International Journal of Innovative Computing, Information and Control
Volume **12**, Number **4**, August **2016**

ICIC International ©2016 ISSN 1349-4198
pp. 1179–1195

ECONOMIC ORDER QUANTITY FOR TWO-LEVEL TRADE CREDIT FINANCING UNDER CONDITIONALLY PERMISSIBLE DELAY IN PAYMENT

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Received February 2016; revised June 2016

Abstract. *Almost all previously published articles about optimal order quantity under two-level trade credit assumed that the supplier offers the retailer a full trade credit period, while the retailer offers the customer a partial trade credit period. However, this paper extends this extreme case by assuming that the supplier offers the retailer conditionally permissible delay in payments and then the retailer also provides his or her customer with fully permissible delay in payments. In addition, this paper explores Huang's model to that of two-level trade credit policy, but the interest earned and interest payable are calculated in a different way. Under these conditions, this article probes into the retailer's inventory system of up-stream partial trade credit and down-stream full trade credit as a cost minimization problem. Six theorems are developed to determine retailer's optimal ordering policies. Finally, some numerical examples serve to demonstrate that: (1) a higher value of unit purchasing price brings about a smaller order quantity and smaller annual total relevant costs; (2) the retailer's optimal order quantity and annual total relevant costs will remain unchanged when the fraction of the delay payments permitted varies.*

Keywords: Supply chain, EOQ, Trade credit, Partial trade credit, Conditionally permissible delay in payments

1. **Introduction.** The basic EOQ model is based on the implicit assumption that the retailer must pay for the items as soon as he or she receives them from a supplier. However, this may not be true. In today's business transactions, it is more and more common to see that the supplier will allow a certain fixed time period for settling the amount that the supplier owes to retailer for the items supplied. We term this period as trade credit period. Trade credit (i.e., the permissible delay in payments) allows a buyer to accumulate revenue and earn interest during the credit period. However, beyond this credit period the seller charges the buyer interest on the unpaid balance. Hence, from the buyer's perspective, trade credit policy can reduce his or her holding cost, and thus is a powerful promotional tool to attract new customers, who consider such a strategy as an alternative incentive policy to price discounts. On the other hand, from the seller's perspective, although offering trade credit increases his or her opportunity cost due to interest loss during the credit period, it reduces his or her buyer's holding cost, attracts new customers, and in turn increases his or her profit.

The remainder of this paper is organized as follows. We briefly review the related literature in Section 2. Section 3 focuses on the notation and assumption. Then, we formulate our mathematical model. In Sections 4 and 5, we derive several theoretical results to determine the optimal solution under various situations. In Section 6, we provide some numerical examples to illustrate our results. Finally, the conclusions are shown in the last section.

2. **Literature Review.** Goyal [1] first established an economic order quantity (EOQ) model under the conditions of permissible delay in payments to fit the real-life situation. Many research discusses this model under various conditions. Aggarwal and Jaggi [2] extended Goyal's model to the case of deterioration. Jamal and Wang [3] analyzed Aggarwal and Jaggi's model to allow for shortages. Chung and Huang [4] extended Goyal's model to the economic production quantity (EPQ) model. Chung [5], Chung and Lin [6] discussed the inventory model for trade credit in economic ordering policies of deteriorating items under the different circumstances in a supply chain system. In addition, Yen et al. [7] found the optimal retailer's ordering policies with trade credit financing and limited storage capacity in the supply chain system. All the above articles mentioned are studied under the assumption that the supplier would offer the retailer full trade credit but not partial trade credit. However, this paper will explore the inventory model under supplier's partial trade credit to relax the above assumption.

In practice, many firm's endurance capacity in debt is limited and non-payment risks from retailers may happen. Certainly, one remedy for this is to let retailer jointly bear the financial pressure in the trade credit model. Based on this, suppliers/retailers frequently offer partial trade credit to their retailers/customers who must make partial payment of the purchase amount at the time of placing an order and settle the unpaid balance at the end of credit period. Huang [8] and Chung and Lin [9] extended Goyal's work to allow the supplier to offer the retailer partial trade credit, not full trade credit, from different points of view, respectively. In addition, Mahata [10] studied an EPQ-based inventory model for exponentially deteriorating items under the retailer's partial trade credit policy in the supply chain. Chung [11] presented the EOQ model with defective items and partially permissible delay in payments linked to order quantity derived analytically in the supply chain management. Chen et al. [12] proposed an economic order quantity by considering the strategy that suppliers offer retailers a fully permissible delay of some periods if a retailer's order is more than or equal to some predetermined quantities; if the retailer's order quantity is less than the predetermined quantity, then the retailer must make a partial payment to the supplier, and enjoy a permissible delay of the same periods for remaining balances. Chung and Ting [13] incorporated a real payment mode in which the retailers could still gradually settle the partial payment in the inventory EOQ model under the supplier's partial trade credit.

All of the above mentioned inventory models were developed only based on single level trade credit policy. In many recent research, it is assumed that the supplier offers the retailer a full trade credit period, and the retailer offers the customer a partial/full trade credit period. In this direction and perhaps, Huang [14] first explored the EOQ model under two-level trade credit policy when the supplier offers the retailer a permissible delay of M periods, and then the retailer also provides his or her customer a permissible delay of N periods, respectively. Kreng and Tan [15] modified Huang's model [14] by developing optimal wholesaler's replenishment decisions in the EOQ model under two-level trade credit policy depending on the order quantity. Huang and Hsu [16] studied the EOQ model under the situation in which the retailer has the powerful decision right such that the retailer can obtain full trade credit offered by the supplier and just offer partial trade credit to the customers. Chung [17] extended Huang and Hsu's work to present a complete solution procedure to remove logical shortcomings of mathematics. Teng [18] explored optimal ordering policies for a retailer who offers distinct trade credits to his or her good and bad credit customers. Min et al. [19] developed an inventory model for deteriorating items under stock-dependent demand and two-level trade credit. Chung and Ting [20] established a new economic production quantity (EPQ) inventory model for deteriorating items under two-level trade credit, in which the supplier offers the retailer a permissible delay period and simultaneously the retailer in turn provides a maximal trade credit period for his or her customers in a supply chain system composed of three stages. Chen et al. [21] built an appropriate EPQ model for deteriorating items in which the seller receives an up-stream full trade credit from his or her supplier while offers a down-stream partial trade credit to his or her credit-risk customers. Wu and Chan [22] established optimal lot-sizing policies for a retailer who sells a deteriorating item to credit-risk customers by offering partial trade credit to reduce his or her risk. Sarkar and Saren [23] introduced the deterioration function as a time dependent function for fixed lifetime products, and assumed that the supplier offers the retailer a full trade credit period but the retailer offers the customer a partial trade credit period. Shah [24] analyzed the retailer's decision for ordering and credit policies when a supplier offers his or her retailer either a cash discount or a fixed credit period if the order quantity is greater than or equal to that of regular order policy.

