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FUZZY ADAPTIVE PRESCRIBED PERFORMANCE CONTROL FOR A CLASS OF UNCERTAIN CHAOTIC SYSTEMS WITH UNKNOWN CONTROL GAINS

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Abstract. *This paper proposes a fuzzy adaptive control method for uncertain chaotic systems with unknown control gains. Firstly, an error transformation is introduced to transform the original constrained system into an equivalent unconstrained one. Then, based on the error transformation technique and the predefined performance technique, a fuzzy adaptive feedback control method is developed. It is shown that all the signals of the resulting closed-loop system are bounded. Finally, two illustrative examples are given to demonstrate the effectiveness and usefulness of the proposed technique.*

Keywords: Chaotic system, Fuzzy control, Predefined performance technique, Uncertain

1. **Introduction.** Controlling chaos has attracted increasing interests in recent years. Various theories and applications have been investigated. Nowadays, chaos has lots of useful applications in information processing, secure communication, biological engineering, lasers, chemical processing, and many other areas [1,2]. However, chaotic behavior can also result in destructive effects; therefore, the undesired chaotic phenomenon needs to be suppressed. Many control techniques have been successfully applied for the control of chaotic systems, including adaptive feedback control, sliding mode control, adaptive backstepping control [3-5]. However, most of the aforementioned methods have assumed that the model of the chaotic system is known in advance. In practice, most of the chaotic systems are disturbed by external disturbances and model uncertainties. It is worth mentioning that the existence of uncertainties may lead to notable performance degradations or even instability of the control system. So it is more advisable to take the effects of the system uncertainties and external disturbances into account.

In recent years, adaptive fuzzy control design for uncertain chaotic systems has received increasing attention; based on Lyapunov stability theory and backstepping design technique, many adaptive fuzzy control design approaches have been developed for uncertain chaotic systems, see [6-12]. Li et al. [6] proposed an adaptive control strategy to increase the efficiency of adaptive control by combining T-S fuzzy modeling and the GYC partial region stability theory. Yu et al. [7] proposed an adaptive fuzzy control method to suppress chaos in the permanent magnet synchronous motor drive system via backstepping technology. Boulkroune et al. [11] investigated fuzzy adaptive control schemes for a class of MIMO unknown nonlinear systems with known and unknown sign of the control gain matrix, while Boulkroune and Saad [12] developed a fuzzy adaptive variable-structure control scheme for a class of uncertain MIMO chaotic systems with both sector nonlinearities and dead-zones. In order to realize the robust compensator, most of the aforementioned control schemes are obtained with the restriction that the control gains are known in advance. However, this assumption does not appear to be realistic in a general case [13]. In practice, most of the chaotic systems are disturbed by model uncertainties. It is worth mentioning that the existence of uncertainties may lead to notable performance degradations or even instability of the control system. When there is no a priori knowledge of control gain, the fuzzy control technique is a valid solution for solving the problem of unknown gain. For example, Xiang et al. [14] proposed an adaptive fuzzy sliding mode control scheme for a class of uncertain chaotic systems with mismatched uncertainties and unknown control gains. However, the general chaotic systems with unknown control gains are not considered in [14]. So it is more advisable to take the effects of the unknown control gains into account for uncertain chaotic systems.

Recently, a design solution called prescribed performance control (PPC) for the problem has been proposed in [15]. Utilizing a transformation function that incorporates the desirable performance characteristics, PPC suggests transforming the original controlled system into a new one. Guaranteeing the uniform boundedness of the states of the latter, through proper control action, proves necessary and sufficient to solve the problem for the former. [16] established a control scheme to control unknown pure feedback systems of known high relative degree, exhibiting prescribed performance with respect to trajectory oriented metrics. Na et al. [17] proposed an adaptive control for a class of nonlinear mechanisms with guaranteed transient and steady-state performance. Sun and Liu [18] presented a fuzzy adaptive control method for MIMO uncertain chaotic systems, which is capable of guaranteeing the prescribed performance bounds. However, the main limitation in [17,18] is that the effect of the unknown control gains for uncertain chaotic systems has not been taken into account.

To the author's best knowledge, there are few studies dealing with the prescribed performance control problem with unknown control gains. Inspired by the works in [17,18], we investigate the tracking control with guaranteed prescribed performance for uncertain chaotic systems. Compared with related works, there are four main contributions that are worth to be emphasized.

(1) Compared with the results in [18], the uncertain chaotic systems with unknown control gains are considered.

(2) The prescribed performance function (PPF) is incorporated into the control design.

(3) An adaptation law is proposed to update the fuzzy parameters.

(4) The system we considered is nonstrict feedback form.

So, the prescribed performance adaptive fuzzy output feedback control design methodology still remains an open problem for MIMO uncertain chaotic systems with unknown control gains, which is important and more practical, and thus has motivated us for this study.

Motivated by the aforementioned works, this paper focuses on the problem of adaptive fuzzy control for a class of uncertain chaotic systems with unknown control gains. Based on Lyapunov function, it is proved that all the signals of the closed-loop system are bounded and that the tracking error remains an adjustable neighborhood of the origin with the prescribed performance bounds.

