International Journal of Innovative Computing, Information and Control
Volume **12**, Number **1**, February **2016**

ICIC International ©2016 ISSN 1349-4198
pp. 321–331

DESIGN OF A ROBUST H_∞ CONTROLLER FOR A LOAD-FREQUENCY POWER SYSTEM

Betania G. da S. Filha[1,2], Alexandre C. de Castro[3]
Fernando A. Moreira[1] and José Mário Araújo[4]

[1]Programa de Pós-Graduação em Engenharia Elétrica
Universidade Federal da Bahia
R. Prof Aristides Nóvis, 2 – Federação, 40210-630, Salvador, Brasil
betaniafilha@ifba.edu.br; moreiraf@ufba.br

[2]Grupo de Pesquisa no Desempenho dos Sistemas Elétricos de Potência
Departamento de Eltrotécnica
[4]Grupo de Pesquisa em Sinais e Sistemas
Departamento de Automação e Sistemas
Instituto Federal da Bahia
Campus Salvador, Brasil
jomario@ifba.edu.br

[3]Departamento de Engenharia Elétrica
Universidade Federal da Paraíba
Campus universitário I, João Pessoa – PB, Brasil
castro@cear.ufpb.br

Received June 2015; revised December 2015

Abstract. *Techniques for the analysis and design of multivariable system controllers in the frequency domain are used to verify the possibility of achieving robust control using reduced-order decentralized controllers with the objective of robust stability of a power system. In order to achieve robustness, the selection of the most effective signals and inputs for application to the controllers is essential. Accordingly, two frequency analysis techniques were used: relative gain matrix (RGM) and singular value analysis. The proposed techniques were applied to a three-area load-frequency system. Finally, it was possible to realize a first-order efficient controller that remains robust, even in the presence of faults or disturbances. The controller used was an H_∞ controller and the optimization method applied was based on genetic algorithms.*
Keywords: Power systems, Load-frequency, Robust control, Genetic algorithms

1. **Introduction.** The stability of an electric power system refers to the ability of the components of the system to remain in equilibrium under normal operating conditions and to achieve an acceptable steady state after being subjected to disturbances [1,2]. Electric power generation systems can lose stability owing to a lack of synchronism of the generators. As the exchange power increases, electrical systems become vulnerable to instability due to the resulting oscillations [3]. Such oscillations are harmful because they not only limit the transmitted power but also increase the risk of unscheduled breaks in the power supply. In addition, system security has a significant effect on electricity prices, directly affecting the competitiveness of the system in the market [4].

Oscillatory stability is reflected by the existence of positive damping for all natural oscillation modes of the system when they are excited by a small disturbance or even normal load fluctuations [5]. Therefore, the damping of these oscillations is a prerequisite for safe operation of an electric system, and thus, a major concern for engineers and operators [6,7].

For effective control of these oscillations, knowledge and analysis of factors such as the nature, types, and frequencies of the most disturbing oscillations are essential. Linear techniques for designing most controllers exhibit poor performance because they consider neither the variations in operating conditions (e.g., variations in system parameters owing to failures) nor the system dynamics. In recent decades, some researchers have adopted robust control techniques that consider the inherent complexity of a system [8-11]. The use of robust controllers and filters, especially the H_∞ based design, enables us to realize control that is close to that of a real system, with all its specificity in different segments, and reduces the problem to a simple optimization problem [12,13]. However, in power systems, such controllers are of a higher order (equivalent to the number of variables). To address this issue, the model can be pre-reduced; thus, the order of the controller is also reduced [9]. Hence, there is a need to improve control techniques for obtaining additional models of controllers with lower order and higher efficiency in order to maintain system stability.

2. **Materials and Methods.**

The approach for developing the controllers can be described as follows.

2.1. **Signal analysis for decentralized control.**

A power system with n units, m control inputs, and r output signals is expressed as

$$y(j\omega) = G(j\omega)u(j\omega)$$

where $G(j\omega)$ is the transfer function matrix of the frequency responses (MFTfr).

One can achieve acceptable performance with limited inputs and outputs. Similarly, one can define the "observability" of an OM as the contribution of the OM to the system response.

For analysis of modal controllability and observability of multivariable systems in the frequency domain, "singular values" of MFTfr are used; these are defined by

$$\sigma_i(G) = \sqrt{\lambda_i(G^H G)} = \sqrt{\lambda_i(G G^H)}, \quad i = 1, \dots, k \tag{1}$$

where λ_i is the i-th eigenvalue of the matrix, G^H is the conjugated and transposed matrix, and $k = \min(m, r)$. Setting $\bar{\sigma}$ as the supremum singular value, $\underline{\sigma}$ as the infimum, and the relation $\gamma = \bar{\sigma}/\underline{\sigma}$ as the condition number, the following interesting properties are described [14,15]:

- $\bar{\sigma}$ in the frequency of an EOM represents the degree of observability of the mode in the system response and $\underline{\sigma}$ represents the degree of controllability of the mode. Weakly damped and strongly observable EOM in the response signals shows large peaks in the graphs of $\bar{\sigma}$;
- the peaks of $\bar{\sigma}$ are associated with system robustness. Robust systems have small peaks of $\bar{\sigma}$.

