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SINGLE INPUT FUZZY LOGIC CONTROLLER FOR FLEXIBLE JOINT MANIPULATOR

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Abstract. *Joint elasticity in the dynamics of robots manipulator makes the conventional model-based control strategies complex and difficult to synthesize. This paper presents investigations into the development of single input fuzzy logic controller (SIFLC) for tip angular position tracking and deflection angle reduction of a flexible joint manipulator system. A Quanser flexible joint manipulator system is considered and the dynamic model of the system is derived using the Euler-Lagrange formulation. The proposed method, known as the SIFLC, reduces the conventional two-input FLC (CFLC) to a single input single output (SISO) controller. Two parallel SIFLC are developed for both tip angular position and deflection angle control. The proposed control scheme is also compared with existing results by Ahmad et al., which are hybrid proportional-derivative (PD) with low-pass filter (LPF) and PD with non-collocated fuzzy logic control schemes. The performances of the control schemes are assessed in terms of tip angular tracking capability, level of deflection angle reduction and time response specifications. Finally, a comparative assessment of the control techniques is presented and discussed.*
Keywords: Flexible joint, Vibration control, Intelligent controller, Classical controller

1. **Introduction.** Recently, flexible joint manipulators have received a growing number of attention from many researchers due to its light weight, high manoeuvrability, flexibility, high power efficiency, and large number of applications. Nevertheless, controlling such systems still faces numerous degrees of difficulties that need to be addressed before they can be used in abundance in everyday real-life applications. The control issue of the flexible joint is to design the controller so that link of robot can reach a desired position or track a prescribed trajectory precisely with minimum deflection to the link. In order to achieve these objectives, various methods using different technique have been proposed, such as adaptive output-feedback controller based on a back stepping design [1-3], nonlinear control approach using namely feedback linearization technique and the integral manifold technique [4,5], robust control design [6,7]. However, their work requires a nonlinear control theory which needs a complicated mathematical analysis. To reduce the complexity of the controller design, hybrid control scheme is introduced which can be realized by utilizing control strategies consisting of either non-collocated with collocated feedback controllers [8] or feed-forward with feedback controllers [9,10].

On the other hand, intelligent schemes approaches including fuzzy logic controller (FLC) have also been proposed for controlling the flexible joint manipulator system by several researchers [11-13]. Although those modern control methods are very promising for flexible joint applications, they require substantial computational power because of complex decision making processes. However, it is possible to take full advantages of FLC for flexible joint application if the computational time of FLC is minimized. In this paper, the Single Input Fuzzy Logic Controller (SIFLC) is proposed. The SIFLC is a simplification of the conventional Fuzzy Logic Controller (CFLC). It is achieved by applying the signed distance method [14] where the input to SIFLC is only one variable known as "distance". This is in contrast to the CFLC which requires an error and the derivative (change) of the error as its inputs. The reduction in the number of inputs simplifies the rule table to one-dimensional, allowing it to be treated as a single input single output (SISO) controller. As SIFLC can be treated as SISO controller, it can be a practical controller for flexible joint manipulator system. As the objective of the controlling flexible joint system is to perform a horizontal rotary motion with very minimal vibration, two parallel SIFLC structures (comprise of two feedback loops) are considered. (1) The position error and the velocity of the tip angular position will be the input of the first SIFLC. (2) The deflection angle error and the velocity of the deflection angle will be the input to the second SIFLC. The objective of the design is to actuate the system to a certain angular position with zero deflection angle. To evaluate the effectiveness of the proposed controller, the performance results are compared with hybrid PD-LPF and hybrid PD with non-collocated fuzzy logic as reported in [10].

This paper is organized as follows. The next section provides a description of the linear model of flexible joint manipulator system in a state-space form. Section 3 is devoted to developing a tip angular tracking and deflection angle reduction control schemes for flexible joint manipulator system. Implementation results are shown in Section 4 and conclusions are drawn in Section 5.

2. **Modeling of Flexible Joint Manipulator.** The flexible joint manipulator system considered in this work is shown in Figure 1, where θ is the tip angular position and α is the deflection angle of the flexible joint. The base of the elastic joint manipulator which determines the tip angular position of the flexible link is driven by servomotor, while the flexible link will respond based on base movement. The deflection of link will be determined by the flexibility of the spring as their intrinsic physical characteristics.

