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ADAPTIVE ACTUATOR FAULT COMPENSATION FOR DISCRETE-TIME T-S FUZZY SYSTEMS WITH MULTIPLE INPUT-OUTPUT DELAYS

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Abstract. *This paper develops a novel adaptive actuator fault compensation scheme for discrete-time Takagi-Sugeno (T-S) fuzzy systems with multiple input-output delays and redundant actuators. Based on a newly developed fuzzy prediction model, a new framework on system parametrization and controller structures has been proposed for an adaptive fault compensation control scheme to accommodate uncertain actuator faults for T-S fuzzy systems. A stable adaptive law is derived to estimate unknown model parameters and unknown actuator fault in the fault-tolerant control. Detailed design procedures and complete stability analysis are provided. Closed stability and asymptotic tracking performance can be guaranteed. A mass-spring-damper system is used to demonstrate the desired performance of the new adaptive fuzzy fault-tolerant control scheme.*
Keywords: T-S fuzzy system, Fault-tolerant control, Actuator fault, Prediction model, Adaptive fuzzy control

1. **Introduction.** At present, control problems of complex nonlinear systems are attracting more and more attention with the need of performance requirements increasing in nonlinear systems. In recent years, T-S fuzzy systems have gained much interest in nonlinear system identification and control due to their universal approximation capability and excellent system parametric and structural modeling properties that are suitable for various parameter learning algorithms and rigorous feedback control design techniques [1, 2]. There have been many research developments on T-S fuzzy systems including stability analysis [3-5], stabilization control [6, 7] and tracking control designs [8-10]. Adaptive control using T-S fuzzy model also receives much attention due to its capability to deal with uncertain parameters and achieve asymptotic tracking performance [11-14]. However, systematic studies on adaptive tracking control and fault-tolerant control of T-S fuzzy systems are not rich, especially for T-S fuzzy systems in input-output forms. An adaptive T-S fuzzy control scheme for MIMO nonlinear systems has been provided in [14]. A rigorously designed adaptive control scheme has been studied in [1] for single-input single-output fuzzy systems with single-delay. For T-S fuzzy system with multiple input-output delays, two different prediction models are developed in [16] and [13], respectively, where the former builds the prediction model from local linear models and the latter from the global nonlinear model. Based on the prediction models, novel system parameterization and adaptive control designs are proposed.

Some catastrophic faults such as an unknown actuator fault or component impairment could generally occur in every complex system. Therefore, it is important to enhance the system reliability by designing control systems to compensate the impact of faults to the nonlinear systems. T-S fuzzy models have been employed in fault detection and Fault-Tolerant Control (FTC) of nonlinear systems [15-21], including T-S model based fault detection [15], online T-S identification based FTC [17], integrated fault estimation and accommodation for discrete-time T-S fuzzy models with actuator faults [18], T-S fuzzy FTC for hypersonic vehicles [19], FTC for T-S systems via delta operator approach [20] and fault-tolerant saturation control for T-S fuzzy systems [21].

Many control schemes have been developed for handling nonlinear systems with redundant actuators. How to ensure the stability and tracking performance when adjusting redundant actuators with adaptive control techniques to compensate the failed actuators has become an important issue. In reality, many plants may have multiple input-output delays and redundant actuators. When T-S fuzzy systems are employed to approximate those plants, they also have multiple input-output delays and redundant actuators. Therefore, it is interesting and meaningful to investigate how to design an adaptive fault-tolerant control scheme for T-S fuzzy systems with multiple input-output delays and redundant actuators in the presence of system parameter uncertainties and actuator faults.

In [13], a new prediction model of the global discrete-time input-output multiple-delay T-S fuzzy systems with multiple delays was proposed and employed for adaptive fuzzy control in the presence of system parameter uncertainties. The system considered in [13] only has one actuator and the proposed controller cannot handle actuator fault. Our work in this paper adopts the method in [13] to derive a prediction model for T-S fuzzy systems with redundant actuators. Then, based on the prediction model, an actuator fault-tolerant controller is designed to effectively deal with both system parameter uncertainties and actuator fault uncertainties.

The rest of this paper is organized as follows. In Section 2, a discrete-time input-output form T-S fuzzy system model with multiple delays and redundant actuators are introduced. The actuator fault model and different faulty cases are described. In Section 3, based on a d-step prediction nonlinear T-S fuzzy model, a new framework with detailed design and analysis for adaptive compensation of uncertain actuator faults has been developed. We also show its desired stability and tracking properties in this section. In Section 4, a simulation study has been presented to demonstrate the desired performance of the developed adaptive control systems. Concluding remarks are in Section 5.

2. **Problem Formulation.**

This section derives a discrete-time input-output multiple-delay T-S fuzzy system model and its minimum phase definition. We also give the actuator fault model in this section.

2.1. **T-S fuzzy system models.**

Consider a multiple-input single-output nonlinear system in its discrete-time input-output form

$$y(t)=f(y(t-1),\ldots,y(t-n),u_1(t-d),\ldots,u_1(t-n),\ldots,u_m(t-d),\ldots,u_m(t-n)), \quad (1)$$

where $f(\cdot,\ldots,\cdot)$ is a nonlinear function, $y(\cdot)$ is the system output signal, $u_i(\cdot)$, $i = 1,2,\ldots,m$ are the input signals whose actuators may fail during the system operation, $t = 0,1,2,\ldots,$ is the discrete-time time variable, n is the system order, and d is the number of system input-output delays ($1 \leq d \leq n$).

The system would be a single-input single-output system for the case when $m = 1$ [13] (the non-redundant actuator case).

We first look for a prediction model for (1) in the form

$$\begin{aligned} y(t+d)=f_d(&y(t),y(t-1),\ldots,y(t-n+1),u_1(t),u_1(t-1),\ldots,u_1(t-n+1),\ldots,\\ &u_m(t),u_m(t-1),\ldots,u_m(t-n+1)), \end{aligned} \quad (2)$$

Then a nominal control law can be chosen to satisfy: $y_m(t+d) = f_d(y(t), y(t-1), \ldots, y(t-n+1), u_1(t), u_1(t-1), \ldots, u_1(t-n+1), \ldots, u_m(t), u_m(t-1), \ldots, u_m(t-n+1))$.

In this paper, we consider the system (1) to be in the form of a global T-S system model based on a set of general local system models for approximation. This leads to the following discrete-time T-S fuzzy system model for the case when $m = 2$:

$$\begin{aligned} &\text{IF} \quad \xi_1 \quad \text{is} \quad F_1^i \quad \text{and} \quad \ldots \quad \text{and} \quad \xi_L \quad \text{is} \quad F_L^i \quad \text{THEN} \\ &\quad y(t+d) + a_{i,1}y(t+d-1) + \cdots + a_{i,n}y(t+d-n) \\ &= b_{1,i,0}u_1(t) + b_{1,i,1}u_1(t-1) + \cdots + b_{1,i,n-d}u_1(t+d-n) + b_{2,i,0}u_2(t) \\ &\quad + b_{2,i,1}u_2(t-1) + \cdots + b_{2,i,n-d}u_2(t+d-n), \end{aligned} \tag{3}$$

where $b_{1,i,0} \neq 0$, $b_{2,i,0} \neq 0$, $i = 1, 2, \ldots, N$, N is the number of fuzzy rules, $d > 0$ is the system delay, $u_1(t) \in R$, $u_2(t) \in R$ and $y(t) \in R$ are the input and output variables, and F_j^i is an interval of real numbers, associated with a membership function $F_j^i(\xi_j(t))$ which indicates the degree of membership of $\xi_j(t)$ in F_j^i.

Introducing the polynomials in z^{-1} (which denotes the delay operator: $z^{-1}[y](t) = y(t-1)$): $A_i(z^{-1}) = -a_{i,1}z^{-1} - \cdots - a_{i,n}z^{-n}$, $B_{1,i}(z^{-1}) = b_{1,i,0} + b_{1,i,1}z^{-1} + \cdots + b_{1,i,n-d}z^{-n+d}$, and $B_{2,i}(z^{-1}) = b_{2,i,0} + b_{2,i,1}z^{-1} + \cdots + b_{2,i,n-d}z^{-n+d}$.