Consider the power system $G(j\omega)$ with controllers $H(j\omega)$, reference inputs R, and disturbances d, as shown in Figure 1.

The following relation is obtained from Figure 1:

$$(I + GH)^{-1} GR + (I + GH)^{-1} G_d d \tag{2}$$

where $S = (I + GH)^{-1}$ is the sensitivity matrix and $T = SG$ is the matrix transfer function of the closed-loop system. These matrices are used for analysis of the controlled system performance.

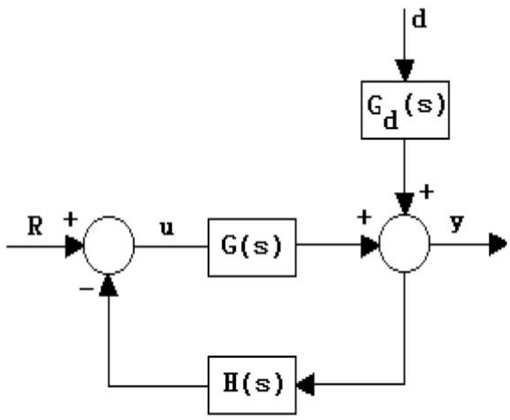

FIGURE 1. Power system with controller

2.2. **Frequency interactions.** The matrix of relative gains (MRG) is important for the analysis of multivariable systems; it is used with a previous selection of inputs and outputs for decentralized control. The MRG is defined by

$$\Lambda(G(j\omega)) = \begin{bmatrix} \lambda_{11} & \cdots & \lambda_{1m} \\ \vdots & \dots & \vdots \\ \lambda_{r1} & \cdots & \lambda_{rm} \end{bmatrix} \tag{3}$$

where $\lambda_{ij} = g_{ij}b_{ji}$, with b_{ji} being the ji element of the generalized inverse matrix G, defined as $G^+ = (G^H G)^{-1} G^H$ for $m < r$, $rank(G) = m$ or $G^+ = G^H(GG^H)^{-1}$ for $r \leq m$, $rank(G) = r$. It is known that λ_{ij} is a measure of interaction between the input j and the output i [15].

The MRG can be used for selecting the most effective input-output pairs. However, the use of MRG alone for such a selection has some limitations. The main limitation is the inability of the MRG to select the most effective output signals from a single unit, for example, speed and electrical power in a generator [16], or, more generally, any signal with respect to another.

A technique that combines the MRG and singular values for selecting the most effective input-output pairs has been proposed for application to decentralized controllers [17]. This technique has been proved to be very efficient and reliable for signal selection.

2.3. **Decentralization.** A set of inputs and outputs is completely decentralized if $\Lambda(G) = I$. However, this equality only occurs if the matrix G is triangular, which is not the case in power systems. However, we can accept as decentralized the set that yields the result $\Lambda(G(j\omega)) \approx I$ for $\omega = \omega_c$ [13]. The *crossover frequency* ω_c is defined as the frequency at which $\bar{\sigma} = 1$, when $\bar{\sigma}$ is decreasing.

The closer the result $\Lambda(G(j\omega_c))$ is to the identity matrix, the more independent the input-output pairs are; consequently, smaller interactions occur between controllers.

2.4. **Input and output selection.** Initially, all the input and output signals are used to determine the MRG frequency $\omega = 0$. Using this matrix, the ineffective signals or entries that cause undesirable interactions are removed.

Next, as p controllers are sufficient to dampen the oscillation modes with a robust control system, sets of p inputs and p outputs are created. Thus, these sets are tested to verify the decentralization at the frequency $\omega = \omega_c$. Sets with strong interactions between units (weak decentralization) are discarded.

Finally, the remaining sets are compared using singular values in order to select a set with good decentralization and the highest controllability (largest $\underline{\sigma}$) in the range of the OM. This set is used for implementing the decentralized controllers.

2.5. **Robust decentralized controller design.** The controllers will be designed by considering modeling errors. These errors, called uncertainties, occur because of the exclusion of non-linear properties in the system model, changes in the parameters with the load variations, and exclusion of dynamics of the generators, excitation systems, etc. In this model, these errors are considered by taking multiplicative uncertainty reflected in the output, as shown in Figure 2 [15,16], where $\Delta_o W_o(s) = (G' - G)G^{-1}$ is the matrix of related uncertainties and G' is the real system transfer matrix. The diagonal matrix $W_o(s)$ represents the upper limits of uncertainties in the control channels corresponding to $W_2 W_1$.

