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SELF-REPAIRING PI/PID CONTROL AGAINST SENSOR FAILURES

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Abstract. *This paper presents a new design method for a self-repairing PI/PID control system against unknown sensor failures. This active fault tolerant control system can automatically replace the failed sensor with the healthy backup if the failure occurs. One of the advantages is that sensor failures of various types can be detected with the integral controller (integrator) and the switching term, and so the structure of the fault detector becomes quite simple independently of plant models. Thus, the system has high robustness with respect to both failures and changes in plant parameters. Furthermore, in order to reduce an influence of the injected switching term on the control performance, the PI and PID controllers are designed based on high-gain feedback. This paper also shows a theoretical analysis of the control performance and explores a numerical simulation to confirm the effectiveness.*

Keywords: Self-repairing, Sensor failure, PID control, High-gain feedback

1. **Introduction.** A wrong signal measured by a faulty sensor, often cuts a feedback-loop to destabilize a control system. Hence, sensor failure is one of the most fatal and critical issues on system stability and safety. As a remedy, based on dynamic redundancy, active fault-tolerant control (AFTC) has been considered that automatically replaces the failed sensor with the healthy backup if failure occurs [1]. In general, this kind of AFTC exploits a fault detector. Unfortunately, many existing deterministic design methods for detectors, require *a priori* information about the plants to find failures exactly. However, this means that those AFTCs cannot work in the presence of variation in plant parameters.

Recently, in the framework of AFTC based on dynamic redundancy, our previous works [4, 5, 6], have focused on feedback-loop-cutting by sensor failure, and a simple detection filter, whose structure does not depend on the mathematical model of the plant, has been developed. An unstable filter is utilized as the detection filter, but the controller is designed to stabilize the feedback system including both the plant and the filter. If the sensor fails, then the feedback-loop opens and the output of the filter behaves unstably. Therefore, monitoring the detection filter can make it possible to find a sensor failure. After the detection, the feedback-loop is repaired by replacing the sensors, and the control system can recover its stability and performance. So, we call this AFTC system as the self-repairing control system (SRCS). The feature is that no *a priori* information about the plant is required to design the preceding detection filter for the SRCS. Thus, exact fault detection can be robustly achieved regardless of unknown plant parameters. However, failures to be found by the detection filter, are limited to stuck and/or slowly floating types only [6]. From a practical viewpoint, a class of detectable failures should be expanded.

Meanwhile, it is well-known that PI/PID control is widely used in various industrial fields, and is applied to more than 90 percent of practical systems, e.g., chemical processes, mechanical systems [2, 3]. Despite its simple control structure, reasonable performance can be obtained easily. This is the reason why PI/PID control is common and popular. However, from social demands on safety and reliability, sensor failure in PI/PID control is still an open problem to be solved. Fortunately, according to [4], an integrator can be exploited as a detection filter for the SRCS.

Then, in this paper, a new design method for the self-repairing PI control system is presented against sensor failures, that utilizes the I (integral) controller for detection. In order to expand a class of detectable failures, instead of the test signal used in [4], a switching signal is introduced to the I controller, and it also contributes early fault detection. Because the switching signal sometimes degrades the control performance, the PI controller is designed based on the concept of high-gain feedback [7, 8] so as to make an influence from the switching signal small. Furthermore, it is possible to extend to PID control by adding the D (derivative) controller to the self-repairing PI control system.

Thus, this paper presents the concrete design methods for the self-repairing PI and PID control systems, and also shows the theoretical analysis of their self-repairing and control performances. Moreover, several numerical simulations are explored to confirm the effectiveness of the proposed SRCSs.

This paper is organized as follows. Section 2 describes the SRC problem, and Section 3 shows the basic design of the self-repairing PI control system and its theoretical analysis. In Section 4, the self-repairing PID control system is discussed. Section 5 demonstrates the numerical examples by the proposed SRCS. Finally, we conclude in Section 6.

Throughout this paper, with $x \in \mathbb{R}$ we redefine the "sgn" function by

$$\mathrm{sgn}[x] = \begin{cases} 1 & (x \geq 0) \\ -1 & (x < 0) \end{cases}$$

The above function is slightly different from the ordinary one.

