

## OPTIMAL NODES COMBINATION OF A WAVELET PACKET TREE FOR PHONOCARDIOGRAM SIGNAL ANALYSIS

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**ABSTRACT.** *In order to extract pathological features of phonocardiogram signal (PCG) accurately, a study was proposed in this paper to select the most informative nodes combination of a wavelet packet tree as a basis for feature extraction. The basic function of this work computes the optimal sub tree of an initial wavelet packet tree with respect to an entropy type criterion. The resulting tree may be much smaller than the initial one and use this selection of a wavelet packet tree to generate an adaptive filter for PCG signal. The proposed function could be utilized in different signal processing applications by adapting the exclusion criteria based on the signals characteristics. It could be indicated particularly for PCG signals and all signals of a small range of frequencies.*

**Keywords:** Phonocardiogram signal (PCG), Heart murmur, Wavelet packet transform (PWT), Optimal sub tree, Entropic cost function, Adaptive filtering algorithm

**1. Introduction.** Cardiac activity is accompanied by the appearance of a set of sounds. All these sounds are the phonocardiogram signal (PCG). In normal conditions, PCG contains two sounds (S1 and S2) for each cardiac cycle. Two other sounds (S3 and S4) with amplitudes are definitely less important than the two first appearing sometimes on the level of the cardiac cycle by the effect of pathology or age (childhood or old age or like precursory signs of any pathology) [1].

PCG signal is an inexpensive way to convey cardiac physiological information in the form of acoustic vibration. With a judicious analysis of signal processing techniques, this signal can then be an effective tool to aid medical diagnosis.

Valvular pathologies induce considerable modifications on the morphology of the normal PCG signal [2]. Murmurs are usually generated from the turbulence in the blood circulation through contracted heart valves or reflow through the valves between atria and ventricles.

These modifications could be captured through extracting suitable features and used for heart sound classification. Focus of this study is on classifying murmurs which are the common pathological heart sounds.

The wavelet packet decomposition is a generalization of wavelet decomposition that offers a richer signal analysis. Wavelet packet atoms are waveforms indexed by three naturally interpreted parameters: position, scale and frequency.

For a given orthogonal wavelet function, we generate a library of bases called wavelet packet bases. Each of these bases offers a particular way of coding signals, preserving global energy, and reconstructing exact features [3]. The wavelet packets can be used for

numerous expansions of a given signal. We then select the most suitable decomposition of a given signal with respect to an entropy-based criterion.

There exist simple and efficient algorithms for both wavelet packet decomposition and optimal decomposition selection. We can then produce adaptive filtering algorithms with direct applications in optimal signal coding [4].

In recent researches to differentiate murmurs from normal heart sounds a number of features have been extracted based on wavelet transform. For instance, the study presented by Say et al. [5] regarding the classification of heart sounds using discrete wavelet transform (DWT) has shown that the decomposition levels D2 and D6 gave the best results in this classification [6]. Applying techniques such as energy, entropy, and fractal dimension to extracting features from PCG signals for classifying systolic murmurs. A series of studies were also conducted by Choi [7] introduced meanWPE and stdWPE, based on wavelet packet energy to distinguish normal and abnormal heart sounds. New entropy was defined on PWT and multi-level basis selection (MLBS) is proposed to preserve the most informative bases of a wavelet packet decomposition tree by Safara et al. [8,9] for classification of heart murmurs. L. Hamza Cherif et al. [10] found that the PWT affects the morphology of the S1 and S2 sounds of PCG signal much more than the DWT, which is confirmed by a more important error between the original signal and the synthesis signal. If we want to make a filtering a base of the PWT, it is wise to choose the analyzing wavelet “db10”.

On the basis of these results, our work will be to find the optimal nodes combination of a wavelet packet tree by selecting the most informative nodes. This selection of a wavelet packet tree can be used to generate an adaptive filter for PCG signal. Details of each process are provided in Section 1. Section 2 includes the procedure of the heart sound recording followed by experimental results. In Section 3, results achieved were compared and discussed. Conclusions and prospective extensions are drawn in Section 4.