This section provides a brief description on the modelling of the elastic joint manipulator system, as a basis of a simulation environment for development and assessment of the SIFLC control technique. The Euler-Lagrange formulation is considered in characterizing the dynamic behaviour of the system.

The linear model of the uncontrolled system can be represented in a state-space form [15] as shown in Equation (1), that is

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

with the vector $x = \begin{bmatrix} \theta & \alpha & \dot{\theta} & \dot{\alpha} \end{bmatrix}^T$ and the matrices A, B and C are given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{K_{stiff}}{J_{eq}} & \dfrac{-\eta_m \eta_g K_t K_m K_g^2 + B_{eq} R_m}{J_{eq} R_m} & 0 \\ 0 & \dfrac{-K_{stiff}\left(J_{eq} + J_{arm}\right)}{J_{eq} J_{arm}} & \dfrac{\eta_m \eta_g K_t K_m K_g^2 + B_{eq} R_m}{J_{eq} R_m} & 0 \end{bmatrix} \tag{2}$$

$$B = \begin{bmatrix} 0 & 0 & \dfrac{\eta_m \eta_g K_t K_g}{J_{eq} R_m} & \dfrac{-\eta_m \eta_g K_t K_g}{J_{eq} R_m} \end{bmatrix}, \quad C = [1 \quad 0 \quad 0 \quad 0]$$

In Equation (1), the input u is the input voltage of the servomotor, V_m which determines the elastic joint manipulator base movement. In this study, the values of the parameters are defined in Table 1.

TABLE 1. System parameters

Symbol	Quantity	Value
R_m	Armature Resistance (Ohm)	2.6
K_m	Motor Back-EMF Constant (V.s/rad)	0.00767
K_t	Motor Torque Constant (N.m/A)	0.00767
J_{link}	Total Arm Inertia (kg.m^2)	0.0035
J_{eq}	Equivalent Inertia (kg.m^2)	0.0026
K_g	High gear ratio	14:5
K_{stiff}	Joint Stiffness	1.2485
B_{eq}	Equivalent Viscous Damping (N.m.s/rad)	0.004
η_g	Gearbox Efficiency	0.9
η_m	Motor Efficiency	0.69

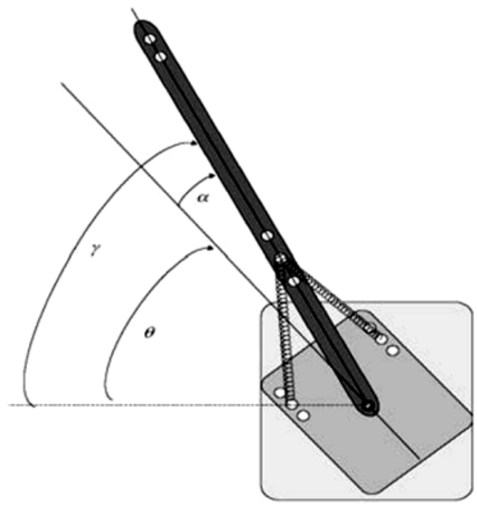


FIGURE 1. Flexible joint manipulator system

3. **Controller Design.** This section provides a description on the SIFLC, hybrid PD with LPF and hybrid PD with non-collocated fuzzy logic control design for flexible joint manipulator system. The main objective of those controllers is to achieve good performance in input tracking of tip angular position with minimal deflection angle.

3.1. **Single input fuzzy logic controller.** Fuzzy Logic controller (FLC) is a linguistic-based controller that tries to emulate the way human thinking in solving a particular problem by means of rule inferences. Typically, an FLC has two controlled inputs, namely error (e) and the change of error ($\dot{e}$). Its rule table can be created on a two-dimensional space of the phase-plane ($e, \dot{e}$) as shown in Table 2. It is common for the rule table to have the same output membership in a diagonal direction. Additionally, each point on the particular diagonal lines has a magnitude that is proportional to the distance from its main diagonal line L_Z. This is known as the Toeplitz structure. The Toeplitz property is true for all FLC types which use the error and its derivative terms, namely $\dot{e}$, $\ddot{e}$ ... and $e^{(n-1)}$ as input variables [16].

