmatrix} 0 & -N_1^T B_0 F C + P_0 \\ * & -N_2^T B_0 F C - (B_0 F C)^T N_2 \end{bmatrix}$$

$$W_{03} = \begin{bmatrix} 0 & 0 \\ 0 & C^T F^T R F C \end{bmatrix}$$

$$W_{i1} = \begin{bmatrix} 0 & -N_1^T A_i + P_i \\ P_i - A_i^T N_1 & -N_2^T A_i - A_i^T N_2 \end{bmatrix}$$

$$W_{i2} = \begin{bmatrix} 0 & -N_1^T B_i F C \\ -(B_i F C)^T N_1 & -N_2^T B_i F C - (B_i F C)^T N_2 \end{bmatrix}$$

$i = 1, 2, \ldots, N$.

The first main results for design of robust controller which ensure the Affine Quadratic Stability with input constraints are summarized in the next theorem.

Theorem 3.1. *Consider the uncertain systems governed by (1) with robust control algorithm (2) and hard input constraints* $|u_j| \leq u_{Mj}$*,* $j = 1, 2, \ldots, m$*. Closed loop system (3) is robust affine quadratically stable with guaranteed cost if there exist auxiliary matrices* N_1*,* N_2*,* $p+2$ *symmetric matrices* $P, P_0, P_1, \ldots, P_p$ *and output feedback gain matrix* F *such that inequality*

$$W \leq 0 \tag{13}$$

holds for all $\theta \in \Omega$*,* $\dot{\theta} \in \Omega_t$ *and* $\left(\theta_0, \dot{\theta}_0\right) \in \Phi$*.*

Variable θ_0 will guarantee the hard input constraints $|u_j| \leq u_{Mj}$, $j = 1, 2, \ldots, m$ if it is calculated as

$$\theta_{0cj} = \begin{cases} 1 & \text{if } |u_j| < u_{Mj}, \text{ for all } j = 1, 2, \ldots, m \\ \dfrac{u_{Mj}}{\max_j |u_j|} & \text{if for some } |u_j| > u_{Mj},\ j = 1, 2, \ldots, m \end{cases} \tag{14}$$

To ensure that rate of θ_0 change within $\dot{\theta}_0 \in \left\langle \underline{\dot{\theta}_0}, \overline{\dot{\theta}_0} \right\rangle$ and to obtain the real value of θ_0 it is recommended to use a first order filter with transfer function $\theta_{0j} = \frac{\theta_{0cj}}{\left(\dot{\theta}_{0j}\right)^{-1} s + 1}$.

Proof: The proof of sufficient conditions are based on Equations (6), (8), (9) and (10).

Because matrix $W_{03} \geq 0$, matrix W with respect to θ_0, θ_i, $i = 1, 2, \ldots, p$ is convex, matrix W is negative definite (semidefinite) if and only if it takes negative definite (semidefinite) at the corners of θ_0 and θ. For robust stability analysis (13) reduces to LMI; for robust control synthesis we obtain BMI.

3.2. **Parameter Dependent Quadratic Stability.** This subsection formulates the theoretical approach to the robust static output feedback controller design for system (1) which ensures closed loop system parameter dependent quadratic stability and guaranteed cost (4), for all uncertain plant parameters $\Pi \in \Omega$ and hard input constraints. When one substitutes to Equation (1) maximal and minimal value of θ a polytopic system model with $N = 2^p$ vertices is obtained in the form.

$$\{(A_{1v}, B_{1v}, C), (A_{2v}, B_{2v}, C), \dots, (A_{Nv}, B_{Nv}, C)\}, \quad N = 2^p \tag{15}$$

or

$$\{A(\alpha), B(\alpha)\} = \sum_{i=1}^{N} (A_{iv}, B_{iv})\alpha_i, \quad \sum_{i=1}^{N} \alpha_i = 1, \quad \alpha_i \geq 0$$

$$\sum_{i=1}^{N} \dot{\alpha}_i = 0$$

where $\alpha_i \in \langle 0 \quad 1 \rangle$, $i = 1, 2, \dots, N$ is uncertain constant or time varying parameter. The parameter dependent Lyapunov function is given as follows

$$V(\alpha) = x^T P(\alpha) x, \quad P(\alpha) = P_0\theta_0 + \sum_{i=1}^{N} P_{iv}\alpha_i \tag{16}$$