We then form a global d-step prediction T-S fuzzy system model using the local models in (3), based on a standard fuzzy system modeling technique of singleton fuzzification, product inference and weighted-average defuzzification. Thus, we express a global d-step prediction T-S fuzzy system model as:

$$\begin{aligned} y(t+d) &= \sum_{i=1}^{N} \mu_i(\xi(t))A_i(z^{-1})[y](t+d) + \sum_{i=1}^{N} \mu_i(\xi(t))B_{1,i}(z^{-1})[u_1](t) \\ &\quad + \sum_{i=1}^{N} \mu_i(\xi(t))B_{2,i}(z^{-1})[u_2](t), \end{aligned} \tag{4}$$

where $\mu_i(\xi(t))$ is the normalized membership function:

$$\begin{aligned} &\mu_i(\xi(t)) = \frac{\lambda_i(\xi(t))}{\sum_{i=1}^{N} \lambda_i(\xi(t))}, \quad \lambda_i(\xi(t)) = \prod_{j=1}^{L} F_j^i(\xi_j(t)), \\ &\mu_i(\xi(t)) \geq 0, \quad \sum_{i=1}^{N} \mu_i(\xi(t)) = 1. \end{aligned} \tag{5}$$

The approximation error from such a standard fuzzy system modeling technique can be made small by increasing the number of membership base functions [2]. Therefore, we can get an approximate model for the original nonlinear system model (1) with d-step delays by expressing the fuzzy system model (4).

2.2. **Minimum phase fuzzy system definition.** In order to ensure closed-loop signal boundedness and asymptotic tracking of a bounded reference output $y_m(t)$ by the system output $y(t)$, we need the following assumptions [13]:

(A.1a): The individual subsystems (u_1, y) and (u_2, y) are minimum phase.

(A.2a): $\sum_{i=1}^{N} \mu_i(\xi(t))b_{m,i,0} \neq 0$, for all $t \geq 0$, $m = 1, 2$; $i = 1, 2, \ldots, N$.

To specify a minimum phase fuzzy system, we use the following condition: the global fuzzy system (4) is minimum phase if

$$|u_m(t-d)| \leq c_1|y(t)| + c_2 \sum_{\tau=0}^{t-1} \lambda^{t-\tau-1}|y(\tau)|, \quad t \geq d, \tag{6}$$

for $c_1 > 0$, $c_2 > 0$, $m = 1, 2$ and $\lambda \in (0, 1)$.

2.3. **Actuator fault model.** The actuator faults for this system model can be described by

$$u_j = \bar{u}_j, \quad t \geq t_j. \quad j = 1, 2 \tag{7}$$

for some unknown index $j \in \{1, 2\}$, unknown time instant t_j and unknown constant $\bar{u}_j$.

For the system (4) with two actuators, there are only four possible cases regarding to the healthy conditions (normal or stuck) of the two actuators.

- *Case 1*: both the actuators u_1 and u_2 are normal,
- *Case 2*: the actuator u_1 is normal while u_2 is stuck,
- *Case 3*: the actuator u_2 is normal while u_1 is stuck,
- *Case 4*: both the actuators u_1 and u_2 are stuck.

In Case 4, the system will lose the control authority completely and fault-tolerant control cannot be achieved. Therefore, only Cases 1-3 are considered in this paper. We want to design an adaptive fault-tolerant controller which can handle all these three cases without an explicit fault detection unit. That is, when both actuators are normal (Case 1), there is a need of actuator coordination to meet a desired system output performance; when one of the actuators is stuck, the other normal actuator can compensate the adverse effects caused by the stuck actuator.

Case 2 and Case 3 are parallel. There are no substantial differences between them from the viewpoint of control design. That is, the same fault controller structure can be applied to both cases. However, Case 2 and Case 3 lead to different parameter values in the fault-tolerant controller, which needs to be considered in designing the composite fault-tolerant controller to handle all three cases together.

2.4. **Control objective.** Throughout this paper, our goal is to develop one controller structure which is suitable for all three cases and guarantee stability and achieve desirable control performance for the system (4) with unknown parameters and subject to actuator faults.

3. Adaptive Actuator Fault Compensation Design.

This section aims at designing an actuator fault compensation controller based on a prediction model of T-S fuzzy system (4) with d-step delay. In [13], a new prediction model of the global discrete-time input-output multiple-delay T-S fuzzy systems with multiple delays was proposed. In this paper, we employ the similar procedure as in [13] to derive a prediction model for T-S fuzzy system with redundant actuators.

Based on the prediction model, we first give a nominal control scheme for the system with actuator faults, assuming all system parameters and fault information are known. We then formulate a parametrization of the system with unknown parameters and unknown faults, and design an adaptive parameter estimation algorithm to estimate those unknown parameters. Furthermore, we develop an adaptive control law and analyze the closed-loop system performance.

3.1. **Prediction of T-S fuzzy systems.** In (4), we have the term $A_i(z^{-1})[y](t+d)$ containing $y(t+d-1)$, $y(t+d-2), \ldots, y(t+1)$, $y(t)$, $y(t-1), \ldots, y(t+d-n)$, of which $y(t+d-1)$, $y(t+d-2), \ldots, y(t+1)$ are not available at time t. By following the similar procedure as [13], we use (4) backward to get expressions of $y(t+d-1)$, $y(t+d-2), \ldots,$ $y(t+1)$ of available signals $y(t)$, $y(t-1), \ldots, y(t-n+1)$. Thus, the prediction model of a discrete-time system with a d-step delay can be expressed independent of unknown signals $y(t+d-1)$, $y(t+d-2)$, …, $y(t+1)$.

The new prediction model derived from the global T-S fuzzy system (4) can be denoted as

$$y(t+d) = f_y\left(\mu_i(\cdot), y(\cdot)\right) + f_{u_1}\left(\mu_i(\cdot), u_1(\cdot)\right) + f_{u_2}\left(\mu_i(\cdot), u_2(\cdot)\right), \tag{8}$$

for some functions

$$f_y\left(\mu_i(\cdot), y(\cdot)\right) = f_y(\mu_i(t), \ldots, \mu_i(t-d+1), y(t), \ldots, y(t-n+1)), \tag{9}$$

$$\begin{aligned} f_{u_1}\left(\mu_i(\cdot), u_1(\cdot)\right) = & \sum_{i=1}^{N} \mu_i(t) b_{1,i,0} u_1(t) + f_{u_{11}}(\mu_i(t), \ldots, \mu_i(t-d+1), \\ & u_1(t-1), \ldots, u_1(t-n+1)), \end{aligned} \tag{10}$$

$$\begin{aligned} f_{u_2}\left(\mu_i(\cdot), u_2(\cdot)\right) = & \sum_{i=1}^{N} \mu_i(t) b_{2,i,0} u_2(t) + f_{u_{21}}(\mu_i(t), \ldots, \mu_i(t-d+1), \\ & u_2(t-1), \ldots, u_2(t-n+1)), \end{aligned} \tag{11}$$

where $f_{u_{11}}(\mu_i(t), \ldots, \mu_i(t-d+1), u_1(t-1), \ldots, u_1(t-n+1))$ is independent of $u_1(t)$, and $f_{u_{21}}(\mu_i(t), \ldots, \mu_i(t-d+1), u_2(t-1), \ldots, u_2(t-n+1))$ is independent of $u_2(t)$.

Comparing the original T-S fuzzy system (4) with its prediction model (8), it can be observed that the right side of (8) does not contain the future values of y, i.e., $y(t+1)$, $y(t+2), \ldots, y(t+d-1)$, which makes the model (8) more suitable for controller design.