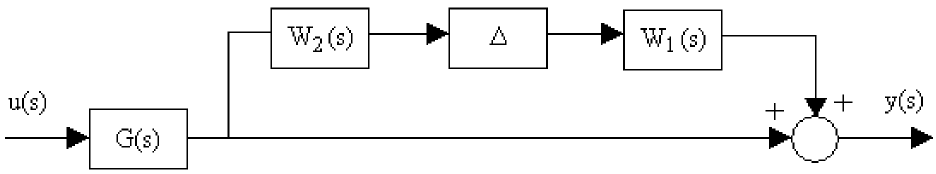

FIGURE 2. Block diagram of the real power system

The goal is to design controllers to stabilize not only the nominal plant $G(s)$ but also the set of all plants defined by $G'(s) = (I + \Delta_o W_o)G(s)S' = (I + G'H)^{-1}$, which is the actual system sensitivity matrix.

Uncertainty is introduced into the block diagram shown in Figure 1, and a separate block $\mathbf{\Delta}_o$ representing uncertainty is created; the resultant diagram is shown in Figure 3, where $M(s) = -W_o(s)T(s)H(s)$, with T being equivalent to the product of the sensitivity matrix system.

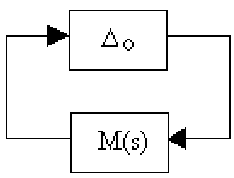

FIGURE 3. $\mathbf{M\Delta}$ structure of the system

In general, the $W_o(s)$ matrix is represented by $\omega_o(s)I$, where $\omega_o(s)$ is a weight considering a single upper limit, representing the worst case, for all the associated control channels. This weight is described by $\omega_o(s) = (\zeta s + \zeta_o)/[(\zeta/\zeta_\infty)s + 1]$, where ζ_o is the uncertainty in the steady state, $1/\zeta$ is the frequency where the uncertainty reaches 100%, and ζ_∞ is the magnitude of the weight at high frequencies.

Assuming that the matrix $\mathbf{M}$ and $\mathbf{\Delta}$ disorders are stable, the system in Figure 3, $\mathbf{M\Delta}$, is stable for all disturbances if and only if [13]

$$\mu(\mathbf{M}(j\omega)) < 1, \quad \forall\omega \tag{4}$$

where $\mu(\mathbf{M})$ is the structured singular value of $\mathbf{M}$.

It is known that $\mu(\mathbf{M}) \leq \bar{\sigma}(\mathbf{M})$ and that equality occurs when the matrix of uncertainty, $\mathbf{\Delta}$, is full, which should occur in the case of modeling errors or exclusion of the dynamics of generators, transmission networks, etc. Therefore, the necessary and sufficient condition for a robust stable system with $\bar{\sigma}(\Delta) \leq 1$, $\forall\omega$, is given by

$$\bar{\sigma}(\mathbf{M}(j\omega)) < 1 \quad \forall\omega \tag{5}$$

It is assumed that the controller is of a known structure (centralized and reduced order). To achieve robustness, the parameters of the controller $\mathbf{H}$(s) are set to solve the following optimization problem:

$$\min[\sup(\bar{\sigma}(\mathbf{M}(j\omega)))] \tag{6}$$

If, after minimization, the robustness established by (6) is not met, i.e., if the minimum value of the result is equal to or greater than one, then the order of each is increased and the optimization problem is repeated. It is noteworthy that $\sup(\bar{\sigma}(\mathbf{M}))$ implies a higher value or peak maximum of $\bar{\sigma}(\mathbf{M})$.

Some constraints may be introduced in the optimization problem. For example, the parameters of the controllers should be adjusted between practical limits. In this study, the parameters are always positive, and their relationship in the advance-delay stages should satisfy $0.1 \leq T1/T2 \leq 10$.

Consider $\mathbf{M} = \omega_o\mathbf{TH}$ (negative sign does not affect the result). Then, (6) reduces to $\bar{\sigma}(\mathbf{M}) = \omega_o\bar{\sigma}(\mathbf{TH}) \leq \omega_o\bar{\sigma}(\mathbf{H})\bar{\sigma}(\mathbf{T}) < 1$, or

$$\bar{\sigma}(T) < \frac{1}{\omega_o} \cdot \frac{1}{\bar{\sigma}(\mathbf{H})} \quad \forall\omega \tag{7}$$

which is the easiest condition to be verified.

Note that for any controller, a robustness check is performed by plotting different graphs on the same scale and ensuring that the graph of $\bar{\sigma}(\mathbf{T})$ remains below the graphs of $\frac{1}{\omega_o\bar{\sigma}(\mathbf{H})}$ for all ω.

In terms of computational resources, the most practical procedure for designing the decentralized robust controller is to adjust the parameters for the minimization of (6). Then, it checks if (7) is satisfied. If so, we achieve robust decentralized control; otherwise, we increase the order. In the present study, genetic algorithms were adopted to search for the best results that fit perfectly to the conditions proposed in the problem, and the possibility of obtaining a global minimum for the objective function ensures greater robustness. While the method was carried out, the necessity to impose a constraint on the value of the gain K was noted; such a constraint is defined as $0.4 < K < 1$ to ensure that all eigenvalues are negative, resulting in closed-loop system stability.