In Huang's work [14], if the customer purchases one item from the retailer at time t belonging to $[0, N]$, then the customer has a trade credit period $N - t$ and must make the payment at time N. Thus, the retailer allows a maximal trade credit period N for the customer to settle the account. In fact, the trade credit periods offered by the retailer to the customer are different. The customer's trade credit period N offered by the retailer in Huang's work [14] should mean the due day customer makes his or her payment to the retailer. In Teng's work [18], if the customer purchases one item from the retailer at time t belonging to $[0, T]$, then the customer has a trade credit period N and must make the payment at time $N + t$. So, the retailer allows each customer fixed trade period N. The viewpoint of Teng [18] can be used in any general business transactions and this is not restricted to business cards as it was hypothesized in Huang's work [14]. This paper proposes an EOQ model in a supply chain under two-level trade credit policy from the viewpoint of Teng [18].

In all above cited articles, based on the two-level trade credit policy, most of the inventory models are formulated based on the assumption that the supplier offers the retailer a full trade credit period and the retailer offers the customer a partial/full trade credit period. However, two-level trade credit policy when the supplier offers the retailer conditionally permissible delay in payments link to order quantity and then the retailer also provides his or her customer with a full trade credit period, a hypothesis that has not been taken into consideration.

In this paper, an EOQ model with up-stream partial trade credit and down-stream full trade credit is proposed. The main purpose of this paper is to amend the paper Huang [8,14], Chung et al. [25], and Chung and Ting [13] to make their model more relevant and so applicable to practice. Here, we are taking account of the following factors: (1) the supplier is willing to provide the retailer a fully permissible delay of some periods if the retailer's order is more than or equal to some predetermined quantities. On the other hand, if the retailers order quantity is less than the predetermined quantity, then the retailer must make a partial payment to the supplier; (2) the retailer's trade credit period (M) offered by the supplier is not shorter than the customer's trade credit period (N) offered by the retailer; (3) the retailer's selling price per unit is higher than his or her purchase unit cost; (4) the model is based on two-level trade credit policy from the viewpoint of Teng [18]; (5) this model is closely related to that of Huang [8], but the interest earned and interest payable have been calculated in a different and, according to our opinion, more properly way, etc. Under these conditions, we model the retailer's inventory system as a cost minimization problem. Some theorems are developed to determine a retailer's optimal ordering policies and numerical examples are given to illustrate some theorems. In addition, some managerial insights from the numerical examples are also concluded.

3. Problem Statement and Preliminaries.

3.1. Notation and assumption.

For simplicity, the notation and the assumption used through the paper are presented below.

3.1.1. *Notation.*

- D the annual demand rate
- s the ordering cost in dollars per order
- W the pre-determined quantity in units by the supplier at which the fully permissible delay in payments is granted
- c the purchasing cost in dollars per unit
- p the selling price in dollars per unit, which is greater than the unit purchase cost c
- h the unit holding cost in dollars per year excluding interest charge
- I_e the interest earned per dollar per year
- I_c the interest charged per dollar in stocks per year, and $I_c > I_e$
- M the length of the permissible delay in years offered by the supplier
- N the customer's length of permissible delay in years offered by the retailer
- α the percentage of the purchase amount that is granted permissible delay in payments by the supplier
- T the retailer's replenishment cycle time in years
- Q the retailer's order quantity in units
- T_W the time interval that W units are depleted to zero due to annual demand rate D, hence $T_W = W/D$
- $TC(T)$ the retailer's annual total relevant cost in dollars, which is a function of T
- T^* the retailer's optimal replenishment cycle time of $TC(T)$
- Q^* the retailer's optimal order quantity

3.1.2. *Assumptions.*

(1) Demand rate is known and constant. (2) Replenishments are instantaneous. (3) Shortages are not allowed. (4) Only one type of product is considered. (5) The time horizon is infinite. (6) While the account is not settled, the revenue is placed in an interest bank account in order to earn interests. (7) The retailer starts earning revenue from time N to time $(T + N)$, not from time 0 to time T.

(8) The retailer's trade credit period offered by the supplier (M) must be greater and equal than the customer's trade credit period offered by the retailer (N); i.e., $M \geq N$. When $M \geq N$, then the seller deposits the sales revenue into an interest bearing account. If $M \geq T + N$ (i.e., the permissible delay period is longer than the time at which the retailer receives the last payment from his or her customers), then the seller receives all revenue and pays off the entire purchase cost at the end of the permissible delay M. Otherwise (if $M \leq T + N$), the seller pays the supplier the sum of all units sold by $M - N$ and the collateral deposit received from N to M, keeps the profit for the use of the other activities, and starts paying for the interest charges on the items sold after $M - N$.

(9) If $Q \geq W$ (i.e., $T \geq T_W$), then fully delayed payment is permitted. Otherwise, the partially delayed payment is permitted. Hence, if $Q < W$ (i.e., $T < T_W$), then the retailer must take a loan (with the interest charged of I_c) to pay the supplier the partial payment of $(1 - \alpha) cQ$ when the order is filled at time 0. From the constant sales revenue pD, the

retailer pays off the loan $(1-\alpha)\,cQ$ at time $(1-\alpha)\,cT/p$. Set $t_0=(1-\alpha)\,cT/p$. This assumption constitutes the major difference of the proposed model from previous ones.

Notice that Huang [8] and Chung et al. [25] assume that, if $Q<W$, then the retailer must make a partial payment of $(1-\alpha)\,cQ$ when the order is filled at time 0, and the retailer will pay off the loan $(1-\alpha)\,cQ$ at time $(1-\alpha)\,T$, which is significantly different from the assumption in this paper.

Next, we are ready to build up the mathematical model for the retailer to obtain his or her optimal order quantity.

3.2. **Mathematical formulation.** For the derivation of the retailer's annual total relevant cost, the mathematical expressions of ordering cost, holding cost (excluding interest charges), the interest payable and the interest earned are required.

(a) Annual setup cost is s/T.

(b) Annual holding cost excluding interest charges is $hDT/2$.

Not that (1) if $T\geq T_W$, then the fully delayed payment is permitted, and (2) if the payment time of the partial payment at $(1-\alpha)\,cT/p$ is shorter or equal to the permissible delay M, then $T\leq T_0\equiv\frac{(M-N)p}{(1-\alpha)c}$, and vice versa. Note that $T_0\geq M-N$. Consequently, based on the values of $M-N$, T_W and T_0, there are three possible cases in terms of annual opportunity cost of the capital:

1) $T_0>M-N\geq T_W$, 2) $T_0\geq T_W>M-N$, and 3) $T_W>T_0>M-N$.