TABLE 2. Rule table with Toeplitz structure

$\dot{e}$ \ e	**PL**	**PM**	**PS**	**Z**	**NS**	**NM**	**NL**
NL	Z	NS	NM	***NL***	***NL***	***NL***	***NL***
NM	PS	Z	NS	NM	***NL***	***NL***	***NL***
NS	PM	PS	Z	NS	NM	***NL***	***NL***
Z	***PL***	PM	PS	Z	NS	NM	***NL***
PS	***PL***	***PL***	PM	PS	Z	NS	NM
PM	***PL***	***PL***	***PL***	PM	PS	Z	NS
PL	***PL***	***PL***	***PL***	***PL***	PM	PS	Z

By observing the consistent patterns of the output memberships in Table 2, there is an opportunity to simplify the table considerably. Instead of using two-variable input sets ($e, \dot{e}$), it is possible to obtain the corresponding output, u_0 using a single variable input only. The significance of the reduction was first realised by Choi et al. and is known as the signed distance method [14]. The method simplifies the number of inputs into a single input variable known as distance, d. The distance represents the absolute distance magnitude of the parallel diagonal lines (in which the input set of e and $\dot{e}$ lies) from the main diagonal line L_Z. To derive the distance, d variable, let $Q(e_0, \dot{e}_0)$ be an intersection point of the main diagonal line and the line perpendicular to it from a known operating point $P(e_1, \dot{e}_1)$, as illustrated in Figure 2.

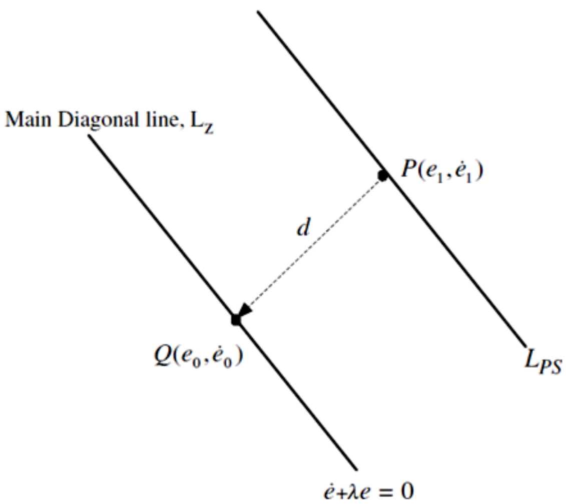


FIGURE 2. Derivation of distance variable

It can be noted that the main diagonal line can be represented as a straight line function, i.e.,

$$\dot{e} + \lambda e = 0 \tag{3}$$

In Equation (4), variable λ is the slope magnitude of the main diagonal line L_Z. The distance d from point $P(e_1, \dot{e}_1)$ to point $Q(e_0, \dot{e}_0)$, can be obtained as [16]:

$$d = \frac{\dot{e} + \lambda e}{\sqrt{1 + \lambda^2}} \tag{4}$$

The derivation of distance input variable resulted in a one-dimensional rule table, in contrast to a two-dimension table required by the conventional FLC. The reduced rule table is depicted in Table 3, where L_{NL}, L_{NM}, L_{NS}, L_Z, L_{PS}, L_{PM} and L_{PL} are the diagonal lines of Table 2. The diagonal lines correspond to the new input of this rule table, while NL, NM, NS, Z, PS, PM and PL represent the output of corresponding diagonal lines. As can be realized, the control action of FLC is now exclusively determined by d. It is therefore appropriate to call it the Single Input FLC (SIFLC).

The structure of SIFLC, derived from the signed distance method can be depicted as a block diagram in Figure 3. Two system state variables e (error) and $\dot{x}$ (velocity) are selected as the feedback signal. The input to the FLC block is the distance variable d, while the output from FLC block is the change of control output $\dot{u}_0$. The final output of this FLC is obtained by multiplying $\dot{u}_0$ with the output scaling factor, denoted as r. The output equation can be written as:

$$u = \dot{u}_0 r \tag{5}$$

Accordingly, the slope magnitude, λ was deduced as -1 while the output scaling factor, r for tip angular tracking and deflection angle were deduced as 78 and -5 respectively. Figure 4 shows the complete control structure of the proposed controller which consists of two parallel SIFLC.

