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NONLINEAR ADAPTIVE IMMERSION AND INVARIANCE CONTROL FOR POWER SYSTEMS WITH SMES

Adirak Kanchanaharuthai

Department of Electrical Engineering
Rangsit University
Patumthai 12000, Thailand
adirak@rsu.ac.th

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Abstract. *This paper centers on the design of an adaptive immersion and invariance (I&I) controller of an electrical power system to enhance transient stability and to achieve the desired transient response performance. A single machine infinite bus (SMIB) power system including the coordinated excitation and superconducting magnetic energy storage (SMES) controller is developed. Practically, the nonlinear power system is unavoidably affected by some unknown parameters. In particular, the damping coefficient and the change in the mechanical input power are considered as unknown constant parameters in this work. Using this strategy, the adaptive control law and parameter update law are derived and used to achieve transient stability improvement when a large disturbance occurs. The simulation results indicate the effectiveness of the proposed method. In this study, three nonlinear controller, that is, the proposed adaptive controller, an I&I controller and an adaptive backstepping controller, are compared.*

Keywords: Transient stability, Generator excitation, SMES, Immersion and invariance, Adaptive control

1. **Introduction.** Power system stability enhancement is of great importance due to a rapid increase of the size and complexity in power systems. In particular, a difficult task in power system operation during an occurrence of disturbances in the power system is to maintain power system stability. Even though there have recently been various studies for improving power system stability, an excitation control of synchronous generators is one of the most effective control design techniques for improving the power system stability.

Recent studies concentrate on the use of storage energy to provide the benefit of improving power quality, dynamic and transient stability, and power system operation reliability. There are currently various types of storage energy such as super conducting magnetic energy storage (SMES) [1-11], and battery storage system (BESS) [12, 13]. These have the potential to significantly improve power flow, voltage control, inter-area and system oscillation in interconnected and long-distance transmission systems. Accordingly, much attention has been recently paid to using energy storage devices to enhance power transfer capability and to augment small-signal and transient stability of power systems. The superconducting magnetic energy storage (SMES) is an important member of energy storage technologies on account of its ability to inject and absorb active and reactive power to increase grid transfer capability through enhanced dynamic voltage stability, to provide smooth and rapid reactive power compensation for voltage support, and to improve both damping oscillations and transient stability [2].

There is currently considerable research devoted to the application of SMES while less attention has been paid to the coordination of generator excitation and SMES for nonlinear power system control in power system engineering literature for years. In [5], using the Hamiltonian function method, an adaptive $\mathcal{L}_2$ disturbance attenuation controller of synchronous generators with SMES unit for multi-machine power systems was developed to improve the system performance. The work in [6] proposed an extended backstepping strategy used to design the generator excitation and SMES controller for improvement of both transient and steady-state performances. Under a large sudden fault, a robust nonlinear excitation and SMES controller [8] was presented for transient stability improvement of an SMIB power system. In [9], the feedback linearization and linear $\mathcal{H}_\infty$ controller was applied to the SME unit to achieve the desired transient stability of power systems and evaluated with experiment results. Based on a Hamiltonian function methodology, Li and Wang [10] proposed a robust adaptive controller of synchronous generators with SMES for the stability improvement faced with disturbances and unknown parameters. Recently, Kanchanaharuthai et al. [11] have proposed an immersion and invariance (I&I) for the design of a nonlinear coordinated generator excitation and SMES controller, based on all measurable state variables, to improve transient stability of power systems. In these work, an evident fact that system with SMES has better dynamic performance, has been validated via simulation or experimental results.

This paper continues this line of investigation and further extends our previous work reported in [11]. There are usually uncertain parameters that cannot be measured accurately in power systems [14]. Thus, an adaptive control design capable of countering the effect of the uncertain parameters is developed for SMIB systems with SMES. This developed approach is more practical and suitable for the system model including uncertain parameters, particularly the damping coefficient and a change of mechanical input power. Following the work proposed in [15], the control objective of the I&I method is to find a stabilizing feedback controller not only to ensure that the closed-loop system behaves asymptotically the same as the pre-specified target system, i.e., achieve asymptotical model matching, but also to achieve the desired closed-loop system performance. Furthermore, the I&I method [15, 16] was further extended to an adaptive I&I one for the solution of nonlinear adaptive control problems [17]. The corresponding adaptive controller can counter the effect of the uncertain parameters adopting a robust perspective [18]. In this paper, the adaptive I&I scheme is applied on the power control system including SMES units. In particular, the damping coefficient and the perturbation in mechanical input power are considered as unknown constant parameters. Despite having two unknown parameters mentioned, the proposed controller is designed to not only simultaneously achieve power angle stability along with frequency and voltage regulation, but also keep the system transiently stable.

The remainder of this paper is organized as follows. A simplified dynamic model of an SMIB power system with generator excitation and SMES is briefly described in Section 2. Adaptive I&I control strategy and design are given in Sections 3 and 4, respectively. Simulation results are given in Section 5 while a conclusion is drawn in Section 6.

2. **System Model.** As discussed in [11], we have the dynamic models of an SMIB power system consisting of generator excitation of a synchronous generator and SMES as follows:

$$\begin{cases} \dot{\delta} &= \omega - \omega_s \\ \dot{\omega} &= \frac{1}{M}\left(P_m - P_e - P_d - P_q - D(\omega - \omega_s)\right) \\ \dot{P}_e &= (-a + (\omega - \omega_s)\cot\delta)P_e + \frac{bV_\infty \sin 2\delta}{2X'_{d\Sigma}} + \frac{V_\infty \sin\delta}{X'_{d\Sigma}} \cdot \frac{u_f}{T'_0} \\ \dot{P}_d &= \frac{P_d}{P_e}\dot{P}_e + \frac{P_e X_2 \cot\delta}{V_\infty} \cdot \frac{1}{T_d}\left(-\left(\frac{P_d V_\infty}{P_e X_2 \cot\delta} - I_{de}\right) + u_d\right) + \frac{I_d P_e X_2}{V_\infty}(\cot^2\delta + 1)(\omega - \omega_s) \\ \dot{P}_q &= \frac{P_q}{P_e}\dot{P}_e + \frac{P_e X_2}{V_\infty} \cdot \frac{1}{T_q}\left(-\left(\frac{P_q V_\infty}{P_e X_2} - I_{qe}\right) + u_q\right) \end{cases} \tag{1}$$