The prediction Equation (8) is linear in the products of μ_i and y, the products of μ_i and u_1 and the products of μ_i and u_2, which can be formulated into a linearly parametrized model:

$$y(t+d) = \theta_y^T \phi_y(t) + \theta_{u_{10}}^T \phi_{u_{10}}(t) + \theta_{u_{11}}^T \phi_{u_{11}}(t) + \theta_{u_{20}}^T \phi_{u_{20}}(t) + \theta_{u_{21}}^T \phi_{u_{21}}(t), \tag{12}$$

where θ_y, $\theta_{u_{10}}$, $\theta_{u_{11}}$, $\theta_{u_{20}}$ and $\theta_{u_{21}}$ are parameter vectors and $\phi_y(t)$, $\phi_{u_{10}}(t)$, $\phi_{u_{11}}(t)$, $\phi_{u_{20}}(t)$ and $\phi_{u_{21}}(t)$ are vector signals, satisfying

$$\theta_y^T \phi_y(y) = f_y\left(\mu_i(\cdot), y(\cdot)\right), \tag{13}$$

$$\theta_{u_{10}}^T \phi_{u_{10}}(t) = \sum_{i=1}^{N} \mu_i(t) b_{1,i,0} u_1(t),\ \theta_{u_{20}}^T \phi_{u_{20}}(t) = \sum_{i=1}^{N} \mu_i(t) b_{2,i,0} u_2(t), \tag{14}$$

$$\theta_{u_{11}}^T \phi_{u_{11}}(t) = f_{u_{11}}\left(\mu_i(\cdot), u_1(\cdot)\right),\ \theta_{u_{21}}^T \phi_{u_{21}}(t) = f_{u_{21}}\left(\mu_i(\cdot), u_2(\cdot)\right). \tag{15}$$

The exact forms of $\theta_{u_{10}}$, $\theta_{u_{20}}$, $\phi_{u_{10}}(t)$ and $\phi_{u_{20}}(t)$ are

$$\theta_{u_{10}} = [b_{1,1,0}, b_{1,2,0}, \ldots, b_{1,N,0}]^T,\ \theta_{u_{20}} = [b_{2,1,0}, b_{2,2,0}, \ldots, b_{2,N,0}]^T, \tag{16}$$

$$\phi_{u_{10}}(t) = \phi_{\mu 0} u_1(t),\ \phi_{u_{20}}(t) = \phi_{\mu 0} u_2(t),\ \phi_{\mu 0} = [\mu_1(t), \mu_2(t), \ldots, \mu_N(t)]^T. \tag{17}$$

The vector signals $\phi_y(t)$, $\phi_{u_{11}}(t)$ and $\phi_{u_{21}}(t)$ may have high dimensions and complicated forms due to the procedure used to express the unavailable future signals $y(t+1)$, $y(t+2), \ldots, y(t+d-1)$ with available signals $y(t)$, $y(t-1), \ldots, y(t+d-n)$. The elements in $\phi_y(t)$ are products of various combinations of $\mu_i(t)$, $\mu_i(t-1), \ldots, \mu_i(t-d+1)$ and $y(t)$, $y(t-1), \ldots, y(t-n+1)$. The elements in $\phi_{u_{11}}(t)$ are products of different combinations of $\mu_i(t)$, $\mu_i(t-1), \ldots, \mu_i(t-d+1)$ and $u_1(t-1)$, $u_1(t-2), \ldots, u_1(t-n+1)$. $\phi_{u_{21}}$ has a form similar to $\phi_{u_{11}}$ but contains u_2 instead of u_1.

The parameters in θ_y are products of different combinations of a_{ij}, $i = 1, 2, \ldots, N$; $j = 1, 2, \ldots, n$, $\theta_{u_{k1}}$ contains products of different combinations of a_{ij} and $b_{m,i,l}$; $m = 1, 2$, $l = 0, 1, \ldots, n-d$.

It is difficult to write the general forms of $\phi_y(t)$, $\phi_{u_{11}}(t)$, $\phi_{u_{21}}(t)$, θ_y, $\theta_{u_{11}}$ and $\theta_{u_{21}}$ in a concise way. However, for a particular system, they can be formulated exactly. In the simulation, we will show how to formulate those signals and parameter vectors through an example.

3.2. **Nominal fault compensation design.** We first consider the system parameters are all known, then the control problem can be solved by the following nominal control schemes.

Design for the no fault case:

For the first case in Section 2.3 with no fault (both actuators u_1 and u_2 are healthy), we need to design an actuator coordination to meet a desired system output performance. One possible choice is the following actuation scheme

$$u_i(t) = \delta_i v_0(t), \quad \delta_i > 0, \ i = 1, 2 \tag{18}$$

for an applied input signal $v_0(t)$ to be designed. Then the system (12) becomes

$$y(t+d) = \theta_y^T \phi_y(t) + \left(\delta_1 \theta_{u_{10}}^T + \delta_2 \theta_{u_{20}}^T\right) \phi_{\mu_0}(t) v_0(t) + \theta_{v_{01}}^T \phi_{v_{01}}(t), \tag{19}$$

where $\theta_{v_{01}} = \left[\theta_{u_{11}}^T, \ \theta_{u_{21}}^T\right]^T$ and $\phi_{v_{01}}(t) = \left[\phi_{u_{11}}(t), \ \phi_{u_{21}}(t)\right]^T$.

Based on Assumptions (A.1a) and (A.2a), the assumption for the coordinated system (19) becomes:

(A.1b): The coordinated system (19) is minimum phase for the chosen $\delta_m > 0$, $m = 1, 2$.

(A.2b): $\delta_m \sum_{i=1}^N \mu_i(\xi(t)) b_{m,i,0} \neq 0$, for all $t \geq 0$, $m = 1, 2$, $i = 1, 2, \ldots, N$.

We design the control law as

$$v_0(t) = \frac{1}{(\delta_1 \theta_{u_{10}}^T + \delta_2 \theta_{u_{20}}^T) \phi_{\mu_0}} \left[-\theta_y^T \phi_y(y) - \theta_{v_{01}}^T \phi_{v_{01}}(t) + y_m(t+d)\right]. \tag{20}$$

Design for the u_2 fault case:

In this case, we have $u_2 = \bar{u}_2$ and $u_1 = \delta_1 v_0(t)$. The system (12) becomes

$$y(t+d) = \theta_y^T \phi_y(t) + \theta_{v_{01}}^T \phi_{v_{01}}(t) + \theta_{\bar{u}_{20}}^T \phi_{\mu_0}(t) + \delta_1 \theta_{u_{10}}^T \phi_{\mu_0}(t) v_0(t), \tag{21}$$

where $\theta_{\bar{u}_{20}} = \bar{u}_2 \theta_{u_{20}}$ and $\phi_{\mu_0} = [\mu_1(t), \mu_2(t)]$. The nominal control law is designed as

$$v_0(t) = \frac{1}{\delta_1 \theta_{u_{10}}^T \phi_{\mu_0}} \left[-\theta_y^T \phi_y(y) - \theta_{\bar{u}_{20}}^T \phi_{\mu_0}(t) - \theta_{v_{01}}^T \phi_{v_{01}}(t) + y_m(t+d)\right]. \tag{22}$$

Design for the u_1 fault case:

In this case, we have $u_1 = \bar{u}_1$ and $u_2 = \delta_2 v_0(t)$. The system (12) becomes

$$y(t+d) = \theta_y^T \phi_y(t) + \theta_{v_{01}}^T \phi_{v_{01}}(t) + \theta_{\bar{u}_{10}}^T \phi_{\mu_0}(t) + \delta_2 \theta_{u_{20}}^T \phi_{\mu_0}(t) v_0(t), \tag{23}$$

where $\theta_{\bar{u}_{10}} = \bar{u}_1 \theta_{u_{10}}$. The nominal control law is designed as

$$v_0(t) = \frac{1}{\delta_2 \theta_{u_{20}}^T \phi_{\mu_0}} \left[-\theta_y^T \phi_y(y) - \theta_{\bar{u}_{10}}^T \phi_{\mu_0}(t) - \theta_{v_{01}}^T \phi_{v_{01}}(t) + y_m(t+d)\right]. \tag{24}$$