3.2.1. *Case 1.* $T_0>M-N\geq T_W$.

Sub-case 1.1: $T\geq M-N$.

Since $T_W\leq M-N\leq T$, the retailer receives the fully permissible delay as shown in Figure 1. The interest earned per cycle is the return rate I_e multiplied by the area of the triangle on the interval $[N,M]$ (i.e., $pI_eD(M-N)^2/2$). Therefore, the annual interest earned is $\frac{pI_eD(M-N)^2}{2T}$. Likewise, the interest charged is the interest rate I_c multiplied by the area of the triangle on the interval $[M,T+N]$ $\left(\frac{cI_cD(T+N-M)^2}{2}\right)$. Hence, the annual interest charged is $\frac{cI_cD(T+N-M)^2}{2T}$, and the annual opportunity cost of capital is $\frac{cI_cD(T+N-M)^2-pI_eD(M-N)^2}{2T}$.

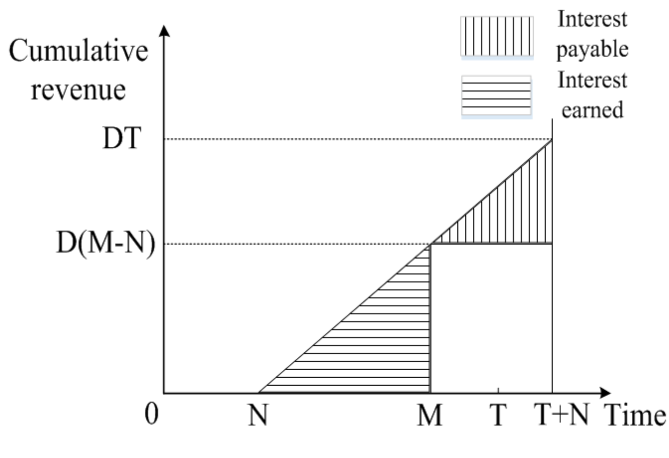

FIGURE 1. Total amount of interest earned and payable when $T\geq M-N$

Sub-case 1.2: $T_W\leq T\leq M-N$

Since $T_W\leq T$, the retailer receives the fully permissible delay as shown in Figure 2. There is no interest charge because of $T\leq M-N$. The interest earned per cycle is the return rate I_e multiplied by the area of the trapezoid on the interval $[N,M]$. Therefore,

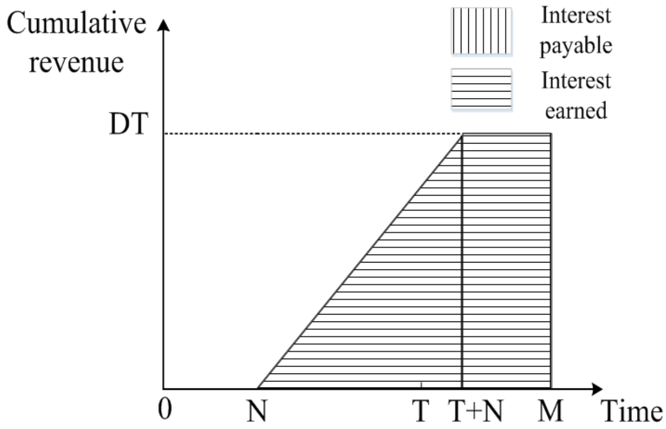

FIGURE 2. Total amount of interest earned and payable when $T_W \le T \le M - N$

the annual interest earned in this case is $pI_eD[T/2 + M - (T + N)]$, and the annual opportunity cost of capital is $-pI_eD[T/2 + M - (T + N)]$.

Sub-case 1.3: $0 < T < T_W$

Since $0 < T < T_W$, the retailer receives the partially permissible delay as shown in Figure 3. For the immediate payment, the interest charged per cycle is the interest rate I_c multiplied by the area of the rectangle on the interval $[0, N]$ and the triangle on the interval $\left[N, \frac{(1-\alpha)cT}{p} + N\right]$. Hence, the annual interest charged is $(1-\alpha)cI_cD\left[N + \frac{(1-\alpha)c}{2p}T\right]$.

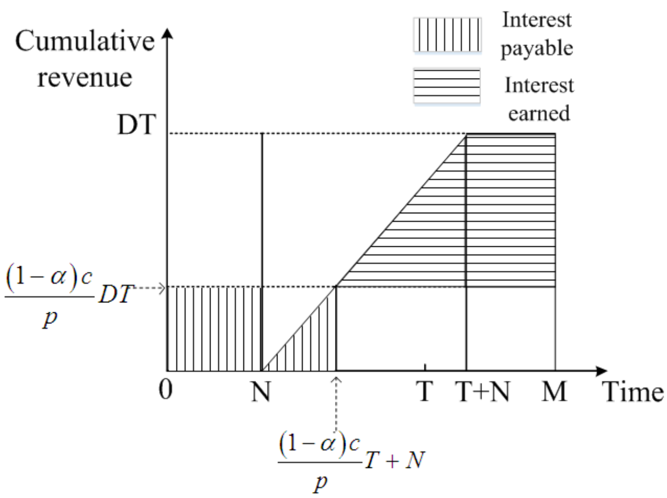

FIGURE 3. Total amount of interest earned and payable when $0 < T < T_W$

As to the delayed payment, there is no interest charged while the interest earned per cycle is the return rate I_e multiplied by the area of the trapezoid on the interval $\left[\frac{(1-\alpha)cT}{p} + N, N\right]$. Consequently, the annual interest earned is

$$\frac{pI_eDT\left[1 - \frac{(1-\alpha)c}{p}\right]^2}{2} + pI_eD\left[1 - \frac{(1-\alpha)c}{p}\right][M - (T + N)].$$

As a result, the annual opportunity cost of capital is

$$(1-\alpha)cI_cD\left[N + \frac{(1-\alpha)c}{2p}T\right] - \frac{I_eD\left[p - (1-\alpha)c\right]\left[T - \frac{(1-\alpha)c}{p}T - 2(T + N - M)\right]}{2}.$$

From the above results, we have the annual total relevant cost for the retailer in Case 1 is

$$TC(T) = \begin{cases} TC_1(T) & T \geq M - N \\ TC_2(T) & T_W \leq T \leq M - N \\ TC_3(T) & 0 < T < T_W \end{cases} \quad (1)$$

where

$$TC_1(T) = \frac{s}{T} + \frac{hDT}{2} + \frac{cI_cD(T+N-M)^2}{2T} - \frac{pI_eD(M-N)^2}{2T} \quad (2)$$

$$TC_2(T) = \frac{s}{T} + \frac{hDT}{2} - pI_eD\left(M - N - \frac{T}{2}\right) \quad (3)$$

$$\begin{aligned} TC_3(T) = {} & \frac{s}{T} + \frac{hDT}{2} + (1-\alpha)cI_cD\left[N + \frac{(1-\alpha)c}{2p}T\right] \\ & - \frac{I_eD[p-(1-\alpha)c]\left[T - \frac{(1-\alpha)c}{p}T - 2(T+N-M)\right]}{2} \end{aligned} \quad (4)$$

3.2.2. *Case 2.* $T_0 \geq T_W > M - N$.