where δ is the power angle of the generator, ω denotes the relative speed of the generator, $D \geq 0$ is a damping constant, P_m is the mechanical input power, P_e is the electrical power, without SMES, delivered by the generator to the voltage at the infinite bus V_∞, P_d and P_q are the electrical power from SMES, ω_s is the synchronous machine speed, $\omega_s = 2\pi f$, H represents the per unit inertial constant, and f is the system frequency and $M = 2H/\omega_s$. $X'_{d\Sigma} = X'_d + X_T + X_L$ is the reactance consisting of the direct axis transient reactance of SG, the reactance of the transformer, and the reactance of the transmission line X_L. Similarly, $X_{d\Sigma} = X_d + X_T + X_L$ is identical to $X'_{d\Sigma}$ except that X_d denotes the direct axis reactance of SG. T'_0 is the direct axis transient short-circuit time constant. u_f is the field voltage control input to be designed. u_d and u_q are the SMES control input to be designed. T_d and T_q are a time constant of SMES models.

Consider the dynamic equations of a power system with generator excitation and SMES in (1) and define the state variable $x = [x_1, x_2, x_3, x_4, x_5]^T = [\delta, \omega - \omega_s, P_e, P_d, P_q]^T$. Therefore, the dynamic model of the power system with SMES can be expressed as an affine nonlinear system as follows:

$$\dot{x} = f(x) + g(x)u(x) \tag{2}$$

where

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \\ f_5(x) \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{1}{M}\left(P_m - Dx_2 - x_3 - x_4 - x_5\right) \\ (-a + x_2 \cot x_1)x_3 + \frac{bV_\infty \sin 2x_1}{2X'_{d\Sigma}} \\ \frac{x_4}{x_3} f_3(x) - x_3 \tan x_1(\cot^2 x_1 + 1)x_2 - \frac{x_4}{T_d} + \frac{x_3 X_2 \cot x_1}{V_\infty T_d} I_{de} \\ \frac{x_5}{x_3} f_3(x) - \frac{x_5}{T_q} + \frac{x_3 X_2}{V_\infty T_q} I_{qe} \end{bmatrix}$$

$$g(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31}(x) & 0 & 0 \\ g_{41}(x) & g_{42}(x) & 0 \\ g_{51}(x) & 0 & g_{53}(x) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{V_\infty \sin x_1}{X'_{d\Sigma}} & 0 & 0 \\ \frac{x_4 V_\infty \sin x_1}{x_3 X'_{d\Sigma}} & \frac{x_3 X_2 \cot x_1}{V_\infty} & 0 \\ \frac{x_5 V_\infty \sin x_1}{x_3 X'_{d\Sigma}} & 0 & \frac{x_3 X_2}{V_\infty} \end{bmatrix}, u(x) = \begin{bmatrix} \frac{u_f}{T'_0} \\ \frac{u_d}{T_d} \\ \frac{u_q}{T_q} \end{bmatrix} \tag{3}$$

The region of operation is defined as the set $\mathcal{D} = \{x \in \mathcal{S} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} |\ 0 < x_1 < \frac{\pi}{2}\}$. The open loop operating equilibrium is denoted by $x_e = [x_{1e}, 0, P_{ee}, P_{de}, P_{qe}]^T = [x_{1e}, 0, P_m, 0, 0]^T$. Considering the model stated above, one can see that the power (rotor) angle can be measured by using phasor measurement units (PMUs). The state variables such as ω, P_e, P_d and P_q can be measured or identified. Thus, all state variables can be measured in this control system. In (2), there is also a nonlinear controller $u = [u_f/T'_0, u_d/T_d, u_q/T_q]^T$.

It is known well that unknown parameters in the system model unavoidably exist in reality. They happen from the following facts. In general, the damping coefficient cannot be measured accurately in practical engineering applications [14]; thus the estimation online and real time of this unknown and/or uncertain constant parameter is needed. Additionally, an unknown mechanical input power and a sudden mechanical perturbation are important uncertainties in the power system; therefore, it may destabilize the operating conditions. In particular, as the mechanical power is perturbed to a new constant value, this causes the equilibrium of the system that will be shifted to a new corresponding point. Further, the new point will be unknown provided that the mechanical power is unknown [22-25]. However, no uncertain parameters in the system model (2) were taken into account.

For notational convenience, let us define $\theta = [\theta_1, \theta_2]^T = [-D, P_m]^T$ as two unknown constant parameters of interest, then the system (2) and (3) can be rewritten as follows.

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{M}(\theta_2 + \theta_1 x_2 - x_3 - x_4 - x_5) \\ \dot{x}_3 &= f_3(x) + g_{31}(x)\frac{u_f}{T_0'} \\ \dot{x}_4 &= f_4(x) + g_{41}(x)\frac{u_f}{T_0'} + g_{42}(x)\frac{u_d}{T_d} \\ \dot{x}_5 &= f_5(x) + g_{51}(x)\frac{u_f}{T_0'} + g_{53}(x)\frac{u_q}{T_q} \end{cases} \tag{4}$$

Thus, the objective of this paper is to solve the problem of the transient stabilization of system (4) with unknown constant parameters θ, which can be formulated as follows: with the help of the adaptive immersion and invariance methodology, to design an adaptive (state) feedback controller:

$$u = \varphi\left(x, \hat{\theta}\right), \quad \dot{\hat{\theta}} = \varpi\left(x, \hat{\theta}\right) \tag{5}$$

such that the resulting closed-loop system is asymptotically stable at the only equilibrium (x_e, θ) and $x \to x_e$, $\hat{\theta} \to \theta$ as $t \to \infty$ where $\hat{\theta}$ is the estimate of $\theta = [\theta_1, \theta_2]^T$.