## 2. Methods.

**2.1. Building wavelet packets.** Compared to the decomposition by the DWT, the PWT provides a richer decomposition, since even the detail signals from the high-pass filtering are also broken again contrary to the DWT or they are left aside at every step. Thus, each level of decomposition presents different information. The computation scheme for wavelet packets generation is easy when we use an orthogonal wavelet. We start with the two filters of length  $2N$ , where  $h(n)$  and  $g(n)$ , correspond to the wavelet [11], (Wavelet Methods for Time Series Analysis by Percival and Walden, 2000). By induction we define the following sequence of functions ( $W_n(x), n \in N$ ):

$$W_{2n}(x) = \sqrt{2} \sum_{k=0}^{2N-1} h(k)W_n(2x - k) \quad (1)$$

$$W_{2n+1}(x) = \sqrt{2} \sum_{k=0}^{2N-1} g(k)W_n(2x - k) \quad (2)$$

where  $W_0(x) = \Phi(x)$  is the scaling function and  $W_1(x) = \Psi(x)$  is the wavelet function.

From the functions ( $W_n(x), n \in N$ ) and following the same line leading to orthogonal wavelets, we consider the three-indexed family of analyzing functions:

$$W_{j,n,k}(x) = 2^{-j/2}W_n(2^{-j}x - k) \quad (3)$$

where  $n \in N$  and  $(j, k) \in Z^2$ .  $k$  can be interpreted as a time-localization parameter and  $j$  as a scale parameter. For fixed values of  $j$  and  $k$ ,  $W_{j,n,k}$  analyzes the fluctuations of the

signal roughly around the position  $2^j \cdot k$  at the scale  $2^j$  and at various frequencies for the different admissible values of the last parameter  $n$  (times).

The set of functions  $W_{j,n} = (W_{j,n}, k(x), k \in \mathbb{Z})$  is the  $(j, n)$  wavelet packet. For positive values of integer's  $j$  and  $n$ , wavelet packets are organized in trees. The tree in Figure 1 is created to give a maximum level decomposition equal to 3. For each scale  $j$ , the possible values of parameter  $n$  are  $0, 1, \dots, 2^j - 1$ .

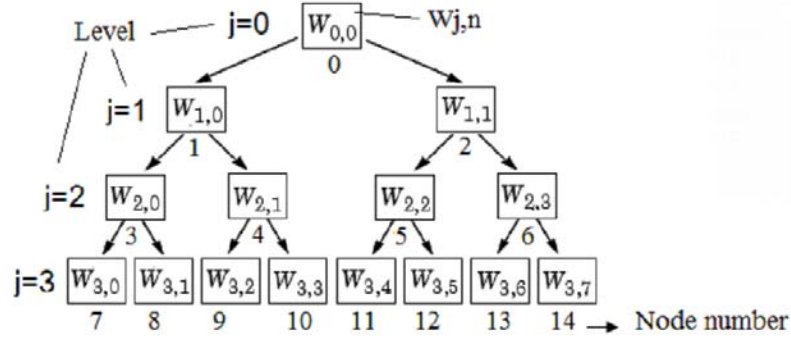


FIGURE 1. Wavelet packets organized in a tree; scale  $j$  defines depth and frequency  $n$  defines position in the tree

Unfortunately, the number of possible combinations is astronomically when the signal is longer than a few samples. Therefore, we need a test that can tell us whether such an intermediate node of the tree should be selected as it is, or whether it is better to keep its mean and its details. This criterion is called cost function.

So we will calculate the cost of the signal of each node of the tree, and then we will select the combination of these signals giving the minimum total cost. Different types of entropy such as log, norm, sure, and threshold can be used to characterize the heart sounds [12]. We propose in this study the Shannon entropy.