Similar to Case 1 of $T_0 > M - N \geq T_W$, we know that the annual ordering cost is s/T and the annual holding cost excluding interest charges is $hDT/2$. Likewise, the annual opportunity cost of capital has three Sub-cases.

Sub-case 2.1: $T_W \leq T$

Similar to the previous Sub-case 1.1, both the interest charged and the interest earned here are the same as those in Sub-case 1.1.

Sub-case 2.2: $M - N \leq T < T_W$

Since $M - N \leq T < T_W$, the retailer receives the partially permissible delay as shown in Figure 4. For the immediate payment, the annual interest charged is the same as $(1-\alpha)cI_cD\left[N + \frac{(1-\alpha)c}{2p}T\right]$ in Sub-case 1.3. As to the delayed payment, the interest charged per cycle is the interest rate I_c multiplied by the area of the triangle on the interval $[M, T+N]$. Hence, the annual interest charged is $\frac{cI_cD(T+N-M)^2}{2T}$. The interest earned per cycle is the return rate I_e multiplied by the area of the triangle on the interval

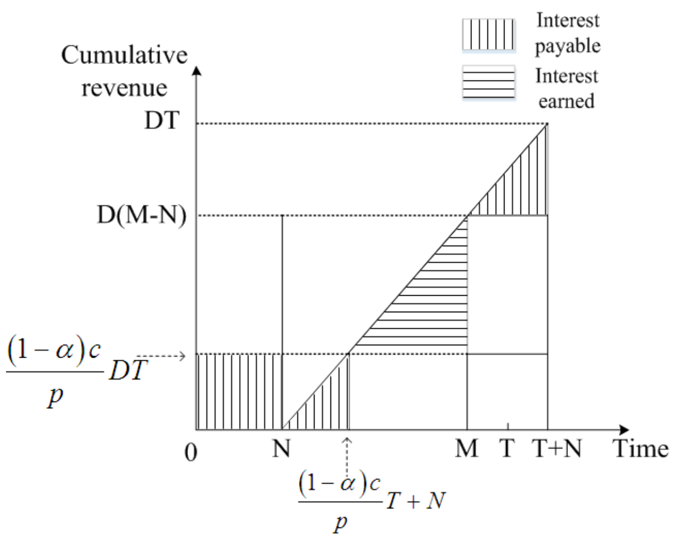

FIGURE 4. Total amount of interest earned and payable when $M - N \leq T < T_W$

$\left[\frac{(1-\alpha)cT}{p}+N,M\right]$. So, the annual interest earned is $\frac{pI_eD}{2T}\left\{M-\left(\frac{(1-\alpha)c}{p}T+N\right)\right\}^2$, and the annual opportunity cost of capital is

$$(1-\alpha)cI_cD\left[N+\frac{(1-\alpha)c}{2p}T\right]+\frac{cI_cD(T+N-M)^2}{2T}-\frac{pI_eD}{2T}\left\{M-\left(\frac{(1-\alpha)c}{p}T+N\right)\right\}^2.$$

Sub-case 2.3: $0<T\le M-N$

Similar to the previous Sub-case 1.3, both the interest charged and the interest earned here are the same as those in Sub-case 1.3.

From the above results, we have the annual total relevant cost for the retailer in Case 2 is

$$TC(T)=\begin{cases}TC_1(T) & T\ge T_W\\ TC_4(T) & M-N\le T<T_W\\ TC_3(T) & 0<T\le M-N\end{cases}\tag{5}$$

where

$$\begin{aligned}TC_4(T)=&\frac{s}{T}+\frac{hDT}{2}+(1-\alpha)cI_cD\left[N+\frac{(1-\alpha)c}{2p}T\right]\\&+\frac{cI_cD(T+N-M)^2}{2T}-\frac{pI_eD}{2T}\left\{M-\left(\frac{(1-\alpha)c}{p}T+N\right)\right\}^2\end{aligned}\tag{6}$$

3.2.3. *Case 3.* $T_W>T_0>M-N$.

Similar to Case 1 of $T_0>M-N\ge T_W$, we know that the annual ordering cost is s/T and the annual holding cost excluding interest charges is $hDT/2$. The annual opportunity cost of capital has four Sub-cases.

Sub-case 3.1: $T\ge T_W$

Similar to the previous Sub-case 1.1, both the interest charged and the interest earned here are the same as those in Sub-case 1.1.

Sub-case 3.2: $T_0\le T<T_W$

Since $T_0\le T<T_W$, the retailer receives the partially permissible delay as shown in Figure 5. For the immediate payment, the annual interest charged is the same as

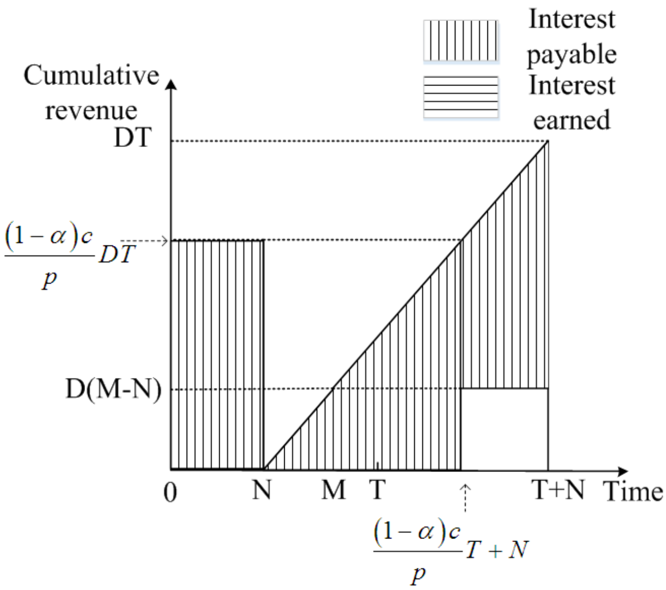

FIGURE 5. Total amount of interest earned and payable when $T_0\le T<T_W$

$(1-\alpha)cI_cD\left[N+\frac{(1-\alpha)c}{2p}T\right]$ in Sub-case 1.3. As to the delayed payment, there is no interest earned while the interest charged per cycle is the return rate I_c multiplied by the area of the trapezoid on the interval $\left[\frac{(1-\alpha)cT}{p}+N, T+N\right]$. Consequently, the annual interest charged is $cI_cDT\left[1-\frac{(1-\alpha)c}{p}\right]^2\Big/2+cI_cD\left[\frac{(1-\alpha)c}{p}T+N-M\right]\left[1-\frac{(1-\alpha)c}{p}\right]$. As a result, the annual opportunity cost of capital is

$$(1-\alpha)cI_cD\left[N+\frac{(1-\alpha)c}{2p}T\right]+\frac{cI_cDT[p-(1-\alpha)c]^2}{2p^2}+\frac{cI_cD[p-(1-\alpha)c][T(1-\alpha)cT+p\,(N-M)]}{p^2}.$$

Sub-case 3.3: $M-N\le T\le T_0$

Similar to the previous Sub-case 2.2, both the interest charged and the interest earned here are the same as those in Sub-case 2.2.