Composite control law:

To accommodate all the three cases, we design the composite control law $v_0(t)$ as

$$\begin{aligned} v_0(t) = & \frac{1}{\beta_0^* \left(\delta_1 \theta_{u_{10}}^T + \delta_2 \theta_{u_{20}}^T\right) \phi_{\mu_0} + \beta_1^* \delta_2 \theta_{u_{20}}^T \phi_{\mu_0} + \beta_2^* \delta_1 \theta_{u_{10}}^T \phi_{\mu_0}} \\ & \cdot \left[-\theta_y^T \phi_y(y) - \beta_2^* \theta_{\bar{u}_{10}}^T \phi_{\mu_0}(t) - \beta_1^* \theta_{\bar{u}_{20}}^T \phi_{\mu_0}(t) - \theta_{v_{01}}^T \phi_{v_{01}}(t) + y_m(t+d)\right], \end{aligned} \tag{25}$$

where β_0^*, β_1^* and β_2^* are defined as

$$(\beta_0^*,\ \beta_1^*,\ \beta_2^*) = \begin{cases} (1,\ 0,\ 0) & \text{no fault} \\ (0,\ 1,\ 0) & u_2 = \bar{u}_2 \\ (0,\ 0,\ 1) & u_1 = \bar{u}_1. \end{cases}$$

Define

$$\begin{aligned} \theta_{v_0}^T &= \beta_0^* \left(\delta_1 \theta_{u_{10}}^T + \delta_2 \theta_{u_{20}}^T\right) + \beta_1^* \delta_1 \theta_{u_{10}}^T + \beta_2^* \delta_2 \theta_{u_{20}}^T, \\ \theta_{p_{10}}^T &= \beta_2^* \theta_{\bar{u}_{10}}^T,\ \theta_{p_{20}}^T = \beta_1^* \theta_{\bar{u}_{20}}^T. \end{aligned} \tag{26}$$

Then we can write $v_0(t)$ in (25) as

$$v_0(t) = \frac{1}{\theta_{v_0}^T \phi_{\mu_0}(t)} \left[-\theta_y^T \phi_y(t) - \theta_{p_{10}}^T \phi_{\mu_0}(t) - \theta_{p_{20}}^T \phi_{\mu_0}(t) - \theta_{v_{01}}^T \phi_{v_{01}}(t) + y_m(t+d)\right]. \tag{27}$$

However, since both system parameters and fault parameter are unknown, the control law (27) cannot be implemented. In the following section, we will develop an adaptive fault compensation control law to handle both system parameter and fault uncertainties.

3.3. **Adaptive fault compensation.** To develop an adaptive actuator fault compensation scheme, we need to obtain the estimates of θ_y^T, $\theta_{p_{m0}}^T$, $\theta_{u_{m1}}^T$, and $\theta_{v_0}^T$, $m = 1, 2$. Using the parameter definitions in (26), the systems (19), (21) and (23) can be written into the following unified form:

$$y(t+d) \ = \ \theta_y^T \phi_y(t) + \theta_{v_0}^T \phi_{v_0}(t) + \left(\theta_{p_{10}}^T + \theta_{p_{20}}^T\right) \phi_{\mu_0}(t) + \theta_{v_{01}}^T \phi_{v_{01}}(t), \tag{28}$$

where $\phi_{v_0}(t) = \phi_{\mu_0}(t) v_0(t)$.

3.3.1. *Parameter estimation.* The parametrized model (28) for the fuzzy system prediction model (12) can be further written into a more concise form:

$$y(t+d) = \theta^T \phi(t), \tag{29}$$

where

$$\begin{aligned} \theta \ &= \ \left[\theta_y^T,\ \theta_{v_0}^T,\ \theta_{v_{01}}^T,\ \theta_{p_{10}}^T,\ \theta_{p_{20}}^T\right]^T \\ \phi(t) \ &= \ \left[\phi_y^T(t),\ \phi_{v_0}^T(t),\ \phi_{v_{01}}^T(t),\ \phi_{\mu_0}^T(t),\ \phi_{\mu_0}^T(t)\right]^T. \end{aligned}$$

The expression (29), with θ^T unknown and $\phi(t)$ known, is a regression form with a linear parametrization. Many parameter adaptation algorithms can be adopted to estimate the unknown parameters in θ^T. We chose the following adaptive law to obtain the estimate $\hat{\theta}$ of θ:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\gamma(t)\phi(t-d)\varepsilon(t)}{c + \phi^T(t-d)\phi(t-d)}, \tag{30}$$

where

$$\phi(t-d) \ = \ \left[\phi_y^T(t-d), \phi_{v_0}^T(t-d), \phi_{v_{01}}(t-d), \phi_{\mu_0}^T(t-d),\ \phi_{\mu_0}^T(t-d)\right]^T, \tag{31}$$

and $\gamma(t) \in (\gamma_0, 2-\gamma_0)$ is an adaptation gain for some constant $\gamma_0 \in (0,1)$, $c > 0$ is a small design parameter, and the estimation error is

$$\varepsilon(t) = y(t) - \hat{\theta}^T(t-1)\phi(t-d). \tag{32}$$

For this parameter estimation algorithm, we have the following lemma.

Lemma 3.1. *When applied to the prediction of fuzzy system (29), the parameter adaptation law (30) has the standard properties:*

(i) $\left\|\hat{\theta}(t)-\theta\right\| \leq \left\|\hat{\theta}(t-1)-\theta\right\| \leq \left\|\hat{\theta}(0)-\theta\right\|$; *(ii)* $\frac{\varepsilon(t)}{\sqrt{c+\phi^T(t-d)\phi(t-d)}} \in L^2$;
(iii) $\lim_{t\to\infty} \frac{\varepsilon(t)}{\sqrt{c+\phi^T(t-d)\phi(t-d)}} = 0$; *(iv)* $\left\|\hat{\theta}(t)-\hat{\theta}(t-t_1)\right\| \in L^2$; *and*
(vi) $\lim_{t\to\infty} \left\|\hat{\theta}(t)-\hat{\theta}(t-t_1)\right\| = 0$, $\forall t_1 > 0$.

The proof of this lemma follows from the linear error equation $\varepsilon(t) = y(t) - \hat{\theta}^T(t-1)\phi(t-d)$ and the adaptive law (30), which lead to the squared error norm equation

$$\begin{aligned} &\left\|\hat{\theta}(t)-\theta\right\|^2 - \left\|\hat{\theta}(t-1)-\theta\right\|^2 \\ &= -\gamma(t)\left[2-\frac{\gamma(t)\phi^T(t-d)\phi(t-d)}{c+\phi^T(t-d)\phi(t-d)}\right]\cdot\frac{\varepsilon^2(t)}{c+\phi^T(t-d)\phi(t-d)}, \end{aligned} \tag{33}$$

where

$$\begin{aligned} \gamma(t)\left[2-\frac{\gamma(t)\phi^T(t-d)\phi(t-d)}{c+\phi^T(t-d)\phi(t-d)}\right] &= \gamma(t)\left[2-\gamma(t)+\frac{\gamma(t)c}{c+\phi^T(t-d)\phi(t-d)}\right] \\ &\geq \gamma_0(2-\gamma_0) > 0. \end{aligned} \tag{34}$$

Hence, we have the desired inequality

$$\left\|\hat{\theta}(t)-\theta\right\|^2 - \left\|\hat{\theta}(t-1)-\theta\right\|^2 \leq -\gamma_0(2-\gamma_0)\frac{\varepsilon^2(t)}{c+\phi^T(t-d)\phi(t-d)}. \tag{35}$$

Thus, the lemma's properties can be readily derived.

This adaptive law generates online estimate $\hat{\theta}(t)$ of the unknown parameter θ, with desired stability and L^2 properties, to be used for implementing an adaptive control law.