The organization of this paper is described as follows. In the next section, system model is derived, and the assumptions are also given. In Section 3, the design of the proposed control strategies is discussed. The simulation results are presented to demonstrate the effectiveness of proposed control scheme in Section 4. Conclusion is presented in Section 5.

2. **System Descriptions and Problem Formulations.** Consider the following chaotic system:

$$\begin{cases} \dot{x}_1 = f_1(t,x) + g_1(t,x)u_1(t), \\ \dot{x}_2 = f_2(t,x) + g_2(t,x)u_2(t), \\ \cdots, \\ \dot{x}_n = f_n(t,x) + g_n(t,x)u_n(t), \end{cases} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T$ is the system state vector which is assumed to be available for measurement. $u = [u_1, u_2, \ldots, u_n]^T$ is the control input and $f(t,x) = [f_1(t,x), f_2(t,x), \ldots, f_n(t,x)]^T$ is the unknown continuous nonlinear function with uncertainty and disturbance, and $g(t,x) = [g_1(t,x), g_2(t,x), \ldots, g_n(t,x)]^T$ is unknown control gain.

Then, system (1) can be rewritten as

$$\dot{x} = f(t,x) + \mathrm{diag}(g(t,x))u, \tag{2}$$

where $\mathrm{diag}(g(t,x)) = \mathrm{diag}(g_1(t,x), g_2(t,x), \ldots, g_n(t,x))$.

Remark 2.1. *There have been many chaotic systems that belong to the proposed system (1), for example, Lorenz system, and Lü system.*

The objective of this paper is to construct a fuzzy adaptive controller for system (2) such that the system state x tracks the reference signal $x_d \in R^n$ and all the signals in the closed-loop system remain bounded.

To meet the objective, the following assumptions are made for the system (2).

Assumption 2.1. *$f(t,x)$ and $\mathrm{diag}(g(t,x))$ are unknown but bounded. And there exists $\delta > 0$ such that $\mathrm{diag}(g(t,x)) > \delta I_n$, where I_n is an $n \times n$ identity matrix.*

Assumption 2.2. *The desired trajectory x_d is a known bounded differentiable function.*

2.1. **Prescribed performance.** The prescribed performance is achieved by ensuring that tracking error $e = x - x_d = [e_1, e_2, \ldots, e_n]^T$ evolves strictly within predefined decaying bounds as follows [14,15]:

$$-\delta_{i\min}\mu_i(t) < e_i(t) < \delta_{i\max}\mu_i(t), \ t \geq 0, \ i = 1, 2, \ldots, n, \tag{3}$$

where $\delta_{i\min}$ and $\delta_{i\max}$ are design constants, and the performance functions $\mu_i(t)$ are bounded and strictly positive decreasing smooth functions and $\lim_{t\to\infty} \mu_i(t) = \mu_{i\infty} > 0$. Choosing the performance function $\mu_i(t)$ and the constants $\delta_{i\min}$ and $\delta_{i\max}$ appropriately determines the performance bounds of the error e_i, $i = 1, 2, \ldots, n$.

To represent (3) by an equality form, we employ an error transformation as

$$e_i(t) = \mu_i(t)s_i(z_i), \quad i = 1, 2, \ldots, n, \tag{4}$$

where z_i is the transformed error, and $s_i(\cdot)$ is smooth, strictly increasing function, and satisfies the following condition

$$\begin{array}{l} -\delta_{i\min} \leq s_i(z_i) \leq \delta_{i\max}, \\ \lim_{z_i\to-\infty} s_i(z_i) = -\delta_{i\min}, \\ \lim_{z_i\to+\infty} s_i(z_i) = \delta_{i\max}. \end{array} \tag{5}$$

Note that $s_i(z_i)$ are strictly increasing functions, and we have

$$z_i = s_i^{-1}\left(\frac{e_i(t)}{\mu_i(t)}\right), \quad i = 1, 2, \ldots, n. \tag{6}$$

Differentiating (6) with respect to time yields

$$\dot{z}_i = \frac{\partial s_i^{-1}}{\partial\left(\frac{e_i(t)}{\mu_i(t)}\right)} \frac{1}{\mu_i(t)} \left[f_i(t,x) + g_i(t,x)u_i - \dot{x}_{di} - \frac{e_i(t)\dot{\mu}_i(t)}{\mu_i(t)}\right]. \tag{7}$$

Let

$$r_i = \frac{\partial s_i^{-1}}{\partial\left(\frac{e_i(t)}{\mu_i(t)}\right)} \frac{1}{\mu_i(t)} > 0,\ h_i = -\dot{x}_{di} - \frac{e_i(t)\dot{\mu}_i(t)}{\mu_i(t)}.$$

Then (7) can be rewritten as

$$\dot{z}_i = r_i[f_i(t,x) + g_i(t,x)u_i + h_i],\ i = 1,2,\ldots,n. \tag{8}$$

Let $z = [z_1, z_2, \ldots, z_n]^T$, $h = [h_1, h_2, \ldots, h_n]^T$, then (8) can be written into the following form:

$$\dot{z} = \mathrm{diag}(r)[f(t,x) + \mathrm{diag}(g(t,x))u + h], \tag{9}$$

where $\mathrm{diag}(r) = diag(r_1, r_2, \cdots, r_n)$.