Sub-case 3.4: $0<T\le M-N$

Similar to the previous Sub-case 1.3, both the interest charged and the interest earned here are the same as those in Sub-case 1.3.

From the above results, we have the annual total relevant cost for the retailer in Case 3 is

$$TC\,(T)=\begin{cases} TC_1\,(T) & T\ge T_W \\ TC_5\,(T) & T_0\le T<T_W \\ TC_4\,(T) & M-N\le T\le T_0 \\ TC_3\,(T) & T\le M-N \end{cases} \tag{7}$$

where

$$TC_5=\frac{s}{T}+\frac{hDT}{2}+(1-\alpha)cI_cD\left[N+\frac{(1-\alpha)c}{2p}T\right]+\frac{cI_cDT[p-(1-\alpha)c]^2}{2p^2}+\frac{cI_cD[p-(1-\alpha)c][(1-\alpha)cT+p\,(N-M)]}{p^2} \tag{8}$$

4. Theoretical Results.

4.1. The convexity of the functions $TC_i\,(T)$ $(i=1,\dots,5)$.

For convenience, we treat the convexity properties of all of the functions $TC_i\,(T)$ $(i=1,\dots,5)$ defined on $T>0$.

$$TC_1'\,(T)=-\frac{2s+cI_cD\,(M-N)^2-pI_eD\,(M-N)^2}{2T^2}+\frac{D\,(h+cI_c)}{2} \tag{9}$$

$$TC_1''\,(T)=\frac{2s+cI_cD\,(M-N)^2-pI_eD\,(M-N)^2}{T^3} \tag{10}$$

$$TC_2'\,(T)=-\frac{s}{T^2}+\frac{D\,(h+pI_e)}{2} \tag{11}$$

$$TC_2''\,(T)=\frac{2S}{T^3}>0 \tag{12}$$

$$TC_3'\,(T)=-\frac{s}{T^2}+\frac{D\,(h+pI_e)}{2}+\frac{(1-\alpha)^2\,c^2D\,(I_c-I_e)}{2p} \tag{13}$$

$$TC_3''\,(T)=\frac{2s}{T^3}>0 \tag{14}$$

$$TC_4'(T) = -\frac{2s + cI_cD(M-N)^2 - pI_eD(M-N)^2}{2T^2} + \frac{D(h+cI_c)}{2} + \frac{(1-\alpha)^2 c^2 D(I_c - I_e)}{2p} \tag{15}$$

$$TC_4''(T) = \frac{2s + cI_cD(N-M)^2 - pI_eD(M-N)^2}{T^3} \tag{16}$$

$$TC_5'(T) = -\frac{s}{T^2} + \frac{D(h+cI_c)}{2} + \frac{(1-\alpha)^2 c^2 DI_c(p-c)}{2p^2} \tag{17}$$

$$TC_5''(T) = \frac{2s}{T^3} > 0 \tag{18}$$

Equations (12), (14) and (18) imply that $TC_2(T)$, $TC_3(T)$ and $TC_5(T)$ are convex on $T > 0$. However, $TC_1(T)$ and $TC_4(T)$ are convex on $T > 0$, if $2s + cI_cD(M-N)^2 - pI_eD(M-N)^2 > 0$.

Now, we let $\beta = 2s + cI_cD(N-M)^2 - pI_eD(M-N)^2$. Then we can obtain following results.

Lemma 4.1.

Case 1. $T_0 > M - N \geq T_W$

(A) If $\beta \leq 0$*, then* $TC(T)$ *is convex on* $(0, M-N]$*, and concave on* $[M-N, +\infty)$.

(B) If $\beta > 0$*, then* $TC(T)$ *is convex on* $(0, +\infty)$.

Case 2. $T_0 \geq T_w > M - N$

(A) If $\beta \leq 0$*, then* $TC(T)$ *is convex on* $(0, M-N]$*, and concave on* $[M-N, +\infty)$.

(B) If $\beta > 0$*, then* $TC(T)$ *is convex on* $(0, +\infty)$.

Case 3. $T_W > T_0 > M - N$

(A) If $\beta \leq 0$*, then* $TC(T)$ *is convex on* $(0, M-N]$ *and* $[T_0, T_W)$*, and concave on* $[M-N, T_0)$ *and* $[T_W, +\infty)$.

(B) If $\beta > 0$*, then* $TC(T)$ *is convex on* $(0, +\infty)$.

Let $TC_i'(T_i^*) = 0$ for all $i = 1, \ldots, 5$. We can obtain

$$T_1^* = \sqrt{\frac{2s + cI_cD(M-N)^2 - pI_eD(M-N)^2}{D(h+cI_c)}} \quad \text{If } \beta > 0 \tag{19}$$

$$T_2^* = \sqrt{\frac{2s}{hD + pI_eD}} \tag{20}$$

$$T_3^* = \sqrt{\frac{2ps}{pD(h+pI_e) + (1-\alpha)^2 c^2 D(I_c - I_e)}} \tag{21}$$

$$T_4^* = \sqrt{\frac{[2s + cI_cD(M-N)^2 - pI_eD(M-N)^2]\,p}{pD(h+cI_c) + (1-\alpha)^2c^2D(I_c - I_e)}} \quad \text{If } \beta > 0 \tag{22}$$

$$T_5^* = \sqrt{\frac{2ps}{pD(h+cI_c) + (1-\alpha)^2 c^2 DI_c\left(1 - {}^{c}\!/_{p}\right)}} \tag{23}$$

If T_i^* exists, then we find (for $i = 1, \ldots, 5$) that

$$TC_i'(T) \begin{cases} < 0 & (0 < T < T_i^*) \\ = 0 & (T = T_i^*) \\ > 0 & (T > T_i^*) \end{cases} \tag{24}$$

Therefore, it follows that (for $i = 1, \ldots, 5$) the function $TC_i(T)$ is decreasing on $(0, T_i^*]$ and increasing on $[T_i^*, +\infty)$.