3.3.2. *Adaptive control law.* To develop an adaptive actuator fault compensation scheme, we need to obtain the estimates of θ_y, θ_{v_0}, $\theta_{v_{01}}$, $\theta_{p_{10}}$ and $\theta_{p_{20}}$. With the parameter estimates $\hat{\theta}_y$, $\hat{\theta}_{v_0}$, $\hat{\theta}_{v_{01}}$, $\hat{\theta}_{p_{10}}$ and $\hat{\theta}_{p_{20}}$, the nominal control law (27) can be implemented as:

$$v_0(t) = \frac{1}{\hat{\theta}_{v_0}^T\phi_{\mu_0}(t)}\left[-\hat{\theta}_y^T\phi_y(t) - \hat{\theta}_{p_{10}}^T\phi_{\mu_0}(t) - \hat{\theta}_{p_{20}}^T\phi_{\mu_0}(t) - \hat{\theta}_{v_{01}}^T\phi_{v_{01}}(t) + y_m(t+d)\right], \tag{36}$$

where the parameters $\hat{\theta}_y$, $\hat{\theta}_{v_0}$, $\hat{\theta}_{v_{01}}$, $\hat{\theta}_{p_{10}}$ and $\hat{\theta}_{p_{20}}$ can be obtained directly from corresponding terms in $\hat{\theta}$ obtained from (30).

3.3.3. *Performance analysis.* We now show that the adaptive control system has desired stability and tracking properties. Substituting (36) into (28), we obtain the closed-loop system as

$$\begin{aligned} y(t+d) = {}& \theta_y^T\phi_y(t) - \hat{\theta}_y^T\phi_y(t) + \theta_{v_{01}}^T\phi_{v_{01}}(t) - \hat{\theta}_{v_{01}}^T\phi_{v_{01}}(t) + \theta_{p_{10}}^T\phi_{\mu_0}(t) - \hat{\theta}_{p_{10}}^T\phi_{\mu_0}(t) \\ & +\theta_{p_{20}}^T\phi_{\mu_0}(t) - \hat{\theta}_{p_{20}}^T\phi_{\mu_0}(t) + y_m(t+d). \end{aligned} \tag{37}$$

Defining $e(t) = y(t) - y_m(t)$ and $\tilde{\theta} = \hat{\theta}(t) - \theta$, we obtain

$$e(t+d) = -\tilde{\theta}(t)\phi(t). \tag{38}$$

We first present a desired property for $\phi(t)$.

Lemma 3.2. *Under Assumption (A.1b), the regressor $\phi(t)$ defined in (31) satisfies*

$$\|\phi(t-d)\| \leq \rho_1 + \rho_2 \max_{\tau=0,1,\ldots,t} |e(\tau)| \tag{39}$$

for some positive constants ρ_1 and ρ_2.

The proof of Lemma 3.2 is based on Assumption (A.1b) and minimum phase definition.
Proof: From the definition of $\phi(t)$ in (31) and μ_i in (5), we have

$$\|\phi(t)\| \leq \kappa_1 \|\psi(t)\|, \tag{40}$$

for some constant $\kappa_1 > 0$, where

$$\psi(t) = [y(t), y(t-1), \ldots, y(t-n+1), v_0(t), v_0(t-1), \ldots, v_0(t-n+1), 1\ldots, 1]^T. \tag{41}$$

Since $y(t) = e(t) + y_m(t)$, $\psi(t)$ can be expressed as

$$\begin{aligned}\psi(t) \;=\; & [e(t), e(t-1), \ldots, e(t-n+1), v_0(t), v_0(t-1), \ldots, v_0(t-n+1), 1\ldots, 1]^T \\ & + [y_m(t), y_m(t-1), \ldots, y_m(t-n+1), 0, \ldots, 0]^T. \end{aligned} \tag{42}$$

Then we have

$$\|\psi(t)\| \leq \kappa_2 \max_{\tau=t-n+1,\ldots,t} |e(\tau)| + \kappa_3 \max_{\tau=t-n+1,\ldots,t} |v_0(\tau)| + \kappa_4, \tag{43}$$

where κ_i, $i = 2, 3, 4$, are some positive constants.
Again with $y(t) = e(t) + y_m(t)$, the system (28) can be formulated as

$$y(t+d) \;=\; \theta_y^T \phi_e(t) + \theta_y^T \phi_{y_m}(t) \theta_{v_0}^T \phi_{v_0}(t) + \theta_{v01}^T \phi_{v01}(t) + \left(\theta_{p10}^T + \theta_{p20}^T\right) \phi_{\mu_0}(t), \tag{44}$$

where $\phi_{\mu_0}(t)$ is bounded. With Assumption (A.1b) and minimum phase fuzzy system definition, we obtain

$$\begin{aligned}|v_0(t)| \;\leq\; & \kappa_5 |e(t+d)| + \kappa_6 \sum_{\tau=0,1,\ldots,t-1} \lambda^{t-\tau-1} |e(\tau+d)| + \kappa_7 \\ \leq\; & \kappa_8 \max_{\tau=0,1,\ldots,t} |e(\tau+d)| + \kappa_7, \end{aligned} \tag{45}$$

where κ_i, $i = 5, 6, 7, 8$, are some positive constants.
Finally, using (40), (43) and (45), we obtain

$$\|\phi(t)\| \leq \rho_1 + \rho_2 \max_{\tau=0,1,\ldots,t} |e(\tau+d)|, \tag{46}$$

where ρ_1 and ρ_2 are some positive constants.
We now show the desired closed-loop system properties.

Theorem 3.1. *All signals in the closed-loop system, with the plant (28) satisfying Assumptions (A.1b) and (A.2b), the controller (36) and the adaptive law (30), are bounded, and* $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$.

Proof: From (28) and (30), we have

$$\varepsilon(t) = -\left(\hat{\theta}(t-1) - \theta\right)^T \phi(t-d), \tag{47}$$

and from (38), we express

$$\begin{aligned} e(t) \;=\; & \bar{\varepsilon}(t)\sqrt{c + \phi^T(t-d)\phi(t-d)} \\ & - \left(\hat{\theta}(t-d) - \hat{\theta}(t-1)\right)^T \bar{\phi}(t-d)\sqrt{c + \phi^T(t-d)\phi(t-d)}, \end{aligned} \tag{48}$$

where $\hat{\theta}(t-d) - \hat{\theta}(t-1) \in L^2 \cap L^\infty$,

$$\bar{\varepsilon}(t) = \frac{\varepsilon(t)}{\sqrt{c + \phi^T(t-d)\phi(t-d)}} \in L^2 \cap L^\infty,$$
$$\bar{\phi}(t-d) = \frac{\phi(t-d)}{\sqrt{c + \phi^T(t-d)\phi(t-d)}} \leq 1. \tag{49}$$

Using the inequality: $\sqrt{c + \phi^T(t-d)\phi(t-d)} \leq \sqrt{c} + \|\phi^T(t-d)\|$, we express $e(t)$ from (48) as

$$|e(t)| \leq c_1 + |\bar{\varepsilon}(t)| \left\|\phi^T(t-d)\right\| + \left\|\hat{\theta}(t-d) - \hat{\theta}(t-1)\right\| \left\|\phi^T(t-d)\right\|$$

for some constant $c_1 > 0$. Using Lemma 3.2: $\|\phi(t-d)\| \leq \rho_1 + \rho_2 \max_{\tau=0,1,\dots,t} |e(\tau)|$, we obtain

$$|e(t)| \leq c_2 + c_3|\bar{\varepsilon}(t)| \max_{\tau=0,1,\dots,t} |e(\tau)| + c_4 \left\|\hat{\theta}(t-d) - \hat{\theta}(t-1)\right\| \max_{\tau=0,1,\dots,t} |e(\tau)|, \tag{50}$$

for some constants $c_i > 0$, $i = 2, 3, 4$.