In the following expressions  $x$  is the signal and  $(x_i)$  are the coefficients of  $x$  in an ortho-normal basis. The entropy  $E$  must be an additive cost function such that  $E(0) = 0$  and

$$E(x) = \sum_i E(x_i) \quad (4)$$

The normalized Shannon entropy:

$$E_N(x_i) = -x_i^2 \log(x_i^2) \quad (5)$$

So

$$E_N(x) = -\sum_i x_i^2 \log(x_i^2) \quad (6)$$

with the convention  $0 \log(0) = 0$ .

**2.2. Selection of the optimal combination of node.** To select the combination of nodes that has the lowest total cost, we calculate the cost of the PCG signal corresponding to each node of the tree. Two methods for subtrees selection are presented in this work.

**2.2.1. Best-level tree method.** In this method, we calculate the total cost of each level of decomposition by summing the costs of the nodes at that level. As the final decomposition we note all the signals of the level of lower cost (Figure 2).

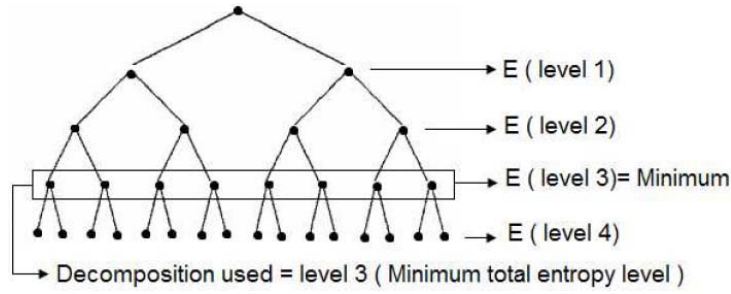


FIGURE 2. Best-level tree method

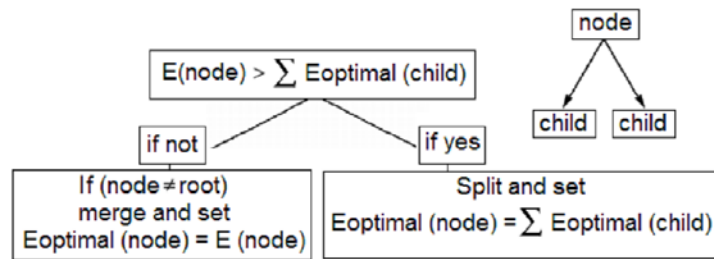


FIGURE 3. Entropy condition and the action on tree and on entropy labeling

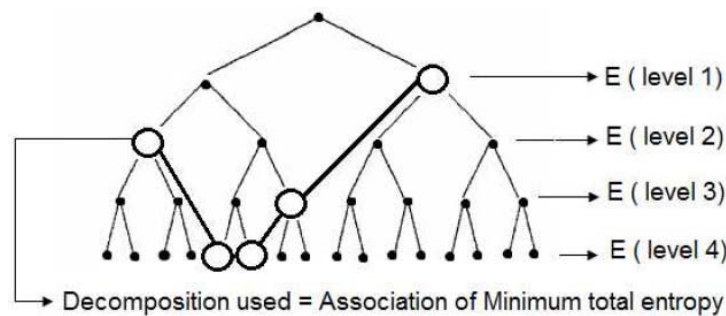


FIGURE 4. Best-basis tree method

2.2.2. *Best-basis tree method.* Instead of proceeding level by level, we will consider each node of the binary tree individually, from the penultimate level of decomposition, and compare its cost to the total cost of his child. For any non-terminal node, we use the following basic step to find the optimal subtree with respect to a given entropy criterion  $E$  (where  $E_{optimal}$  denotes the optimal entropy value), with the natural initial condition on the reference tree,  $E_{optimal}(t) = E(t)$  for each terminal node  $t$  (Figure 3).

If the cost of the father node is lower than that of his two children together, then the decomposition of this father signal is not justified: we keep the father signal as it is, and again with his own father node to the lower level. However, if the total cost of child is lower, it is advantageous to keep the child signals rather than the father signal to reduce the total entropy of the decomposition as shown in Figure 3. We get the combination of signals whose total entropy is as small as possible, such as Figure 4.