4.2. **The meanings of the symbols $\boldsymbol{\Delta_i}$ $\boldsymbol{(i = 1, \ldots, 5)}$.** In this section, we introduce and interpret the symbols Δ_i $(i = 1, \ldots, 5)$ which are defined below.

Case 1. $T_0 > M - N \geq T_W$

Since $TC_1(M-N) = TC_2(M-N)$, $TC_2(T_W) \leq TC_3(T_W)$, the function $TC(T)$ is continuous except at $T = T_W$.

Furthermore, we have $TC_3(T) \geq TC_2(T)$

$$TC_1'(M-N) = TC_2'(M-N) = \frac{-2s + D(h + pI_e)(M-N)^2}{2(M-N)^2},$$

$$TC_2'(T_W) = \frac{-2s + DT_W^2(h + pI_e)}{2T_W^2}$$

$$TC_3'(T_W) = \frac{-2s + T_W^2\left[D(h + pI_e) + \frac{(1-\alpha)^2 c^2 D(I_c - I_e)}{p}\right]}{2T_W^2}.$$

Thus, upon setting

$$\Delta_1 = -2s + D(h + pI_e)(M-N)^2 \tag{25}$$

$$\Delta_2 = -2s + D(h + pI_e)T_W^2 \tag{26}$$

$$\Delta_3 = -2s + \left[D(h + pI_e) + \frac{(1-\alpha)^2 c^2 D(I_c - I_e)}{p}\right]T_W^2 \tag{27}$$

Equations (25) to (27) reveal the fact that $\Delta_1 \geq \Delta_2$, $\Delta_3 \geq \Delta_2$.

Case 2. $T_0 \geq T_W > M - N$

Since $TC_1(T_W) \leq TC_4(T_W)$, $TC_4(M-N) = TC_3(M-N)$, the function $TC(T)$ is continuous except at $T = T_W$.

Furthermore, we have $TC_4(T) \geq TC_1(T)$

$$TC_1'(T_W) = \frac{-2s - cI_cD(M-N)^2 + pI_eD(M-N)^2 + DT_W^2(h + cI_c)}{2T_W^2}$$

$$TC_4'(T_W) = \frac{-2s - cI_cD(M-N)^2 + pI_eD(M-N)^2 + DT_W^2\left[(h + cI_c) + \frac{(1-\alpha)^2 c^2 (I_c - I_e)}{p}\right]}{2T_W^2}$$

$$TC_4'(M-N) = TC_3'(M-N) = \frac{-2s + D(M-N)^2\left[h + pI_e + \frac{(1-\alpha)^2 c^2 (I_c - I_e)}{p}\right]}{2(M-N)^2}$$

Thus, upon setting

$$\Delta_4 = -2s + pI_eD(M-N)^2 - cI_cD(M-N)^2 + DT_W^2(h + cI_c) \tag{28}$$

$$\begin{aligned}\Delta_5 = &-2s + pI_eD(M-N)^2 - cI_cD(M-N)^2 \\ &+ DT_W^2\left[h + cI_c + \frac{(1-\alpha)^2 c^2 (I_c - I_e)}{p}\right]\end{aligned} \tag{29}$$

$$\Delta_6 = -2s + D(M-N)^2\left[h + pI_e + \frac{(1-\alpha)^2 c^2 (I_c - I_e)}{p}\right] \tag{30}$$

Equations (28) to (30) reveal the fact that $\Delta_5 \geq \Delta_4$, $\Delta_5 \geq \Delta_6$.

Case 3. $T_W > T_0 > M - N$

Since $TC_1(T_W) \le TC_5(T_W)$, $TC_5(T_0) = TC_4(T_0)$, $TC_4(M-N) = TC_3(M-N)$, the function $TC(T)$ is continuous except at $T = T_W$.

Furthermore, we have $TC_5(T) \ge TC_1(T)$

$$TC_1'(T_W) = \frac{-2s + pI_eD(M-N)^2 - cI_cD(M-N)^2 + DT_W^2(h + cI_c)}{2T_W^2}$$

$$TC_5'(T_W) = \frac{-2s + DT_W^2\left[h + cI_c + (1-\alpha)^2 c^2 I_c \left(1 - \frac{c}{p}\right)\Big/p\right]}{2T_W^2}$$

$$TC_5'(T_0) = TC_4'(T_0) = \frac{-2s + D\left[h + cI_c + (1-\alpha)^2 c^2 I_c \left(1 - \frac{c}{p}\right)\Big/p\right]\left[\frac{(M-N)p}{(1-\alpha)c}\right]^2}{2\left[\frac{(M-N)p}{(1-\alpha)c}\right]^2}$$

$$TC_4'(M-N) = TC_3'(M-N) = \frac{-2s + D\left[h + pI_e + \frac{(1-\alpha)^2c^2(I_c - I_e)}{p}\right](M-N)^2}{2(M-N)^2}.$$

Thus, upon setting

$$\Delta_7 = -2s + DT_W^2\left[h + cI_c + (1-\alpha)^2 c^2 I_c \left(1 - \frac{c}{p}\right)\Big/p\right] \tag{31}$$

$$\Delta_8 = -2s + D\left[h + cI_c + (1-\alpha)^2 c^2 I_c \left(1 - \frac{c}{p}\right)\Big/p\right]\left[\frac{(M-N)p}{(1-\alpha)c}\right]^2 \tag{32}$$

Equations (28), (30), (31) and (32) reveal the fact that $\Delta_7 \ge \Delta_4$, $\Delta_7 > \Delta_8 > \Delta_6$.

5. **Decision Rules of the Optimal Cycle Time T^*.** In this section, we develop efficient decision rules to find the optimal cycle time for the retailer.

Case 1. $T_0 > M - N \ge T_W$

Suppose that $\beta \le 0$

When $\beta \le 0$, we can find $TC_1(T)$ is increasing on $(M - N, +\infty)$ from Equation (9). In addition, we can obtain $\Delta_1 > 0$ from Equation (25). According to Equation (24), Then, we have the following result to determine the optimal cycle time T^*.

Theorem 5.1.

(A) If $\Delta_2 > 0$, $\Delta_3 > 0$, then $TC(T^*) = \min\{TC_3(T_3^*), TC_2(T_W)\}$, and $T^* = T_3^*$ or T_W.

(B) If $\Delta_2 \le 0$, $\Delta_3 > 0$, then $TC(T^*) = TC_2(T_2^*)$ and $T^* = T_2^*$.

(C) If $\Delta_2 \le 0$, $\Delta_3 \le 0$, then $TC(T^*) = TC_2(T_2^*)$ and $T^* = T_2^*$.

Suppose that $\beta > 0$

When $\beta > 0$, all T_i^* $(i = 1, 2, 3)$ are well-defined. According to Equation (24), we have the following result to determine the optimal cycle time T^*.

Theorem 5.2.