Genetic algorithms are search algorithms based on natural selection and genetic mechanisms; they apply genetic operators such as selection, recombination, and mutation to individuals until the combination solution to the problem is obtained. Such algorithms belong to the class of global optimization heuristic techniques, as opposed to methods such as the gradient method (hill climbing), which adopts the derivative of a function to find its maximum and is easily trapped in local minima [18]. It is a random search method, and as such, it can generate different responses to the same function with the same set of initial conditions. However, random searches that consider historical information to find new search points are more likely to perform well [19].

3. Results and Discussion.

In general, automatic control of a generation system occurs through two channels: load-frequency (PF) and reactive power/voltage (QV). The basic objective of QV control is to maintain the terminal voltage of the generator constant, whereas the PF controller focuses on maintaining the same power output demand of electric power, constant frequency equal to the standard, and constant exchange power equal to the programmed value PF control [20], in contrast with QV control, is performed collectively, and it acts on all the generator control units in a control area [21]. Electric power systems are usually large and complex with separate units distributed over hundreds of kilometers and divided into several areas. We refer to the part of an interconnected power system responsible for absorbing its own load variations as the control area [22]. In general, the borders of the control area coincide with the borders of an electric utility provided that it possesses a good ability to generate and feed a significant load. Thus, a load-frequency power control system of three interconnected equivalent areas was used for illustration purposes.

The linearized model is described in [23], and the system is shown in Figure 4.

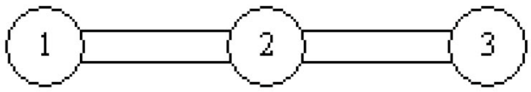

FIGURE 4. Three interconnected equivalent areas

The system state-space model is given by

$$\begin{aligned} \dot{X} &= Ax + Bu \\ y &= CX \end{aligned} \tag{8}$$

where

$$\begin{aligned} X^T &= [f_1\ X_{E1}\ P_{G1}\ P_{tie1}\ f_3\ X_{E3}\ P_{G3}\ P_{tie2}\ f_2\ X_{E2}\ P_{G2}] \\ u^T &= [P_{C1} \quad P_{C3} \quad P_{C2}] \\ y^T &= [f_1\ P_{tie1}\ f_3\ P_{tie2}\ f_2] \end{aligned}$$

where f_i, X_{Ei}, P_{Gi} and P_{tiei} are the frequency, output signal of the velocity regulator, mechanical power of the turbine, and turbo generator exchange power of the i-th area in incremental values. Further, P_{Ci} is the regulator control input speed of the i-th area.

The system has three OMs whose associated eigenvalues are as follows: mode 1: $-0.1759 \pm j3.0010$; mode 2: $-0.1199 \pm j4.0102$; and mode 3: $-0.1893 \pm j4.6410$. It was found that the three modes are of inter-area type; thus, it was recommended that controllers be applied to the three areas to damp the three OMs.

The most natural way to manage a large system, such as a power system, is with the application of decentralized control. Moreover, for the controller to be well accepted by engineers and power system operators, a reduced order and easy adjustment are recommended [24]. To improve the decentralized controller, representation with output compound signals was considered as a resource used in practice, defined as follows:

$$y_2^T = |P_{tie1} + Bf_1 \quad P_{tie2} + Bf_3 \quad -P_{tie2} + Bf_2|$$

where B is the bias factor in MW/Hz and the relationship $P_{tie} + Bf$ with incremental variables is called the area control error (ACE). Both ACE and the bias are widely used in the literature as frequency control operators and for power system exchange. Typically, $0 < B < 1$. Traditionally, one seeks to take the bias natural characteristic equal to the combined area, i.e., $B_i = \frac{1}{R_i} + D_i$ [21,22,25]. Thus, we adopted a typical value $B_1 = B_2 = B = 0.417$ MW/Hz.

The signal MRG composition in $\omega_c = 6$ rad/s results in

$$\mathbf{\Lambda} = \begin{bmatrix} 1.063 - j0.036 & 0 & -0.063 + j0.036 \\ -0.004 - j0.003 & 1.004 - j0.103 & 0 + j0.106 \\ -0.059 + j0.039 & -0.004 + j0.103 & 1.063 - j0.142 \end{bmatrix}$$