2.3. **Choice of the analyzing wavelet.** The analysis of the choice of the wavelet analyzing (mother wavelet) will be carried out on the basis of test of several wavelets analyzing. This will be done on the study of the error existing between the original signal (normal case) and the synthesis signal (signal after reconstruction). In this direction, a parameter of error characterizes the rebuilding (or synthesis). The error that will be calculated each

time in the continuation of the analysis is given by the following expression:

$$E_{r_{moy}} = \left( \sum_{i=1}^N |X_{0i} - X_{ri}| \right) / N \quad (7)$$

with  $X_0$  being the original signal,  $X_{0i}$  being the sample of  $X_0$ ,  $X_r$  being the synthesis signal, and  $X_{ri}$  being the sample of  $X_r$ .

The analyzing wavelet chosen will be the one who will present the lowest error reconstruction respecting the particularities of PCG signal. The analyzing wavelets used are presented in Table 1.

TABLE 1. Wavelets used in the analysis

Orthogonal wavelets	Bi-orthogonal wavelets
Daubechies (db) [db3, db4...db19]	Bi-orthogonal (bior) [bior1.1, bior1.3...bior3.9]
Symelet (sym) [sym3, sym4...sym19]	Reversible bi-orthogonal (rbior)
Coiflet (coif) [coif1...coif5]	[rbio1.1, rbio1.3...rbior3.9]

### 3. Results.

**3.1. Signals used.** Experimental data is a set of 40 PCG signals of pathological PCG, and we present in this paper 6 heart sounds including one normal and 5 Aortic Stenosis (Table 2). Data acquisition was performed in Boston Children's Hospital, Boston, USA. An electronic stethoscope was used for PCG recording and 3-lead ECG for parallel ECG recording (Figure 5). The goal is to make a difference between the severities of each case. Table 2 summarizes these cases.

**3.2. Optimal nodes selected by the best level and best basis tree methods.** Tables 3 and 4 show the resulted nodes from applying the best level and best basis tree selection methods on a wavelet packet tree of a PCG signal (Normal PCG) as a first analysis and the Aortic Stenosis (AS1, AS2, AS3, AS4 and AS5) as a second analysis, with Shannon entropy as a cost function. The size of initial data is the same for all PCG signals used (200000 samples ( $\approx 6$  cycles of the PCG signal) and frequency sampling = 44100 Hz). Testing simulation exceeds 6 decomposition levels. Several analyzing wavelet (Table 1) is used to assess the reconstruction average error  $E_{r_{moy}}$ ; calculating the average error is important for the choice of the best analyzing wavelet to be applied for the PCG signals analysis to determine the optimal nodes decomposition.

The application of the best level method shows that the optimal nodes selected are based on the choice of the decomposition level (PCG signal has a wide frequency band [15-700 Hz]); since all the nodes of each level are kept (Table 3). Compared to the first method, the best basis tree method offers different choices of combinations for each level and for each analyzing wavelet. The optimal nodes selected are the decomposition used by the best basis tree method that represents the smaller quantity of nodes (Table 4).

Tables 5 and 6 summarize the application of all the wavelet analyzing (Table 1).

The obtained results (Table 5) show that the analyzing wavelet **sym16** gives the best decomposition (Optimal Nodes selected by the Best basis tree methods) for all levels (level6: Optimal Nodes [63 64 32 16 8 4 2], level7: Optimal Nodes [127 128 64 32 16 8 4 2], level8: Optimal Nodes [255 256 128 64 32 16 8 4 2] and level9: Optimal Nodes [511 512 256 128 64 32 16 8 4 2]). The other analyzing wavelets (db10, db12, coif2, coif4, bior2.6-3.9, and rbior2.6-3.9) also give acceptable results but at definite levels. For