From (16) and (8) one obtains the time derivative of the Lyapunov function for polytopic systems as follows

$$\frac{dV(\alpha)}{dt} = \begin{bmatrix} \dot{x}^T & x^T \end{bmatrix} \begin{bmatrix} 0 & P_0\theta_0 + \sum_{i=1}^{N} P_{iv}\alpha_i \\ P_0\theta_0 + \sum_{i=1}^{N} P_{iv}\alpha_i & P_0\dot{\theta}_0 + \sum_{i=1}^{N} P_{iv}\dot{\alpha}_i \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \end{bmatrix} \tag{17}$$

The closed-loop system can be obtained from (15) and (2)

$$\dot{x} = (A_{iv} + B_{iv}F\theta_0 C)x = A_{ci}x \quad i = 1, 2, \dots, N \tag{18}$$

Using the same procedure as for affine quadratic stability (9), (10) and (11) one obtains the closed loop system robust parameter-dependent quadratic stability conditions with guaranteed cost in the form

$$B_{ev} = \begin{bmatrix} \dot{x}^T & x^T \end{bmatrix} W_v \begin{bmatrix} \dot{x}^T & x^T \end{bmatrix}^T \leq 0 \tag{19}$$

where

$$W_v = \begin{bmatrix} N_1^T + N_1 + S & -N_1^T A_c(\alpha) + N_2 + P(\alpha) \\ * & \varphi(\alpha, \theta_0) \end{bmatrix}$$

where

$$\varphi(\alpha, \theta_0) = -N_2^T A_c(\alpha) - A_c(\alpha)^T N_2 + Q + P_0\dot{\theta}_0 + \sum_{i=1}^{N} P_{iv}\dot{\alpha}_i + C^T F^T R F C \theta_0^2$$

Inequality (19) is linear with respect to uncertain parameter α_i, and inequality (19) can be split to N inequalities as

$$W_i = W_{iq} + W_{il}\theta_0 + W_{ik}\theta_0^2 \leq 0, \quad i = 1, 2, \dots, N \tag{20}$$

where

$$W_{iq} = \begin{bmatrix} N_1^T + N_1 + S & -N_1^T A_{iv} + N_2 + P_{iv} \\ * & \varphi_i \end{bmatrix}$$

$$W_{il} = \begin{bmatrix} 0 & -N_1^T B_{iv}FC + P_0 \\ P_0 - (B_{iv}FC)^T N_1 & -N_2^T B_{iv}FC - (B_{iv}FC)^T N_2 \end{bmatrix}$$

$$W_{ik} = \begin{bmatrix} 0 & 0 \\ 0 & C^T F^T RFC \end{bmatrix}$$

$$\varphi_i = -N_2^T A_{iv} - A_{iv}^T N_2 + Q + P\dot{\theta}_0 + \sum_{i=1}^{N} P_{iv}\dot{\alpha}_i$$

Robust stability conditions are summarized in the following theorem.

Theorem 3.2. *Consider the uncertain linear system (1) or (15). The closed loop system (18) with control algorithm (2) is parameter dependent quadratically stable with input constraints* $|u_j| \leq u_{Mj}$, $j = 1, 2, \ldots, m$ *if there exist matrices* $N_1, N_2 \in R^{n\times n}$, $N+1$ *symmetric matrices* $P_0, P_{1v}, \ldots, P_{Nv}$ *such that* $P_0\theta_0 + \sum_{i=1}^{N} P_{iv}\alpha_i \geq 0$, $\sum_{i=1}^{N}\alpha_i = 1$, $\alpha_i \geq 0$, $\theta_0 \in \langle \underline{\theta_0} \quad \overline{\theta_0} \rangle$, $\dot{\theta}_0 \in \left\langle \underline{\dot{\theta}_0} \quad \overline{\dot{\theta}_0} \right\rangle$, *symmetric positive definite matrices* Q, R, S *and output feedback gain matrix* F *such that*

$$W_i = W_{iq} + W_{il}\theta_0 + W_{ik}\theta_0^2 \leq 0 \tag{21}$$

for $i = 1, 2, \ldots, N$, $\theta_0 = \underline{\theta}_0$ *and* $\theta_0 = \overline{\theta}_0$. *Variable* θ_0 *is calculated as in (14).*

Because matrix $W_{ik} \geq 0$, matrix W_i with respect to θ_0 is convex, matrix W_i is negative definite (semidefinite) if and only if it takes negative definite (semidefinite) at the corners of θ_0 and N vertices of polytopic system. For robust stability analysis (21) reduces to LMI; for robust control synthesis we obtain BMI.