3. Adaptive I&I Methodology.

The I&I method for stabilizing nonlinear systems was proposed in [15, 16]. The method is based on the notion of invariant manifolds and system immersion. In addition, the method was further extended to an adaptive I&I control one as reported by [17]. The key concept of this method is still based upon the notion of system immersion and invariant manifold. Further, the knowledge of a Lyapunov function is not needed for this method, resulting in the fact that the obtained adaptive control methods counter the effect of uncertain parameters adopting a robust perspective. The following results, revealed in those papers, are used to design this proposed nonlinear adaptive controller for power systems including generator excitation and SMES controllers.

Consider the nonlinear system[1] [17]

$$\dot{x}(t) = f(x, u, \theta) \tag{6}$$

with state $x \in \mathbb{R}^n$ and control input $u \in \mathbb{R}^m$, and an assignable equilibrium point $x_e \in \mathbb{R}^n$ to be stabilized. Define the augmented system

$$\dot{x}(t) = f(x, u, \theta), \quad \dot{\hat{\theta}} = \varpi(x, \hat{\theta}) \tag{7}$$

where $\hat{\theta} \in \mathbb{R}^q$ and $\varpi \in \mathbb{R}^q$ is a new control signal to be designed. The adaptive stabilization problem can be posed, informally, as follows.

Find (if possible) a state feedback control law described by equations of the form (5) such that all trajectories of the closed-loop system (12) and (13) are bounded and $\lim_{t\to\infty} x(t) = x_e$. Note that, since $f(\cdot)$ is only partially known, it is not required that $\hat{\theta}$ converges to any particular equilibrium, but merely that it remains bounded. The following theorem, presented in the major result of [17], not only offers conditions under which the above problem is solvable through the I&I strategy, but also is used to design this proposed adaptive controller for power systems with SMES.

Theorem 3.1. [17] *Consider the nonlinear system and an equilibrium point* $x_e \in \mathbb{R}^n$. *Let* $p \le n$, $\xi \in \mathbb{R}^p$, $\zeta \in \mathbb{R}^{n-p}$, $z \in \mathbb{R}^q$. *Assume that there exist smooth mappings* $\alpha(\xi, \theta) \to \mathbb{R}^p$,

[1] It is assumed that throughout this paper all functions and mappings are $\mathbb{C}^\infty$.

$\pi(\xi,\theta) \to \mathbb{R}^n$, $c(\xi,\theta) \to \mathbb{R}^m$, $\phi(x,\theta) \to \mathbb{R}^{n-p}$, $u(x,\zeta,z+\theta) \to \mathbb{R}^m$, $\varpi(x,\zeta,z+\theta) \to \mathbb{R}^q$ and $\beta(x) \to \mathbb{R}^q$ such that the following conditions hold.

(H1) (Target system) The system

$$\dot{\xi} = \alpha(\xi,\theta) \tag{8}$$

has an asymptotically stable equilibrium at $\xi_e \in \mathbb{R}^p$ and $x_e = \pi(\xi_e,\theta)$.

(H2) (Immersion condition) For all $\xi \in \mathbb{R}^p$

$$f(\pi(\xi,\theta), c(\xi,\theta), \theta) = \frac{\partial \pi}{\partial \xi}\alpha(\xi,\theta) \tag{9}$$

(H3) (Implicit manifold) The following set identity holds

$$\mathcal{M} := \{x \in \mathbb{R}^n \,|\, x = \pi(\xi,\theta) \text{ for some } \xi \in \mathbb{R}^p\} = \{x \in \mathbb{R}^n \,|\, \phi(x,\theta) = 0\} \tag{10}$$

(H4) (Manifold attractivity and trajectory boundedness) All trajectories of the system

$$\begin{cases} \dot{\zeta} &= \frac{\partial \phi}{\partial x} f\left(x, u(x,\zeta,z+\theta),\theta\right) + \frac{\partial \phi(x,\hat{\theta})}{\partial \hat{\theta}} \omega\left(x,\zeta,z+\theta\right) \\ \dot{z} &= \varpi(x,\zeta,z+\theta) + \frac{\partial \beta(x)}{\partial x} f(x, u(x,\zeta,z+\theta),\theta) \\ \dot{x} &= f(x,u(x,\zeta,z+\theta),\theta) \end{cases} \tag{11}$$

are bounded and satisfy

$$\lim_{t\to\infty} \zeta(t) = 0, \quad \lim_{t\to\infty} [\phi(x(t), z(t)+\theta) - \phi(x(t),\theta)] = 0 \tag{12}$$

Then all trajectories of the closed-loop system[2]

$$\dot{x} = f\left(x, u\left(x, \phi\left(x, \hat{\theta}+\beta(x)\right), \hat{\theta}+\beta(x), \theta\right)\right), \quad \dot{\hat{\theta}} = \varpi\left(x, \phi\left(x, \hat{\theta}+\beta(x)\right), \hat{\theta}+\beta(x)\right)$$

are bounded and satisfy (3). Finally, $\lim_{t\to\infty} x(t) - \pi(\xi(t),\theta) = 0$ and, if $\phi\left(x(0), \hat{\theta}(0)\right) = 0$, $\hat{\theta}(0) - \theta + \beta(x(0)) = 0$ and $\xi(0) = \pi(x(0),\theta)$, $x(t) = \pi(\xi(t),\theta)$ for all $t \geq 0$.

Definition 3.1. *System (7) is said to be adaptive I&I stabilizable with target dynamics (8) if (H1)-(H4) of Theorem 3.1 are satisfied.*

4. Adaptive I&I Controller Design.

The proposed adaptive I&I control approach is expressed in the following proposition.

