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THE PARAMETERIZATION OF ALL TWO-DEGREES-OF-FREEDOM SEMISTRONGLY STABILIZING CONTROLLERS

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Abstract. *A semistrongly stabilizing controller is a stabilizing controller that has one pole at the origin and other poles in the open left-half plane. Using semistrongly stabilizing controllers, we can overcome the problem of strong stabilization, that is, if an uncertainty in the plant or a step disturbance occurs, the output of the control system cannot follow the step reference input without steady state error. Accordingly, Hoshikawa et al. proposed parameterizations of all semistrongly stabilizable plants and of all semistrongly stabilizing controllers. In this parameterization, we cannot specify the input-output characteristic and the feedback characteristic separately. One way to achieve this is to use a two-degrees-of-freedom control system. However, the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers has not been examined. The purpose of this paper is to propose such a parameterization for semistrongly stabilizable plants.*
Keywords: Strong stabilization, Robust servo, Semistrong stabilization, Two-degrees-of-freedom control system, Parameterization

1. **Introduction.** In the parameterization problem, all stabilizing controllers for a plant [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] and all plants that can be stabilized [11] are sought. Because this parameterization can successfully search for all proper stabilizing controllers, it is used as a tool for many control problems.

In practical control problems, both the stability of the closed-loop systems and that of the stabilizing controllers are important. In certain cases [12], the instability of stabilizing controllers causes poor overall system sensitivity to variations in plant parameters. On the other hand, from [13], even if a plant is stabilizable, the plant is not necessarily strongly stabilizable. In addition, the achievable control characteristic is restricted in comparison with the case using unstable controllers. It is thus desirable to choose either a stable controller or an unstable one by the required control specification. Since nonstrongly stabilizable plants exist, two necessary and sufficient conditions that a plant is strongly stabilizable have been clarified. One was clarified by Youla et al. and is called the parity interlacing property, which is a condition on the placement of poles and zeros of strongly stabilizable plants [4, 13]. They also proposed a method to find strongly stabilizing controllers using Nevanlinna-Pick interpolation [4, 13]. This result was developed further in several papers about the design method for strongly stabilizing controllers [14, 15, 16, 17]. The other condition was clarified by Hoshikawa et al. and is the parameterization of all strongly stabilizable plants, which shows that strongly stabilizable plants have a common feedback structure [19]. They also proposed the parameterization of all strongly stabilizing controllers, thus enabling the systematic design of strongly stabilizing controllers. The strong stabilization problem has thus been studied extensively.

With strongly stabilizing controllers, when there is an uncertainty in the plant or a step disturbance, the output of the control system cannot follow the step reference input without steady state error. The reason is that strongly stabilizing controllers cannot have a pole at the origin. If the control requires high tracking performance, stabilizing controllers require an integrator. Therefore, it is necessary to examine controller designs that have a pole at the origin and other poles in the open left-half plane. We call such controllers semistrongly stabilizing controllers [19]. Because plants that are unstabilizable by strongly stabilizing controllers exist [13], it is expected that plants that cannot be stabilized by a semistrongly stabilizing controller also exist. Accordingly, Hoshikawa et al. clarified the parameterization of all semistrongly stabilizable plants [19]. In addition, Hoshikawa et al. proposed the parameterization of all semistrongly stabilizing controllers [20].

However, with their parameterization [20], we cannot specify the input-output characteristic and the feedback characteristic, that is, a disturbance attenuation characteristic and robust stability, separately. When we specify one characteristic, other characteristics are also decided. From the practical viewpoint, it is desirable to specify the input-output characteristic and the feedback characteristic separately. One way to achieve this is to use a two-degrees-of-freedom control system. In addition, because a two-degrees-of-freedom control system can have no overshoot for the reference input, more accurate control can be expected.

In this paper, we propose the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers for semistrongly stabilizable plants, in which the output of the control system can follow the step reference input without steady state error even if an uncertainty in the plant or a step disturbance exists.

This paper is organized as follows. In Section 2, we propose the concept of a two-degrees-of-freedom semistrongly stabilizing controller and formulate the problem considered in this study. In Section 3, we propose the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers for semistrongly stabilizable plants. In Section 4, we present a design method for the semistrongly stabilizing controllers presented in Section 3. In Section 5, a numerical example is illustrated to show the effectiveness of the proposed method. Section 6 gives concluding remarks.

Notation

R	The set of real numbers.
$R(s)$	The set of real rational functions with s.
RH_∞	The set of stable proper real rational functions.
$\mathcal{U}$	The set of unimodular functions on RH_∞. That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.