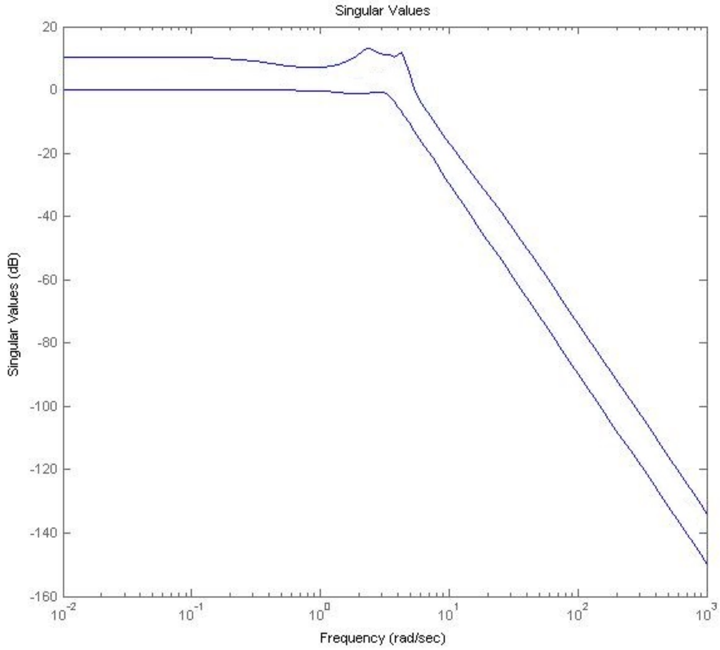

FIGURE 5. $\bar{\sigma}$ and $\underline{\sigma}$ values of $\mathbf{G}_2(j\omega)$

Then, there is a composite signal with the input y_2, which inputs u, resulting in pairs with very good decentralization. The graphs of $\bar{\sigma}$ and $\underline{\sigma}$ for $y_2 = \mathbf{G}_2(j\omega)\mathbf{u}$ are shown in Figure 5.

It is found that $\underline{\sigma}$ for G_2 is superior in all frequency bands of the OM. Thus, the signals y_2 are selected for feedback. Note that even though the signals y_2 are the most effective, it could still be difficult to achieve robust control, because $\underline{\sigma}(G_2) < 1$ across the frequency band. The controller chosen is of the type $h_i(s) = K(1 + sT_1)/(1 + sT_2)$ in each area, with two identical controllers and a third with different parameters, in order to have a generalized system and thus show that the model can be applied, regardless of the chosen controllers, to yield a good response for low-order controllers. The controller parameters are re-adjusted to minimize the M function and thus to reduce $\bar{\sigma}(T)$. For this problem, combinations of 20 individuals in 200 generations were considered; the usual values of MATLAB® were taken. The results for the parameters were $K = 0.4628$, $T_1 = 0.001$ s, and $T_2 = 0.01$ s for the first controller and $K = 0.4$, $T_1 = 0.0081$ s, and $T_2 = 0.0099$ s for the second controller. Thus, the parameters are close owing to the characteristics of the areas that are very similar in a wide range. The graphs of $\bar{\sigma}(T)$ for the obtained controllers and of $\frac{1}{\omega_o \bar{\sigma}(\mathbf{H})}$ for $\omega_{o1} = (0.25s + 0.15)/(0.5s + 1)$ are shown in Figure 6.

Figure 6 shows that robust control can be obtained without high uncertainty using decentralized controllers of the first order. The best results will be obtained with higher-order controllers.

The effectiveness of the controller can also be demonstrated in the time domain. For this purpose, the impulse response was obtained for the system and it was observed that by applying the controller, the system attains the steady state faster. This can be seen by comparing the results of Figure 7 that match the impulse response for the pair of input-output ΔP_{tie1} e $P_{tie1} + Bf_1$, without the controller (open loop) and with the controller.

With regard to the chosen optimization method, genetic algorithms also perform well when used in robust H_∞ control techniques, as shown by [26]. The difference between

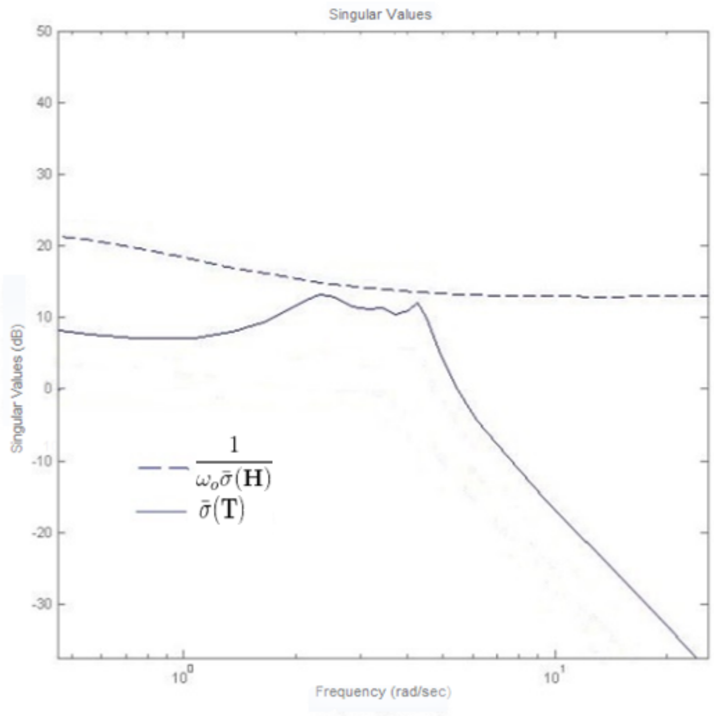

Figure 6. Graph of $\bar{\sigma}(\mathbf{T})$ and $\frac{1}{\omega_o \bar{\sigma}(\mathbf{H})}$ for the proposed controller