Remark 2.2. *In this paper, we choose the function* $s_i(z_i) = \tanh(z_i) = \frac{\delta_{i\max}e^{z_i} - \delta_{i\min}e^{-z_i}}{e^{z_i}+e^{-z_i}}$. *So, we can calculate that* $r_i = \frac{1/(\lambda_i+\delta_{i\min})+1/(\delta_{i\max}-\lambda_i)}{2\mu} > 0$, $\lambda_i = \frac{e_i(t)}{\mu_i(t)}$.

2.2. **Fuzzy logic systems.** The basic configuration of a fuzzy logic system consists of a fuzzifier, some fuzzy IF-THEN rules, a fuzzy inference engine and a defuzzifier. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input vector $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ to an output $\alpha(x) \in R$. The ith fuzzy rule is written as

Rule i: if x_1 is F_1^i and $\cdots$ and x_n is F_n^i then $\alpha(x)$ is α_i.

where F_1^i, F_2^i, ... and F_n^i are fuzzy sets and α_i is the fuzzy singleton for the output in the ith rule. By using the singleton fuzzifier, product inference, and the center-average defuzzifier, the output of the fuzzy system can be expressed as follows:

$$\alpha(x) = \frac{\sum_{j=1}^N \alpha_j \prod_{i=1}^n \mu_{F_i^j}(x_i)}{\sum_{j=1}^N \left[\prod_{i=1}^n \mu_{F_i^j}(x_i)\right]} = \theta^T\psi(x),$$

where $\mu_{F_i^j}(x_i)$ is the degree of membership of x_i to F_i^j, N is the number of fuzzy rules, $\theta = [\alpha_1, \ldots, \alpha_N]^T$ is the adjustable parameter vector, and $\psi(x) = [p_1(x), p_2(x), \ldots, p_N(x)]^T$, where

$$p_j(x) = \frac{\prod_{i=1}^n \mu_{F_i^j}(x_i)}{\sum_{j=1}^N \left[\prod_{i=1}^n \mu_{F_i^j}(x_i)\right]}$$

is the fuzzy basis function. It is assumed that fuzzy basis functions are selected so that there is always at least one active rule.

3. **Main Results.** Due to the fact that the system functions $f(t,x)$ and $\mathrm{diag}(g(t,x))$ are unknown in system (2), we need to use fuzzy logic system to approximate the nonlinear unknown functions.

By applying the introduced fuzzy systems, approximation of function $f_i(t,x)$ and $g_i(t,x)$ can be expressed as follows:

$$\hat{f}_i(x,\theta_{f_i}) = \theta_{f_i}^T\psi_{f_i}(x),\ \hat{g}_i(x,\theta_{g_i}) = \theta_{g_i}^T\psi_{g_i}(x),\ i = 1,2,\ldots,n. \tag{10}$$

Optimal parameters $\theta_{f_i}^*$ and $\theta_{g_i}^*$ can be defined such that

$$\theta_{f_i}^* = \arg\min_{\theta_i}\left[\sup|f_i(t,x) - \hat{f}_i(t,x)|\right],\ \theta_{g_i}^* = \arg\min_{\theta_i}\left[\sup|g_i(t,x) - \hat{g}_i(t,x)|\right], \tag{11}$$

$i = 1, 2, \cdots, n$. Define the parameter estimation errors and the fuzzy approximation errors as follows:

$$\tilde{\theta}_{f_i} = \theta_{f_i} - \theta_{f_i}^*,\ \tilde{\theta}_{g_i} = \theta_{g_i} - \theta_{g_i}^*, \tag{12}$$

and

$$\varepsilon_{f_i}(x) = f_i(t,x) - f_i(x,\theta_{f_i}^*),\ \varepsilon_{g_i}(x) = g_i(t,x) - g_i(x,\theta_{g_i}^*). \tag{13}$$

Let $\hat{f}(x,\theta_f) = [\hat{f}_1(x,\theta_{f_1}), \hat{f}_2(x,\theta_{f_2}), \ldots, \hat{f}_n(x,\theta_{f_n})]^T$, $\hat{g}(x,\theta_g) = [\hat{g}_1(x,\theta_{g_1}), \hat{g}_2(x,\theta_{g_2}), \ldots, \hat{g}_n(x,\theta_{g_n})]^T$, $\varepsilon_f = f(t,x) - f(x,\theta_f^*)$ and $\varepsilon_g = g(t,x) - g(x,\theta_g^*)$. We assume that $||\varepsilon_f|| \le \varepsilon_1$, $||\varepsilon_f|| \le \varepsilon_2$, ε_1 and ε_2 are positive constants.

The controller can be constructed as

$$u = u_{equ} + u_r, \tag{14}$$

where

$$u_{equ} = -\mathrm{diag}(\hat{g}(x,\theta_g))[\epsilon I_n + \mathrm{diag}(\hat{g}(x,\theta_g))^2]^{-1}[h + \hat{f}(x,\theta_f) + \mathrm{diag}(k)z], \tag{15}$$

$$u_r = -\frac{z||z||||\mathrm{diag}(r)||(\varepsilon_1 + \varepsilon_2||u_{equ}|| + ||\epsilon[\epsilon I_n + \mathrm{diag}(\hat{g}(x,\theta_g))^2]^{-1}[h + \hat{f}(x,\theta_f) + \mathrm{diag}(k)z]||)}{\delta||z||^2}, \tag{16}$$

where ϵ is a small positive constant, $\mathrm{diag}(k) = \mathrm{diag}(k_1, k_2, \cdots, k_n)$, $k_i > 0$, $i = 1, 2, \cdots, n$.