2. **Two-Degrees-of-Freedom Semistrongly Stabilizing Controller and Problem Formulation.** Consider the two-degrees-of-freedom control system shown in Figure 1, which can specify the input-output characteristic and the feedback characteristic separately. Here, $G(s) \in R(s)$ is the plant, $C(s)$ is the two-degrees-of-freedom controller:

$$C(s) = \begin{bmatrix} C_1(s) & -C_2(s) \end{bmatrix}, \tag{1}$$

$u(s)$ is the control input:

$$u(s) = C(s)\begin{bmatrix} r(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} C_1(s) & -C_2(s) \end{bmatrix}\begin{bmatrix} r(s) \\ y(s) \end{bmatrix}, \tag{2}$$

$r(s)$ is the reference input, $d_1(s)$ and $d_2(s)$ are disturbances, and $y(s)$ is the output. In the following, we call $C_1(s) \in R(s)$ the feed-forward controller and $C_2(s) \in R(s)$ the feedback

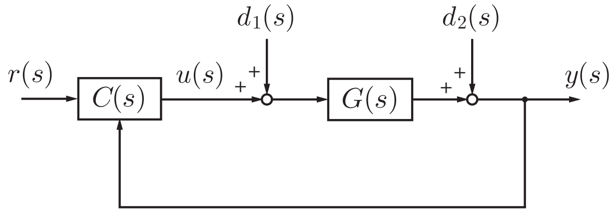

FIGURE 1. Two-degrees-of-freedom control system

controller. From the definition of internal stability [4], when all transfer functions $V_i(s)$ $(i = 1, \dots, 6)$:

$$\begin{bmatrix} u(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} V_1(s) & V_2(s) & V_3(s) \\ V_4(s) & V_5(s) & V_6(s) \end{bmatrix} \begin{bmatrix} r(s) \\ d_1(s) \\ d_2(s) \end{bmatrix} \tag{3}$$

are stable, the two-degrees-of-freedom control system in Figure 1 is stable.

Semistrongly stabilizing controllers were defined in [19] as follows.

Definition 2.1. *(semistrongly stabilizing controllers)* [19]
We call the controller $C(s)$ a "semistrongly stabilizing controller" if the stabilizing controller has only one pole at the origin and other poles in the open left-half plane. That is, if $C(s)$ is:

$$C(s) = \frac{s + \alpha}{s} Q(s), \tag{4}$$

then we call $C(s)$ a semistrongly stabilizing controller, where $\alpha \in R$ is any positive real number and $Q(s) \in RH_\infty$ is any function satisfying $Q(0) \neq 0$.

According to Definition 2.1, the difference between strongly stabilizing controllers and semistrongly stabilizing controllers is whether or not the controllers have only one pole at the origin. That is, the characteristic of semistrongly stabilizing controllers is to make the output of the control system follow the step reference input without steady state error in the presence of an uncertainty in the plant or a step disturbance. In addition, a plant stabilizable by a semistrongly stabilizing controller, so-called semistrongly stabilizable plants, is also defined in [19] as follows.

Definition 2.2. *(semistrongly stabilizable plant)* [19]
We call the plant $G(s)$ a "semistrongly stabilizable plant" if $G(s)$ is stabilizable by a semistrongly stabilizing controller $C(s)$ in (4).

According to [19], the parameterization of all semistrongly stabilizable plants is defined by:

$$G(s) = \frac{\beta + sQ_2(s)}{(s + \alpha)\left(1 + Q_3(s) - Q_1(s)Q_2(s)\right)}, \tag{5}$$

where $\beta \in R$ is given by:

$$\beta = \frac{\alpha}{Q_1(0)}, \tag{6}$$

$Q_3(s) \in RH_\infty$ is given by:

$$Q_3(s) = \frac{\alpha - \beta Q_1(s)}{s}, \tag{7}$$

and $Q_1(s) \in RH_\infty$ and $Q_2(s) \in RH_\infty$ are any functions satisfying $Q_1(0) \neq 0$. That is, the plant $G(s)$ in Figure 1 is described by the form of (5). In addition, Hoshikawa et al. gave the parameterization of all semistrongly stabilizing controllers for semistrongly stabilizable plants in (5) [20].

However, the parameterization of all semistrongly stabilizing controllers [20] was only considered for one-degree-of-freedom control systems. This means that all characteristics are specified for a one-degree-of-freedom semistrongly stabilizing controller. From the practical point of view, it is desirable to specify the input-output characteristic and the feedback characteristic separately. One way to do this is to use a two-degrees-of-freedom control system in Figure 1.

From this viewpoint, we consider a two-degrees-of-freedom semistrongly stabilizing controller that makes the output of the control system follow a step reference input without steady state error, under the existence of an uncertainty in the plant or a step disturbance. The concept of a two-degrees-of-freedom semistrongly stabilizing controller is proposed as follows.

Definition 2.3. *(two-degrees-of-freedom semistrongly stabilizing controller)*
We call the controller $C(s)$ in (1) a "two-degrees-of-freedom semistrongly stabilizing controller" if the following expressions hold true.

1. *The feed-forward controller $C_1(s)$ in (1) has only one pole at the origin. That is, the feed-forward controller $C_1(s)$ is defined by:*
$$C_1(s) = \frac{s+\gamma}{s} Q_f(s), \tag{8}$$
where $\gamma \in R$ is any positive real number and $Q_f(s) \in RH_\infty$ is any function satisfying $Q_f(0) \neq 0$.
2. *The feedback controller $C_2(s)$ in (1) works as a semistrongly stabilizing controller. That is, the feedback controller $C_2(s)$ is defined in the form of (4).*
3. *The two-degrees-of-freedom control system in Figure 1 is stable. That is, all transfer functions $V_i(s)$ $(i = 1, \ldots, 6)$ in (3) are stable.*

From Definition 2.3, the feed-forward controller $C_1(s)$ also has a pole at the origin. This means the transfer function $V_4(s)$ in (3) from the reference input $r(s)$ to the output $y(s)$ in Figure 1 cannot have a zero at the origin with the origin pole of the feedback controller $C_2(s)$, which is to ensure that the output cannot have a steady state error for the step reference input.

