International Journal of Innovative Computing, Information and Control
Volume **12**, Number **4**, August **2016**

ICIC International ©2016 ISSN 1349-4198
pp. 1089–1100

DESIGN OF REPETITIVE CONTROL FOR REJECTING NONLINEAR MULTIPLE-PERIODIC DISTURBANCES

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Received January 2016; revised April 2016

Abstract. *Many vibration signals appearing in engineering are nonlinear multi-periodic, i.e., superposition of finitely periodic signals with time varying harmonics. Such signals are dominant in industrial applications, such as machining tool, electrical power generation and transmission, power converters, and engines. Repetitive control has been developed to achieve disturbance rejection for periodic signals. It uses an internal model and achieves improved control by exploiting the periodic nature of the signals. Its applicability has been limited by the requirement of periodicity, and by the computational difficulty in getting a stable closed-loop system. This paper is aimed at overcoming these difficulties and making repetitive control a valuable technique to reject multiple-periodic vibration. Control performance and stability of the proposed repetitive controller with a multi-periodic disturbance are analyzed and its control scheme can be obtained. The decay rate of the error due to the multiple-periodic disturbance inputs is related to the peak value of a defined regeneration spectrum function. The control performance of the presented method is evaluated in an experimental disturbance rejecting control system. Both computer simulation and experimental results are presented to illustrate the effectiveness of the proposed repetitive controller design.*
Keywords: Multiple-period, Repetitive controller, Regeneration spectrum, Disturbance rejecting control

1. **Introduction.** The rotating machinery is widespread in massive industries fields. Featuring the use of various types of rotating components such as motors, shafts, bearing and gears, it is a typical example of complex multi-degree-of-freedom system such as wind turbine gear boxes having multi-gearing and multi-bearing. These rotating components installed in the rotating machinery can be viewed as multiple vibration sources whose physical vibrations may couple with each other. These vibration signals are often multi-periods with non-linear and non-stationary characteristics (i.e., variable instantaneous frequencies). This makes it difficult to reject the vibration signal using traditional control methods. In general, the vibration signal including many harmonics can be observed by Fourier transformation of the signal. Its fundamental has a certain bandwidth and contains a rich harmonic component. Multiple periodicities can be acquired in sequence by subtracting any periodic components existing in the time series. Therefore, the solution to accurately reject the vibration signal is main point in many applications. Repetitive control has proven to be very effective for a system to reject periodic vibration disturbance signal in practical applications. If the disturbance contains only single fundamental frequency, the repetitive control can be very effective. However, in many cases, the vibration disturbance may contain different fundamental frequencies, whose ratio can be irrational. Since single periodic repetitive control will fail in these situations, a general multi-periodic

repetitive control scheme should be required. Therefore, this paper presents a method for synthesizing repetitive controllers capable of rejecting multi-periodic vibration disturbance.

Based on the internal model principle, the repetitive control design can accomplish zero tracking error in the steady state if the generator for all frequency modes of the periodic signal is included in the control loop. It is known that a periodic signal with a period of T_d can be generated, which has the transfer function of $\frac{1}{1-e^{-sT_d}}$, as shown in Figure 1 with $n = 1$. It is expected from the internal model principle that the asymptotic tracking for exogenous periodic signals can be achieved by incorporating the model into the closed loop system. In practice, with known T_d, a required control performance for rejecting periodic signals can be achieved. The multi-periodic signal is a linear combination of periodic components with different fundamental frequencies, $k = 1, 2, 3, \ldots, n$, and then we have that the transfer function of the generator is given as $\sum_{k=1}^{n} \frac{1}{1-e^{-sT_{dk}}}$ as shown in Figure 1. If the periods of the multi-periodic signal are constant and are also independent on time as shown in Figure 2, then the repetitive controller with the generator $\sum_{k=1}^{n} \frac{1}{1-e^{-sT_{dk}}}$ can be used to regulate the signal. However, in many cases, the periodic disturbance signals may contain different fundamental frequencies and the ratio of these frequencies can be irrational, where the periods are dependent on time, as shown in Figure 3. However, the periodic generator used in the method is restricted to that with a single harmonic. Consequently, in order to regulate the periodic signals with variable instantaneous frequencies, the generator with dead-time length is exported and is included in the proposed repetitive controller. The corresponding stability and error decay rate of the control system are also reviewed in this paper.

In practice, a conventional repetitive controller cannot be directly applied if its reference signal is a general multi-periods signal with variable instantaneous frequencies.