Theorem 4.1. *Consider the system (4), the adaptive I&I control law is as follows:*

$$u\left(x,\hat{\theta}\right) = \begin{bmatrix} \frac{u_f(x,\hat{\theta})}{T_0'} & \frac{u_d(x,\hat{\theta})}{T_d} & \frac{u_q(x,\hat{\theta})}{T_q} \end{bmatrix}^T \tag{13}$$

where

$$\begin{cases} \frac{u_f(x,\hat{\theta})}{T_0'} &= \frac{1}{g_{31}(x)}\left[-f_3(x) + \left(\beta M\cos(x_1 - x_{1e}) + \dot{\hat{\theta}}_1\right)x_2 + \dot{\hat{\theta}}_2 + \delta\left(x,\hat{\theta}\right)\dot{\bar{x}}_2 - \sigma_1\left(x,\hat{\theta}\right)\zeta_1\right] \\ \frac{u_d(x,\hat{\theta})}{T_d} &= \frac{1}{g_{42}(x)}\left[-f_4(x) - g_{41}(x)\frac{u_f}{T_f} - \gamma_d\dot{x}_2 - \sigma_2\zeta_2\right] \\ \frac{u_q(x,\hat{\theta})}{T_q} &= \frac{1}{g_{53}(x)}\left[-f_5(x) - g_{51}(x)\frac{u_f}{T_f} + \gamma_d\dot{x}_2 - \sigma_3\zeta_3\right] \end{cases} \tag{14}$$

Also, the parameter update law is given by:

$$\dot{\hat{\theta}}_1 = \varpi_1\left(x,\hat{\theta}\right) = \frac{\gamma_1 x_2 \dot{\bar{x}}_2}{M}, \quad \dot{\hat{\theta}}_2 = \varpi_2\left(x,\hat{\theta}\right) = \frac{\gamma_2 \dot{\bar{x}}_2}{M} \tag{15}$$

[2]Note that the function $\varpi(\cdot)$ in the $\hat{\theta}$ equation is a function of x and $\hat{\theta}$ consistently with (7).

where $\dot{x}_2 = \frac{1}{M}\left(\hat{\theta}_2 + \frac{\gamma_2 x_2}{M} - x_3 - x_4 - x_5 + \left(\hat{\theta}_1 + \frac{\gamma_1 x_2^2}{2M}\right)x_2\right)$, $\delta\left(x, \hat{\delta}\right) = \frac{\gamma_2}{M} + \gamma_d + \hat{\theta}_1 + \frac{3\gamma_2 x_2^2}{2M}$, *and* $\sigma_1\left(x, \hat{\theta}\right) = \lambda_1 + \epsilon\delta\left(x, \hat{\theta}\right)^2$, $\sigma_2 = \lambda_2$, $\sigma_3 = \lambda_3$, $\lambda_i > 0$, $(i = 1, 2, 3)$ *and* $\left(\gamma_1, \gamma_2, \gamma_d, \epsilon, \bar{\beta}\right)$ *are positive design parameters. The control law and the parameter update law are such that all trajectories of the closed-loop system are bounded together with* $\lim_{t\to\infty}\left(\hat{\theta}_1 + \frac{\gamma_1 x_2^2}{2M} - \theta_1\right) = 0$ *and* $\lim_{t\to\infty}\left(\hat{\theta}_2 + \frac{\gamma_2 x_2}{2M} - \theta_2\right) = 0$. *Further, the closed-loop adaptive system is asymptotically stable at the equilibrium point* (x_e, θ).

Proof: According to Theorem 3.1, we can proceed from checking the four conditions via adaptive I&I approach as follows.

(H1) Target system: specification of the target system. In order to design a stabilizing controller and verify the condition according to Theorem 3.1, we start with selecting the target dynamics $\left(\dot{\xi} = \alpha(\xi, \theta)\right)$ as the mechanical subsystems (e.g., a simple damped pendulum system)

$$\dot{\xi}_1 = \xi_2, \quad \dot{\xi}_2 = -\frac{\partial V(\xi_1)}{\partial \xi_1} - R(\xi)\xi_2 \tag{16}$$

where $V(\xi_1)$ and $R(\xi)$ represent the potential energy and a damping function of the pendulum systems, respectively, which both are to be selected. The pendulum system considered with a stable equilibrium point $\xi_e = (\xi_{1e}, 0)^T$ has the potential energy $V(\xi_i)$ satisfying the following two assumptions (i) $\frac{\partial V(\xi_{1e})}{\partial \xi} = 0$ (ii) $\frac{\partial^2 V(\xi_{1e})}{\partial^2 \xi} > 0$, the damping function verifying $R(\xi_e) \geq 0$, and the energy function $H(\xi) = \frac{1}{2}\xi_2^2 + V(\xi_1)$.

(H2) Immersion condition: computation of the mapping $\boldsymbol{\pi(\xi, \theta)}$. As the desired target systems have been selected, a mapping $\pi : \mathcal{S} \times \mathbb{R} \times \mathbb{R} \to \mathcal{S} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ is the following.

$$\pi(\xi_1, \xi_2, \theta) := \left[\ \xi_1, \quad \xi_2, \quad \pi_3(\xi, \theta), \quad \pi_4(\xi, \theta), \quad \pi_5(\xi, \theta)\ \right]^T \tag{17}$$