Proof: The proof of Theorem sufficient conditions are based on the Equations (10), (15), (17) and (18).

4. **Examples.** In order to prove applicability of the proposed robust controller design procedure with input constraints, two examples are presented. The first example is simulation of randomly generated system near to the boundary of stability and the second one is slightly modified first example to make the system unstable. Two control algorithms are calculated for each of the two examples: for the first algorithm the affine quadratic stability approach is used and for the second one the parameter dependent quadratic stability. In the both cases the problem is to design two robust decentralized PI controllers which ensure the affine (parameter dependent) quadratic stability and guaranteed cost with input constraints. A level of hard input constraints is equal to 200 percent value of nominal input variable. The rate of θ_0 changes need to be more than 3. The states of third order system due to PI controller design need to be increased to five states.

Example 4.1. *Parameters of randomly generated example (1) are given as follows:*

$$A_0 = \begin{bmatrix} -1.1 & 1 & .5 & 0 & 0 \\ 0.4 & -0.435 & 0.2 & 0 & 0 \\ 0.3 & .25 & -1.85 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad B_0 = \begin{bmatrix} 0.1 & 1 \\ 1 & .1 \\ 0 & .1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

with eigenvalues $Eig(A_0) = 0; 0; 0.0301; -2.0152; -1.4$.

$$A_1 = \begin{bmatrix} -0.2 & 0 & 0.2 & 0 & 0 \\ 0.028 & 0 & 0.02 & 0 & 0 \\ 0.03 & 0.075 & -0.05 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0.02 & 0.2 \\ 0.05 & 0.03 \\ 0.025 & 0.01 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.1 & 0.05 & 0.07 & 0 & 0 \\ 0.1 & 0 & 0.03 & 0 & 0 \\ 0.025 & 0.1 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0.035 & 0.03 \\ 0.01 & 0.2 \\ 0.02 & 0.05 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and output matrix C

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For the parameters: performance $Q = qI$, $q = 0.01$, $R = rI_r$, $r = 1$, $S = sI$, $s = 0.001$; rates of uncertain parameters changes $\dot{\theta}_i = 0$, $\dot{\alpha}_i = 0$; input constraints $\theta_0 \in \langle .5, 1\rangle$, $\dot{\theta}_0 \in \langle -8, 8\rangle$; Lyapunov matrices constraints $P \in \langle 0.1 * I, ro * I\rangle$, $ro = 1000$; uncertainties $\theta_i \in \langle -1, 1\rangle$, $i = 1, 2$ the following two robust PI controllers are obtained
AQS

$$R_{11}(s) = -6.0907 - \frac{1.2702}{s}; \quad R_{12}(s) = -1.4507 + \frac{0.9607}{s}$$

PDQS

$$R_{11}(s) = -5.0674 - \frac{1.3597}{s}; \quad R_{12}(s) = -1.7537 + \frac{1.1603}{s}$$

For simulation we have used the nominal plant model and robust controller designed with AQS. Simulation results (Figures 1-5) confirm, that Theorem 3.1 holds. Figures 1 and 2 show the simulation results without input constraints and Figures 3-5 show simulation results with hard input constraints $|u_1| \leq 4$ and $|u_2| \leq 6$ which proves that Theorem 3.1 holds and guarantees hard input constraints. Figure 5 shows the calculated parameters θ_{01} and θ_{02} which change the corresponding controller gains for guarantee of the hard input constraints.