TABLE 3. The reduced rule table using the signed distance method

d	L_{NL}	L_{NM}	L_{NS}	L_Z	L_{PS}	L_{PM}	L_{PL}
u_0	NL	NM	NS	Z	PS	PM	PL

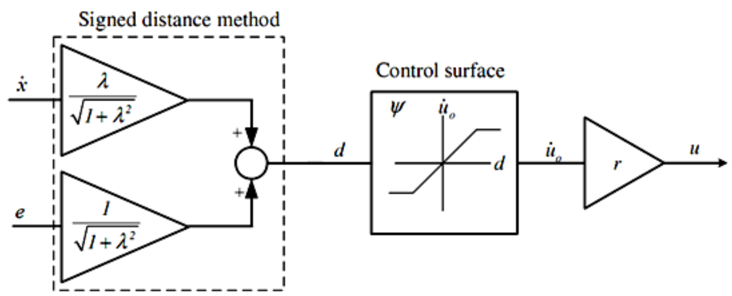


FIGURE 3. SIFLC structure for flexible joint manipulator with linear control surface

3.2. **Hybrid PD with non-collocated fuzzy logic controller.** A combination of collocated PD and non-collocated fuzzy logic control scheme for control of tip angular position and vibration suppression of the system respectively is presented in this section [10]. The control structure comprises two feedback loops. (1) The hub angle is as input to collocated PD for tip angular position control. (2) The deflection angle is as input

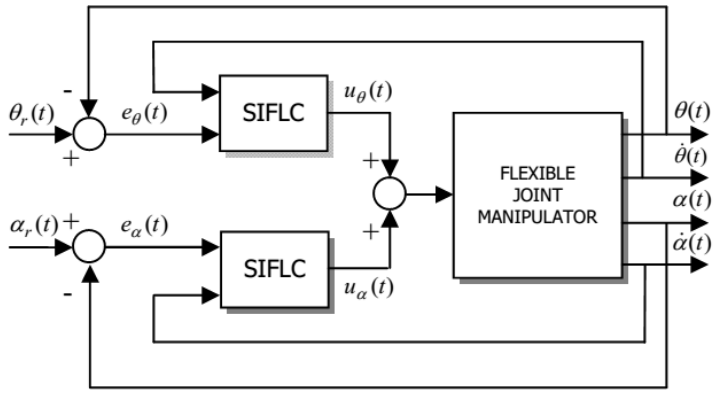


FIGURE 4. Block diagram of SIFLC structure for flexible joint manipulator

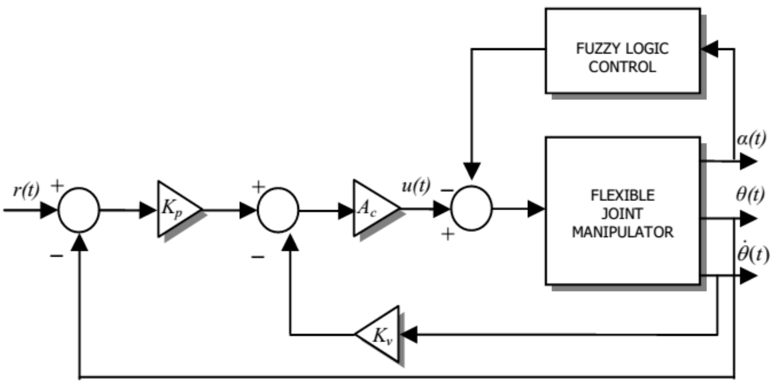


FIGURE 5. Collocated PD with non-collocated fuzzy logic control structure [10]

to a separate non-collocated control law for vibration control. These two loops are then summed together to give a torque input to the system. A block diagram of the composite PD and fuzzy logic control scheme is shown in Figure 5, where K_p and K_v are proportional and derivative gains, respectively, θ and $\dot{\theta}$ represent tip angular position and its velocity, respectively, $r(t)$ is the reference tip angular position and A_c is the gain of the motor amplifier. Here the motor/amplifier gain set is considered as a linear gain.