where $\pi_3(\xi, \theta), \pi_4(\xi, \theta)$ and $\pi_5(\xi, \theta)$ are to be selected. Besides, the condition of Theorem 3.1 gives the constraints, namely $\xi_{1e} = x_{1e}$, $\xi_{2e} = x_{2e} = 0$, $\pi_3(\xi_e, \theta) = x_{3e}$, $\pi_4(\xi_e, \theta) = x_{4e} = 0$, $\pi_5(\xi_e, \theta) = x_{5e} = 0$. We can choose $\pi_3(\xi, \theta)$, $\pi_4(\xi, \theta)$ and $\pi_5(\xi, \theta)$ to satisfy the condition (9). Firstly, we can choose the potential energy $V(\xi_1)$ along two assumptions given above as $V(\xi_1) = -\bar{\beta}\cos\tilde{\xi}_1$, $\tilde{\xi}_1 = \xi_1 - \xi_{1e}$, for some $\bar{\beta} > 0$ and $R(\xi) = \frac{\gamma_d}{M}$. Then, $\pi_3(\xi, \theta)$ and $c(\pi(\xi, \theta))$ are chosen to satisfy the condition (H2) as follows: the first row of (9) is already satisfied $\left(\dot{\xi}_1 = \xi_2\right)$. After considering the second row of (9), we obtain

$$\begin{aligned} &\frac{1}{M}\left(\theta_2 - \pi_3(\xi, \theta) - \pi_4(\xi, \theta) - \pi_5(\xi, \theta) + \theta_1\xi_2\right) \\ &= -\frac{\partial V(\xi_1)}{\partial \xi_1} - R(\xi)\xi_2 = -\bar{\beta}\sin\left(\tilde{\xi}_1\right) - \frac{\gamma_d}{M}\xi_2 \end{aligned}$$

From the expression above, in order to simplify our derivations, we choose $\pi_4(\xi_2, \theta) = x_{4e} - \gamma_d\xi_2$ and $\pi_5(\xi_2, \theta) = x_e + \gamma_d\xi_2$. Consequently, $\pi_3(\xi, \theta) = \theta_2 + \bar{\beta}M\sin\tilde{\xi}_1 + (\gamma_d + \theta_1)\xi_2 - \pi_4(\xi, \theta) - \pi_5(\xi, \theta)$. It is obvious that π_3 is a function of both ξ_1, ξ_2 and θ, and defined in $\mathcal{D}$. As the mapping $\pi(\xi)$ has been chosen, by using some straightforward calculation from the third row to the fifth row of (9), respectively, we have the control input $c(\pi(\xi, \theta))$ that renders the manifold $\mathcal{M}$ invariant.

(H3) Implicit manifold: derivation of the manifold $\phi(x,\theta)$. From the result above, the mapping $\pi(\xi,\theta)$ has been defined and then the condition in (10) is verified. This subsection is to find an implicit definition of the manifold $\mathcal{M}$ that can be implicitly described by $\mathcal{M} = \{x \in \mathcal{S}\times\mathbb{R}\times\mathbb{R}\times\mathbb{R} | \phi(x,\theta)=0\}$ and the mapping $\phi(x,\theta)$ can be defined as $\phi(x,\theta) = \left[\begin{array}{ccc}\phi_1(x,\theta) & \phi_2(x,\theta) & \phi_3(x,\theta)\end{array}\right]^T$. With the direct correspondances of $\pi_1(\xi)=x_1$, $\pi_2(\xi)=x_2$, the manifold $\phi(\xi,\theta)$ is constructed by

$$\phi_1(x,\theta) = x_3 - \pi_3(\xi,\theta), \quad \phi_2(x,\theta) = x_4 - \pi_4(\xi,\theta), \quad \phi_3(x,\theta) = x_5 - \pi_5(\xi,\theta) \tag{18}$$

(H4) Manifold attractivity and trajectory boundedness. In this subsection, a control law $u\left(x,\hat{\theta}\right)$ and a parameter adaptation law $\varpi\left(x,\hat{\theta}\right)$ are designed to ensure that all trajectories of the closed-loop system are bounded and converge to the manifold $\mathcal{M}$.

We define $\zeta_i = \phi_{i+2}\left(x,\hat{\theta}\right)$, $(i=1,2,3)$ as the off-the-line manifold coordinate. The estimate of θ_j are defined as $\hat{\theta}_j + \beta_j(x)$, $(j=1,2)$ and the error variables are described as $z_j = \hat{\theta}_j + \beta_j(x) - \theta_j$. According to condition (H4) in (11), straightforward calculations show that

$$\begin{cases}
\dot{\zeta}_1 &= f_3(x) + g_{31}(x)\frac{u_f}{T_0'} - \left[\left(\bar{\beta}M\cos(x_1 - x_{1e}) + \dot{\hat{\theta}}_1\right)x_2\right. \\
&\quad \left. + \dot{\hat{\theta}}_2 + \frac{1}{M}\left(\frac{\partial\beta_2}{\partial x_2} + \gamma_d + \hat{\theta}_1 + \beta_1(x) + \frac{\partial\beta_1}{\partial x_2}x_2\right)\dot{\bar{x}}_2\right], \\
\dot{\zeta}_2 &= f_4(x) + g_{41}(x)\frac{u_f}{T_0'} + g_{42}(x)\frac{u_d}{T_q} + \gamma_d\dot{x}_2, \\
\dot{\zeta}_3 &= f_5(x) + g_{51}(x)\frac{u_f}{T_0'} + g_{53}(x)\frac{u_q}{T_q} - \gamma_d\dot{x}_2, \\
\dot{z}_1 &= \dot{\hat{\theta}}_1 + \frac{\partial\beta_1}{\partial x_1}x_2 + \frac{\partial\beta_1}{\partial x_2}\dot{\bar{x}}_2 - \frac{1}{M}\frac{\partial\beta_1}{\partial x_2}(z_2 + z_1x_2), \\
\dot{z}_2 &= \dot{\hat{\theta}}_2 + \frac{\partial\beta_2}{\partial x_2}x_2 + \frac{\partial\beta_2}{\partial x_2}\dot{\bar{x}}_2 - \frac{1}{M}\frac{\partial\beta_2}{\partial x_2}(z_2 + z_1x_2), \\
\dot{x}_1 &= x_2,\ \dot{x}_2 = \frac{1}{M}(-z_2 - (z_1+\gamma_d)x_2 - \bar{\beta}M\sin(x_1 - x_{1e})),
\end{cases} \tag{19}$$

where $\dot{\bar{x}}_2 = \frac{1}{M}\left(\hat{\theta}_2 + \beta_2(x) - x_3 - x_4 - x_5 + \left(\hat{\theta}_1 + \beta_1(x)\right)x_2\right)$.