2. **Problem Statement.** Consider the following LTI system of the $n \in \mathbb{I}^+$-th order [7].

$$\begin{aligned} \Sigma_P \ : \ \dot{y} &= ay + bu + \boldsymbol{h}^T \boldsymbol{z} \\ \boldsymbol{z} &= \boldsymbol{F}\boldsymbol{z} + \boldsymbol{g}y \end{aligned} \tag{1}$$

where $y \in \mathbb{R}$ is the actual output, $u : \mathbb{R}^+ \to \mathbb{R}$ is the control input, and $\boldsymbol{z} \in \mathbb{R}^{n-1}$ is the state of the plant. In the form (1), $a \in \mathbb{R}$ and $b \in \mathbb{R}$ are constants, and $\boldsymbol{F} \in \mathbb{R}^{(n-1)\times(n-1)}$, $\boldsymbol{g} \in \mathbb{R}^{n-1}$ and $\boldsymbol{h} \in \mathbb{R}^{n-1}$ are a matrix and vectors respectively. Here, we assume that the plant Σ_P has a minimum-phase characteristic, and the sign of the high-frequency gain is supposed to be positive, i.e., $b > 0$.

To measure the output y, the two sensors $\sharp 1$ (primary) and $\sharp 2$ (backup) are exploited. Thus, the measured signal $y_S : \mathbb{R}^+ \to \mathbb{R}$ is given by

$$y_S(t) = \begin{cases} y_1(t) & (t \leq t_D) \\ y_2(t) & (t > t_D) \end{cases} \tag{2}$$

where $t_D \in \mathbb{R}^+$ is a failure detection time, which will be defined later. Each $y_i \in \mathbb{R}$, $i \in \{1, 2\}$ is the output of the sensor $\sharp i$. If the sensors are healthy, then we have $y_i = y$. From (2), if the failure of the primary sensor $\sharp 1$ is detected, then the backup $\sharp 2$ is activated.

The failure scenario to be considered here, is given as follows.

$$y_1(t) = \varphi(t), \quad t \geq t_F \tag{3}$$

where $t_F \in \mathbb{R}^+$ is an unknown failure time, and $\varphi : [t_F, \infty) \to \mathbb{R}$ is an unknown bounded function, and we assume that the sign of the function φ will not change after the failure time, that is,

$$\mathrm{sgn}[y_1(t)] = \mathrm{sgn}[\varphi(t)] = \mathrm{sgn}[\varphi(t_F)], \quad t \geq t_F \tag{4}$$

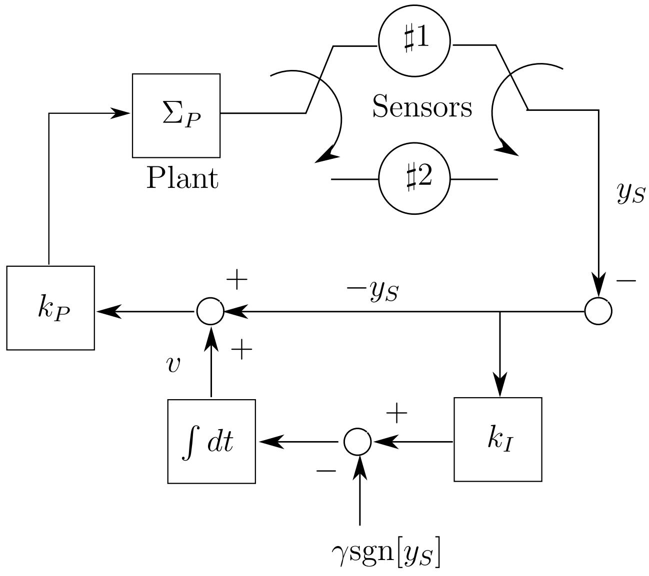

Figure 1. Block diagram of the self-repairing PI control system

Comparing with the assumptions on φ in [6], we should notice that a wider class of failures, φ, can be considered here. In the previous works, the condition on the boundedness of the time derivative $\dot{\varphi}$ has been imposed, that can be alleviated in this paper.

The SRC problem is to replace the failed sensor automatically so as to maintain the control system stability. The following sections will show the concrete design procedures for the SRCSs, and analyze the performances theoretically.