(A) If $\Delta_1 > 0$, $\Delta_2 > 0$, $\Delta_3 > 0$, then $TC(T^*) = \min\{TC_3(T_3^*), TC_2(T_W)\}$ and $T^* = T_3^*$ or T_W.

(B) If $\Delta_1 > 0$, $\Delta_2 \le 0$, $\Delta_3 > 0$, then $TC(T^*) = TC_2(T_2^*)$ and $T^* = T_2^*$.

(C) If $\Delta_1 > 0$, $\Delta_2 \le 0$, $\Delta_3 \le 0$, then $TC(T^*) = TC_2(T_2^*)$ and $T^* = T_2^*$.

(D) If $\Delta_1 \le 0$, $\Delta_2 \le 0$, $\Delta_3 > 0$, then $TC(T^*) = TC_1(T_1^*)$ and $T^* = T_1^*$.

(E) If $\Delta_1 \le 0$, $\Delta_2 \le 0$, $\Delta_3 \le 0$, then $TC(T^*) = TC_1(T_1^*)$ and $T^* = T_1^*$.

Case 2. $T_0 \ge T_W > M - N$

Suppose that $\beta \le 0$

When $\beta \leq 0$, we can find $TC_1(T)$ is increasing on $[T_W, +\infty)$ from Equation (9). TC_4 is increasing on $[M - N, T_W)$ from Equation (15). In addition, we can obtain $\Delta_5 \geq \Delta_4 > 0$, $\Delta_5 \geq \Delta_6 > 0$ from Equations (28) to (30). According to Equation (24), we have the following result to determine the optimal cycle time T^*.

Theorem 5.3. *If $\Delta_5 \geq \Delta_4 > 0$, $\Delta_5 \geq \Delta_6 > 0$, then $TC(T^*) = \min\{TC_3(T_3^*), TC_1(T_W)\}$ and $T^* = T_3^*$ or T_W.*

Suppose that $\beta > 0$

When $\beta > 0$, all T_i^* $(i = 1, 3, 4)$ are well-defined. According to Equation (24), we have the following result to determine the optimal cycle time T^*.

Theorem 5.4.

(A) If $\Delta_4 > 0$, $\Delta_5 > 0$, $\Delta_6 \geq 0$, then $TC(T^*) = \min\{TC_3(T_3^*), TC_1(T_W)\}$ and $T^* = T_3^*$ or T_W.

(B) If $\Delta_4 > 0$, $\Delta_5 > 0$, $\Delta_6 < 0$, then $TC(T^*) = \min\{TC_4(T_4^*), TC_1(T_W)\}$ and $T^* = T_4^*$ or T_W.

(C) If $\Delta_4 \leq 0$, $\Delta_5 > 0$, $\Delta_6 \geq 0$, then $TC(T^*) = \min\{TC_3(T_3^*), TC_1(T_1^*)\}$ and $T^* = T_3^*$ or T_1^*.

(D) If $\Delta_4 \leq 0$, $\Delta_5 > 0$, $\Delta_6 < 0$, then $TC(T^*) = TC_1(T_1^*)$ and $T^* = T_1^*$.

(E) If $\Delta_4 \leq 0$, $\Delta_5 \leq 0$, $\Delta_6 < 0$, then $TC(T^*) = TC_1(T_1^*)$ and $T^* = T_1^*$.

Case 3. $T_W > T_0 > M - N$

Suppose that $\beta \leq 0$

When $\beta \leq 0$, we can find $TC_1(T)$ is increasing on $[T_W, +\infty)$ from Equation (9). TC_4 is increasing on $[M - N, T_0]$ from Equation (15). In addition, we can obtain $\Delta_7 \geq \Delta_4 > 0$, $\Delta_7 > \Delta_8 > \Delta_6 > 0$ from Equations (28), (30), (31) and (32). According to Equation (24), we have the following result to determine the optimal cycle time T^*.

Theorem 5.5. *If $\Delta_7 \geq \Delta_4 > 0$, $\Delta_7 > \Delta_8 > \Delta_6 > 0$, then $TC(T^*) = \min\{TC_3(T_3^*), TC_1(T_W)\}$ and $T^* = T_3^*$ or T_W.*

Suppose that $\beta > 0$

When $\beta > 0$, all T_i^* $(i = 1, 3, 4, 5)$ are well-defined. According to Equation (24), we have the following result to determine the optimal cycle time T^*.

Theorem 5.6.

(A) If $\Delta_4 > 0$, $\Delta_6 \geq 0$, $\Delta_7 > 0$, $\Delta_8 \geq 0$, then $TC(T^*) = \min\{TC_3(T_3^*), TC_1(T_W)\}$ and $T^* = T_3^*$ or T_W.

(B) If $\Delta_4 > 0$, $\Delta_6 < 0$, $\Delta_7 > 0$, $\Delta_8 \geq 0$, then $TC(T^*) = \min\{TC_4(T_4^*), TC_1(T_W)\}$ and $T^* = T_4^*$ or T_W.

(C) If $\Delta_4 > 0$, $\Delta_6 < 0$, $\Delta_7 > 0$, $\Delta_8 < 0$, then $TC(T^*) = \min\{TC_5(T_5^*), TC_1(T_W)\}$ and $T^* = T_5^*$ or T_W.

(D) If $\Delta_4 \leq 0$, $\Delta_6 \geq 0$, $\Delta_7 > 0$, $\Delta_8 \geq 0$, then $TC(T^*) = \min\{TC_3(T_3^*), TC_1(T_W)\}$ and $T^* = T_3^*$ or T_W.

(E) If $\Delta_4 \leq 0$, $\Delta_6 < 0$, $\Delta_7 > 0$, $\Delta_8 \geq 0$, then $TC(T^*) = TC_1(T_1^*)$ and $T^* = T_1^*$.

(F) If $\Delta_4 \leq 0$, $\Delta_6 < 0$, $\Delta_7 > 0$, $\Delta_8 < 0$, then $TC(T^*) = TC_1(T_1^*)$ and $T^* = T_1^*$.

(G) If $\Delta_4 \leq 0$, $\Delta_6 < 0$, $\Delta_7 \leq 0$, $\Delta_8 < 0$, then $TC(T^*) = TC_1(T_1^*)$ and $T^* = T_1^*$.

6. **Numerical Examples.** In this section, we provide some numerical examples to illustrate several distinct theoretical results as well as to gain some managerial insights.