By an appropriate definition of the control law $u\left(x,\hat{\theta}\right)$ in (13) and (14) with the parameter update law $\dot{\hat{\theta}}_1, \dot{\hat{\theta}}_2$ as given in (15) and selecting the nonlinear functions $\beta_1(\cdot)$ and $\beta_2(\cdot)$ as

$$\beta_1(x) = \frac{\gamma_1 x_2^2}{2M}, \quad \beta_2(x) = \frac{\gamma_2 x_2}{2M} \tag{20}$$

then, the system (19) can be expressed in the (ζ,z,x)-coordinate as:

$$\begin{cases}
\dot{\zeta}_1 &= -\sigma_1\zeta_1 + \frac{1}{M}\delta\left(x,\hat{\theta}\right)(z_2 + z_1x_2), \\
\dot{\zeta}_2 &= -\sigma_2\zeta_2, \\
\dot{\zeta}_3 &= -\sigma_3\zeta_3, \\
\dot{z}_1 &= -\frac{\gamma_1 x_2}{M^2}(z_2 + z_1x_2), \\
\dot{z}_2 &= -\frac{\gamma_2}{M^2}(z_2 + z_1x_2), \\
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \frac{1}{M}\left(-z_2 - (z_1+\gamma_d)x_2 - \bar{\beta}M\sin(x_1 - x_{1e})\right),
\end{cases} \tag{21}$$

where $\delta\left(x,\hat{\theta}\right) = \frac{\gamma_2}{M} + \gamma_d + \hat{\theta}_1 + \frac{3\gamma_1 x_2^2}{2M}$, $\sigma_1\left(x,\hat{\theta}\right) = \lambda_1 + \epsilon\delta\left(x,\hat{\theta}\right)^2$, $\sigma_2 = \lambda_2$, $\sigma_3 = \lambda_3$, $\lambda_i > 0$, $(i=1,2,3)$ and $(\gamma_1,\gamma_2,\gamma_d,\epsilon,\bar{\beta})$ are positive design parameters.

In accordance with condition (H4), we need to prove that boundedness of the trajectories of the closed-loop behavior of the state Equation (11) with the control law (13)-(15) and the off-the-manifold coordinate ζ. From the target system in (16), it can be seen that x_1, x_2 are bounded and settle to the equilibrium point $(x_{1e}, 0)$. Note now that the nonlinear functions $\beta_1(\cdot)$ and $\beta_2(\cdot)$ in (20) are chosen to guarantee that the dynamics of z_1 and z_2 are stable. Consider the Lyapunov function $W = \frac{1}{2\gamma_1}z_1^2 + \frac{1}{2\gamma_2}z_2^2$ and its time-derivative along the system trajectory (21) turns into $\dot{W} = -\frac{1}{M^2}(z_2 + z_1 x_2)^2$. Notably, $\dot{W}$ is negative-definite, hence $z_1, z_2 \in \mathcal{L}_2$. This ensures all trajectories of the dynamics (21) are bounded and z_1, z_2 converge to zero. Additionally, $\lim_{t\to\infty} \beta_1(x) \to 0$, $\lim_{t\to\infty} \beta_2(x) \to 0$; thus, we obtain $\lim_{t\to\infty}\left(\hat{\theta}_1 + \beta_1(x_2) - z_1\right) \to \theta_1$ and $\lim_{t\to\infty}\left(\hat{\theta}_2 + \beta_2(x_2) - z_2\right) \to \theta_2$. It is obvious that the off-the-manifold coordinate ζ is bounded and from (21) $\lim_{t\to\infty} \zeta(t) = 0$. Apart from this, boundedness of x_3, x_4 and x_5 immediately follows from the fact that $x_3 = \zeta_1 + \pi_3(x_1, x_2, \theta)$, $x_4 = \zeta_2 + \pi_4(x_2)$ and $x_5 = \zeta_3 + \pi_5(x_2)$. From (4), it is easy to conclude boundedness of x_3, x_4 and x_5, respectively. Finally, boundedness of all trajectories of (21) has been shown; thus, we can deduce that system (21) has an asymptotically stable equilibrium at (x_e, θ). We can summarize the adaptive I&I controller design in Theorem 4.1. This completes the proof.

5. **Simulation Results.** In this section, the proposed control methodology is implemented on an SMIB power system with SMES and the closed-loop performance of the system is evaluated using computer simulation studies under disturbances. The time domain simulations are carried out to investigate the damping performance of the designed controller and the parameter adaptive law, as given in (13)-(15), in the system under study. The performance of the proposed controller (adaptive I&I controller) is compared with that of the following existing nonlinear controllers: (1) an standard I&I controller (full-information) [11], which is obtained by assuming that the two constant parameters (θ_1, θ_2) are precisely known and (2) an adaptive backstepping controller [20]. In the simulations, the fault of interest is a symmetrical three phase short circuit occurring on one of the transmission lines as shown in Figure 1. The following two cases with a temporary fault sequence and an unknown, but bounded, small perturbation to mechanical power to synchronous generators in the system with unknown constant parameters are discussed.

Case 1: temporary fault. The system is in a pre-fault steady state, a fault occurs at $t = 0.5$ sec., the fault is isolated by opening the breaker of the faulted line at $t = 0.7$ sec., and the transmission line is recovered without the fault at $t = 2$ sec. Afterward the system is in a post-fault state.