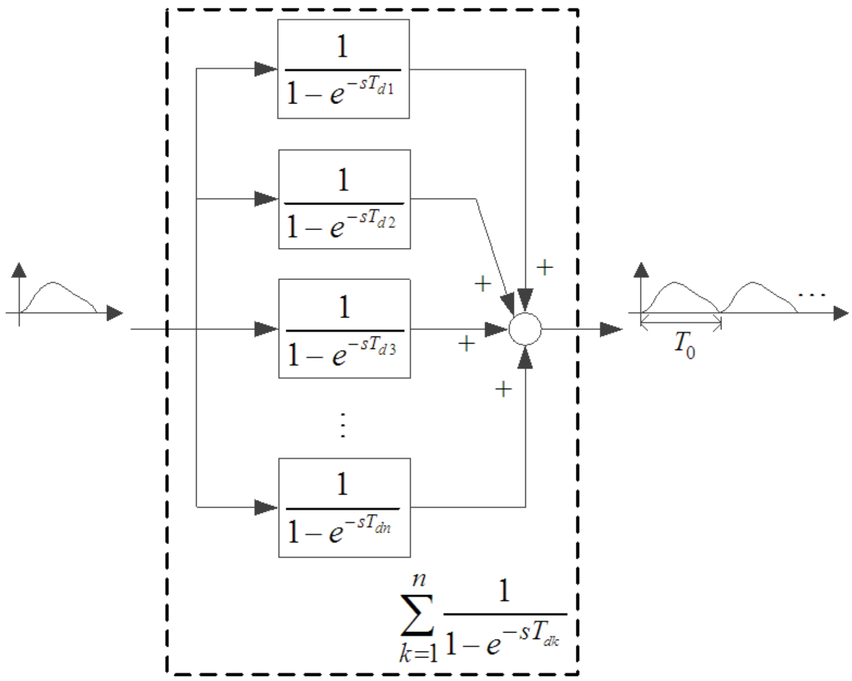
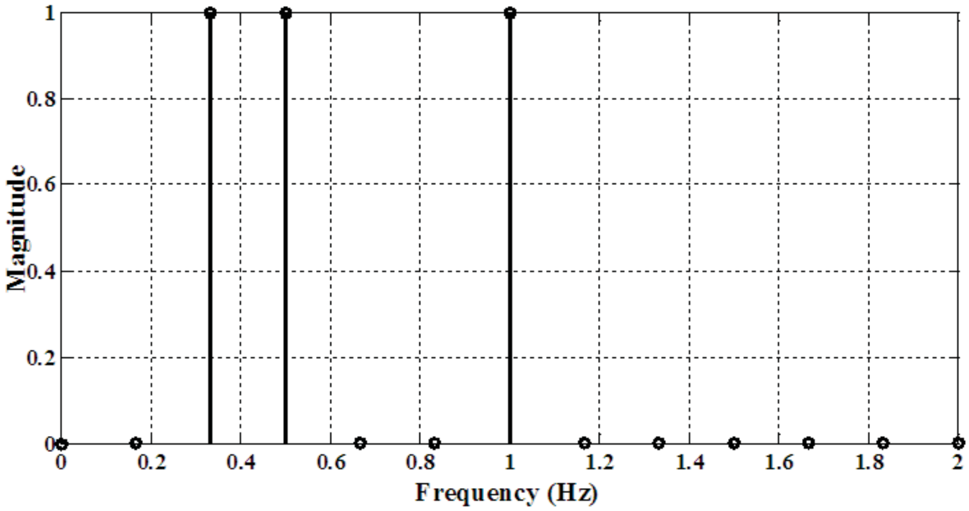
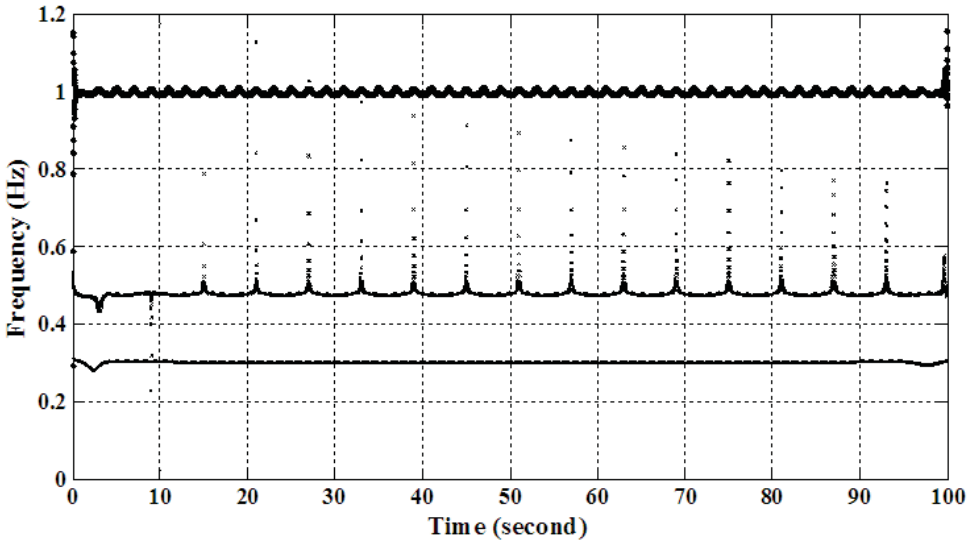


Figure 1. Generator for multi-period signal



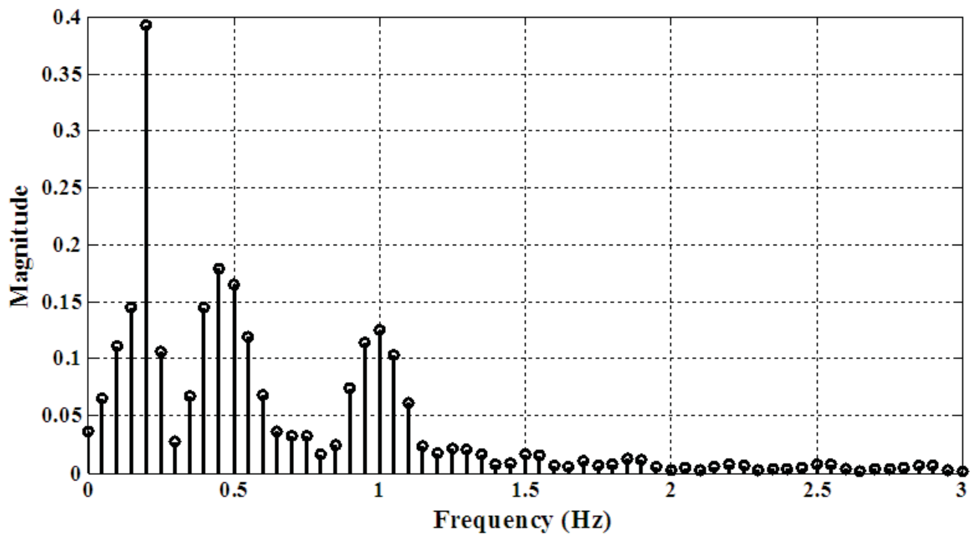
(a)