To generate the approximations $f(t,x)$ and $\mathrm{diag}(g(t,x))$ online, we choose the following adaptation laws:

$$\dot{\theta}_{f_i} = \kappa_{f_i}\psi_{f_i} z_i r_i,\ \dot{\theta}_{g_i} = \kappa_{g_i}\psi_{g_i} u_{equ\ i} z_i r_i, \tag{17}$$

where $\kappa_{f_i}, \kappa_{g_i}$ are positive constants, $i = 1, 2, \ldots, n$.

So, we obtain the following theorem.

Theorem 3.1. *Consider the system (2). Suppose that Assumptions 2.1 and 2.2 are satisfied. Then the controller (14) with the adaption law given by (17) can guarantee all signals in the closed-loop system are bounded in probability, and the tracking error remains in a neighborhood of the origin within the prescribed performance bounds for all* $t \ge 0$.

Proof: Consider a Lyapunov function as

$$V = \frac{1}{2}\left[z^T z + \sum_{i=1}^n \frac{\tilde{\theta}_{f_i}^2}{\kappa_{f_i}} + \sum_{i=1}^n \frac{\tilde{\theta}_{g_i}^2}{\kappa_{g_i}}\right]. \tag{18}$$

The time derivative of V is given by

$$\begin{aligned}\dot{V} &= z^T\mathrm{diag}(r)[f(t,x) + \mathrm{diag}(g(t,x))u + h] + \sum_{i=1}^n \frac{\tilde{\theta}_{f_i}\dot{\theta}_{f_i}}{\lambda_{f_i}} + \sum_{i=1}^n \frac{\tilde{\theta}_{g_i}\dot{\theta}_{g_i}}{\lambda_{g_i}} \\ &= z^T\mathrm{diag}(r)[f(t,x) + (\mathrm{diag}(g(t,x)) - \mathrm{diag}(\hat{g}(x,\theta_g)))u_{equ} + h \\ &\quad + \mathrm{diag}(\hat{g}(x,\theta_g))u_{equ} + \mathrm{diag}(g(t,x))u_r] + \sum_{i=1}^n \frac{\tilde{\theta}_{f_i}\dot{\theta}_{f_i}}{\lambda_{f_i}} + \sum_{i=1}^n \frac{\tilde{\theta}_{g_i}\dot{\theta}_{g_i}}{\lambda_{g_i}}.\end{aligned}$$

Notice that

$$\mathrm{diag}(\hat{g}(x,\theta_g))^2[\epsilon I_n + \mathrm{diag}(\hat{g}(x,\theta_g))^2]^{-1} = I_n - \epsilon[\epsilon I_n + \mathrm{diag}(\hat{g}(x,\theta_g))^2]^{-1}.$$

One can obtain

$$\begin{aligned}\dot{V} &= z^T\mathrm{diag}(r)\Big[f(x,\theta_f^*) - \hat{f}(x,\theta_f) + \varepsilon_f + \big(\mathrm{diag}(g(x,\theta_g^*)) - \mathrm{diag}(\hat{g}(x,\theta_g))\big)u_{equ} \\ &\quad - \mathrm{diag}(k)z + \varepsilon_g u_{equ} + \epsilon\big[\epsilon I_n + \mathrm{diag}(\hat{g}(x,\theta_g))^2\big]^{-1}\Big[h + \hat{f}(x,\theta_f) \\ &\quad + \mathrm{diag}(k)z\Big] + \mathrm{diag}(g(t,x))u_r\Big] + \sum_{i=1}^n \frac{\tilde{\theta}_{f_i}\dot{\theta}_{f_i}}{\lambda_{f_i}} + \sum_{i=1}^n \frac{\tilde{\theta}_{g_i}\dot{\theta}_{g_i}}{\lambda_{g_i}}.\end{aligned}$$

$$= -\sum_{i=1}^{n}\psi_{f_i}^T\tilde{\theta}_{f_i}r_iz_i - \sum_{i=1}^{n}\psi_{g_i}^T\tilde{\theta}_{g_i}r_iz_iu_{equ\ i} + ||z||||\mathrm{diag}(r)||\Big(\varepsilon_1 + \varepsilon_2||u_{equ}||$$
$$+ ||\epsilon\left[\epsilon I_n + \mathrm{diag}(\hat{g}(x,\theta_g))^2\right]^{-1}\left[h + \hat{f}(x,\theta_f) + \mathrm{diag}(k)z\right]||\Big)$$
$$- \sum_{i=1}^{n}k_iz_i^2 + z^T\mathrm{diag}(r)\mathrm{diag}(g(t,x))u_r + \sum_{i=1}^{n}\frac{\tilde{\theta}_{f_i}\dot{\theta}_{f_i}}{\lambda_{f_i}} + \sum_{i=1}^{n}\frac{\tilde{\theta}_{g_i}\dot{\theta}_{g_i}}{\lambda_{g_i}}.$$