3. Design of the Self-Repairing PI Control System.

3.1. Basic structure.

The PI control input u is designed as follows.

$$u = k_P\left(-y_S + v\right) \tag{5}$$

where $k_P \in \mathbb{R}^+$ is the P (proportional) controller parameter. The signal $v \in \mathbb{R}$ is the output of the integrator:

$$\dot{v} = -k_I y_S - \gamma\mathrm{sgn}\left[y_S\right] \tag{6}$$

where $k_I \in \mathbb{R}^+$ is the I controller parameter. The details of k_P and k_I will be discussed later. Furthermore, in (6), the switching signal $\gamma\mathrm{sgn}\left[y_S\right]$ is introduced to detect the failure, and $\gamma \in \mathbb{R}^+$ is an arbitrary constant. Larger γ makes it possible to shorten detection time (see the next subsection).

Here, consider the case when the sensor ♯1 is healthy. In this case, we have $y_S(t) = y(t)$, $t \in [0, t_F)$. For analysis, define the new augmented signal $\varepsilon : \mathbb{R}^+ \to \mathbb{R}$ as

$$\varepsilon \triangleq -y + v \tag{7}$$

Then, the overall control system can be expressed as follows.

$$\begin{aligned}
\dot{\varepsilon} &= -(bk_P - a - k_I)\varepsilon - (k_I + a)v - \boldsymbol{h}^T\boldsymbol{z} - \gamma\mathrm{sgn}[y] \\
\dot{\boldsymbol{z}} &= \boldsymbol{F}\boldsymbol{z} - \boldsymbol{g}\varepsilon + \boldsymbol{g}v \\
\dot{v} &= -k_I v + k_I\varepsilon - \gamma\mathrm{sgn}[y]
\end{aligned} \tag{8}$$

From this result, we obtain the following lemma on the system stability.

Lemma 3.1. *For sufficiently large k_P and k_I, all the signals, y, $\boldsymbol{z}$ and v in the overall closed control system are bounded on the time period $[0, t_F)$ where the sensor is healthy.*

Proof: On the period, $[0, t_F)$, we consider the following positive function $S : \mathbb{R}^+ \to \mathbb{R}^+$:

$$S = \frac{1}{2}\left\{\varepsilon^2 + \boldsymbol{z}^T \boldsymbol{P} \boldsymbol{z} + v^2\right\} \tag{9}$$

where $\boldsymbol{P} \in \mathbb{R}^{(n-1)\times(n-1)}$ is a positive definite matrix satisfying

$$\boldsymbol{F}^T \boldsymbol{P} + \boldsymbol{P}\boldsymbol{F} = -2\boldsymbol{Q} \tag{10}$$

for arbitrarily given, positive definite, $\boldsymbol{Q} \in \mathbb{R}^{(n-1)\times(n-1)}$. The detail of $\boldsymbol{Q}$ will be determined later.

From (8), the time derivative of S can be expressed as follows.

$$\begin{aligned}\dot{S} =& -(bk_P - a - k_I)\varepsilon^2 - (k_I + a)v\varepsilon - \boldsymbol{h}^T \boldsymbol{z}\varepsilon - \gamma\mathrm{sgn}[y]\varepsilon \\ & - \boldsymbol{z}^T \boldsymbol{Q}\boldsymbol{z} - \boldsymbol{z}^T \boldsymbol{P}\boldsymbol{g}\varepsilon + \boldsymbol{z}^T \boldsymbol{P}\boldsymbol{g}v - k_I v^2 + k_I \varepsilon v - \gamma\mathrm{sgn}[y]v\end{aligned} \tag{11}$$

Furthermore, we have

$$\begin{aligned}\dot{S} \le & -\frac{1}{2}\underbrace{\left(bk_P - 3|a| - 2k_I - \|\boldsymbol{h}\|^2 - \|\boldsymbol{P}\boldsymbol{g}\|^2\right)}_{\alpha_1}\varepsilon^2 \\ & -\frac{1}{2}\underbrace{\left(2\lambda_{\min}[\boldsymbol{Q}] - 3\right)}_{\alpha_2}\|\boldsymbol{z}\|^2 - \frac{1}{2}\underbrace{\left(k_I - |a| - \|\boldsymbol{P}\boldsymbol{g}\|^2\right)}_{\alpha_3}v^2 \\ & +\frac{\gamma^2}{2}\underbrace{\left(\frac{1}{bk_P} + \frac{1}{k_I}\right)}_{\beta}\end{aligned} \tag{12}$$