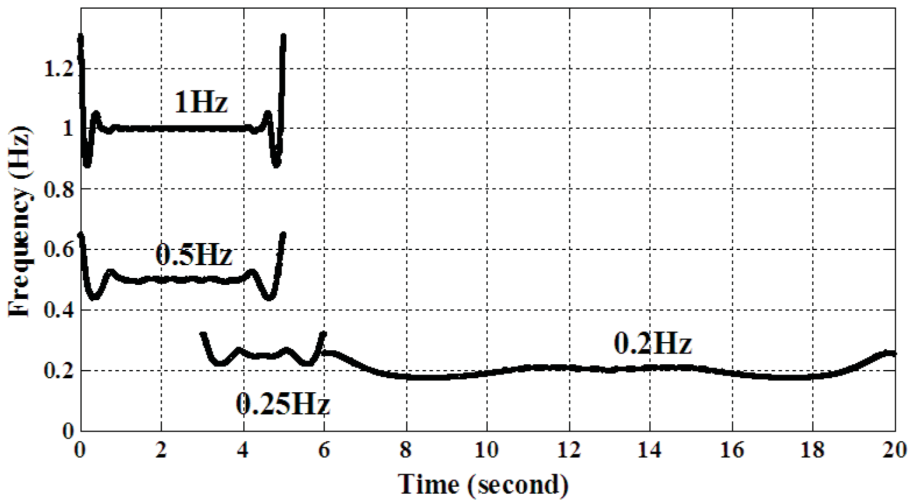
(b)

FIGURE 2. (a) Spectrum and (b) time-frequency characteristic of the multiple periods with the periods of $T_{d1} = 1$s, $T_{d2} = 2$s, and $T_{d3} = 3$s

A number of adaptive repetitive control algorithms have been developed for instantaneous frequencies signals. In [1], the parameterization of all robust stabilizing simple multi-period repetitive controllers is proposed for time-delay plants with the specified input-output characteristic such that the input-output characteristic can be specified beforehand. Cao and Ledwich [2] presented a new adaptive repetitive control, which deals with the non-integer samples per period due to the fixed sampling rate. Interpolations are utilized to generate the fictitious samples required for the repetitive learning. The nearly perfect tracking has been achieved for non-integer samples per period. In [3,11], the parameterization of all stabilizing two-degree-of-freedom multi-period repetitive controllers was proposed with the specified frequency characteristic. Chen et al. [4] proposed the parameterization of all robust stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants. In [5], a closed-loop high-order modified repetitive control system was developed, and the proposed control system can be robustly stable with high control precision for periodic reference inputs with uncertain period-time. In



(a)



(b)

FIGURE 3. (a) Spectrum and (b) time-frequency characteristic of the multiple period's disturbance signal with the periods of $T_{d1} = 1$s and $T_{d2} = 2$s being within $t = 0 \sim 5$s, $T_{d3} = 4$s being at $t = 3 \sim 6$s, and $T_{d4} = 5$s being for $t \geq 6$s

[6], the characteristic of the signal from the time domain to the position domain was transformed in order to eliminate angular, position-dependent disturbances in constant-speed, rotation control systems. Han et al. [7] proposed a new method for treating the computational problem in discrete time linear multi-periodic repetitive control systems, where the reference signal includes several periodic components with already known periods. A lower order multi-periodic repetitive controller is designed which assures BIBO stability of the closed-loop system, and approximate tracking can be achieved. In [8,9], the parameterization of all robust stabilizing simple repetitive controllers for time-delay plants with the specified input-output characteristic was proposed such that the input-output characteristic can be specified beforehand.

In this paper, an approach to regulate quickly general multiple-periodic disturbance is proposed and an appropriate repetitive control scheme is presented from the practical viewpoint. The generator of the multi-period signal with dead-time length is exported.

The stability and error decay rate of the control system are also reviewed. The control performance of the presented method is evaluated in an experimental disturbance rejecting control system. Both computer simulation and experimental results are presented to illustrate the effectiveness of the proposed repetitive controller design. This paper is organized as follows. The proposed repetitive control schemes are derived in Section 2. The repetitive controller is implemented for multi-periodic signals in Section 3 where rejecting responses from the given example due to multi-periodic disturbances are illustrated. Finally, some concluding remarks are given.

2. **Multi-Periodic Repetitive Control.** Figure 4 shows a repetitive control system where r is the reference input, d is the disturbance, y is the system output, u is control output, and e is the error. For a controlled plant, denoted as $G(s)$, the unity feedback system $\frac{G(s)}{1+G(s)}$ should be internally stable. Assume that the regeneration spectrum satisfies $RS(\omega) < 1$, i.e., the control system of Figure 4 is stable, where $RS(\omega) := \left|K_q\left[1 - \frac{K_b G}{1+G}\right]\right|$. The repetitive controller of Figure 4 can be obtained as $RC_1 = \frac{1-K_q e^{-sT_d}+K_b K_q e^{-sT_d}}{1-K_q e^{-sT_d}}$. In practice, with known T_d of the input r, a required control performance for rejecting periodic signals can be achieved by the repetitive controller in Figure 4.