According to Assumption 2.1 and (16), we have $z^T\mathrm{diag}(r)\mathrm{diag}(g(t,x))u_r < -L||z||$, where $L = ||\mathrm{diag}(r)||[\varepsilon_1 + \varepsilon_2||u_{equ}|| + ||\epsilon[\epsilon I_n + \mathrm{diag}(\hat{g}(x,\theta_g))^2]^{-1}[h + \hat{f}(x,\theta_f) + \mathrm{diag}(k)z]||]$. So, by using adaptation laws (17), we obtain

$$\dot{V} \le -\sum_{i=1}^{n}k_iz_i^2 < 0.$$

Therefore, V is always negative, which implies that $z_i \in L_\infty$. Then, according to the properties of function $s_i(z_i)$, we know that $-\delta_{i\min} < s_i(z_i) < \delta_{i\max}$. Then, one can conclude that tracking control of system (2) with prescribed error performance (4) is achieved. This completes the proof.

Remark 3.1. *Compared with the results in [16], the unknown control gains are considered in the paper. Meanwhile, the system we considered is nonstrict feedback form.*

Remark 3.2. *In order to improve the effect of the controller (16), we can modify u_r as follows:*

$$u_r = -\frac{z||z||||\mathrm{diag}(r)||(\varepsilon_1 + \varepsilon_2||u_{equ}|| + ||\epsilon[\epsilon I_n + \mathrm{diag}(\hat{g}(x,\theta_g))^2]^{-1}[h + \hat{f}(x,\theta_f) + \mathrm{diag}(k)z]||)}{\delta||z||^2 + \xi}, \quad (19)$$

where ξ is a design time-varying parameter defined as:

$$\dot{\xi} = -\frac{\eta_0||z||||\mathrm{diag}(r)||(\varepsilon_1 + \varepsilon_2||u_{equ}|| + ||\epsilon[\epsilon I_n + \mathrm{diag}(\hat{g}(x,\theta_g))^2]^{-1}[h + \hat{f}(x,\theta_f) + \mathrm{diag}(k)z]||)}{\delta||z||^2 + \xi}, \quad (20)$$

where η_0 is a positive constant.

4. Numerical Example.

In this section, two examples are presented to demonstrate the effectiveness and applicability of our main results.

Example 4.1. *In this section, the Chen system [19] is used to illustrate the effectiveness of the proposed control scheme. The initial values of the chaotic system are $[x_1(0), x_2(0), x_3(0)]^T = [2, -1, 2]^T$. We describe the Chen system as follows:*

$$\begin{cases} \dot{x}_1 = \underbrace{35x_2 - 35x_1 + 3\sin(x_1)}_{f_1(t,x)} + g_1(t,x)u_1(t), \\ \dot{x}_2 = \underbrace{-7x_1 - x_1x_3 + 28x_2 + 4\cos(t)}_{f_2(t,x)} + g_2(t,x)u_2(t), \\ \dot{x}_3 = \underbrace{x_1x_2 - 3x_3 + 3\sin(2t)}_{f_3(t,x)} + g_3(t,x)u_3(t), \end{cases} \quad (21)$$

where $g_1(t,x) = 5 - 3\cos(t)$, $g_2(t,x) = 4 - 2\cos(t)$, $g_3(t,x) = 2 - \sin(x_2)\cos(x_3)$. The transient and steady state error are prescribed through the performance functions $\mu_i(t) = 3.1e^{-0.7t} + 0.05$, $i = 1,2,3$, and the transformation functions are $s_i = \frac{2}{\pi}\arctan(z_i)$, $i = 1,2,3$. The desired trajectory is $x_d = [\sin(2t), \sin(2t), \sin(2t)]^T$. We define seven Gaussian membership functions uniformly distributed on the interval $[-10, 10]$. We choose the initial values of parameters of the fuzzy systems as $\theta_{f_i} = \theta_{g_i} = 0$, $i = 1,2,3$. $\delta_{i\min} = \delta_{i\max} = 1$, and the other parameters $\kappa_{f_i} = \kappa_{g_i} = 3$, $i = 1,2,3$.