To design the PD controller, a linear state-space model of the flexible joint manipulator in (1) was utilized. The control signal $u(s)$ in Figure 5 can be written as

$$u(s) = A_c \left[K_p \left\{R_f(s) - \theta(s)\right\} - K_v s\theta(s)\right] \tag{6}$$

where s is the Laplace variable. The closed-loop transfer function is, therefore, obtained as

$$\frac{\theta(s)}{R_f(s)} = \frac{K_p H(s) A_c}{1 + A_c K_v \left(s + K_p/K_v\right) H(s)} \tag{7}$$

where $H(s)$ is the open-loop transfer function from the input torque to tip angular position, given by

$$H(s) = C(sI - A)^{-1}B \tag{8}$$

where A, B, and C are the characteristic matrix, input matrix and output matrix of the system, respectively, and I is the identity matrix. The closed-loop poles of the system are, thus, given by the closed-loop characteristics equation as

$$1 + A_c K_v \left(s + K_p/K_v\right) H(s) \tag{9}$$

TABLE 4. Fuzzy rules for vibration control

Deflection angle rate / Deflection angle		$\dot{\alpha}$				
		PB	PS	Z	NS	NB
α	PB	PB	PB	PB	NB	NB
	PS	PB	PS	PS	NS	NB
	Z	PB	PS	Z	NS	NB
	NS	PB	PS	NS	NS	NB
	NB	PB	PB	NB	NB	NB

where $Z = K_p/K_v$ represents the compensator zero which determines the control performance and characterizes the shape of root locus of the closed-loop system. In this study, the root locus approach is utilized to design the PD controller where the controller parameters are deduced as $K_p = 645.52$, $K_v = 150.77$ and $A_c = 2.22$.

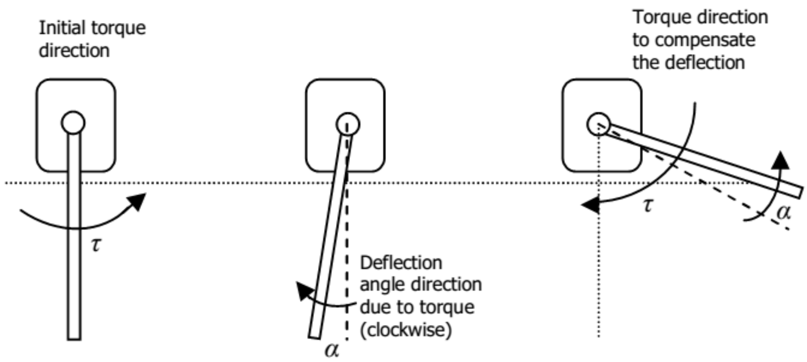
For tip angular position control, the collocated PD strategy developed in the previous section is adopted whereas for the vibration control loop, the deflection angle feedback through a non-collocated fuzzy logic control scheme is utilised. In designing the non-collocated fuzzy logic control, a basic triangle and trapezoidal forms are chosen for input and output membership functions. Please refer to [10] for the detail diagram of the membership functions.

Table 4 lists the generated linguistic rules for vibration control. It consists of Negative Big (NB), Negative Small (NS), Zero (Z), Positive Small (PS) and Positive Big (PB) as shown in the diagram. The rules are designed based on the condition of the deflection angle and the deflection angle rate as illustrated in Figure 6. Consider the joint of the manipulator rotates to anti-clockwise direction and the link deflects on clockwise direction. As illustrated in Figure 6(a), at this condition intuitively the torque should be applied to clockwise direction in order to compensate the deflection. In this case, the relation between input voltage and torque per inertia is shown in Equation (10), where V_m is an input $u(s)$.