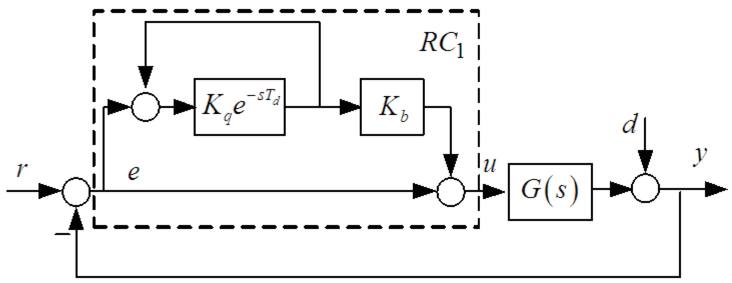


FIGURE 4. Repetitive control system with single periodic input

2.1. **Repetitive control for multi-periodic signal.** For disturbance rejecting control, a multiple period's disturbance input $d = \sum_{i=1}^{n} d_i(t)$ is given where d_i is a fundamental harmonics and its period is given as T_{di}, $\forall i = 1, 2, \ldots, n$. Therefore, we have a multi-periodic repetitive control system as shown in Figure 5 and its repetitive controller is given as

$$RC_n = \sum_{i=1}^{n} \alpha_i \frac{1 - K_{qi}e^{-sT_{di}} + K_{bi}K_{qi}e^{-sT_{di}}}{1 - K_{qi}e^{-sT_{di}}}, \tag{1}$$

where $\sum_{i=1}^{n} \alpha_i = 1$ is selected.

Due to the designed bandwidth of the repetitive control system in Figure 5, the parameters of $K_{qi} = K_q$ and $K_{bi} = K_b$ can be obtained. The characteristic equation of Figure 5 is determined by $\sum_{i=1}^{n} \alpha_i \left(1 + G(s)\frac{1-K_q e^{-sT_{di}}+K_b K_q e^{-sT_{di}}}{1-K_q e^{-sT_{di}}}\right) = 0$. Therefore, the regeneration spectrum of Figure 5 can be given as

$$\begin{aligned} RS_n(\omega) &:= \sum_{i=1}^{n} \left|\alpha_i K_q(j\omega)\left[1 - \frac{K_b(j\omega)G(j\omega)}{1+G(j\omega)}\right]\right| \\ &= \left|K_q(j\omega)\left[1 - \frac{K_b(j\omega)G(j\omega)}{1+G(j\omega)}\right]\right| \sum_{i=1}^{n} |\alpha_i|. \end{aligned} \tag{2}$$

Based on $\sum_{i=1}^{n} |\alpha_i| = 1$, we can have $RS_n(\omega) < 1$. Therefore, the control system of Figure 5 is stable for all ω. To verify the control performance of Figure 5, an example of the control system in [10] is given, where a multiple period's input signal is given as Figure 2 and its time domain response is shown in Figure 6(a). Note that a period 6s of the input signal in Figure 6(a) can be found.

The control parameters of Figure 5 are given as $G(s) = \frac{27450525}{s^3+742.32s^2+202662.4s}$, $K_q(s) = \frac{e^{0.011s}}{\frac{s^2}{\omega_q^2}+\frac{2\zeta_q s}{\omega_q}+1}$, $\omega_q = 21\text{Hz}$, $\zeta_q = \sqrt{2}/2$, and $K_b(s) = e^{0.008s}$. The regeneration spectrum of Figure 5 is calculated as shown in Figure 7 to verify the required stability of Figure 5.

Therefore, the repetitive controller of Figure 5 is obtained as

$$RC_3 = \sum_{i=1}^{3} \alpha_i \frac{1 - K_q e^{-sT_{di}} + K_b K_{qi} e^{-sT_{di}}}{1 - K_q e^{-sT_{di}}}.$$

The error rejecting response of Figure 5 is shown in Figure 6(b). It can be found that the rejecting error decays rapidly within the first two cycles of the input signal.