The problem considered in this paper is to obtain the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers $C(s)$ defined in Definition 2.3.

3. **The Parameterization of All Two-Degrees-of-Freedom Semistrongly Stabilizing Controllers.** In this section, we propose the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers $C(s)$ for semistrongly stabilizable plants $G(s)$ in the form of (5).

This parameterization is summarized in the following theorem.

Theorem 3.1. *The parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers $C(s)$ for semistrongly stabilizable plants $G(s)$ in the form of (5) is:*
$$C_1(s) = \frac{s+\gamma}{s} \frac{Q_{c1}(s)}{Q_u(s)} \tag{9}$$

and

$$C_2(s) = \frac{s+\alpha}{s}\left\{Q_1(s) + \frac{Q_{c2}(s)}{1-\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q_{c2}(s)}\right\}. \tag{10}$$

Here, $\alpha \in R$ *and* $\gamma \in R$ *are any positive real numbers,* $Q_{c1}(s) \in RH_\infty$ *is any function,* $Q_{c2}(s) \in RH_\infty$ *is given by:*

$$Q_{c2}(s) = \frac{1-Q_u(s)}{\dfrac{\beta}{s+\alpha}+\dfrac{s}{s+\alpha}Q_2(s)}, \tag{11}$$

$Q_u(s) \in \mathcal{U}$ *is any function that makes* $Q_{c2}(s)$ *in (11) proper and satisfies:*

$$\left.\frac{1}{(s-s_i)^{m_i-1}}\left(1-Q_u(s)\right)\right|_{s=s_i} = 0 \;\; (\forall i = 1,\dots,n), \tag{12}$$

s_i $(i=1,\dots,n)$ *are unstable zeros of* $\beta + sQ_2(s)$*, and the multiplicities of* s_i $(i=1,\dots,n)$ *are denoted by* m_i $(i=1,\dots,n)$*.*

Proof: First, the necessity is shown. That is, we show that if the controllers $C_1(s)$ and $C_2(s)$ make the control system in Figure 1 stable, that is all transfer functions $V_i(s)$ $(i = 1,\dots,6)$ in (3) are stable, then $C_1(s)$ and $C_2(s)$ are defined by (9) and (10), respectively. The transfer functions $V_i(s)$ $(i=1,\dots,6)$ in (3) are:

$$V_1(s) = \frac{C_1(s)}{1+C_2(s)G(s)}, \tag{13}$$

$$V_2(s) = -\frac{C_2(s)G(s)}{1+C_2(s)G(s)}, \tag{14}$$

$$V_3(s) = -\frac{C_2(s)}{1+C_2(s)G(s)}, \tag{15}$$

$$V_4(s) = \frac{C_1(s)G(s)}{1+C_2(s)G(s)}, \tag{16}$$

$$V_5(s) = \frac{G(s)}{1+C_2(s)G(s)}, \tag{17}$$

and

$$V_6(s) = \frac{1}{1+C_2(s)G(s)}. \tag{18}$$

From the assumption that all transfer functions in (13) to (18) are stable, $C_2(s)$ makes $G(s)$ stable. From [4], the parameterization of all stabilizing feedback controllers is:

$$C_2(s) = \frac{X(s)+D(s)\tilde{Q}(s)}{Y(s)-N(s)\tilde{Q}(s)}, \tag{19}$$

where $N(s)$ and $D(s)$ are coprime factors of $G(s)$ on RH_∞ satisfying:

$$G(s) = \frac{N(s)}{D(s)}, \tag{20}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are any functions satisfying:

$$N(s)X(s) + D(s)Y(s) = 1, \tag{21}$$

and $\tilde{Q}(s) \in RH_\infty$ is any function. Therefore, we must consider the condition to make $C_2(s)$ in (19) work as a semistrongly stabilizing controller. Since the semistrongly stabilizable plant $G(s)$ is defined by the form of (5), when $G(s)$ in (5) is factorized by (20):

$$N(s) = \frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s) \tag{22}$$

and

$$D(s) = 1 + Q_3(s) - Q_1(s)Q_2(s). \tag{23}$$

From (22) and (23), a pair of $X(s)$ and $Y(s)$ satisfying (21) are defined by:

$$X(s) = Q_1(s) \tag{24}$$

and

$$Y(s) = \frac{s}{s+\alpha}. \tag{25}$$

Substituting (22), (23), (24) and (25) for (19), we have:

$$\begin{aligned} C_2(s) &= \frac{Q_1(s) + (1 + Q_3(s) - Q_1(s)Q_2(s))\,\tilde{Q}(s)}{\dfrac{s}{s+\alpha} - \left(\dfrac{\beta}{s+\alpha} + \dfrac{s}{s+\alpha}Q_2(s)\right)\tilde{Q}(s)} \\ &= \frac{s+\alpha}{s}\left\{\frac{Q_1(s) + (1 + Q_3(s) - Q_1(s)Q_2(s))\,\tilde{Q}(s)}{1 - \left(\dfrac{\beta}{s+\alpha} + \dfrac{s}{s+\alpha}Q_2(s)\right)\dfrac{s+\alpha}{s}\tilde{Q}(s)}\right\}. \end{aligned} \tag{26}$$