TABLE 2. PCG signals (Aortic Stenosis and Normal PCG) used in the analysis

PCG	Information about the PCG signals (Aortic Stenosis and Normal PCG)
AS1	PCG recorded at the apex in a 28-year-old male with bicuspid aortic valve, mild-moderate aortic stenosis and moderate aortic regurgitation. There is a prominent early, constant systolic ejection click and a blowing diastolic murmur of aortic regurgitation.
AS2	PCG recorded at the apex in a 14-year-old adolescent with bicuspid aortic valve and mild-moderate aortic regurgitation. There is a prominent early, constant systolic ejection click and a blowing diastolic murmur of aortic regurgitation.
AS3	PCG recorded at the right upper sternal border in a 3-year-old child with moderate valvar aortic stenosis. There is a harsh systolic ejection murmur. At catheterization, there was a 45 mm Hg peak systolic gradient across the aortic valve.
AS4	PCG recorded at the right upper sternal border in a 14-year-old adolescent with bicuspid aortic valve and severe aortic stenosis. At catheterization, there was a 62 mm Hg peak systolic gradient across the aortic valve and no aortic regurgitation.
AS5	PCG recorded at the right upper sternal border in a 28-year-old male with bicuspid aortic valve, mild-moderate aortic stenosis and moderate aortic regurgitation. There is a systolic ejection murmur of aortic stenosis and diastolic blowing murmur of aortic regurgitation.
Normal	PCG recorded at the apex in a 12-year-old asymptomatic patient showing. An echocardiogram demonstrated normal cardiac anatomy and function. The tracing shows an S3, which occasionally occurs in the absence of cardiac dysfunction in the pediatric age group. There is a gallop rhythm that has the cadence of the syllables "Ken-túc-ky". The last component of this sequence represents the third heart sound.

TABLE 3. Optimal nodes selected by the best level tree methods for level 6, 7, 8 and 9 with the Daubechies (db) analyzing wavelet

PCG	Level	Analyzing wavelet	Terminal nodes (Decomposition used) by the best level tree methods
Normal PCG	6	db3... db14	[63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 ...]
	7	db3... db14	[127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 ...]
	8	db3... db14	[255 256 257 258 259 260 261 262 263 264 265 266 267 ...]
	9	db3... db14	[511 512 513 514 515 516 517 518 519 520 521 522 523 ...]

example, db12 for levels 7, 8 and 9; db 10 for level9; bior 2.6, 2.8, 3.1, 3.3, 3.5, 3.7, 3.9; rbio3.3, 3.5, 3.7, 3.9 at levels 6, 7 and 8.

To check the strength of the method, we compare the frequency range of selection nodes obtained with frequency range of the PCG signals covered 15-700 Hz [13]. At level1 we have two nodes (1 and 2), level2 (3, 4, 5, 6)... level7 (127... 254), level8 (255... 510). Every node has a frequency range. For example, frequency range of node 255 or node (8, 0) is [0 to f1 Hz] (f1 = frequency sampling .0.5/255) and the frequency range of node 256 or node (8, 1) is [f1 to f2 Hz] (f2 = frequency sampling .0.5/256). If frequency range of node is out of the frequency range of PCG signal, the node is removed.