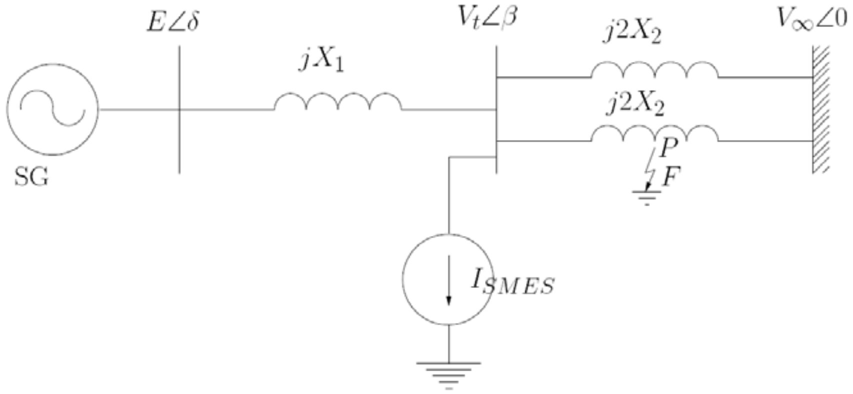

FIGURE 1. A single line diagram of SMIB model with SMES

Case 2: unknown perturbation in mechanical input power. The system is in a pre-fault steady state, and there is an unknown constant perturbation in the mechanical power between $t = 0.5$ sec. and $t = 2.5$ sec. Afterward the system is in a post-fault state.

The physical parameters (pu.) and initial conditions $\left(\delta_0, \omega_0, P_{e0}, P_{s0}, \hat{\theta}_{10}, \hat{\theta}_{20}\right)$ for this power system model are given in [11] and $\hat{\theta}_{10} = -0.3$, $\hat{\theta}_{20} = 0.8$. The tuning parameters of the proposed adaptive controller are $\bar{\beta} = 200$, $\gamma_d = 0.2$; $\gamma_1 = 0.5$, $\gamma_2 = 0.1$, $\lambda_1 = \lambda_2 = \lambda_3 = 100$, $\epsilon = 0.001$. The SMIB power system consisting of generator excitation and SMES has been simulated using the the physical parameters and initial conditions above.

For Case 1, Figure 2 shows the time responses of power angle (δ), frequency ($\omega - \omega_s$), and terminal voltage (V_t), eventually returning to the pre-fault state values, under three controllers. Figure 3 illustrates the parameter estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ under three controllers. It can be observed that the proposed controller and the standard I&I controller perform better transient behavior compared with the adaptive backstepping scheme. In particular,

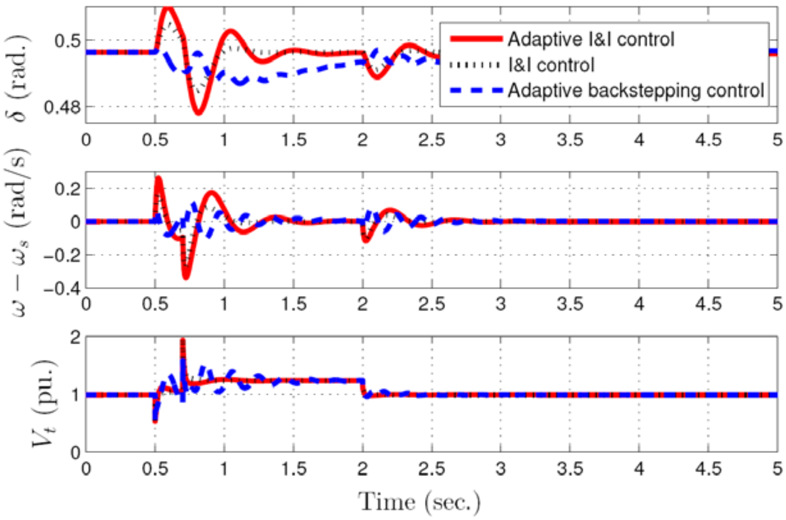

FIGURE 2. Controller performance in Case 1 – power angles (δ) (rad.), frequency ($\omega - \omega_s$) rad/s and terminal voltage (V_t) pu.

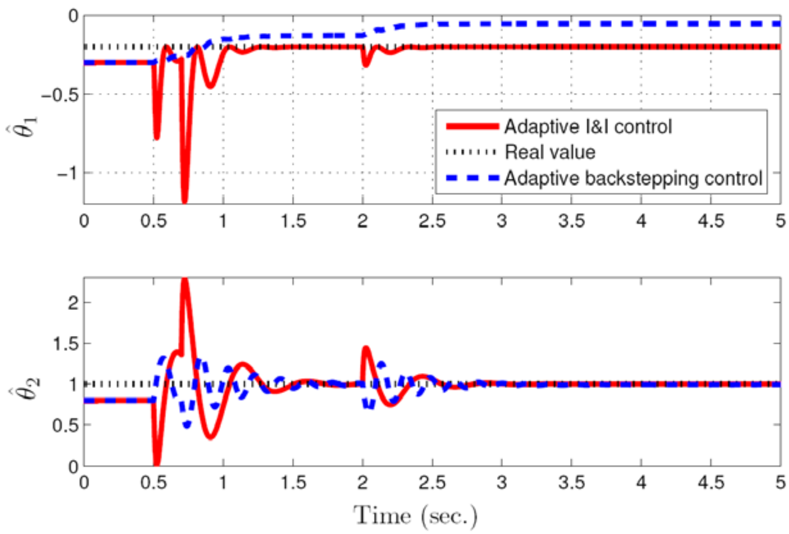

FIGURE 3. Unknown constant paramters in Case 1 – damping coefficient estimate $\left(\hat{\theta}_1\right)$ and mechanical input power estimate $\left(\hat{\theta}_2\right)$