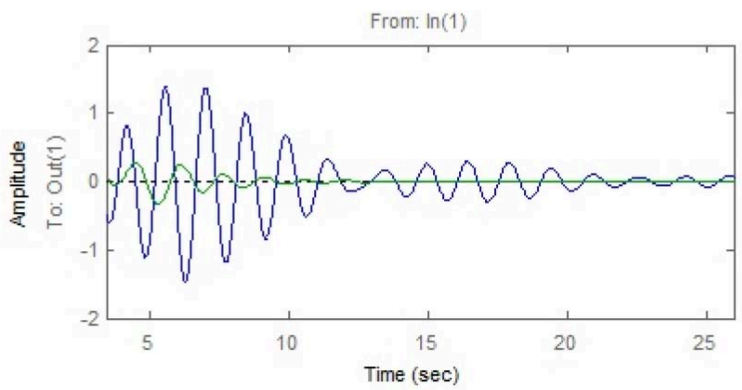

Figure 7. Impulse response

these methods and the proposed method is that the latter focuses on realizing a low-order controller for an arrangement capable of maintaining acceptable performance even with defects in one of the controllers or in the presence of faults in the system.

The proposed method is also compared with that described in [3], where the pattern search method developed by Hooke and Jeeves [27] is used in an optimization problem similar to the one presented in this paper. The results were quite similar, as shown by the singular values in Figure 8. However, the genetic method has a much higher probability of finding the global minimum; thus, it is a more reliable method for use in other systems. Furthermore, the pattern search method is a search method with slower convergence as compared to the genetic algorithm because it involves varying one variable at a time; thus, it may become impracticable for larger systems.

For further comparison, the study was repeated using pure integrators $u_i = (K_i/s)y_i$ as controllers, as in the case of traditional control exchange. However, it was found that

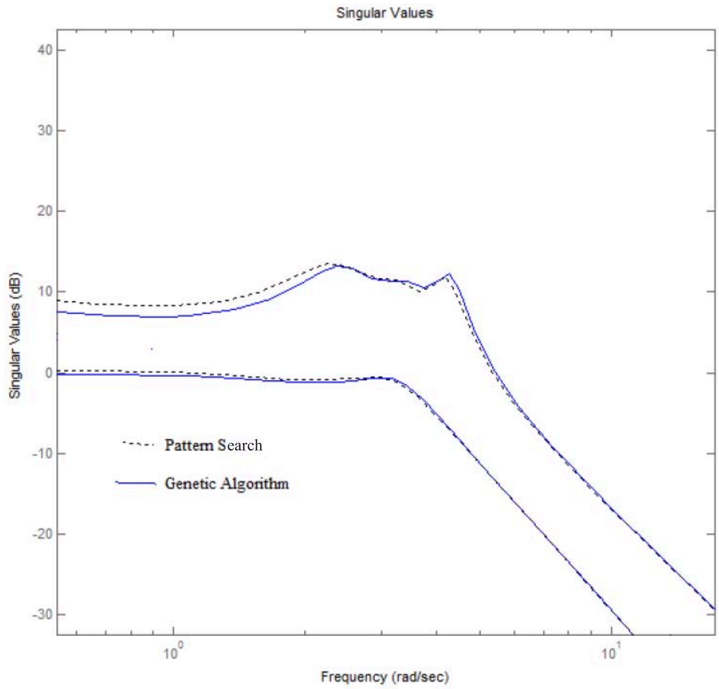

FIGURE 8. Singular values of the system for the obtained controllers using pattern search with genetic algorithms