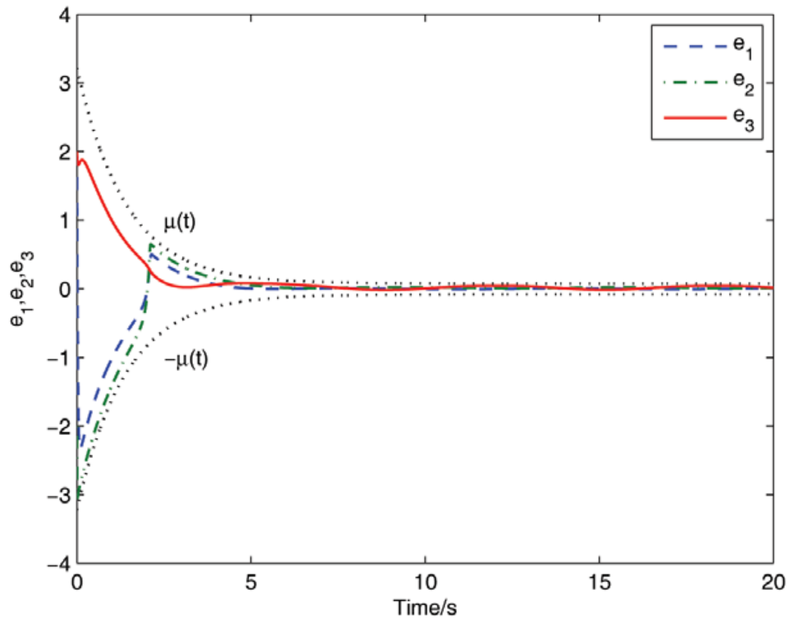

FIGURE 1. Time response e_1, e_2 and e_3 of system (21) by using the present control scheme (14)

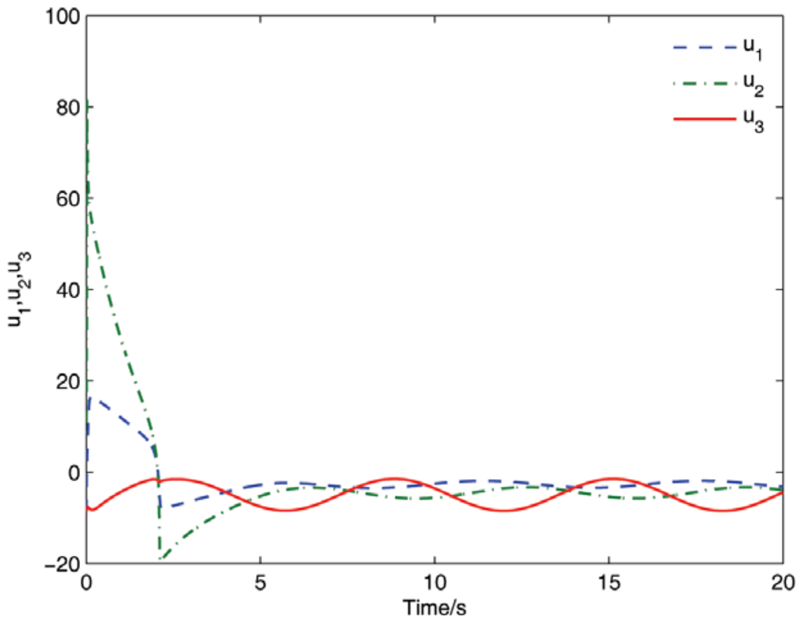

FIGURE 2. Time response u_1, u_2 and u_3 of system (21) by using the present control scheme (14)

Now, by using the present control scheme (14), the simulation results are shown in Figure 1 and Figure 2. From Figures 1 and 2, we know that the tracking errors keep the prescribed error bounds and can achieve the good performances of both transient error and steady-error.

Example 4.2. *Now, the modified Genesio's chaotic system [20] is also used to illustrate the effectiveness of the proposed control scheme. The modified Genesio's chaotic system* *is described as follows:*

$$\begin{cases} \dot{x}_1 = \underbrace{x_2 + d_1(t,x)}_{f_1(t,x)} + g_1(t,x)u_1(t), \\ \dot{x}_2 = \underbrace{x_3 + d_2(t,x)}_{f_2(t,x)} + g_2(t,x)u_2(t), \\ \dot{x}_3 = \underbrace{-6x_1 - 2.92x_2 - 1.2x_3 + x_1^2(3+\sin(0.1t)) + d_3(t,x)}_{f_3(t,x)} + g_3(t,x)u_3(t), \end{cases} \tag{22}$$

where $d_1(t,x) = 3\sin(x_1) + 2\sin(2t)$, $d_2(t,x) = -3\cos(x_2) + 5\sin(t)$, $d_3(t,x) = 0.2x_1 + 6\sin(4t)$, $g_1(t,x) = 3 - \cos(t)$, $g_2(t,x) = 3 - \cos(2t)$, $g_3(t,x) = 3 - \sin(x_2)$.

Firstly, we employ the sliding mode control scheme to control system (20). We assume that the desired trajectory is $x_d = [x_{1d}, x_{2d}, x_{3d}] = [\sin(2t), \sin(2t), \sin(2t)]^T$. Let $e_1 = x_1 - x_{1d}$, $e_2 = x_2 - x_{2d}$ and $e_3 = x_3 - x_{3d}$. So, the error dynamic system can rewrite as follows:

$$\begin{cases} \dot{e}_1 = f_1(t,x) - \dot{x}_{1d} + g_1(t,x)u_1(t), \\ \dot{e}_2 = f_2(t,x) - \dot{x}_{2d} + g_2(t,x)u_2(t), \\ \dot{e}_3 = f_3(t,x) - \dot{x}_{3d} + g_n(t,x)u_3(t). \end{cases} \tag{23}$$

The sliding surfaces are designed as follows:

$$\begin{cases} s_1 = e_1 + \int_0^t e_1(\tau)\mathrm{d}\tau, \\ s_2 = e_2 + \int_0^t e_2(\tau)\mathrm{d}\tau, \\ s_3 = e_3 + \int_0^t e_3(\tau)\mathrm{d}\tau. \end{cases} \tag{24}$$