Here, we choose $\boldsymbol{Q}$, k_P and k_I so that $\lambda_{\min}[\boldsymbol{Q}] > 3/2$ and

$$k_P > \frac{3|a| + 2k_I + \|\boldsymbol{h}\|^2 + \|\boldsymbol{P}\boldsymbol{g}\|^2}{b}, \quad k_I > |a| + \|\boldsymbol{P}\boldsymbol{g}\|^2 \tag{13}$$

Then, all α_i are positive. So, we can get

$$\dot{S} \le -\alpha S + \frac{\gamma^2\beta}{2}, \quad \alpha = \min\left\{\alpha_1, \frac{\alpha_2}{\lambda_{\max}[\boldsymbol{P}]}, \alpha_3\right\} \tag{14}$$

which yields

$$S(t) \le S(0)\exp(-\alpha t) + \frac{\gamma^2\beta}{2\alpha}, \quad t \in [0, t_F) \tag{15}$$

This means that all the signals are bounded on the time period $[0, t_F)$. ■

Remark 3.1. *The PI controller constructed by (5) and (6) can be rewritten in the following well-known form.*

$$u = -k_P\left\{y_S + \frac{1}{T_I}\int_0^t \left(y_S + T_I\gamma\mathrm{sgn}\left[y_S\right]\right)d\tau\right\}$$

where $T_I \in \mathbb{R}^+$ is called as the integral time constant. Comparing with (5), we can find that $T_I = 1/k_I$. Hence, based on high-gain feedback, for sufficiently large k_I, the integral time T_I would become a small value.

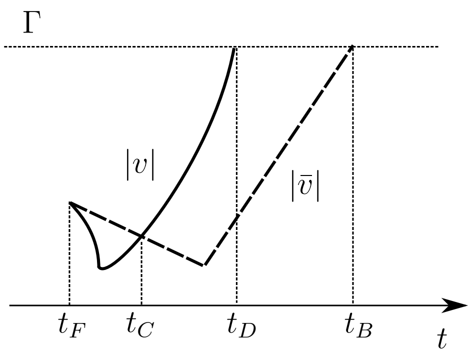

FIGURE 2. The unstable behavior of the absolute value of v for fault detection

3.2. **Failure detection.** From Lemma 3.1, we can see that if no failure occurs, then there is a finite constant $\Gamma \in \mathbb{R}^+$ so that $|v(t)| < \Gamma$, $t \in [0, t_F)$ because of boundedness of v. For example, from (15), a candidate of Γ can be selected as $\Gamma = \sqrt{2S(0) + \gamma^2\beta/\alpha}$.

However, if the sensor $\sharp 1$ fails, then the I controller (6) can be expressed as

$$\dot{v} = -k_I\varphi - \gamma\mathrm{sgn}[\varphi] = -\mathrm{sgn}[\varphi]\,(k_I|\varphi| + \gamma) \tag{16}$$

Hence, in the case of $\mathrm{sgn}[\varphi] = 1$, we have $v < -\gamma(t - t_F) + v(t_F) = \bar{v}$, and in the case of $\mathrm{sgn}[\varphi] = -1$, we have $v > \gamma(t - t_F) + v(t_F) = \bar{v}$. In the both cases, a time $t_C \geq t_F$ exists so that $|v(t)| > |\bar{v}(t)|$, $t > t_C$, and thus $|v|$ tends to infinity as time approaches infinity because $|\bar{v}|$ diverges. Therefore, as shown in Figure 2, $|v|$ hits the threshold Γ whenever the sensor $\sharp 1$ fails.

Then, taking this unstable behavior of v into consideration, we define the detection time t_D as follows.

$$t_D \triangleq \min\{t|\ |v(t)| \geq \Gamma\} \tag{17}$$

Of course, after the detection time, the boundedness of v can be guaranteed again by replacing sensors.

3.3. **Main results.** The control performances of the self-repairing PI control system can be summarized in the following theorem.