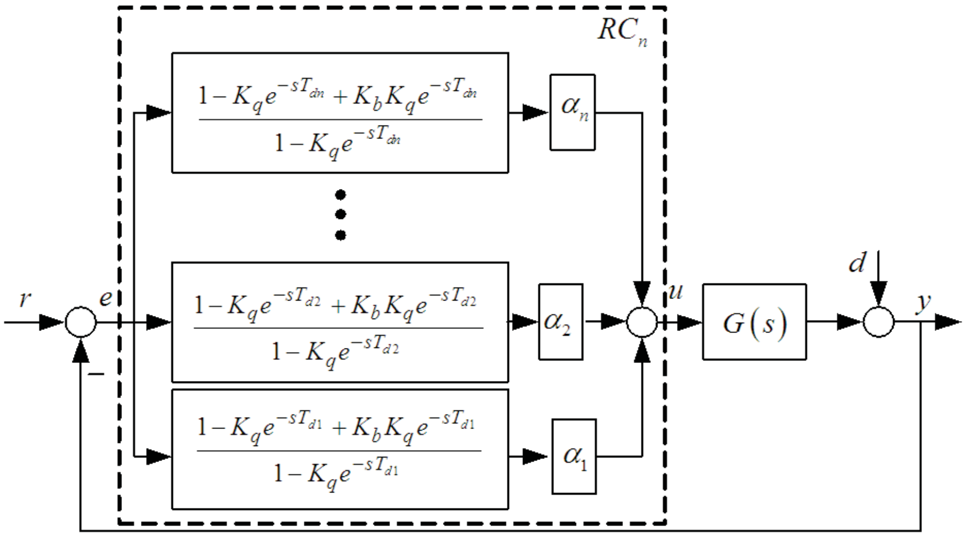


FIGURE 5. Repetitive control system with multiple period's signal

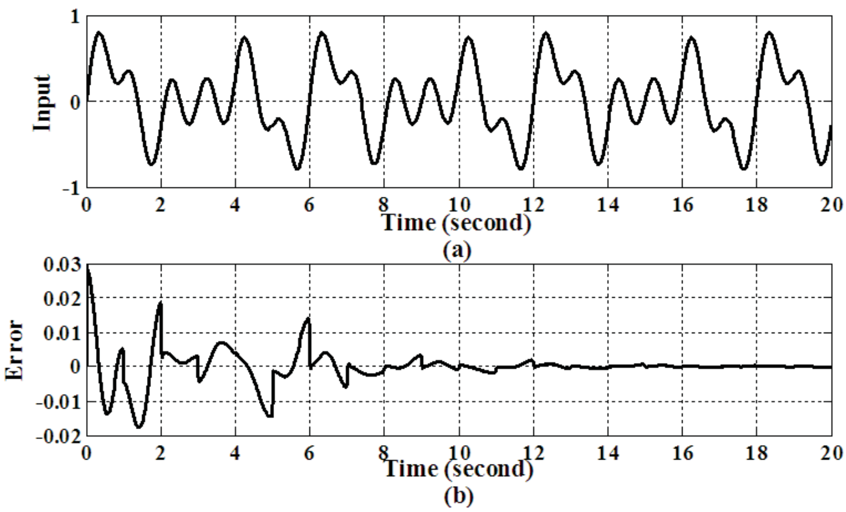


FIGURE 6. (a) The multiple period's signal and (b) error response

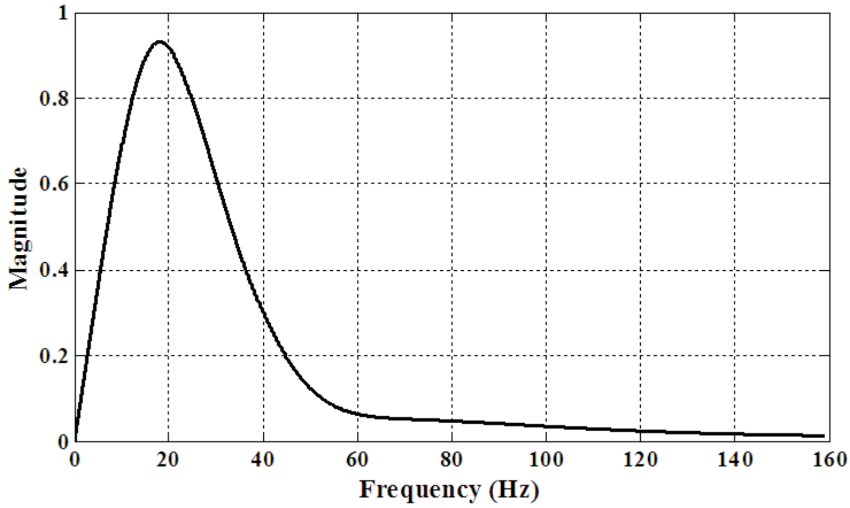


FIGURE 7. Regeneration spectrum of the control system in Figure 5

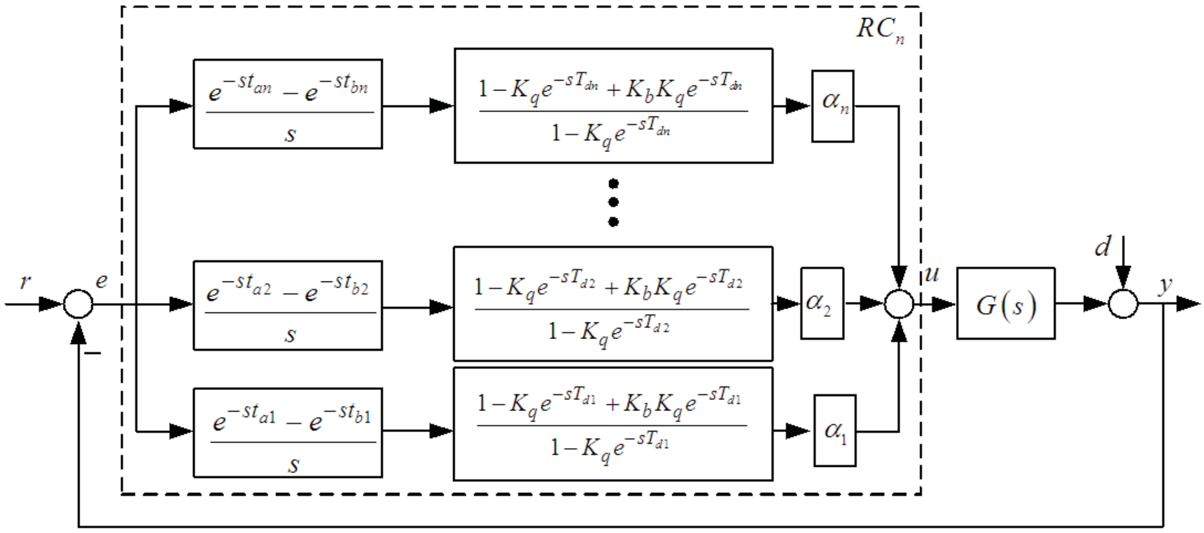


FIGURE 8. Repetitive control system with variable instantaneous frequencies multi-period's signal

2.2. **Repetitive control for multi-periodic signal with variable instantaneous frequencies.** However, in many cases, the periodic disturbance signals may contain different fundamental frequencies and the ratio of these frequencies can be irrational (i.e., variable instantaneous frequencies), where the fundamental harmonics are defined in certain time-period, i.e., $d = \sum_{i=1}^{n} d_i(t)\,[u(t - t_{ai}) - u(t - t_{bi})]$. Note that $u(t)$ is step function with $u(t) = 1$, $t > 0$ and $u(t) = 0$, $t < 0$. Therefore, we have the repetitive control system with the multiple period's input as shown in Figure 8. The repetitive controller is given as

$$RC_n = \sum_{i=1}^{n} \alpha_i \frac{1 - K_q e^{-sT_{di}} + K_b K_q e^{-sT_{di}}}{1 - K_q e^{-sT_{di}}} H_i, \tag{3}$$

where $H_i = \frac{e^{-st_{ai}} - e^{-st_{bi}}}{s}$.