From Lemma 3.1, we have that $\lim_{t\to\infty} \bar{\varepsilon}(t) = 0$ and $\lim_{t\to\infty} \left\|\hat{\theta}(t) - \hat{\theta}(t-d)\right\| = 0$, and with these results, following with (50) that $e(t)$ is bounded, which implies $y(t)$ is bounded. $u(t)$ is bounded with the system's minimum phase property. All signals in the closed-loop system are bounded, considering $\bar{\varepsilon}(t) \in L^2$ and $\hat{\theta}(t-d) - \hat{\theta}(t-1) \in L^2$ from (48), we have $e(t) \in L^2$ so that $\lim_{t\to\infty} e(t) = 0$. Thus, the adaptive fault compensation scheme has been designed and analyzed for fuzzy systems with two redundant actuators. The results obtained can be readily extended to systems with more actuators.

4. Simulation Study.

In this section, we present an illustrative example with simulation results to show the control design and evaluation. The plant is a mass-spring-damper mechanical system [6]:

$$M\ddot{x} + c_1\dot{x} + c_2 x = \left(1 + c_3\dot{x}^3\right) u_1 + \left(1 + c_4\dot{x}^3\right) u_2, \tag{51}$$

where M denotes the mass, x is the displacement (in meters) of the mass, u_1, u_2 are the force (in Newtons) applied to the spring, c_1 is the damping constant, c_2 is the spring constant, c_3, c_4 are a constant related to the nonlinear term $\dot{x}^3$. For simulation, the parameters are set as $M = 1$kg, $c_1 = 150$N·s/m, $c_2 = 200$N/m, $c_3 = 0.13$N/(m/s)^3, $c_4 = 0.13$N/(m/s)^3.

Construction of T-S Fuzzy Model. Choose the output $y = x$. To use the method developed in this paper, we need a discrete-time Takagi-Sugeno fuzzy model to approximate the dynamics of the nonlinear plant (51). Assuming that $\dot{y} \in [-1.5, 1.5]$ and using the same approach as that in [6], a two-rule continuous-time Takagi-Sugeno fuzzy model to approximate (51) is given as

$$\text{IF } \dot{y} \text{ is } F_1^1, \text{ THEN } \ddot{y} = -150\dot{y} - 200y + 1.4387u_1 + 1.4387u_2,$$
$$\text{IF } \dot{y} \text{ is } F_1^2, \text{ THEN } \ddot{y} = -150\dot{y} - 200y + 0.5613u_1 + 0.5613u_1$$

with the membership functions describing "F_1^1" and "F_1^2" chosen as

$$F_1^1(\dot{y}) = 0.5 + \dot{y}^3/6.75, \quad F_1^2(\dot{y}) = 0.5 - \dot{y}^3/6.75. \tag{52}$$

If the sampling time T is chosen small enough, we can approximate $\dot{y}$ and $\ddot{y}$ with $\dot{y} = [y(t+1) - y(t)]/T$ and $\ddot{y} = [y(t+2) - 2y(t+1) + y(t)]/T^2$. Then a discrete-time model can be derived as

$$R^i : \text{IF } \xi_1(t) \text{ is } F_1^i, \text{ THEN } y(t+2) + a_{i1}y(t+1) + a_{i2}y(t) = b_{1,i,0}u_1(t) + b_{2,i,0}u_2(t), \tag{53}$$

where $\xi_1(t) = [y(t+1) - y(t)]/T$, $a_{i1} = 150T - 2$, $a_{i2} = 1 - 150T + 200T^2$, $i = 1, 2$ and $b_{1,1,0} = 1.4387T^2$, $b_{1,2,0} = 0.5613T^2$, $b_{2,1,0} = 1.4387T^2$, $b_{2,2,0} = 0.5613T^2$.

Based on a standard fuzzy modeling technique, we obtain the global fuzzy model of (53) as

$$y(t+2) = \sum_{i=1}^{2} \mu_i(t) \left[-a_{i1} y(t+1) - a_{i2} y(t)\right] + \sum_{i=1}^{2} \mu_i(t) b_{1,i,0} u_1(t) + \sum_{i=1}^{2} \mu_i(t) b_{2,i,0} u_1(t), \quad (54)$$

where $\mu_i(t)$ is the normalized membership function.

T-S Fuzzy Prediction Model. If the jth actuator fails at $t = t_j$, it can be characterized by $u_j(t) = \bar{u}_j(t)$; $t \geq t_j$, where the fault $\bar{u}_j(t)$ is described by (7). In the simulation, we assume $u_1(t) = \bar{u}_1(t)$ and consider a constant fault: $\bar{u}_1(t) = 0.7$. The fault occurs at $t_j = 200$. We consider output tracking of a sinusoidal signal $y_m(t) = 0.05 \sin(0.1t)$ set under the actuator fault.

Since $y(t+1)$ in the model (54) is not available at time t, we need to develop a prediction model in the form (12) for the control design. We can express the 2-step prediction fuzzy system model as:

$$y(t+2) = \theta_y^T \phi_y(t) + \theta_{v_{01}}^T \phi_{v_{01}}(t) + \theta_{v_0}^T \phi_{\mu_0}(t) v_0(t) + \left(\theta_{p_{10}}^T + \theta_{p_{20}}^T\right) \phi_{\mu_0}(t), \quad (55)$$

where

$$\begin{aligned}
\theta_y &= \left[\theta_{y1}^T,\ \theta_{y2}^T\right]^T \in R^{10},\ \phi_y(t) = \left[\phi_{y1}^T(t), \phi_{y2}^T(t)\right]^T \in R^{10} \\
\theta_{y1} &= \left[a_{11}^2, a_{11}a_{12}, a_{11}a_{21}, a_{11}a_{22}, -a_{12}\right]^T,\ \theta_{y2} = \left[a_{21}^2, a_{21}a_{22}, a_{21}a_{11}, a_{21}a_{12}, -a_{22}\right]^T \\
\phi_{y1}(t) &= \mu_1(t)\left[\mu_1(t-1)y(t), \mu_1(t-1)y(t-1), \mu_2(t-1)y(t), \mu_2(t-1)y(t-1), y(t)\right]^T \\
\phi_{y2}(t) &= \mu_2(t)\left[\mu_2(t-1)y(t), \mu_2(t-1)y(t-1), \mu_1(t-1)y(t), \mu_1(t-1)y(t-1), y(t)\right]^T \\
\theta_{v_{01}} &= \left[\theta_{u_{11}}^T,\ \theta_{u_{21}}^T\right]^T \in R^8,\ \phi_{v_{01}} = \left[\phi_{u_{11}}^T,\ \phi_{u_{21}}^T\right]^T \in R^8 \\
\theta_{u_{11}} &= \left[-a_{11}b_{1,1,0}, -a_{11}b_{1,2,0}, -a_{21}b_{1,1,0}, -a_{21}b_{1,2,0}\right]^T \\
\theta_{u_{21}} &= \left[-a_{11}b_{2,1,0}, -a_{11}b_{2,2,0}, -a_{21}b_{2,1,0}, -a_{21}b_{2,2,0}\right]^T \\
\phi_{u_{11}}(t) &= u_1(t-1)\left[\mu_1(t)\mu_1(t-1), \mu_1(t)\mu_2(t-1), \mu_2(t)\mu_1(t-1), \mu_2(t)\mu_2(t-1)\right]^T \\
\phi_{u_{21}}(t) &= u_2(t-1)\left[\mu_1(t)\mu_1(t-1), \mu_1(t)\mu_2(t-1), \mu_2(t)\mu_1(t-1), \mu_2(t)\mu_2(t-1)\right]^T \\
\theta_{v_0} &= \beta_0^*\left(\delta_1\theta_{u_{10}} + \delta_2\theta_{u_{20}}\right) + \beta_1^*\delta_1\theta_{u_{10}} + \beta_2^*\delta_2\theta_{u_{20}} \in R^2,\ \delta_1 = \delta_2 = 1 \\
\theta_{u_{10}} &= \left[b_{1,1,0}, b_{1,2,0}\right]^T,\ \theta_{u_{20}} = \left[b_{2,1,0}, b_{2,2,0}\right]^T,\ \theta_{\bar{u}_{10}} = \bar{u}_1\theta_{u_{10}},\ \theta_{\bar{u}_{20}} = \bar{u}_2\theta_{u_{20}} \\
\theta_{p_{10}} &= \beta_2^*\theta_{\bar{u}_{10}} \in R^2,\ \theta_{p_{20}} = \beta_1^*\theta_{\bar{u}_{20}} \in R^2,\ \phi_{\mu_0}(t) = \left[\mu_1(t), \mu_2(t)\right]^T.
\end{aligned}$$

The model (55) can be further written into the estimation form (29):

$$y(t+2) = \theta^T \phi(t), \quad (56)$$

where $\theta \in R^{24}$ and $\phi(t) \in R^{24}$.