Example 6.1. *Given D = 3800unit/year, h = \$0.8/unit/year, W = 280units, c = \$10/unit, I_e = \$0.12/\$/year and M = 0.1year, we obtain the optimal cycle time and the* *optimal order quantity for different parameters of* $p(11, 12, 12.5, 13, 14)$*,* $\alpha(0.1, 0.3, 0.5, 0.7, 0.9)$*,* $I_c(0.13, 0.14, 0.15, 0.16, 0.17)$*,* $s(20, 25, 30, 35, 40)$ *and* $N(0.01, 0.03, 0.05)$ *as shown in Tables 1-5.*

Table 1 reveals that all the values of T^*, Q^* and $TC(T^*)$ decrease as the selling price per unit p increases. And retailers will obtain more benefits by increasing sales price to reduce holding costs and interest charged cost, so as to reduce the replenishment cycle time. In other words, selling price per unit p has negative effects on the length of replenishment cycle, order quantity and the annual total relevant cost.

Table 2 illustrates that the retailer's optimal replenishment cycle time T^* and the annual total relevant cost $TC(T^*)$ will remain unchanged when the fraction of the delay payments permitted by the supplier per order α varies. Therefore, for a fixed sales price of goods, if retailers have determined the downstream customers deferred payment deadline, retailers can make the optimal order strategy. In other words, the retailers choose the

TABLE 1. Optimal solutions under different p

	p	T^*	Q^*	$TC(T^*)$
$s = 25$, $I_c = 0.15$, $N = 0.03$, $\alpha = 0.1$	11	$T_1^* = 0.07813$	296.894	$TC_1 = 283.856$
	12	$T_1^* = 0.07647$	290.611	$TC_1 = 269.404$
	12.5	$T_1^* = 0.07563$	287.417	$TC_1 = 262.059$
	13	$T_1^* = 0.07478$	284.188	$TC_1 = 254.632$
	14	$T_W = 0.07368$	280	$TC_1 = 239.542$

TABLE 2. Optimal solutions under different α

	α	T^*	Q^*	$TC(T^*)$
$s = 25$, $I_c = 0.15$, $N = 0.01$, $p = 12$	0.1	$T_2^* = 0.07664$	291.241	$TC_2 = 159.9$
	0.3	$T_2^* = 0.07664$	291.241	$TC_2 = 159.9$
	0.5	$T_2^* = 0.07664$	291.241	$TC_2 = 159.9$
	0.7	$T_2^* = 0.07664$	291.241	$TC_2 = 159.9$
	0.9	$T_2^* = 0.07664$	291.241	$TC_2 = 159.9$

TABLE 3. Optimal solutions under different I_c

	I_c	T^*	Q^*	$TC(T^*)$
$s = 25$, $p = 12$, $N = 0.03$, $\alpha = 0.3$	0.13	$T_1^* = 0.07706$	292.846	$TC_1 = 269.177$
	0.14	$T_1^* = 0.07675$	291.679	$TC_1 = 269.295$
	0.15	$T_1^* = 0.07647$	290.611	$TC_1 = 269.404$
	0.16	$T_1^* = 0.07621$	289.626	$TC_1 = 269.504$
	0.17	$T_1^* = 0.07597$	288.718	$TC_1 = 269.597$

TABLE 4. Optimal solutions under different s

	s	T^*	Q^*	$TC(T^*)$
$p = 12$, $I_c = 0.15$, $N = 0.05$, $\alpha = 0.1$	20	$T_W = 0.07368$	280	$TC_1 = 312.296$
	25	$T_1^* = 0.07606$	289.051	$TC_1 = 379.817$
	30	$T_1^* = 0.08324$	316.341	$TC_1 = 442.586$
	35	$T_1^* = 0.08985$	341.458	$TC_1 = 500.354$
	40	$T_1^* = 0.09601$	364.85	$TC_1 = 554.155$

TABLE 5. Optimal solutions under different N

	N	T^*	Q^*	$TC(T^*)$
$s = 25$, $p = 13$, $I_c = 0.15$, $\alpha = 0.5$	0.01	$T_2^* = 0.07466$	283.74	$TC_2 = 136.106$
	0.03	$T_1^* = 0.07478$	284.188	$TC_1 = 254.632$
	0.05	$T_1^* = 0.0752$	285.774	$TC_1 = 372.281$
	0.07	$T_1^* = 0.07548$	286.826	$TC_1 = 488.701$
	0.09	$T_1^* = 0.07561$	287.351	$TC_1 = 603.909$

same order of frequency, and each time order the same amount of goods, which can make the annual total cost remain unchanged.

Table 3 reveals that a higher value of I_c implies lower values of the optimal production cycle time T^*, but a higher value of the retailer's optimal annual total relevant cost in dollars $TC(T^*)$.

Table 4 suggests that a higher value of s implies higher values of the optimal production cycle time T^* and the retailer's optimal annual total relevant cost in dollars $TC(T^*)$.

Table 5 reveals that a higher value of N implies higher values of the optimal production cycle time T^* and the retailer's optimal annual total relevant cost in dollars $TC(T^*)$. Obviously, when retailers extend the downstream customer's length of permissible delay, customers can put the delay payment in the bank to obtain interest earned or do a short-term investment within the period of permissible delay. As a result, retailers lose this income, so the annual total relevant cost increases.

7. **Conclusions.**

The supplier offers the permissible delay in payments to the retailer in order to stimulate the demand. Hence, the assumption in previously published results that the fully permissible delay in payments is permitted under a sufficient quantity is practical. On the other hand, the permissible delay in payments will not be permitted when the order quantity is smaller than a predetermined quantity obviously. Such is an extreme case. In this paper, the proposed model allows the supplier to offer an alternative policy, i.e., partially permissible delay in payments, when the retailer's order quantity is not large enough to get the fully permissible delay in payments, and simultaneously the retailer in turn provides a trade credit period (i.e., a down-stream trade credit) for his or her customers. We then have extended Huang's model [8] to two-level trade credit policy, but the interest earned and interest payable have been calculated in a different way, according to our opinion, more properly way, etc. In addition, we establish six effective and easy-to-use theorems to help the retailer to find the optimal replenishment policy. The part of solution procedure involves mathematical analytic tools and techniques in order to solve the inventory problem to get the optimal solution. Finally, some numerical examples are provided to illustrate all the theorems, and to obtain the some managerial insights: (1) a higher value of the interest charged per dollar in stocks per year brings about a lower order quantity and higher annual total relevant costs; (2) a higher value of customer's length of permissible delay and the ordering cost per order bring about a larger order quantity and larger annual total relevant costs.

For future research, the study presented in this paper can be extended in several ways. For instance, we may consider increasing demand, ramp-type demand or stock-dependent demand. Also, we can generalize the model to allow for shortages and deteriorating items. Finally, we may consider non-cooperative and cooperative solutions to this supply chain supplier-retailer-buyer model, such as Nash solution, Stackelberg solution, Pareto solution, and integrated solution.

Acknowledgements. The research has been supported by the National Natural Science Foundation of China (71301079). The authors greatly appreciate the editor and the anonymous referees for their insightful comments and suggestions that have significantly improved the paper.

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