From the assumption that $C_2(s)$ has one pole at the origin, $\tilde{Q}(s)$ becomes:

$$\tilde{Q}(s) = \frac{s}{s+\alpha}Q_{c2}(s), \tag{27}$$

where $Q_{c2}(s) \in RH_\infty$ is any function. Substituting (7) and (27) for (26), we have:

$$C_2(s) = \frac{Q_1(s) + \left(1 + \dfrac{\alpha - \beta Q_1(s)}{s} - Q_1(s)Q_2(s)\right)\frac{s}{s+\alpha}Q_{c2}(s)}{\frac{s}{s+\alpha} - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\frac{s}{s+\alpha}Q_{c2}(s)}. \tag{28}$$

By simple manipulation, we have:

$$C_2(s) = \frac{s+\alpha}{s}\left\{Q_1(s) + \frac{Q_{c2}(s)}{1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)Q_{c2}(s)}\right\}. \tag{29}$$

We have therefore shown that $C_2(s)$ is defined by (10). The remaining problem is to confirm that:

$$\bar{C}_2(s) = Q_1(s) + \frac{Q_{c2}(s)}{1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)Q_{c2}(s)} \tag{30}$$

is stable. Since $Q_1(s) \in RH_\infty$ and $Q_{c2}(s) \in RH_\infty$, the condition that (30) is stable if and only if $Q_{c2}(s)$ in (30) results in:

$$1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)Q_{c2}(s) \in \mathcal{U}. \tag{31}$$

That is, using $Q_u(s) \in \mathcal{U}$,

$$1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right) Q_{c2}(s) = Q_u(s). \tag{32}$$

This equation corresponds to (11). Since s_i $(i = 1, \dots, n)$ denote unstable zeros of $\beta + sQ_2(s)$ and the multiplicities of s_i $(i = 1, \dots, n)$ are denoted by m_i $(i = 1, \dots, n)$,

$$\frac{1}{(s-s_i)^{m_i-1}} \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right) Q_{c2}(s)\bigg|_{s=s_i} = 0 \ \ (\forall i = 1, \dots, n) \tag{33}$$

hold true. From (32) and (33), (12) is satisfied. The fact that $Q_{c2}(s)$ in (11) is included in RH_∞ is confirmed as follows: From $Q_u(s) \in \mathcal{U}$, if $Q_{c2}(s)$ in (11) is unstable, then unstable poles of $Q_{c2}(s)$ are equal to unstable zeros s_i $(i = 1, \dots, n)$ of $\beta + sQ_2(s)$. Since $Q_u(s)$ satisfies (12), unstable zeros s_i $(i = 1, \dots, n)$ of $\beta + sQ_2(s)$ are not equal to unstable poles of $Q_{c2}(s)$. Therefore, $(\beta/(s+\alpha) + sQ_2(s)/(s+\alpha))Q_{c2}(s)$ is stable. That is, when we select $Q_u(s)$ to make $Q_{c2}(s)$ proper, $Q_{c2}(s)$ in (11) is included in RH_∞. In addition, using $Q_u(s)$, $\bar{C}_2(s)$ in (30) is rewritten:

$$\bar{C}_2(s) = Q_1(s) + \frac{Q_{c2}(s)}{Q_u(s)}. \tag{34}$$

Since $Q_1(s) \in RH_\infty$, $Q_{c2}(s) \in RH_\infty$ and $Q_u(s) \in \mathcal{U}$, $\bar{C}_2(s) \in RH_\infty$. In this way, the fact that $C_2(s)$ works as a semistrongly stabilizing controller in (4) is shown.

Next, we show that the feed-forward controller $C_1(s)$ is described by (9). Using $C_2(s)$ in (10), the transfer functions in (13) and (16) are:

$$V_1(s) = C_1(s)Q_u(s)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\} \tag{35}$$

and

$$V_4(s) = \frac{s}{s+\alpha}C_1(s)Q_u(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right), \tag{36}$$

respectively. From (6),

$$1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\bigg|_{s=0} = 0 \tag{37}$$

in (35) holds true. For $V_1(s)$ in (35) and $V_4(s)$ in (36) to be stable, $C_1(s) \in RH_\infty$ or $C_1(s)$ can have only one pole at the origin and has other poles in the open left-half plane. Therefore, $C_1(s)$ in (8) works as a stabilizing controller. To specify the input-output characteristic and the feedback characteristic separately, $C_1(s)$ in (8) becomes:

$$C_1(s) = \frac{s+\gamma}{s}\frac{Q_{c1}(s)}{Q_u(s)}, \tag{38}$$

where, $Q_{c1}(s) \in RH_\infty$ is any function. In this way, when the plant $G(s)$ takes the form of (5), then $C_1(s)$ and $C_2(s)$ take the form of (9) and (10). Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, we show that if $C_1(s)$ and $C_2(s)$ are described by (9) and (10), $C_1(s)$ and $C_2(s)$ make the control system in Figure 1 stable. Using $C_1(s)$ in (9) and $C_2(s)$ in (10), transfer functions $V_i(s)$ $(i = 1, \dots, 6)$ are written:

$$V_1(s) = \frac{s+\gamma}{s}Q_{c1}(s)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\}, \tag{39}$$

$$V_2(s) = Q_u(s)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\} - 1, \tag{40}$$

$$V_3(s) = -\frac{s+\alpha}{s}\left(Q_1(s)Q_u(s) + Q_{c2}(s)\right)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\}, \tag{41}$$

$$V_4(s) = \frac{s+\gamma}{s+\alpha}Q_{c1}(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right), \tag{42}$$

$$V_5(s) = \frac{s}{s+\alpha}Q_u(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right), \tag{43}$$

and

$$V_6(s) = Q_u(s)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\}. \tag{44}$$

Since $Q_1(s) \in RH_\infty$, $Q_2(s) \in RH_\infty$, $Q_{c1}(s) \in RH_\infty$, $Q_u(s) \in \mathcal{U}$, and α is a positive real number, (40), (42), (43) and (44) are all stable. In addition, since (37) holds true, (39) and (42) have no pole at the origin. From this and because $Q_{c2}(s) \in RH_\infty$, (39) and (42) are also stable. Thus, the sufficiency has been shown.

We have thus proved Theorem 3.1. □

Next, we explain the control characteristics of the control system in Figure 1 using the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers in (9) and (10). First, the input-output characteristic is shown. The transfer function from the reference input $r(s)$ to the output $y(s)$ is:

$$\frac{y(s)}{r(s)} = \frac{s+\gamma}{s+\alpha}Q_{c1}(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right). \tag{45}$$

For the output $y(s)$ to follow the step reference input $r(s)$ without steady state error:

$$\frac{\gamma}{\alpha}\frac{\beta}{\alpha}Q_{c1}(0) = 1 \tag{46}$$

must be satisfied. From (6), (46) is rewritten:

$$\frac{\gamma}{\alpha}\frac{Q_{c1}(0)}{Q_1(0)} = 1. \tag{47}$$

Therefore, we select $Q_{c1}(s)$ satisfying:

$$Q_{c1}(0) = \frac{\alpha}{\gamma}Q_1(0). \tag{48}$$

Next, the disturbance attenuation characteristic, which is one of the feedback characteristics, is shown. The transfer functions from the disturbance $d_1(s)$ to the output $y(s)$ and from the disturbance $d_2(s)$ to the output $y(s)$ of the control system in Figure 1 are:

$$\frac{y(s)}{d_1(s)} = \frac{s}{s+\alpha}Q_u(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right) \tag{49}$$

and

$$\frac{y(s)}{d_2(s)} = Q_u(s)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\}, \tag{50}$$

respectively. Equation (49) shows that the step disturbance $d_1(s) = 1/s$ is attenuated effectively. In addition, since (37) holds true, the step disturbance $d_2(s) = 1/s$ is also attenuated effectively.

Furthermore, we find that the input-output characteristic is specified by $Q_{c1}(s)$ in (9), and the disturbance attenuation characteristic is specified by $Q_u(s)$ in (12). That is, the proposed two-degrees-of-freedom semistrongly stabilizing controller can specify the input-output characteristic and the disturbance attenuation characteristic separately.

4. **Design Method for $Q_u(s)$.** In this section, we present a design method for $Q_u(s) \in \mathcal{U}$ that satisfies (12) and makes $Q_{c2}(s)$ proper.

1. $\beta/(s+\alpha) + sQ_2(s)/(s+\alpha)$ is factorized:

$$\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha} Q_2(s) = Q_i(s) Q_o(s), \tag{51}$$

   where $Q_i(s) \in RH_\infty$ is the inner function satisfying $Q_i(0) = 1$ and $Q_o(s) \in RH_\infty$ is the outer function.
2. Using $Q_o(s)$, $\bar{Q}(s) \in RH_\infty$ is designed:

$$\bar{Q}(s) = \frac{q(s)}{Q_o(s)}, \tag{52}$$

   where

$$q(s) = \frac{k}{(\tau s + 1)^{\epsilon}}, \tag{53}$$

   $\tau \in R$ is an arbitrary positive number, ϵ is an arbitrary positive integer to make $\bar{Q}(s)$ proper, and $k \in R$ is a real number satisfying $0 < k < 1$.
3. Using $\bar{Q}(s)$, $Q_u(s) \in \mathcal{U}$ is designed:

$$Q_u(s) = 1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha} Q_2(s) \right) \bar{Q}(s). \tag{54}$$