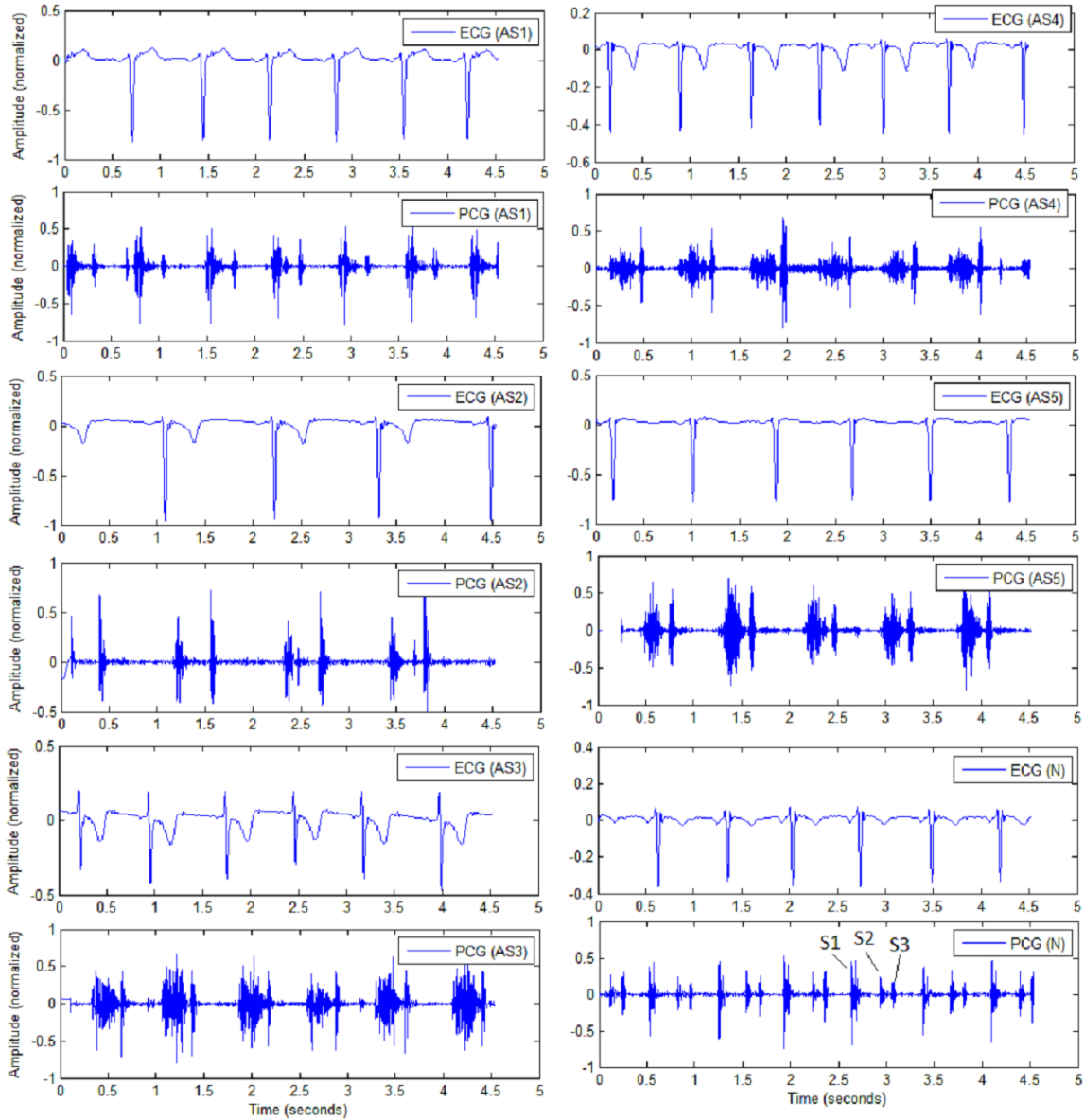


FIGURE 5. PCG signals used (Aortic Stenosis (AS1, AS2, AS3, AS4 and AS5) and Normal PCG (N))

The average error values  $E_{r_{moy}}$  given by Figure 6 make it possible to realize that the values obtained by the application of the optimal nodes selected by the best basis tree methods for levels 6, 7, 8 and 9 with analyzing wavelet sym 16 being lower than those obtained by the other analyzing wavelet in the Aortic Stenosis (AS1, AS2, AS3, AS4 and AS5) PCG signal analysis.

We will be interested here only in nodes: (6.0), (7.0), (8.0) and (9.0) for levels 6, 7, 8 and 9 respectively, and the frequency range contains sufficient information) because in experiments, it better represents the cardiac sounds S1 and S2 and their respective internal components (M1, T1) and (A2, P2) in a very clear way (5, 7, 10).