Adaptive Fault Compensation. In the case $u_1(t) = \bar{u}_1$, we have $\beta_0^* = 0$, $\beta_1^* = 0$, $\beta_2^* = 1$. The nominal control law for $v_0(t)$ is

$$v_0(t) = \frac{1}{\delta_2 \theta_{u_{20}}^T \phi_{\mu_0}} \left[-\theta_y^T \phi_y(y) - \theta_{\bar{u}_{10}}^T \phi_{\mu_0}(t) - \theta_{u_{11}}^T \phi_{u_{11}}(t) - \theta_{u_{21}}^T \phi_{u_{21}}(t) + y_m(t+2)\right]. \quad (57)$$

Since both the system parameters and the fault are unknown, we apply the adaptive fault compensation algorithm derived in Section 3.3 to implementing the fault-tolerant controller in an adaptive way.

When developing the adaptive fault-tolerant control algorithm, we have given the requirements for choosing the parameters, i.e., the parameter adaptation gain $\gamma(t) \in$

$(\gamma_0, 2-\gamma_0)$ for some constant $\gamma_0 \in (0,1)$ and c is a small positive constant. Those requirements are needed to guarantee the properties of parameter adaptation law (35) in Lemma 3.1. In the simulation example, we chose the parameters $\gamma(t) = 0.8$ and $c = 0.01$, which satisfy the requirements on $\gamma(t)$ and c. A bigger value of $\gamma(t)$ can quicken the parameter learning process and makes the tracking error converge faster but may cause more oscillations. A smaller value of $\gamma(t)$ slows down the parameter learning process, making the tracking response go smoother but converge more slowly. Conversely, a bigger value of c slows down the parameter learning process and tracking error convergence.

Theoretically, the initial values of parameter estimates can be chosen arbitrarily. In practice, when the initial values of parameter estimates are set closer to their true values, we have better transient response performance, smoother and quicker. If the initial parameter values are set far away from their true values, the tracking error in the initial stage may be quite big but the stability of the closed-loop system can still be guaranteed. Here, the initial parameter values are set as 65% of their true values.

The system output $y(t)$ and the reference $y_m(t)$ are shown in Figure 1. The control signals are shown in Figure 4. It can be observed from Figure 1 and Figure 4 that the system output tracks the reference accurately after some initial transient with the designed adaptive fuzzy control law. Partial parameter adaptation results of the system (12) are given in Figure 7, which verifies the desired system performance when using an adaptive controller with unknown actuator faults based on the general prediction model.

Comparison. There are two main features of the proposed method: i) it is designed based on a T-S fuzzy prediction model derived from the global T-S fuzzy system; ii) it can guarantee asymptotic output when one actuator is stuck. To highlight the effectiveness and advantage of the work, we have compared the method in this paper to another two methods.

AFC (Local). The first method is also adaptive fault compensation but the controller is designed based on a T-S fuzzy prediction model derived from local model predictions [16]. The approach (AFC (local)) in [16] leads to a prediction model with simpler parametrization, i.e., θ_y, $\theta_{u_{11}}$ and $\theta_{u_{21}}$ have a lower dimension:

$$\begin{aligned}
\theta_y &= [a_{11}^2 - a_{12},\ a_{11}a_{12},\ a_{21}^2 - a_{22},\ a_{21}a_{22}]^T \\
\phi_y &= [\mu_1(t)y(t),\ \mu_1(t)y(t-1),\ \mu_2(t)y(t),\ \mu_2(t)y(t-1)]^T \\
\theta_{u_{11}} &= [-a_{11}b_{1,1,0},\ -a_{21}b_{1,2,0}]^T,\ \theta_{u_{21}} = [-a_{11}b_{2,1,0},\ -a_{21}b_{2,2,0}]^T \\
\phi_{u_{11}} &= [\mu_1(t)u_1(t-1),\ \mu_2(t)u_1(t-1)]^T,\ \phi_{u_{21}} = [\mu_1(t)u_2(t-1),\ \mu_2(t)u_2(t-1)]^T.
\end{aligned}$$

The prediction model [16] in its estimation form ($y(t+2) = \theta^T\phi(t)$) has $\theta \in R^{14}$ and $\phi(t) \in R^{14}$ while our approach leads to $\theta \in R^{24}$ and $\phi(t) \in R^{24}$. However, the prediction model obtained by our approach is an accurate prediction model of the original T-S fuzzy system (54) while the one obtained by [16] is only an approximate prediction model.

We compare adaptive fault compensation performance using our approach with the approach in [16] under the same simulation conditions, i.e., same adaptation gain and same initial conditions. It can be clearly observed from Figure 1 and Figure 2 that although both approaches can adaptively compensate the fault effects and recover the system tracking performance, our approach has a much better transient response performance with a much smaller control signal in the transient process, as illustrated by Figure 4 and Figure 5. Moreover, we also compare the tracking accuracy through Root Mean Square Errors (RMSE) of the last 100 tracking data. It can be seen from Table 1 that our approach achieves a more accurate tracking result.

Robust Fault-tolerant Control (RFC). The second method is a simple robust control approach. No parameter adaptation is required in such a robust controller. However,

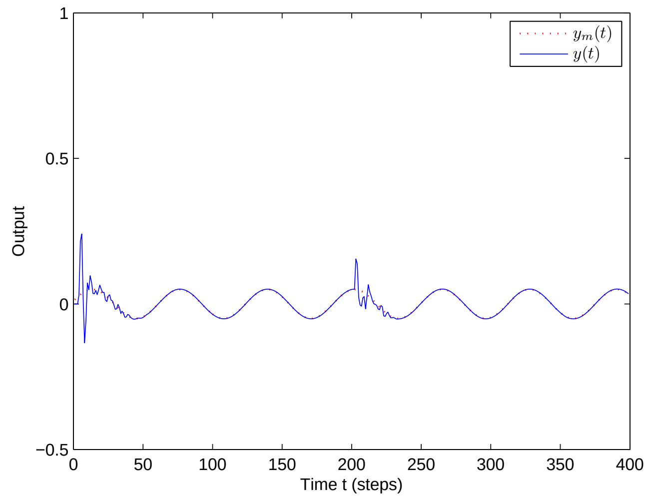

FIGURE 1. FTC tracking response for $u_1(t) = \bar{u}_1$ (this paper's approach)

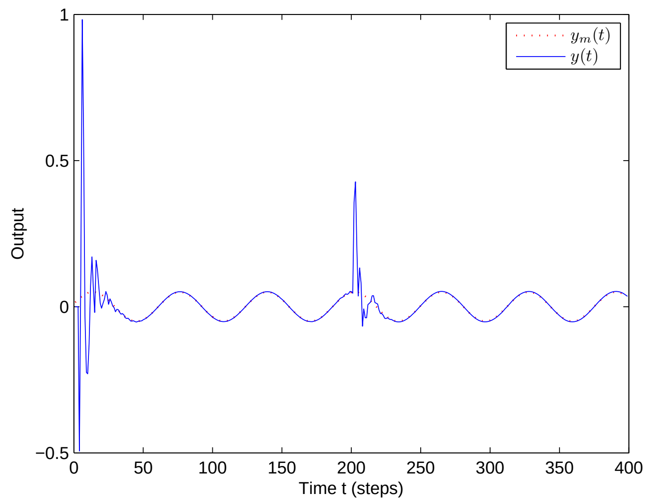

FIGURE 2. FTC tracking response for $u_1(t) = \bar{u}_1$ (the approach in [16])