Next, we show that $Q_u(s)$ in (54) satisfies (12) and makes $Q_{c2}(s)$ proper. First, we show that $Q_u(s)$ in (54) satisfies (12). Substituting (52) for (54), $Q_u(s)$ in (54) is rewritten:

$$Q_u(s) = 1 - Q_i(s) q(s). \tag{55}$$

Since s_i $(i = 1, \ldots, n)$ are unstable zeros of $\beta + sQ_2(s)$, m_i $(i = 1, \ldots, n)$ denote multiplicities of s_i $(i = 1, \ldots, n)$, and $Q_i(s)$ is an inner function of $\beta/(s+\alpha) + sQ_2(s)/(s+\alpha)$:

$$\left. \frac{1}{(s - s_i)^{m_i - 1}} Q_i(s) \right|_{s = s_i} = 0 \quad (\forall i = 1, \ldots, n) \tag{56}$$

holds true. From this equation and (53):

$$\left. \frac{1}{(s - s_i)^{m_i - 1}} Q_i(s) q(s) \right|_{s = s_i} = 0 \quad (\forall i = 1, \ldots, n) \tag{57}$$

are also satisfied. From (55) and (57), $Q_u(s)$ in (54) satisfies (12). Next, we show that $Q_u(s)$ in (54) makes $Q_{c2}(s)$ proper. Substituting (54) for (11), $Q_{c2}(s)$ is rewritten:

$$Q_{c2}(s) = \bar{Q}(s). \tag{58}$$

Since $\bar{Q}(s) \in RH_\infty$, $Q_{c2}(s)$ is proper. Therefore, $Q_u(s)$ in (54) makes $Q_{c2}(s)$ proper.

5. **Numerical Example.** We provide a numerical example to compare responses of a one-degree-of-freedom control system [20] and a two-degrees-of-freedom control system to show the effectiveness of the proposed method.

The plant considered in [20] is the angular velocity control of the two-inertia system in Figure 2. Here, τ_M is the torque of the motor, J_M is the moment of inertia of the motor, D_M is the coefficient of friction of the motor, J_L is the moment of inertia of the load, D_L is the coefficient of friction of the load, K is the torsional spring constant, and ω_L is the angular velocity of the load. In [20], $J_M = 2.0 \cdot 10^{-4}$, $D_M = 0.8 \cdot 10^{-3}$, $J_L = 2.2 \cdot 10^{-2}$, $D_L = 1.8 \cdot 10^{-3}$, and $K = 0.4$. For this plant, we design a two-degrees-of-freedom semistrongly stabilizing controller, and contrast the responses of the one-degree-of-freedom control system in [20] and our two-degrees-of-freedom control system.

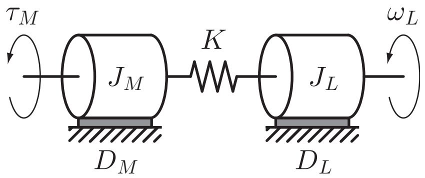

FIGURE 2. Two-inertia system

The plant in Figure 2 is described by:

$$G(s) = \frac{90.9 \cdot 10^3}{(s + 0.117)\left(s^2 + 3.97s + 2.02 \cdot 10^3\right)}. \tag{59}$$

In [20], the plant $G(s)$ in (59) was rewritten in the form of (5). Here, $\alpha = 1$, $\beta = 3.85 \cdot 10^2$:

$$Q_1(s) = 0.26 \cdot 10^{-2}, \tag{60}$$

$$Q_2(s) = -\frac{3.85 \cdot 10^2 \left(s^2 + 4.08s + 1.78 \cdot 10^3\right)}{\left(s^2 + 0.234s + 0.117\right)\left(s^2 + 3.85s + 2.02 \cdot 10^3\right)}, \tag{61}$$

and

$$Q_3(s) = 0. \tag{62}$$

First, we design the feedback controller $C_2(s)$ in (10). To show that the feedback characteristics of the two-degrees-of-freedom control system can be equal to that of the one-degree-of-freedom control system, we set $C_2(s)$ equal to $C(s)$ in [20]. That is, $Q_i(s)$ and $Q_o(s)$ in (51) are:

$$\tilde{Q}_i(s) = 1 \tag{63}$$

and

$$\tilde{Q}_o(s) = \frac{90.9 \cdot 10^3 (s + 1)}{\left(s^2 + 0.234s + 0.117\right)\left(s^2 + 3.85s + 2.02 \cdot 10^3\right)}, \tag{64}$$

respectively. In addition, τ, ϵ, and $k \in R$ in (53) are set to $\tau = 0.02$, $\epsilon = 3$, and $k = 0.99$, respectively. Using these parameters, $Q_u(s)$ in (54) and $C_2(s)$ in (10) are given by:

$$Q_u(s) = \frac{(s + 0.167)\left(s^2 + 1.50 \cdot 10^2 s + 7.48 \cdot 10^3\right)}{(s + 50)^3} \tag{65}$$

and

$$C_2(s) = \frac{1.36\left(s^2 + 0.241s + 0.118\right)\left(s^2 + 4.12s + 2.03 \cdot 10^3\right)}{s(s + 0.167)\left(s^2 + 1.50 \cdot 10^2 s + 7.48 \cdot 10^3\right)}, \tag{66}$$

respectively.