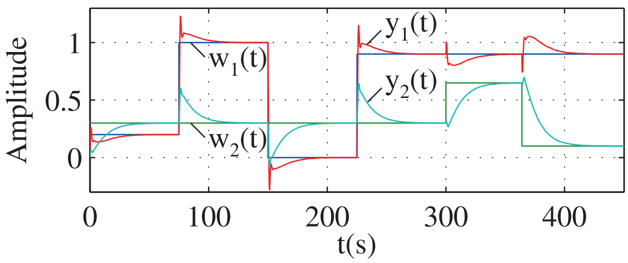

FIGURE 1. Simulations results $w(t)$, $y(t)$ without input constraints

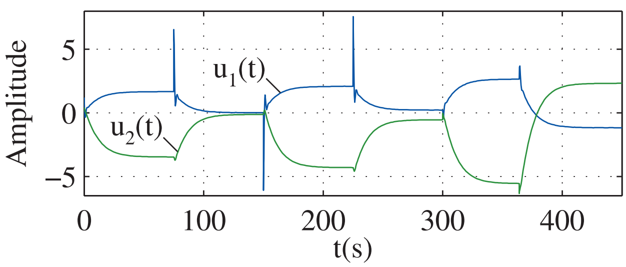

FIGURE 2. Controller output $u(t)$ without input constraints

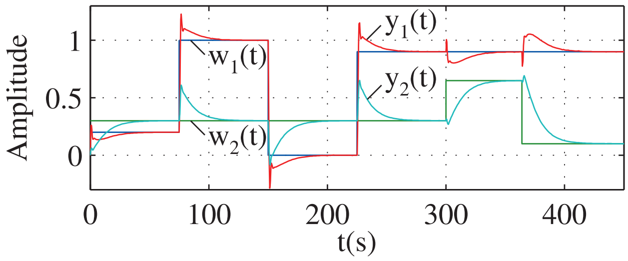

FIGURE 3. Simulations results $w(t)$, $y(t)$ with input constraints

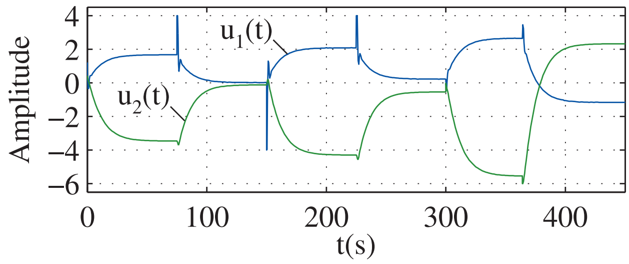

FIGURE 4. Controller output $u(t)$ with input constraints

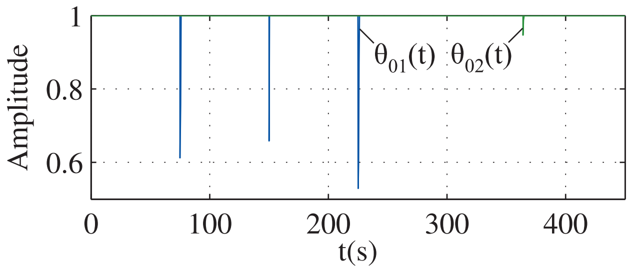

FIGURE 5. Calculated parameters $\theta(t)$

Example 4.2. *To obtain the system parameters for the second example we change the following entries* $A_0(2,2) = -0.135$, $A_0(3,3) = -0.85$ *in matrix* A_0 *(first example). The obtained eigenvalues of matrix* A_0 *for second example are* $Eig(A_0) = 0; 0; 0.3023; -0.8698; -1.5175$. *All other system parameters are the same as for the first example, except for* θ_0 *rates constraints* $\dot{\theta}_0 \in \langle -3, 3 \rangle$. *Parameters of two designed robust controllers are as follows*
AQS

$$R_{21}(s) = -5.1684 - \frac{0.5782}{s}; \quad R_{22}(s) = -0.6081 + \frac{0.6987}{s}$$

PDQS

$$R_{21}(s) = -4.587 - \frac{0.5175}{s}; \quad R_{22}(s) = -0.7636 + \frac{0.6809}{s}$$

For simulation we have used the plant model with $\theta_1 = 1$, $\theta_2 = -1$ *and the robust controller designed with PDQS. Simulation results (Figures 6-10) confirm, that Theorem 3.2 holds. Figures 6 and 7 show the simulation results without input constraints and Figures 8-10 show simulation results with input constraints* $|u_1| \leq 2$ *and* $|u_2| \leq 2$ *which proves that Theorem 3.2 holds and guarantees hard input constraints. Figure 10 shows the calculated scheduled parameters* θ_1 *and* θ_2.