In order to eliminate the influence of $g(t,x)$, we still adopt the same method in this paper. Therefore, the control scheme for error system (23) is designed as:

$$\begin{aligned} u_i &= \bar{u}_{i1} + \bar{u}_{i2}, \\ \bar{u}_{i1} &= -\hat{g}_i(x,\theta_{ig})\left[\epsilon + \hat{g}_i(x,\theta_{ig})^2\right]^{-1}\left[e_i - x_{id} + \hat{f}_i(x,\theta_{if}) + k_i\mathrm{sign}(s_i)\right], \\ \bar{u}_{i2} &= -\frac{\left(\varepsilon_1 + \varepsilon_2|\bar{u}_{i1}| + \epsilon\left[\epsilon + \hat{g}_i(x,\theta_{ig})^2\right]^{-1}\left[e_i - x_{id} + \hat{f}_i(x,\theta_{if}) + k_i\mathrm{sign}(s_i)\right]\right)\mathrm{sign}(s_i)}{\delta}\mathrm{sign}(s_i), \\ \dot{\theta}_{f_i} &= \kappa_{f_i}\psi_{f_i}s_i, \\ \dot{\theta}_{g_i} &= \kappa_{g_i}\psi_{g_i}\bar{u}_{i1}s_i, \ i = 1,2,3. \end{aligned} \tag{25}$$

The initial values of the chaotic system are $[x_1(0), x_2(0), x_3(0)]^T = [2, -2, 1]^T$. The design parameters are chosen as follows: $\kappa_{f_i} = \kappa_{g_i} = 4$, $i = 1,2,3.$, $k_1 = k_2 = k_3 = 2$, The initial conditions for the adaptive parameters are selected as $\theta_{f_i} = \theta_{g_i} = 0.01$, $i = 1,2,3$. By using the sliding mode control scheme (23), the simulation results are shown in Figure 3 and Figure 4.

The transient and steady state error are prescribed through the performance functions $\mu_i(t) = 3.1e^{-0.7t} + 0.05$, $i = 1,2,3$, and the transformation functions are $s_i = \frac{2}{\pi}\arctan(z_i)$, $i = 1,2,3$. We define three membership functions uniformly distributed on the interval $[-2,2]$. $\delta_{i\min} = \delta_{i\max} = 1$. Applying the present control scheme in this paper to control (14), the simulation results are shown by Figure 5 and Figure 6.

From the simulation results in Figures 3-6, we know that the proposed control scheme can guarantee that all the error states are bounded. Moreover, the tracking error remains within the prescribed performance bounds for all time. From Figures 3 and 4, we know that the tracking error e_1, e_2 and e_3 violate the prescribed error bounds and the control inputs appear chatter phenomenon. Through the comparison, we conclude that the control scheme (25) cannot achieve the good performances of both transient error and steady-error as this paper.

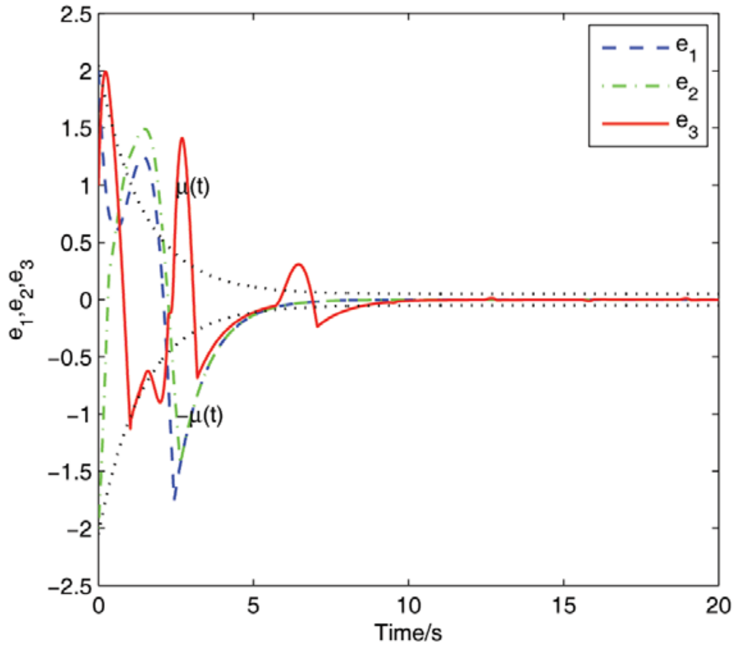

FIGURE 3. Time response e_1, e_2 and e_3 of system (23) by using the control scheme (25)