Theorem 3.1. *Consider the self-repairing PI control system constructed by (1), (2), (5), (6) and (17). Then, the control system has the following properties.*
(P1) If the sensor $\sharp 1$ fails, then the detection time t_D exists, and satisfies

$$t_D \leq t_F + \frac{2\Gamma}{\gamma} \tag{18}$$

(P2) All the signals, y, $\boldsymbol{z}$ and v are bounded over $[0, \infty)$.
(P3) Regarding the output y, for arbitrarily given, small $\lambda \in \mathbb{R}^+$ and large γ, there exists a sufficiently large k_P and k_I such that

$$\limsup_{t\to\infty} |y(t)| \leq \lambda \tag{19}$$

Proof: From the discussion on v in the previous subsection, there is a time $t_B > t_F$ such that $|\bar{v}(t_B)| = \Gamma$ if the sensor failure (3) occurs as shown in Figure 2. Because $|v(t)| > |\bar{v}(t)|$, $t > t_C$, the detection time t_D exists so that

$$t_D \leq t_B \leq t_F + \frac{\Gamma + |v(t_F)|}{\gamma} \tag{20}$$

which means that (P1) is true.

Next, consider the boundedness of all the signals on the anxious period $[t_F, t_D]$ where the failed sensor ♯1 is still activated. On this period, v is bounded because of (17). Hence, the control input u is also bounded. Furthermore, for bounded u, the plant Σ_P does not have a finite escape time. Therefore, all the signals y, $\boldsymbol{z}$ and v are bounded on $[t_F, t_D]$.

After the fault detection, i.e., $t > t_D$, the healthy sensor ♯2 is activated. By the same discussion as Lemma 3.1, all the signals are bounded on the period (t_D, ∞) where the sensor ♯2 is healthy. Thus, (P2) is true.

From (7), it follows that $|y| \le |\varepsilon| + |v| \le 2\sqrt{2S}$. Therefore, from (15), we have

$$|y(t)| \le 2\sqrt{2S(t_D)\exp(-\alpha(t - t_D))} + 2\gamma\sqrt{\frac{\beta}{\alpha}}, \quad t \in (t_D, \infty) \tag{21}$$

which yields

$$\limsup_{t\to\infty} |y(t)| \le 2\gamma\sqrt{\frac{\beta}{\alpha}} \tag{22}$$

From the definitions of α_i and β in (12), it is clear that for sufficiently large k_P and k_I, we can get $2\gamma\sqrt{\beta/\alpha} \le \lambda$. Hence, (P3) holds.

Thus, the proof of Theorem 3.1 is completed. ■

From (18) in the property (P1), it is found that the detection time t_D can be shortened arbitrarily by choosing large γ which is the design parameter of the I controller. Even if larger γ is selected for early detection, from (P3), one can render the output y arbitrarily small in the sense of convergence to the λ-ball [7, 8].

4. **Extension to PID Control.** Introducing the D (derivative) control term to (5), we can construct the PID controller.

$$u = k_P(-y_S + v) - k_D\dot{y}_S \tag{23}$$

where $k_D \in \mathbb{R}^+$ is the D controller parameter.

Now, we consider the case when the sensor ♯1 is healthy. On the time period $[0, t_F)$, we have $y_S = y$. Regarding the augmented error signal ε defined by (7), it follows that

$$\begin{aligned}\dot{\varepsilon} = &\left(\frac{1}{1 + bk_D}\right) \times \{-(bk_P - a - k_I + bk_Dk_I)\varepsilon \\ &- (k_I + a - bk_Dk_I)v - \boldsymbol{h}^T\boldsymbol{z} - (1 - bk_D)\gamma\mathrm{sgn}[y]\}\end{aligned} \tag{24}$$

From this result, we can have the following lemma.

Lemma 4.1. *For sufficiently small k_D, sufficiently large k_P and k_I, all the signals, y, $\boldsymbol{z}$ and v in the overall closed control system are bounded on the time period $[0, t_F)$ where the sensor is healthy.*

Proof: On the period, $[0, t_F)$, we consider the following positive function $\tilde{S} : \mathbb{R}^+ \to \mathbb{R}^+$:

$$\tilde{S} = \frac{1}{2}\left\{(1 + bk_D)\varepsilon^2 + \boldsymbol{z}^T\boldsymbol{P}\boldsymbol{z} + v^2\right\} \tag{25}$$

The time derivative of $\tilde{S}$ can be calculated as follows.