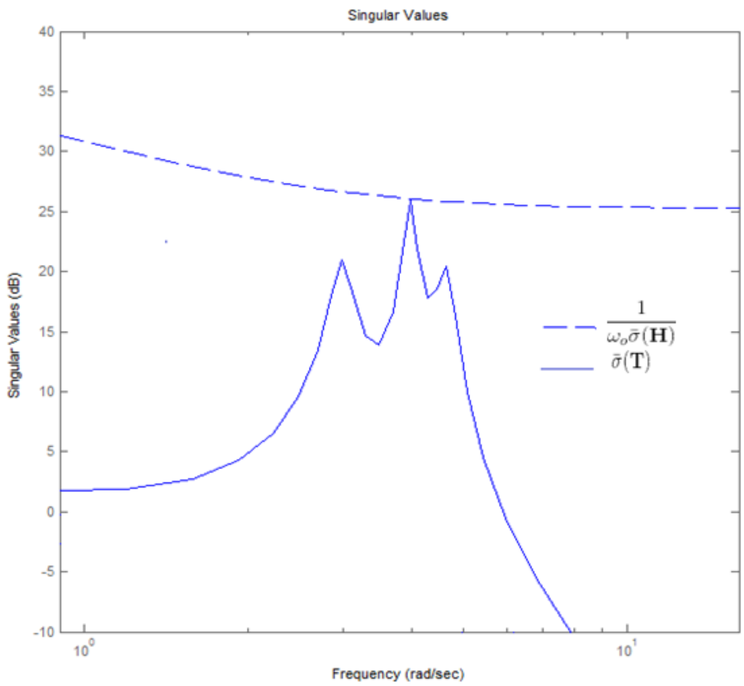

FIGURE 9. Robustness check using traditional control with $K_i = 0.11$

robust control can only be achieved with $K_i < 0.11$ (Figure 9). On the other hand, with small K_i values and non-absorption of the OM, the system yields three real poles very close to the origin and thus has a very slow response.

For other types of disturbances and faults, the methodology also responds satisfactorily. When the power exchanged between the areas is reduced by 20%, it is still possible to

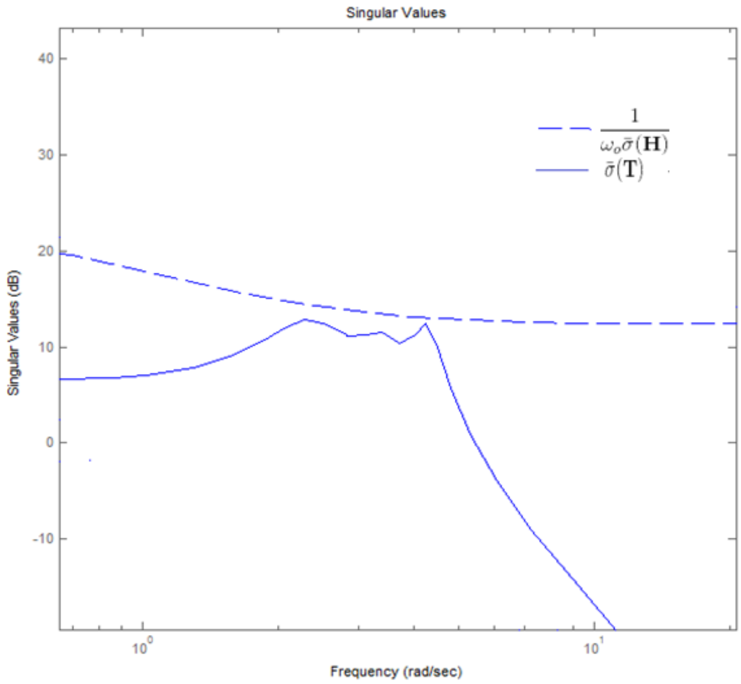

FIGURE 10. Robustness check using $P_{tie} = 0.80$

achieve robustness, as shown in Figure 10. Thus, the model remains robust even if the system is experiencing failures, thereby providing highly reliable control.

4. **Conclusions.** A method was proposed for designing robust controllers and achieving decentralization as well as a low-order for a load-frequency system. In the proposed method, the relative gains are used for pre-selection of inputs and outputs, leaving the final selection to be performed with the use of singular values, which is appropriate and efficient for application to the chosen load-frequency system, and is the key to facilitating efficient choice of a low-order and decentralized controller. The methodology developed in this study can be applied to any system, regardless of its size. This technique outperforms traditional techniques of robust H_∞ control, mainly in large systems, resulting in a lower-order controller that is applied directly into the system without any reduction of the model; moreover, input and output pairs can be selected simultaneously. This methodology is based on bio-inspired optimization; thus, robustness can be achieved even in the presence of disturbances. In addition, the optimization method used to minimize the H_∞ norm ensures a very high probability of finding the global minimum, resulting in the most optimal controller possible. Furthermore, it can adapt to several electric system models and not just a load-frequency system. It is also applicable to different types of controllers. Thus, in summary, it is a highly reliable and competitive approach.

Acknowledgment. The authors wish to thank their institutions for the support to this work development.

REFERENCES

[1] P. Kundur, *Power System Control and Stability*, Editora McGraw-Hill, 1994.
[2] P. Kundur, J. Paserba, V. Ajjarapu et al., IEEE/CIGRE, Joint Task Force on Stability Terms and Definitions, Definition and classification of power system stability, *IEEE Trans. Power System*, vol.19, no.3, pp.1387-1401, 2004.

[3] A. C. Castro, *Projeto de Controladores Robustos Descentralizados de Ordem Reduzida Para Amortecimento de oscilações em Sistemas Elétricos de Potência*, Ph.D. Thesis, Universidade Federal da Paraíba, João Pessoa, PB, Brasil, 2006.