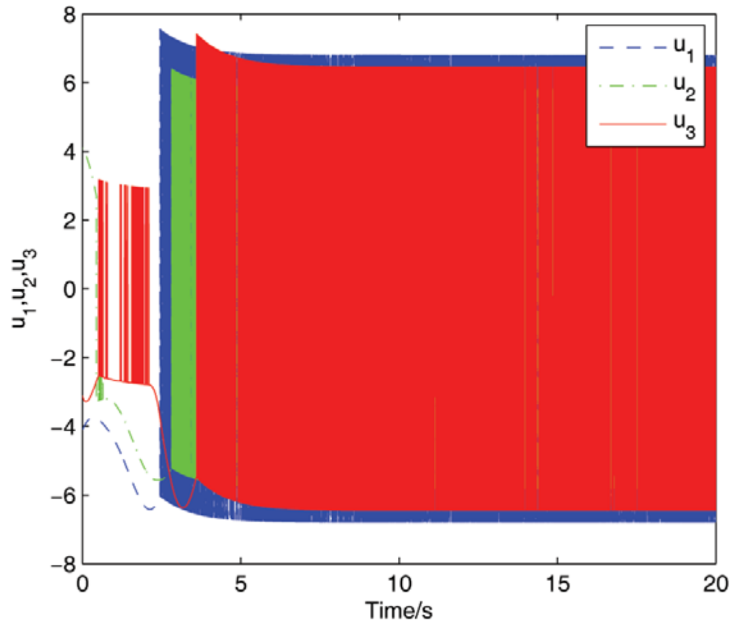

FIGURE 4. Time response u_1, u_2 and u_3 of system (23) by using the control scheme (25)

5. **Conclusions.** For a class of uncertain chaotic systems with unknown control gains, the adaptive fuzzy feedback tracking control problem has been considered. In control design, the fuzzy logic systems are used to identify the unknown nonlinear functions. By using the prescribed performance technique, a new robust fuzzy adaptive feedback control approach has been developed and the stability of the closed-loop system has been proved. Simulation results have shown the effectiveness of the proposed scheme.

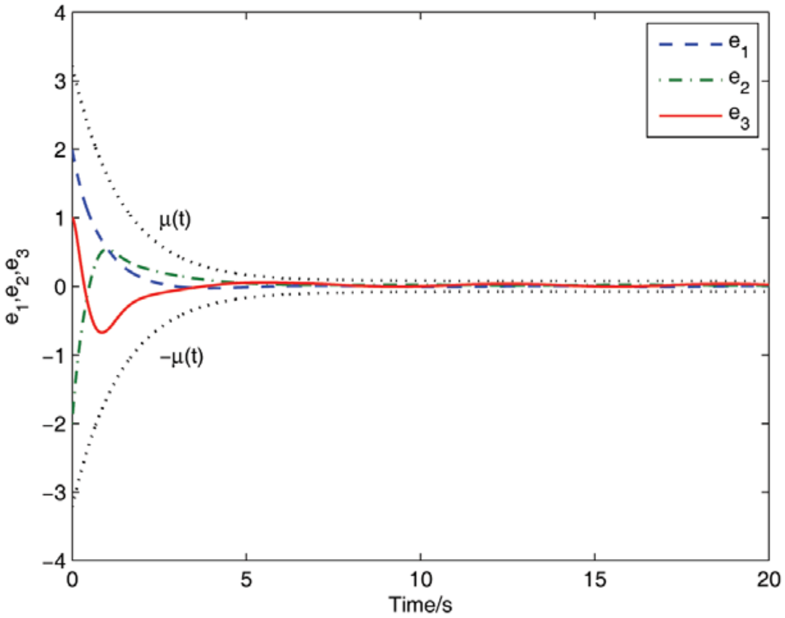

FIGURE 5. Time response e_1, e_2 and e_3 of system (23) by using the present control scheme (14)

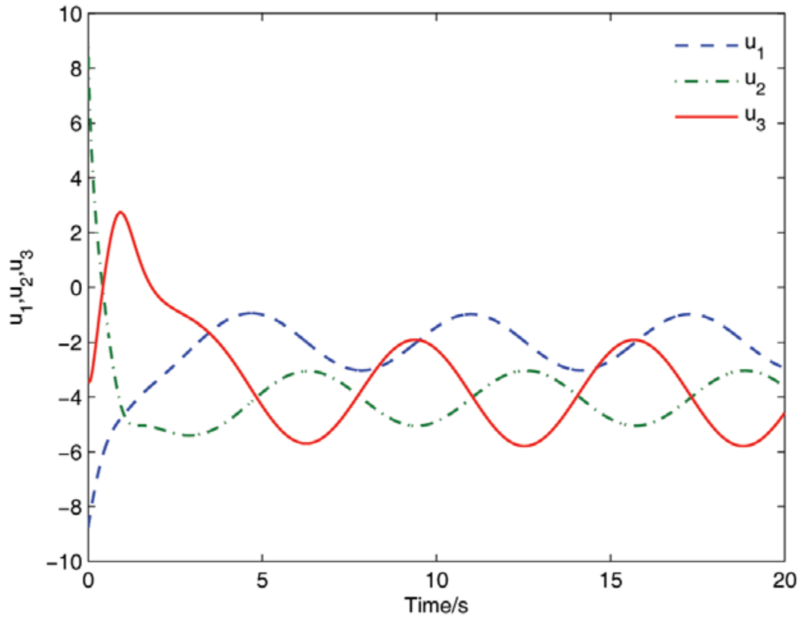

FIGURE 6. Time response u_1, u_2 and u_3 of system (23) by using the present control scheme (14)

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