$$\begin{aligned}\dot{\tilde{S}} = &-(bk_P - a - k_I + bk_Dk_I)\varepsilon^2 \\ &- (k_I + a - bk_Dk_I)v\varepsilon - \boldsymbol{h}^T\boldsymbol{z}\varepsilon - (1 - bk_D)\gamma\mathrm{sgn}[y]\varepsilon \\ &- \boldsymbol{z}^T\boldsymbol{Q}\boldsymbol{z} - \boldsymbol{z}^T\boldsymbol{P}\boldsymbol{g}\varepsilon + \boldsymbol{z}^T\boldsymbol{P}\boldsymbol{g}v - k_Iv^2 + k_I\varepsilon v - \gamma\mathrm{sgn}[y]v\end{aligned} \tag{26}$$

Furthermore, we can evaluate the time derivative of $\tilde{S}$ as

$$\begin{aligned}\dot{\tilde{S}} \leq &-\frac{1}{2}\underbrace{\left(bk_P + bk_Dk_I - 3|a| - 2k_I - \|\boldsymbol{h}\|^2 - \|\boldsymbol{Pg}\|^2\right)}_{\tilde{\alpha}_1}\varepsilon^2 \\ &-\frac{1}{2}\underbrace{(2\lambda_{\min}[\boldsymbol{Q}] - 3)}_{\tilde{\alpha}_2}\|\boldsymbol{z}\|^2 - \frac{1}{2}\underbrace{\left\{(1 - bk_D)k_I - |a| - \|\boldsymbol{Pg}\|^2\right\}}_{\tilde{\alpha}_3}v^2 \\ &+\frac{\gamma^2}{2}\underbrace{\left(\frac{1}{bk_P} + bk_D + \frac{1}{k_I}\right)}_{\tilde{\beta}}\end{aligned} \tag{27}$$

Here, we choose $\boldsymbol{Q}$, k_D, k_P and k_I so that $\lambda_{\min}[\boldsymbol{Q}] > 3/2$ and

$$k_D < \frac{1}{b}, \quad k_P > \frac{3|a| + 2k_I + \|\boldsymbol{h}\|^2 + \|\boldsymbol{Pg}\|^2}{b}, \quad k_I > \frac{|a| + \|\boldsymbol{Pg}\|^2}{1 - bk_D} \tag{28}$$

Then, all $\tilde{\alpha}_i$ are positive. So, we can get

$$\dot{\tilde{S}} \leq -\tilde{\alpha}\tilde{S} + \frac{\gamma^2\tilde{\beta}}{2}, \quad \tilde{\alpha} = \min\left\{\tilde{\alpha}_1, \frac{\tilde{\alpha}_2}{\lambda_{\max}[\boldsymbol{P}]}, \tilde{\alpha}_3\right\} \tag{29}$$

which yields

$$\tilde{S}(t) \leq \tilde{S}(0)\exp(-\tilde{\alpha}t) + \frac{\gamma^2\tilde{\beta}}{2\tilde{\alpha}}, \quad t \in [0, t_F) \tag{30}$$

This means that all the signals are bounded on the time period $[0, t_F)$. ■

From Lemma 4.1, there is a finite constant $\tilde{\Gamma} \in \mathbb{R}^+$ so that $|v(t)| < \tilde{\Gamma}$, $t \in [0, t_F)$. However, if the sensor $\sharp 1$ fails, then $|v|$ tends to infinity because of the integral storage effect of (6). Hence, we define the detection time t_D as follows.

$$t_D \triangleq \min\left\{t \middle| \, |v(t)| \geq \tilde{\Gamma}\right\} \tag{31}$$

Thus, the following theorem about the self-repairing PID control system can be derived.

Theorem 4.1. *Construct the self-repairing PID control system constructed by (1), (2), (6), (23) and (31). Then, the control system has the following properties.*
(P4) If the sensor $\sharp 1$ *fails, then the detection time* t_D *exists, and satisfies*

$$t_D \leq t_F + \frac{2\tilde{\Gamma}}{\gamma} \tag{32}$$

(P5) All the signals, y, $\boldsymbol{z}$ *and* v *are bounded over* $[0, \infty)$.
(P6) Regarding the output y*, for arbitrarily given, small* $\lambda \in \mathbb{R}^+$ *and large* γ*, there exists a sufficiently large* k_P, k_I *and small* k_D *such that*

$$\limsup_{t\to\infty} |y(t)| \leq \lambda \tag{33}$$

Proof: From the same proofs of (P1) and (P2) in Theorem 3.1, it is clear that (P4) and (P5) are true.