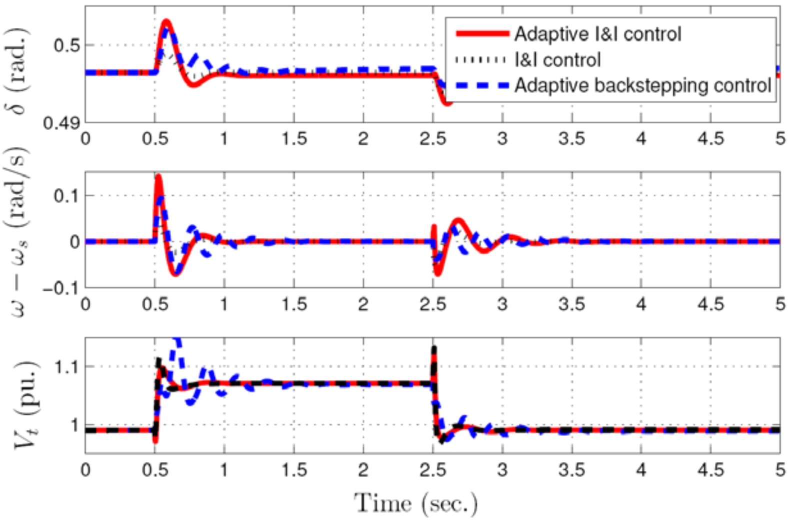

FIGURE 4. Controller performance in Case 2 – power angles (δ) (rad.), frequency ($\omega - \omega_s$) rad/s and terminal voltage (V_t) pu.

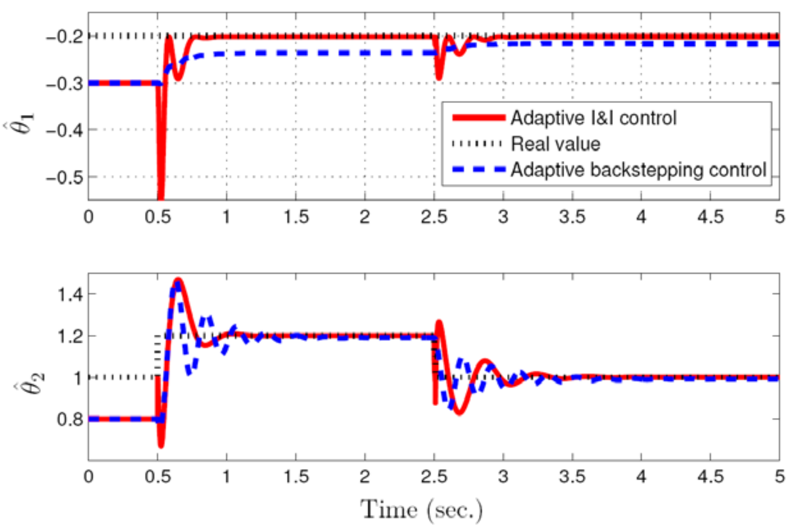

FIGURE 5. Unknown constant paramters in Case 2 – damping coefficient estimate $\left(\hat{\theta}_1\right)$ and mechanical input power estimate $\left(\hat{\theta}_2\right)$

the convergence and damping of the proposed controller are much better compared with those of the adaptive backstepping one. Moreover, the proposed adaptive scheme has an obvious advantage over the adaptive backstepping one. Especially, the parameter estimate of the proposed control converges to the real value of both the damping constant $\left(\hat{\theta}_1 \rightarrow \theta_1\right)$ and mechanical input power $\left(\hat{\theta}_2 \rightarrow \theta_2\right)$. In contrast, the parameter estimate of the adaptive backstepping one converges to the real value of the only mechanical input power $\left(\hat{\theta}_2 \rightarrow \theta_2, \hat{\theta}_1 \nrightarrow \theta_1\right)$.

Similar to Case 1, Figure 4 illustrates time trajectories of power angle, frequency, and terminal voltage setting to the pre-fault steady state despite having an unknown constant perturbation of mechanical input power. For this case, the mechanical power is varied from the unknown normal value to some unknown constant (in simulation $P_m = 1$ pu., $\Delta P_m = 0.2$ pu.). It is clear that the equilibrium can be recovered and the terminal voltage can be regulated to the prescribed value when the system is forced by the proposed adaptive control. Further, as compared with the adaptive backstepping scheme, the proposed adaptive controller not only effectively damps the oscillations of power angle and frequency, but also has superior performance in maintaining the terminal bus voltage magnitude close to its reference voltage values defined for the normal operating condition, and provides effective voltage regulation to the desired pre-fault steady-state values after the occurrence of an unknown perturbation in mechanical power. Apart from this, Figure 5 indicates that identical to Case 1, the parameter estimate of the proposed control converges to the real value of both the damping constant and mechanical input power while the adaptive backstepping one cannot.

As indicated in the simulation results above, it can be, overall, concluded that the proposed adaptive control law is effectively designed for transient stabilization and voltage regulation in two cases. The adaptive I&I control can render the closed-loop system converge quickly to an equilibrium point and the terminal voltage can be quickly regulated to the reference voltage value in spite of having two unknown constant parameters. Even though the system considered has two unknown constant parameters, the time responses of the proposed adaptive scheme are slightly different from those of the standard I&I one and obviously outperform the adaptive backstepping one in terms of fast convergence speed and shorter settling time as well as guaranteed convergence to the true value of two unknown constant parameters in both cases.

6. **Conclusion.** In this paper, we have used the adaptive I&I design tool to design the generator excitation and SMES controller. The resulting controller can be effectively used to enhance transient stability and voltage regulation in the SMIB power system when the mechanical power and the damping constant are unknown constant parameters. Using the adaptive I&I design, the simulation results have demonstrated that the proposed controller can drive the system to a stable equilibrium corresponding to the real value of two unknown constant parameters. Besides, it can be observed that the damping and the closed-loop system dynamics of the proposed control do not differ much from those of the standard I&I one but perform much better than those of the adaptive backstepping one. The values of parameter estimate of the proposed control also converge toward the true parameter values in both cases. In addition, the presented controller simultaneously achieves transient stabilization and accomplishes a good regulation of the SMES terminal voltage. Future studies will be devoted to the extension of this strategy to enhance power system stability for power systems with transfer conductances between buses. The extension of this strategy to control design in the presence of disturbance and more general unknown parameters also deserves further study.

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