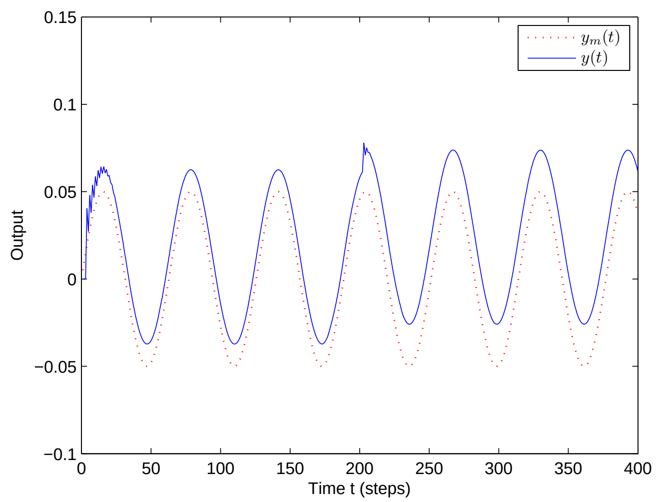

FIGURE 3. FTC tracking response for $u_1(t) = \bar{u}_1$ (a robust control approach)

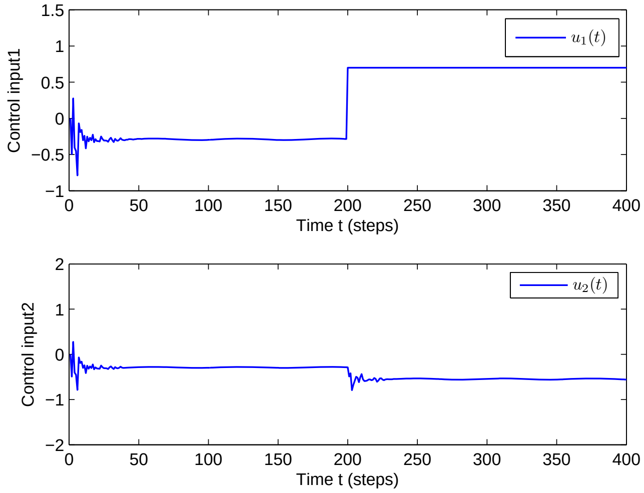

FIGURE 4. FTC control signals for $u_1(t) = \bar{u}_1$ (this paper's approach)

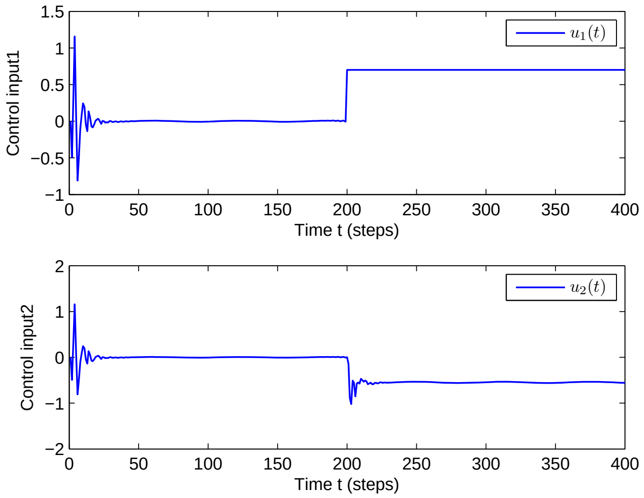

FIGURE 5. FTC control signals for $u_1(t) = \bar{u}_1$ (the approach in [16])

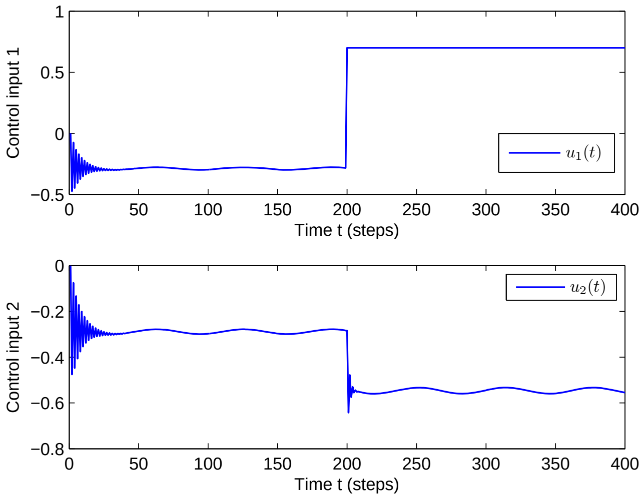

FIGURE 6. FTC control signals for $u_1(t) = \bar{u}_1$ (a robust control approach)

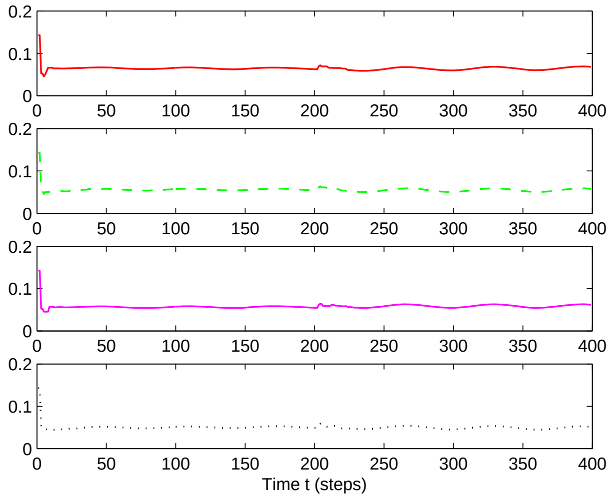

FIGURE 7. Parameter adaptation

TABLE 1. Tracking error comparison

Method	AFC (global)	AFC (local)	RFC
RMSE	4.8471×10^{-4}	8.0631×10^{-4}	0.0117

a typical behavior of such a controller is that even it can stabilize the system before and after u_1 is stuck without adjusting controller parameters, it cannot achieve asymptotic tracking. Such a controller is obtained from the design equation

$$y_m(t+2) = \theta_y^T \phi_y(t) + \theta_{u0}^T \phi_{v_0}(t) v_0(t) + \theta_{u_{11}}^T \phi_{u_{11}}(t) + \theta_{u_{21}}^T \phi_{u_{21}}(t) + \Delta(t) \tag{58}$$

for some parameter vector θ_{u_0}, where $\Delta(t)$ represents a bounded uncertainty term caused by actuator fault. Here, we use a simple trial method to select $\theta_{u_0} = \lambda(\delta_1\theta_{u_{10}} + \delta_2\theta_{u_{20}}) + (1-\lambda)\delta_2\theta_{u_{20}}$ for some chosen parameter $\lambda \in (0,1)$.

When implementing the robust controller, we assume all the system parameters are known and only the fault parameter is unknown. The tracking response and control signal under the robust controller are shown in Figure 3 and Figure 6. It can be seen the closed-loop system is stable under the robust controller for both normal and faulty cases, but asymptotic tracking cannot be achieved even when there is no fault.

5. Concluding Remarks.

This paper has constructed a new control scheme based on the prediction form of the global fuzzy system for a discrete-time multiple-delay T-S fuzzy system in an input-output form with redundant actuators and unknown actuator faults. We provide a multiple-delay prediction model to deal with the fuzzy adaptive control problem. The design and analysis of an adaptive actuator fault compensation scheme for systems with redundant actuators subject to uncertain faults have been developed and analyzed, which have the desired system performance in the presence of parameter and fault uncertainties. Stability and desired system performance have been verified by simulation results.

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