The regeneration spectrum of the control system in Figure 8 can be defined as

$$RS_n(\omega) := \sum_{i=1}^{n} |\alpha_i K_q Q_i|, \tag{4}$$

where $Q_i = 1 - \frac{H_i G K_b}{1+G}$.

Based on (2) and $|H_i| < 1$ for all ω, we have RS_n as $|K_q Q_i| < 1$. Therefore, $RS_n(\omega) < 1$, for all ω, i.e., the stability condition is satisfied. For $r = 0$ of Figure 8, the steady state error can be represented as

$$E = \frac{1}{\sum\limits_{i=1}^{n} \alpha_i \left[1 + G + \frac{K_b K_q e^{-sT_{di}}}{1 - K_q e^{-sT_{di}}} H_i G\right]} D, \tag{5}$$

where E and D are the inverse Laplace transform of e and d respectively.

In the following, the decay rate of the error response is analyzed. Let

$$\bar{E} = \sum_{i=1}^{n} \frac{1}{\alpha_i \left[1 + G + \frac{K_b K_q e^{-sT_{di}}}{1 - K_q e^{-sT_{di}}} H_i G\right]} D,$$

and $|E(j\omega)| \le \left|\bar{E}(j\omega)\right|$ can be found. Define $E_0 = \frac{D}{1+G}$ being the error response without the repetitive control. Then $\bar{E}$ can be rewritten as $\bar{E} = \sum\limits_{i=1}^{n} \frac{1 - K_q e^{-sT_{di}}}{\alpha_i \left[1 - K_q Q_i e^{-sT_{di}}\right]} E_0$. Since $|K_q Q_i| < 1$ for all ω, then the power series expansion of $\bar{E}$ can be obtained as

$$\begin{aligned} \bar{E} = {} & E_0 \sum_{i=1}^{n} \frac{1}{\alpha_i} + \frac{E_0}{\alpha_1} \sum_{k=1}^{\infty} \left\{[Q_1 - 1] Q_1^{k-1} K_q^k e^{-skT_{d1}}\right\} + \frac{E_0}{\alpha_2} \sum_{k=1}^{\infty} \left\{[Q_2 - 1] Q_2^{k-1} K_q^k e^{-skT_{d2}}\right\} \\ & + \cdots + \frac{E_0}{\alpha_n} \sum_{k=1}^{\infty} \left\{[Q_n - 1] Q_n^{k-1} K_q^k e^{-skT_{dn}}\right\} \end{aligned} \tag{6}$$

In practice, the power density of most periodic motion control signals encountered in control systems may concentrate in the first few harmonic due to its velocity and acceleration being continuous. Suppose that K_b can be found such that $Q_i \cong 0$, and then (6) can be rewritten as $\bar{E} = E_0 \sum\limits_{i=1}^{n} \frac{1}{\alpha_i} + \sum\limits_{i=1}^{n} \frac{E_0}{\alpha_i} \left(1 - K_q e^{-sT_{di}}\right)$. Furthermore, based on $K_q(jk\omega_i) e^{-jk\omega_i T_{di}} \cong 1$ for $k\omega_i \,(= 2k\pi/T_{di})$ within a certain frequency bandwidth, $\bar{E}$ can be simplified to $\bar{E} = E_0 \sum\limits_{i=1}^{n} \frac{1}{\alpha_i}$. Then we have $E < \bar{E} < E_0$ with $\|K_q Q\|_\infty < 1$, where $\|\bullet\|_\infty$ is an infinite form. This implies that the steady state error tends to be zero with the decay rate being governed by $\|K_q Q\|_\infty$, i.e., the peak value of the regeneration spectrum. Clearly, if $\|K_q Q\|_\infty$ is made to be smaller, the system has better rejecting response. In fact, the steady-state rejecting errors of the proposed repetitive control system can be effectively eliminated within a few cycles.

3. **An Illustrated Example.** Figure 9 is the illustrated example where a servomotor is given as the target controlled plant and the other one is a disturbance generator to provide a vibration input. Specification of the controlled motor is given in Table 1. Assume that the velocity of the servomotor can be estimated by a rotating encoder. And the motor can be driven in the torque control mode (i.e., the motor's operation of the torque command to the velocity output). The motor velocity output is denoted as ω, J is the inertial of the motor, B is the equivalent damping coefficient, τ_d is the disturbance torque, and K_t is composed of the torque loop control gain and the motor