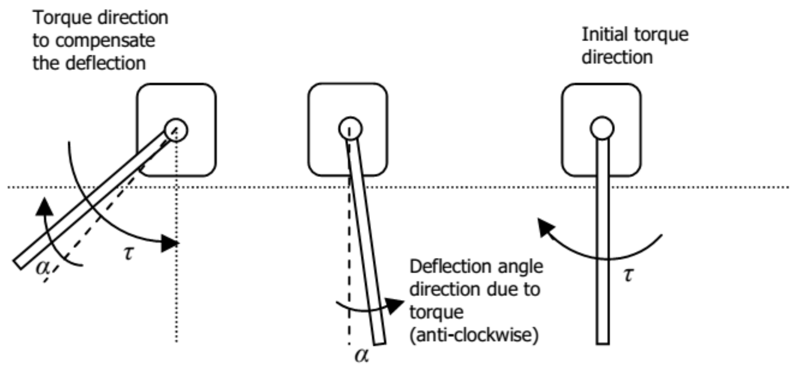
$$\tau = \frac{V_m \eta_m \eta_g K_t K_g}{R_m} \tag{10}$$

Meanwhile, if the joint rotates to clockwise direction as shown in Figure 6(b) and the link deflects to anti-clockwise direction, the torque should be imposed to anti-clockwise direction to suppress the deflection motion. In the case when there is no deflection, no torque should be applied. Furthermore, the proposed fuzzy logic control adopts well-known Mamdani min-max inference and centre of area (COA) methods.

3.3. **Hybrid PD with filtering technique.** In this controller design, the previous collocated PD is combined with a feed-forward scheme [10]. Feed-forward scheme based on filtering technique is developed on the basis of extracting input energy around natural frequencies of the system using filtering techniques. The filter is thus used for pre-processing the input signal so that no energy is fed into the system at the natural frequencies. In this manner, the flexural modes of the system are not excited, leading to a vibration-free motion. This can be realized by employing low-pass filter. The filter is designed with a cut-off frequency lower than the first natural frequency of the system. There are various filter types such as Butterworth, Chebyshev and Elliptic that can be designed and employed. In this investigation, an infinite impulse response (IIR) Butterworth third order



(a) Direction of torque to reduce deflection angle when link moves anti-clockwise



(b) Direction of torque to reduce deflection angle when link moves clockwise

Figure 6. Rules generation based on the motion condition [10]

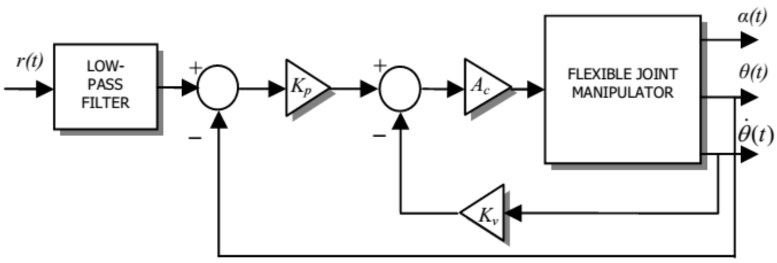


Figure 7. Hybrid PD with low-pass filter control structure

low-pass filter is examined. A block diagram of the composite collocated PD and low-pass filter scheme is shown in Figure 7.

4. **Implementation and Results.** In this section, the proposed control schemes are simulated and tested to the flexible joint manipulator model and the corresponding results are presented. The tip angular position is required to follow a trajectory motion of 50 degree. System responses namely the tip angular position deflection angle and power spectral density of deflection angle are observed. The performances of the control schemes are assessed in terms of input tracking, deflection angle reduction and time response specifications. Finally, a comparative assessment of the performance of the control schemes is presented and discussed.

The response of tip angular position, deflection angle and power spectral density (PSD) of deflection angle of the flexible joint manipulator is depicted in Figures 8 to 10 for hybrid PD with non-collocated fuzzy (PD-Fuzzy), hybrid PD with LPF (PD-LPF) and SIFLC. It shows that those controllers can track the desired trajectory input with zero steady state error and achieve zero vibration from the response of deflection angle. Table 5 summarises the time response specifications of tip angular position. It is noted that PD-Fuzzy and PD-LPF controllers produce a faster settling time as compared to SIFLC controller. However, PD-Fuzzy provides higher overshoot as compared to others controller. On the other hand, the SIFLC controller shows a very minimal oscillation at the deflection angle response with the value of ±1.1 degree as compared to PD fuzzy and PD-LPF with the values of ±6.2 degree and ±3.4 degree, respectively. Moreover, in frequency domain response, SIFLC controller also provides high level of vibration reduction in PSD response as compared to PD-Fuzzy and PD-LPF controllers.