Moreover, we will proceed to calculate existing error between PCG signals of origin and signals represented by the nodes (6.0), (7.0), (8.0) and (9.0). Figure 6 shows that the smallest error represents the lightest severity compared with information about the PCG signals (Aortic Stenosis and Normal PCG) in Section 3.1. The reconstruction error does

TABLE 4. Optimal nodes selected by the best basis tree methods for levels 6, 7, 8 and 9 with the Daubechies (db) analyzing wavelet

PC	Level	Analyzing wavelet	Terminal nodes (Decomposition used) by the best basis tree methods
Normal PCG	6	db7	[63 64 65 66 16 8 4 95 96 48 24 103 104 52 107 108 ...]
		db11	[63 64 65 66 16 8 4 2]
		<b>db12</b>	<b>[63 64 32 16 8 4 2]</b>
		db13	[63 64 65 66 16 8 4 23 99 100 101 102 25 107 108 54 ...]
		db14	[63 64 32 16 8 4 23 24 103 104 52 26 6]
	7	db9	[127 128 64 65 66 16 8 4 95 96 97 197 198 199 200 100 ...]
		db11	[127 128 129 130 65 66 16 8 4 2]
		<b>db12</b>	<b>[127 128 64 32 16 8 4 2]</b>
		db13	[127 128 129 130 65 66 16 8 4 23 199 200 100 203 204 102 ...]
		db14	[127 128 129 130 32 16 8 4 95 96 195 196 98 99 201 202 ...]
	8	db7	[255 256 128 64 131 132 66 16 8 4 191 192 96 195 196 98 ...]
		db10	[255 256 257 258 129 130 65 66 16 8 4 2]
		<b>db12</b>	<b>[255 256 257 258 64 32 16 8 4 2]</b>
		db13	[255 256 257 258 259 260 130 65 66 16 8 4 23 199 200 100 ...]
		db14	[255 256 257 258 259 260 130 32 16 8 4 95 96 195 196 98 ...]
	9	db9	[511 512 513 514 128 259 260 261 525 526 65 66 16 8 4 95 ...]
<b>db10</b>		<b>[511 512 513 514 257 517 518 129 130 65 66 16 8 4 2]</b>	
db12		[511 512 256 257 258 64 32 16 8 4 767 768 384 192 96 97 ...]	

TABLE 5. Optimal nodes selected by the best level tree methods for level 6, 7, 8 and 9 with different analyzing wavelets

PC	Level	Analyzing wavelet	Terminal nodes (Decomposition used) by the best level tree methods
Normal PC	6	db, sym, coif, bior, rbior.	[63 64 65 66 67 68 69 70 71 72 73 74 75 76 ...]
	7	db, sym, coif, bior, rbior.	[127 128 129 130 131 132 133 134 135 136 137 138...]
	8	db, sym, coif, bior, rbior.	[255 256 257 258 259 260 261 262 263 264 ...]
	9	db, sym, coif, bior, rbior.	[511 512 513 514 515 516 517 518 519 520 ...]

not change much between different levels. The PCG signal AS2 is the most moderate cases and the PCG signal AS4 is the most severe cases.

4. **Conclusions.** With the dyadic decomposition of signals, one is guaranteed to get a final decay length exactly  $N$ , length of the starting signal. So there is no loss or creation of information. Simply, the signal was decomposed by a selection of basic functions (the optimal decomposition), chosen from a larger collection (the binary tree in full), so that this choice minimizes the total entropy of the decomposed signal.

In a filter when filtering signal is complex it is difficult to define an optimal level. According to the simulation tests, go at least beyond three levels for acceptable test filtering. Therefore, it is interesting to extend the test field of the number of levels to determine the most appropriate level filtering. Thus, the simulation tests were pushed from level 6 to level 9 to select the most efficient nodes combination.