From (7) and (30), it follows that

$$\begin{aligned} |y(t)| &\le |\varepsilon(t)| + |v(t)| \\ &\le \left(1 + \frac{1}{\sqrt{1+bk_D}}\right)\sqrt{2S(t)} \\ &\le \left(1 + \frac{1}{\sqrt{1+bk_D}}\right)\left\{\sqrt{2S(t_D)\exp(-\tilde{\alpha}(t-t_D))} + \gamma\sqrt{\frac{\tilde{\beta}}{\tilde{\alpha}}}\right\}, \quad t \in (t_D, \infty) \end{aligned} \tag{34}$$

which yields

$$\limsup_{t\to\infty} |y(t)| \le \gamma\left(1 + \frac{1}{\sqrt{1+bk_D}}\right)\sqrt{\frac{\tilde{\beta}}{\tilde{\alpha}}} \tag{35}$$

From the definitions of $\tilde{\alpha}_i$ and $\tilde{\beta}$ in (27), for sufficiently large k_P, k_I and small k_D, we can get $\gamma\left(1 + 1/\sqrt{1+bk_D}\right)\sqrt{\tilde{\beta}/\tilde{\alpha}} \le \lambda$. Hence, (P6) holds.

Thus, the proof of Theorem 4.1 is completed. ■

Remark 4.1. *The PID controller constructed by (6) and (23) can be rewritten as follows.*

$$u = -k_P\left\{y_S + \frac{1}{T_I}\int_0^t \left(y_S + T_I\gamma\mathrm{sgn}\,[y_S]\right)d\tau + T_D\dot{y}_S\right\}$$

where $T_D \in \mathbb{R}^+$ is called as the derivative time constant. Comparing with (5), we can verify that $T_D = k_D/k_P$. Hence, based on high-gain feedback, for sufficiently large k_P and small k_D, the derivative time T_D would become an extremely small value.

5. **Numerical Examples.** In this section, to confirm the effectiveness of the proposed SRCS, several numerical simulations are explored.

Consider the following unstable second order system:

$$\begin{aligned} \Sigma_P : \dot{y} &= y + u + z, \quad y(0) = 1 \\ \dot{z} &= -z + y, \quad z(0) = -0.5 \end{aligned} \tag{36}$$

Suppose that the failure time t_F is set as

$$t_F = 25.0 \text{ [s]}$$

and the function φ is supposed to be

$$\varphi(t) = y_1(t_F) + 0.01\cos t, \quad t \ge t_F$$

Of course, these are unknown.

For this plant Σ_P with the faulty sensor, we construct the controller. The following are the design parameters for the PI/PID controller and fault detection.

$$k_P = 3, \quad k_I = 1, \quad k_D = 0.003, \quad \gamma = 1, \quad \Gamma = 2$$

These parameters are selected by trial and error in the preliminary simulation.

From Theorem 3.1 (and also 4.1), the maximum detection time can be estimated as follows.

$$t_D \le t_F + \frac{2\Gamma}{\gamma} = 29.0 \text{ [s]}$$

The simulation results are shown in Figures 3 and 4. In Figure 3, the time responses of the actual outputs y (top) and the states z (bottom) are shown. The dashed lines are the results by the PI controller, and the solid lines are the ones by the PID controller. Figure 4 shows the time responses of the absolute value of the monitored signal v with the threshold Γ. The dashed line indicates the result by the PI controller, and the solid line represents