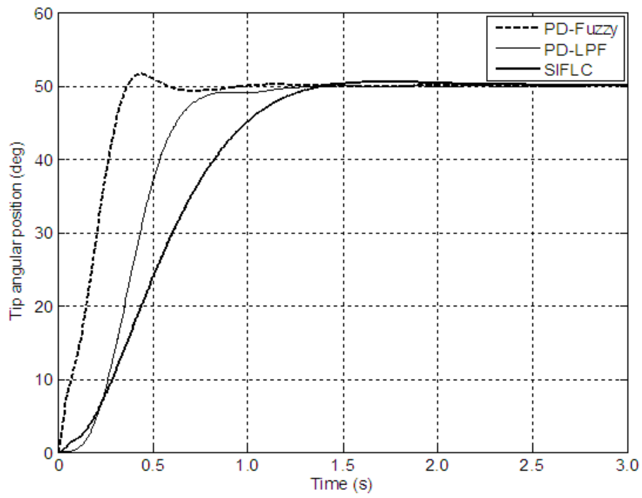


Figure 8. Tip angular position response with PD-fuzzy, PD-LPF and SIFLC

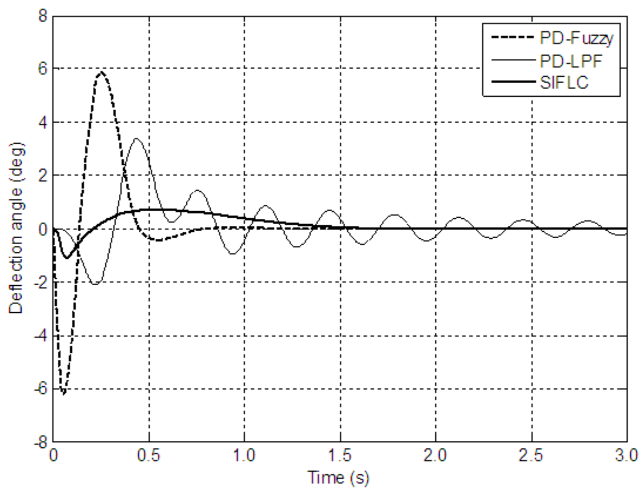


Figure 9. Deflection angle response with PD-fuzzy, PD-LPF and SIFLC

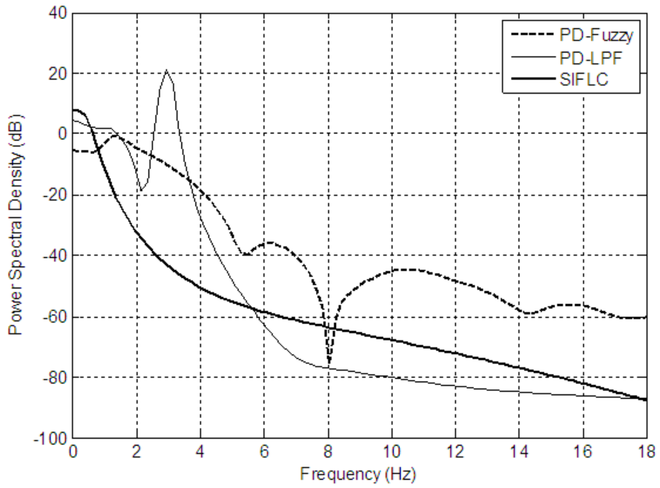


Figure 10. PSD response with PD-fuzzy, PD-LPF and SIFLC

Table 5. Time response specifications of tip angular position

Controller		PD-Fuzzy	PD-LPF	SIFLC
Specifications of tip angular position response	Settling time (s)	0.501	0.842	1.253
	Rise time (s)	0.275	0.417	0.798
	Overshoot (%)	3.24	0.06	1.2

5. **Conclusions.** The control schemes development of single input fuzzy logic controller for tip angular position and deflection angle has been presented. The performances of the proposed controller are compared with PD-fuzzy and PD-LPF controllers [10] and have been evaluated in terms of tip angular tracking capability and deflection angle reduction. The results show that the SIFLC controller provides a very minimal level of vibration as compared to PD-fuzzy and PD-LPF controllers. The work thus developed and reported in this paper forms the basis of design and development of experimental work for trajectory tracking and vibration suppression of others flexible structure system.

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