If we want to make a filtering based on the PWT and best basis tree methods are selected, it is wise to choose the analyzing wavelet “sym16” earlier than the other analyzing wavelets to reduce the rebuilding error of reconstruction and not to distort the sounds S1 and S2. The method used enabled us to distinguish between the severities of each Aortic Stenosis case.



TABLE 6. Optimal nodes selected by the best basis tree methods for level 6, 7, 8 and 9 with different analyzing wavelets

PC	Level	Analyzing wavelet	Terminal nodes (Decomposition used) by the best basis tree methods
Normal PCG	6	db12	[63 64 32 16 8 4 2]
		<b>Sym16</b>	<b>[63 64 32 16 8 4 2]</b>
		Coif2	[63 64 32 33 34 8 4 23 24 12 6]
		Coif4	[63 64 65 66 16 8 4 11 25 26 6]
		bior2.6-3.9	[63 64 32 16 8 4 2]
		rbio3.3-3.9	[63 64 32 16 8 4 2]
	7	db12	[127 128 64 32 16 8 4 2]
		<b>Sym16</b>	<b>[127 128 64 32 16 8 4 2]</b>
		Coif2	[127 128 129 130 32 33 34 8 4 23 24 12 6]
		Coif4	[127 128 129 130 65 66 16 8 4 11 25 26 6]
		bior2.6-3.9	[127 128 64 32 16 8 4 2]
		rbior2.6-3.9	[127 128 64 32 16 8 4 2]
	8	db12	[255 256 257 258 64 32 16 8 4 2]
		<b>Sym16</b>	<b>[255 256 128 64 32 16 8 4 2]</b>
		Coif4	[255 256 257 258 259 260 130 65 66 16 8 4 11 25 26 6]
		bior2.6-3.9	[255 256 128 64 32 16 8 4 2]
		rbior2.6-3.9	[255 256 128 64 32 16 8 4 2]
		9	db10
	<b>Sym16</b>		<b>[511 512 256 128 64 32 16 8 4 2]</b>
	Coif4		[511...516 258 519 520 260 130 65 66 16 8 4 11 25 26 6]
	Bior3.3-3.9		[511 512 513 514 128 64 32 16 8 4 2]
	Rbior3.9		[511 512 513 514 257 517 518 64 32 16 8 4 2]

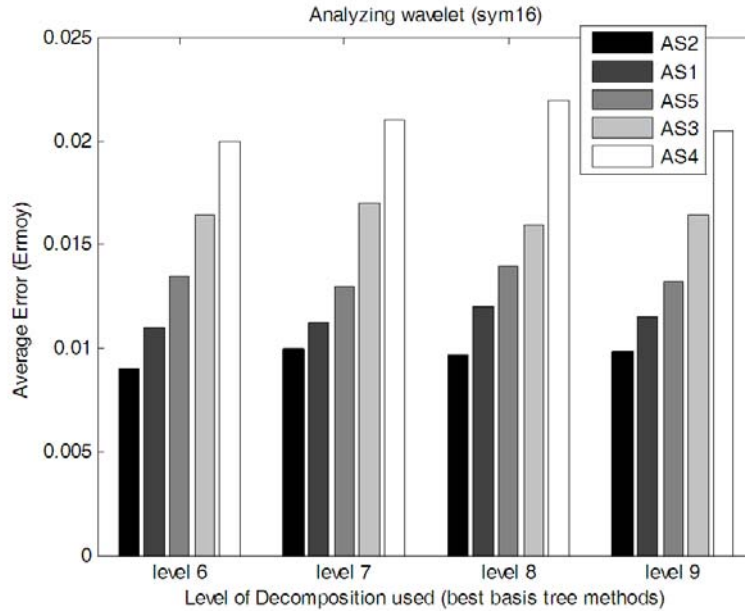


FIGURE 6. Average variation of the rebuilding error calculated over five cycles between original PCG signals (AS1...AS5) and signals represented by the nodes (6.0), (7.0), (8.0) and (9.0) for levels 6, 7, 8 and 9 using analyzing wavelet sym16 and best basis tree methods

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