Next, we design the feed-forward controller $C_1(s)$ in (9). Since the transfer function from the reference input $r(s)$ to the output $y(s)$ is described by $V_4(s)$ in (42), $Q_{c1}(s)$ in (9) is designed as:

$$Q_{c1}(s) = \frac{s+\alpha}{s+\gamma}\frac{1}{(\tau_{c1}s+1)^{\epsilon_{c1}}}\frac{1}{\dfrac{\beta}{s+\alpha}+\dfrac{s}{s+\alpha}Q_2(s)}, \tag{67}$$

where $\tau_{c1} \in R$ is an arbitrary positive number and ϵ_{c1} is an arbitrary positive integer to make $Q_{c1}(s)$ proper. When γ, τ_{c1}, and ϵ_{c1} are set to $\gamma = 1$, $\tau_{c1} = 0.02$, and $\epsilon_{c1} = 3$, $Q_{c1}(s)$ and $C_1(s)$ in (9) are given by:

$$Q_{c1}(s) = \frac{1.38\left(s^2+0.234s+0.117\right)\left(s^2+3.85s+2.02\cdot 10^3\right)}{(s+1)(s+50)^3} \tag{68}$$

and

$$C_1(s) = \frac{1.38\left(s^2+0.234s+0.117\right)\left(s^2+3.85s+2.02\cdot 10^3\right)}{s(s+0.167)\left(s^2+1.50\cdot 10^2 s+7.48\cdot 10^3\right)}, \tag{69}$$

respectively.

Using the designed $C_1(s)$ in (69) and $C_2(s)$ in (66), the responses of the output $y(t)$ for step disturbance $d_2(t) = 1$ of the one-degree-of-freedom control system using $C_2(s)$ and two-degrees-of-freedom control system in Figure 1 are shown in Figure 3 and Figure 4, respectively. The solid line shows the response of the output $y(t)$ and the broken line shows that of the step disturbance $d_2(t) = 1$. Figure 3 and Figure 4 show that the step disturbance $d_2(t) = 1$ is attenuated effectively. In addition, we find that the response of the two-degrees-of-freedom control system is the same as that of the one-degree-of-freedom control system.

On the other hand, the response of the output $y(t)$ for the step reference input $r(t) = 1$ of the one-degree-of-freedom control system using $C_2(s)$ and the two-degrees-of-freedom control system in Figure 1 are shown in Figure 5 and Figure 6, respectively. The solid line shows the response of the output $y(t)$ and the broken line shows that of the step reference input $r(t) = 1$. Figure 5 and Figure 6 show that these control systems are stable and the output $y(t)$ follows the step reference input $r(t) = 1$ without steady state error. In

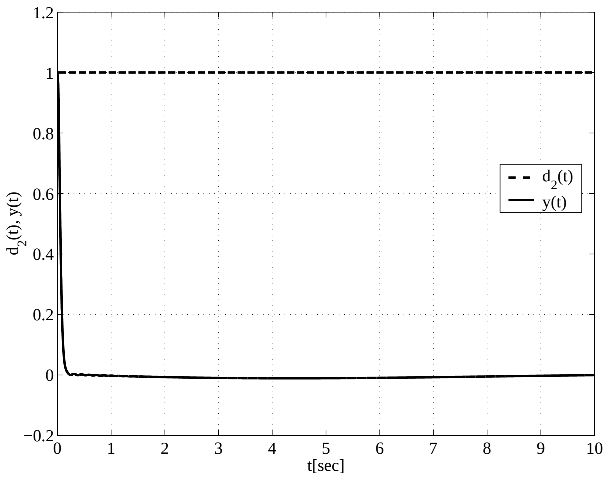

FIGURE 3. Response of $y(t)$ with the one-degree-of-freedom control system for $d_2(t) = 1$

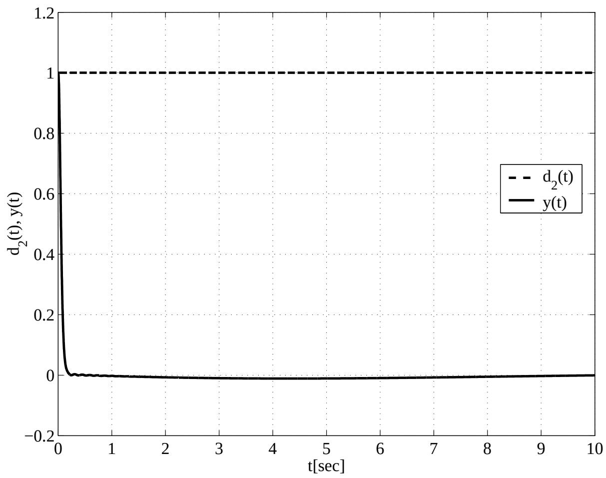

FIGURE 4. Response of $y(t)$ with the two-degrees-of-freedom control system for $d_2(t) = 1$