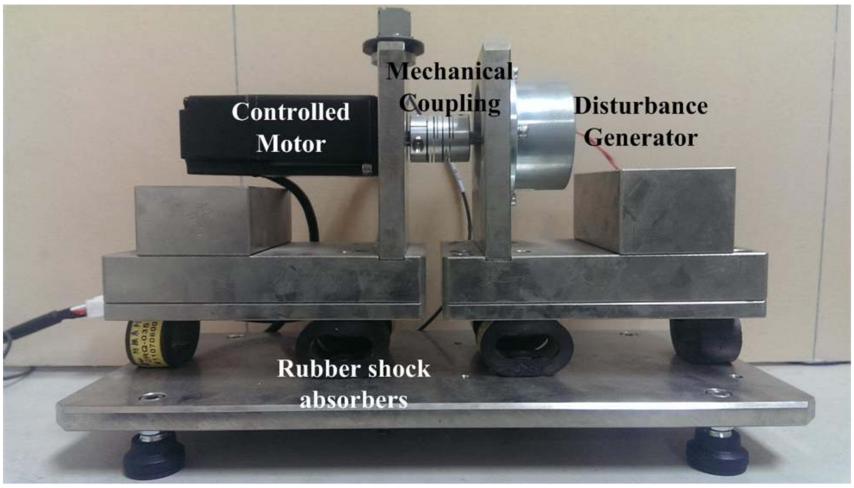


FIGURE 9. Illustrated example of the two motors with the mechanical coupling

TABLE 1. Specification of the controlled motor

Rated output power (kW)	1.5
Rated torque (N-m)	7.16
Maximum torque (N-m)	21.48
Rated speed (r/min)	2000
Rated output power (kW)	1.5

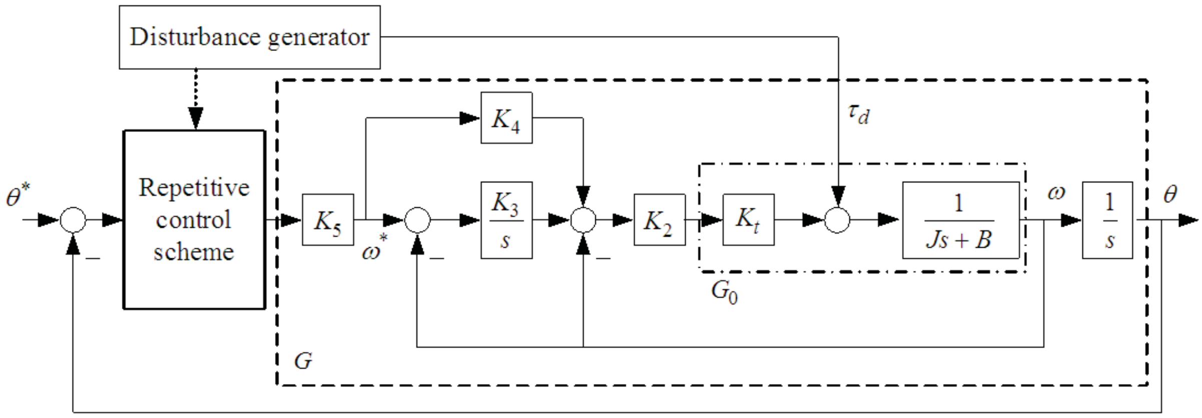


FIGURE 10. Block diagram of disturbance rejection control system

torque constant. The mathematical model of the controlled motor can be obtained as $G_0(s) = \frac{K_t}{Js+B} = \frac{0.87}{0.001118s+0.00055}$.

In the paper, a control strategy of the disturbance rejection control (see Figure 10) is proposed for the illustrated example. The position loop and velocity loop controllers (proportional + pseudo derivative feedback feed-forward) are pre-designed to stabilize the control system, i.e., $G/1+G$ being internally stable. Let the bandwidths of the velocity and position loops be set to 40Hz and 15Hz respectively for the required performance. For the velocity control loop, the parameters of K_2, K_3, and K_4 can be designed in the following. Considering the control system of Figure 12 without repetitive control, we have the transfer function of the velocity control loop with $K_4 = 0$, i.e., $\frac{\omega_1}{\omega_1^*} = \frac{\omega_{nv}^2}{s^2+2\xi\omega_{nv}s+\omega_{nv}^2}$, where $\omega_{nv} = \sqrt{\frac{K_{t1}K_2K_3}{J_1}}$ and $2\xi\omega_{nv} = \frac{B+K_{t1}K_2}{J_1}$ can be found. Therefore, based on the bandwidth of the velocity control loop $\omega_{nv} = 40$Hz and $\xi = \sqrt{2}/2$, the parameters of $K_2 = 0.4561$ and $K_3 = 117.96$ can be obtained. The feedforward control gain $K_4 = 0.5$ (in general $0 < K_4 < 1$) is determined. For the position control loop of Figure 12 without the repetitive control, based on the bandwidth $\omega_{np} = 15$Hz of the position control loop, $K_5 = 70$ can be obtained. Then the controlled plant $G(s) = \frac{12424s+2930700}{s^3+355.4562s^2+41869s}$ can be obtained. The frequency responses of position and velocity control loop in Figure 10 are shown in Figure 11. Based on the design rule of [10], the parameters of the repetitive controller can be calculated as $K_q(s) = \frac{e^{0.016s}}{\frac{s^2}{\omega_q^2}+\frac{2\zeta_q s}{\omega_q}+1}$, $\omega_q = 20$Hz, $\zeta_q = \sqrt{2}/2$, and $K_b(s) = e^{0.02s}$. To verify the control scheme, the multiple period's disturbance signal of Figure 2 is given. The result can be found that the error is gradually decaying as shown in Figure 12. For a variable instantaneous frequencies period's signal, the signal of Figure 3 is given. The result can be found that the tracking error is gradually decaying as shown in Figure 13. The larger errors can be found at 3s, 5s, and 6s, which is generated by the non-continuous of the variable instantaneous frequencies period's signal.