[4] A. G. Exposito, A. Conejo and C. Cañizares, *Sistemas de Energia Elétrica: Análise e Operação*, LTC, Rio de Janeiro, 2011.

[5] H. M. Ayres, *Aplicação do Controlador UPFC para o Amortecimento de Oscilações Eletromecânicas em Sistemas Elétricos de Potência*, Dissertação de Mestrado da Faculdade de Engenharia Elétrica e de Computação, UNICAMP, Campinas, São Paulo, 2005.

[6] G. Rogers, *Power System Oscillations*, Kluwer, Massachusetts, 2000.

[7] O. L. Tortelli, *Alocação e Operação de Controladores FACTS em Sistemas Elétricos de Potência*, Tese de Doutorado apresentada à Faculdade de Engenharia Elétrica e de Computação da Universidade Estadual de Campinas, UNICAMP, Campinas, 2010.

[8] M. Klein, G. J. Rogers, S. Farrokhpay and N. J. Balu, H_∞ damping controller design in large power system, *IEEE Trans. Power Systems*, vol.10, no.1, pp.158-165, 1995.

[9] G. E. Boukarim, S. Wang, J. H. Chow, G. N. Taranto and N. Martins, A comparison of classical, robust, and decentralized control design for multiple power system stabilizers, *IEEE Trans. Power Systems*, 2000.

[10] B. Pal and B. Chaudhuri, *Robust Control in Power Systems*, Springer, 2005.

[11] H. Chen, Z. Guo and H. Bai, Wide-area robust H_2/H_∞ control with pole placement for damping inter-area oscillations, *Proc. IEEE Power Eng. Soc. General Meeting*, 2006.

[12] P. Shi, Y. Yin and F. Liu, Robust control on saturated Markov jump systems with missing information, *Information Sciences*, vol.265, pp.123-138, 2014.

[13] P. Shi, X. Luan and F. Liu, H_∞ filtering for discrete-time systems with stochastic incomplete measurement and mixed delays, *IEEE Trans. Industrial Electronics*, vol.59, no.6, pp.2732-2739, 2012.

[14] J. J. Cruz, *Controle Robusto Multivariável*, Editora Universidade de São Paulo, São Paulo, 1996.

[15] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*, Wiley, Chichester, U.K., 2005.

[16] J. V. Milanovic and A. S. Duque, The use of relative gain array for optimal placement of PSSs, *CD ROM of the IEEE Power Eng. Soc. Winter Meeting*, 2001.

[17] A. C. Castro, C. S. Araujo, J. M. Araujo, E. T. F. Santos and F. A. Moreira, Robust H_∞ control with selection of sites for application of decentralized controllers in power systems, *International Journal of Innovative Computing, Information and Control*, vol.9, no.1, pp.139-152, 2013.

[18] M. S. Saad, H. Jamaluddin and I. Z. M. Darus, Implementation of PID controller tuning using differential evolution and genetic algorithms, *International Journal of Innovative Computing, Information and Control*, vol.8, no.11, pp.7761-7779, 2012.

[19] A. Carvalho, A. Braga and T. Ludermir, Computação evolutiva, in *Rezende, S. O. (Co-ord.), Sistemas Inteligentes – Fundamentos e Aplicações*, 1ª Ed. Cap. 9, Ed. Manole, São Paulo, Brasil, 2003.

[20] W. S. Mota, *Simulação de Transitórios Eletromecânicos em Sistemas de Potência*, Ed. EPGRAF, Campina Grande-PB, Brasil, 2006.

[21] O. I. Elgerd, *Introdução à Teoria de Sistemas de Energia Elétrica*, 1ª Ed., São Paulo, McGraw-Hill do Brasil, 1976.

[22] N. Cohn, Control of interconnected power system, in *Handbook of Automation Computation and Control, Chapter 17*, S. Ramo and D. E. Wooldrige (eds.), John Wiley, 1961.

[23] J. L. Calvet and A. Titli, Overlapping vs. partitioning in block-iteration methods: Application in large-scale system theory, *Automatica*, vol.25, no.1, 1989.

[24] A. C. Castro and C. S. Araújo, Um Estudo sobre Robustez de Controladores Descentralizados em Sistemas de Potência, in *Anais do XV Congresso Brasileiro de Automática*, Gramado, 2004.

[25] J. C. Castro, M. O. Catão and R. Doraiswami, Identification of the generating units to be equipped with stabilizers in multimachine power systems, *Automatica*, vol.24, pp.405-409, 1988.

[26] Y. Morishita, K. Suzuki and S. Iwamoto, Lead-lag PSS design based on H_∞ control theory and genetic algorithm, *IEEE Power and Energy Society General Meeting*, vol.8, pp.1-7, 2012.

[27] B. S. Gottfried and J. Weisman, *Introduction to Optimization Theory*, Prentice Hall, Englewood Cliffs, 1973.