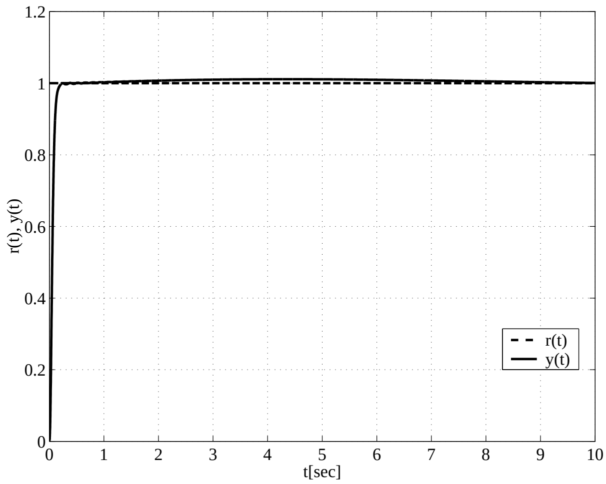

FIGURE 5. Response of $y(t)$ for the one-degree-of-freedom control system for $r(t) = 1$

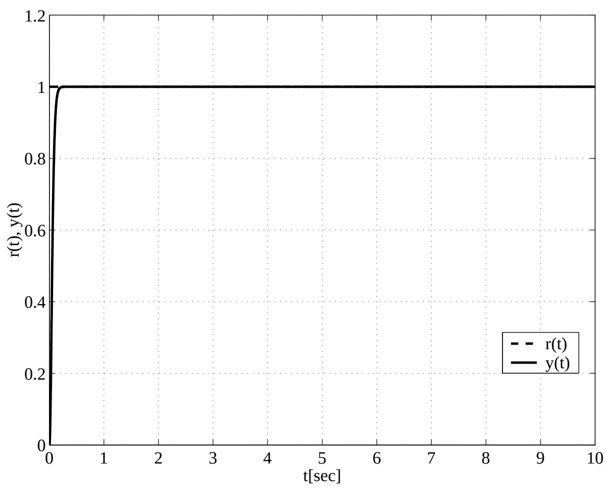

FIGURE 6. Response of $y(t)$ for the two-degrees-of-freedom control system for $r(t) = 1$

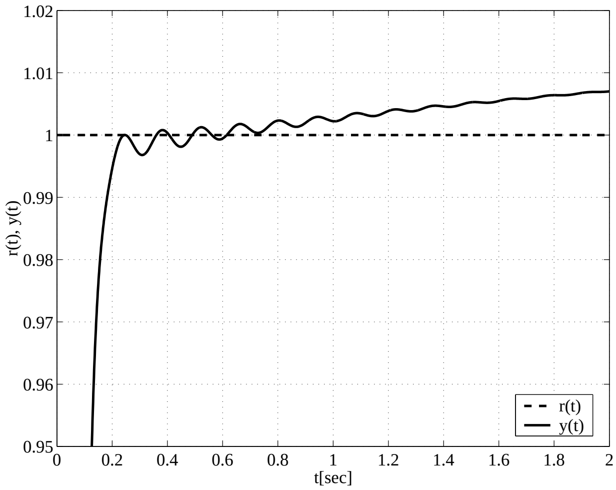

FIGURE 7. Enlarged view from 0[sec] to 2[sec] of Figure 5

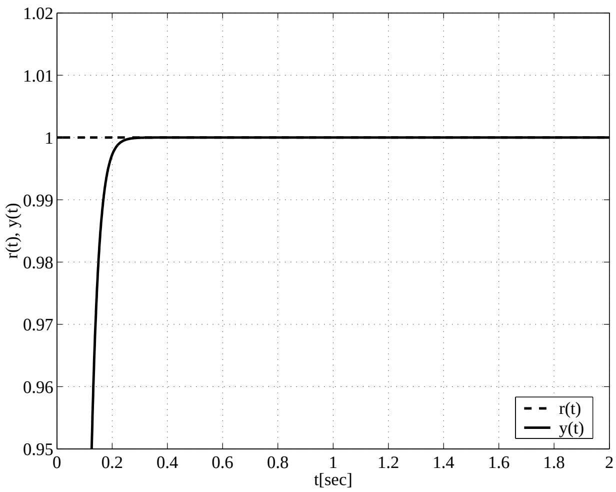

FIGURE 8. Enlarged view from 0[sec] to 2[sec] of Figure 6

addition, to compare the responses of Figure 5 and Figure 6, enlarged views from 0[sec] to 2[sec] are shown in Figure 7 and Figure 8. Figure 7 and Figure 8 show that the response of the two-degrees-of-freedom control system has no overshoot and the settling time of the two-degrees-of-freedom control system is shorter than that of the one-degree-of-freedom control system.

We see that with the proposed two-degrees-of-freedom semistrongly stabilizing controller $C(s)$, the disturbance attenuation characteristic of the two-degrees-of-freedom control system can be the same as that of the one-degree-of-freedom control system and the input-output characteristic of the two-degrees-of-freedom control system can be different from that of the one-degree-of-freedom control system. That is, with the proposed controller, we can realize more accurate control for the reference input.

6. **Conclusions.** In this paper, we have proposed the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers for semistrongly stabilizable plants. We then presented a design method for $Q_u(s) \in \mathcal{U}$ that satisfies (12) and makes $Q_{c2}(s)$

proper. Finally, a numerical example was presented to compare the responses of the one-degree-of-freedom control system [20] and the two-degrees-of-freedom control system to show the effectiveness of the proposed method.

In future work, we will consider two-degrees-of-freedom semistrongly stabilizing controllers for plants with time delay.

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