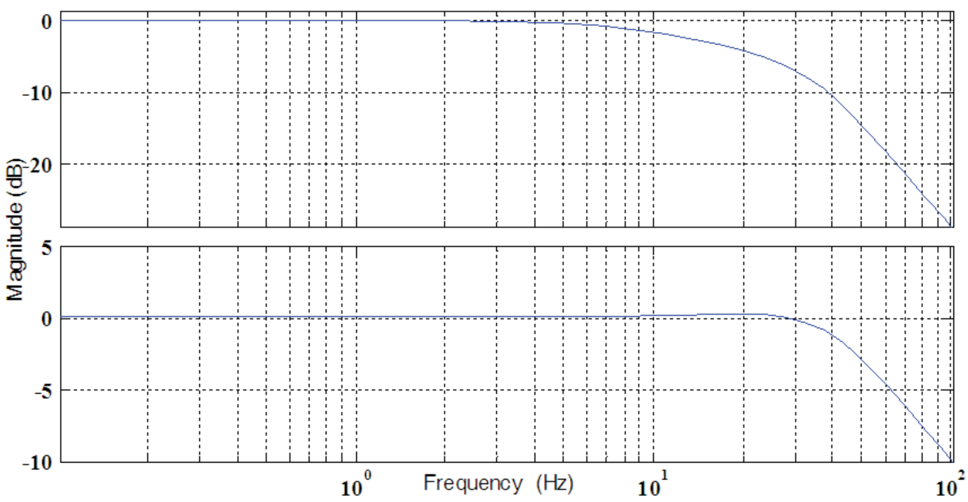


FIGURE 11. Frequency responses of position and velocity control loop

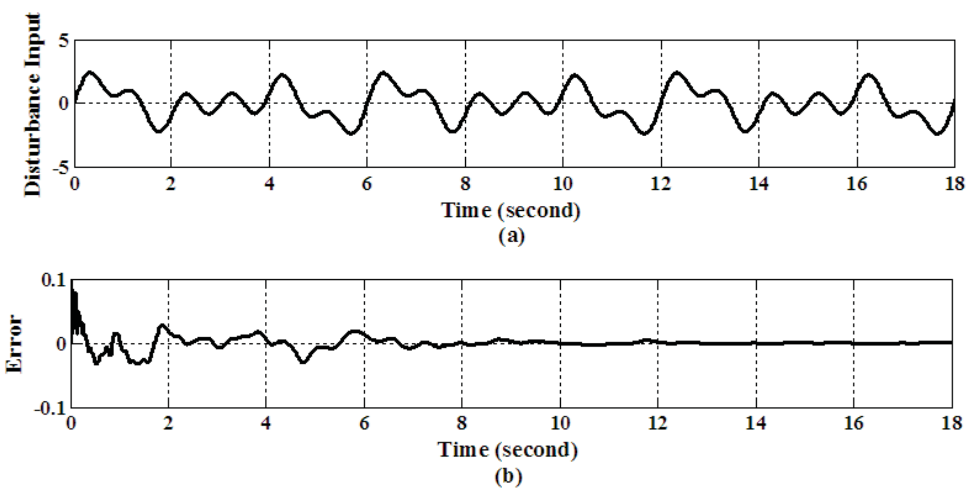


FIGURE 12. Experimental result of Figure 10 with a multiple period's disturbance: (a) the disturbance input and (b) error response

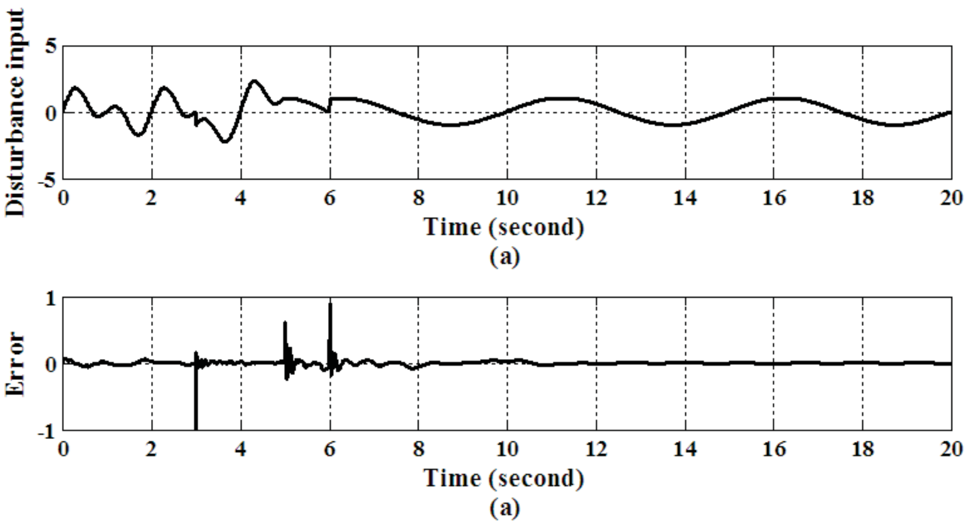


Figure 13. Experimental result of Figure 10 with variable instantaneous frequencies period's signal: (a) the disturbance input and (b) error response

4. **Conclusions.** This paper presents a method for synthesizing repetitive controllers capable of rejecting variable instantaneous frequencies disturbance, where the time-frequency characteristics of the disturbance signal are estimated. The regeneration spectrum to verify the stability of the proposed repetitive control system is redefined. The decay rate of the rejecting error due to the variable instantaneous frequencies disturbance inputs is analyzed, which is related to the peak value of the regeneration spectrum function. An anti-vibration control system with time-varying periodic disturbances is studied to illustrate control performance. The experimental results are given to illustrate that the proposed repetitive control can effectively eliminate steady-state rejecting errors within a few cycles.

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