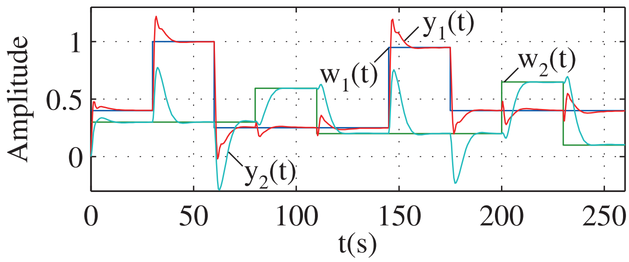

FIGURE 6. Simulations results $w(t)$, $y(t)$ without input constraints

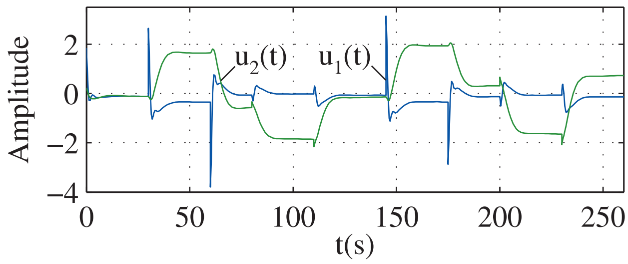

FIGURE 7. Controller output $u(t)$ without input constraints

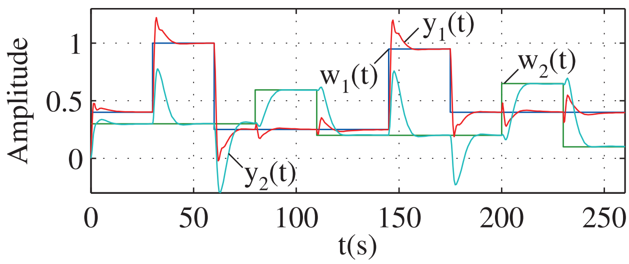

FIGURE 8. Simulations results $w(t)$, $y(t)$ with input constraints

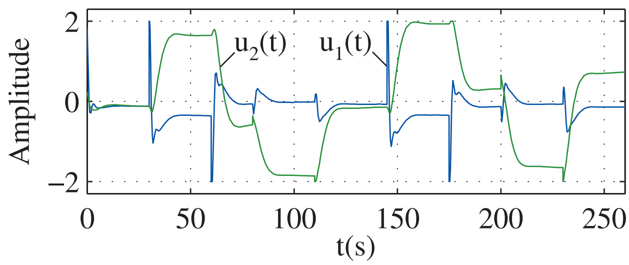

FIGURE 9. Controller output $u(t)$ with input constraints

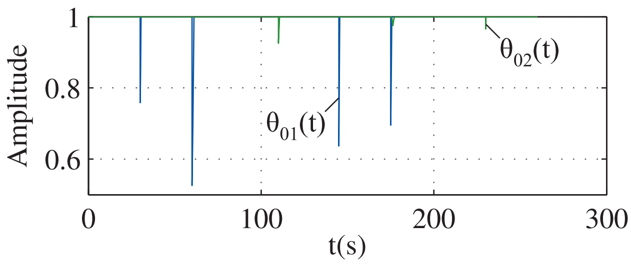

FIGURE 10. Calculated scheduled parameters $\theta(t)$

5. **Conclusions.** A novel robust controller design approach with hard input constraints has been proposed. The obtained results, illustrated on the two examples, show the applicability of the designed robust controller and its ability to cope with model uncertainties. Several forms of parameter dependent/affine quadratic Lyapunov functions are presented and tested by simulations. The proposed robust controller design approach with parameter dependent/affine Lyapunov function consider quick parameter changes to ensure the hard input constraints. Simulation results prove the potential ability of the designed closed-loop to withstand also these changes. The obtained robust controller design procedures are in the form of BMI and LMI approaches. The proposed approach contributes to the design tools for robust controllers which guarantee the affine/parameter dependent quadratic stability, guaranteed performance and hard input constraints.

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