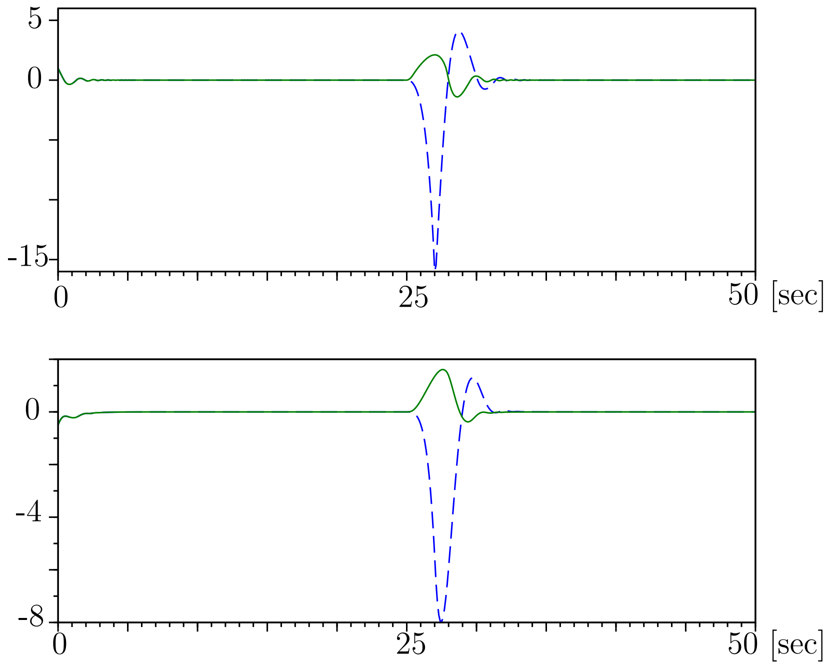

FIGURE 3. Simulation results: the outputs y (top) and the states z (bottom)

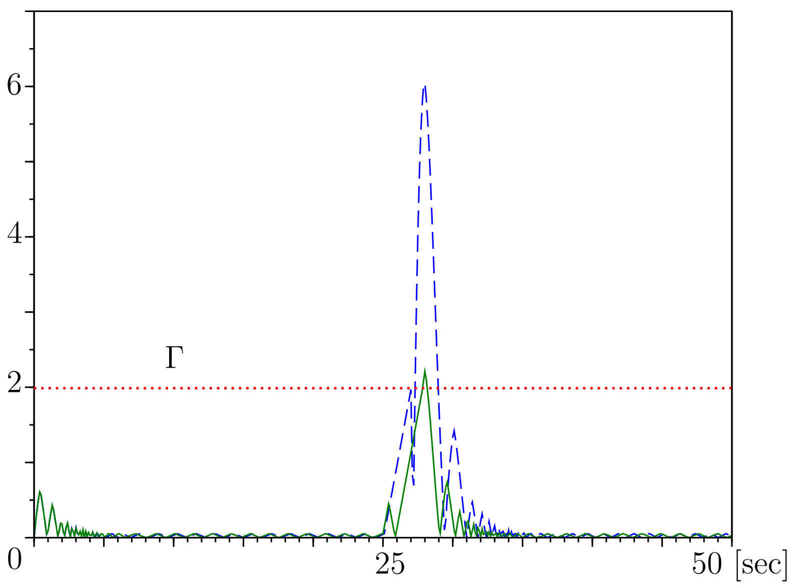

FIGURE 4. Simulation results: the absolute values of v

the one by the PID controller. In both cases, the signal v hits the threshold Γ exactly. From these results, it can be shown that the unstable plant (36) can be stabilized well in spite of the existence of the sensor failure. Also, the failed sensor can be successfully detected and replaced after the failure.

From Figure 3, the control performance can be remarkably improved by the introduction of the D control term. However, in Figure 4, comparing the detection times by PI and

PID controllers, it can be found that the use of the PI controller makes the fault detection much earlier than the PID controller; in the case by the PID controller, $t_D \simeq 27.6$ [s], and in the case by the PI controller, $t_D \simeq 26.8$ [s]. Fortunately, those detection times are much earlier than the above-estimated maximum time 29.0 [s].

6. **Concluding Remarks.** This paper has shown the new design method for the self-repairing PI/PID control against sensor failures, and the theoretical analysis and the numerical simulation are performed to show the control performance and the effectiveness.

Fundamentally, in this method, based on the high-gain feedback [7, 8], the system can be stabilized, and also the influence from the switching signal $\gamma \mathrm{sgn}\,[y_S]$ can be rendered small. This is the reason why the plant Σ_P has to fulfill the high-gain feedback stabilization conditions (see Section 2). Regarding the restriction on the relative degree, the well-known backstepping [9] might solve this issue because this design strategy has treated nonlinear systems with high relative degree. In addition, the introduction of parallel feedforward compensator [10, 11] can alleviate not only the condition of the relative degree but also the requirement of the minimum-phase property because the compensator can make the augmented plant with the desired zeros. Thus, the proposed SRCS is expected to be applied to a wider class of the plants with sensor failures.

Fortunately, if one already has had a stable PI/PID control system, then it is available instead of (6) and (5) or (23). However, in such a case, the property (P3) does not hold, and hence small γ should be chosen so as to avoid degradation of the control performance due to the injection of the switching signal.

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