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A NEW ROBUST INTERNAL MODEL CONTROLLER AND ITS APPLICATIONS IN PMSM DRIVE

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Abstract. *In this paper, using H_∞ control theory, an H_∞ feedback controller is proposed to improve the robustness and servo precision of the internal model control system with external disturbance. Firstly, the nominal model of position plant is established as a part of closed-loop system from the physical laws of practical process. Moreover, based on internal model control method, an H_∞ feedback position controller is designed for the nominal model of position plant to guarantee the stability of the system, which can improve the control performance of internal model controller extremely. Finally, simulation results and the analysis of mechanism on robustness and disturbance rejection both demonstrate that compared with conventional internal model controller, the designed control system has better load disturbance rejection capability and parameter perturbation resisting capability.*

Keywords: Permanent magnet synchronous motor, H_∞ control, Internal model controller, Disturbance rejection, Parameter perturbation

1. **Introduction.** The internal model control (IMC) is a new advanced control strategy based on its process mathematical model, which was introduced by Garcia and Morari [1]. The internal model controller has many advantages including the simple structure and only one parameter to be tuned. Due to the simple design principle and convenient parameter tuning, IMC receives more and more attention [2-4]. From papers above, we know that IMC is based on the nominal model of the system plant. Actually, it is almost impossible to get the accurate model of the system plant, because the system plant is complex and the parameters of the plant can vary slowly. Therefore, the robustness of the IMC against the model uncertainties will get worse. For this reason, some researchers have adopted adaptive control [5-8] and neural control [9-11] design techniques to improve the accuracy of modeling for IMC. In [5], a fuzzy adaptive internal model controller (FAIMC) for open-loop stable plants is presented. [9] proposes a design procedure of neural internal model control systems for stable processes with delay. Those design methods are based on pattern recognition and need mass of the measurement data to establish the model of the plant for IMC; thus it may cause the uncertainty and also make the design difficult and

complex to realize. And some researchers combine robust control and IMC to improve the robustness of IMC [12-16]. In [12], it presents a method to estimate the H_2 optimal and a robust feedback controller by means of subspace model identification using the internal model control approach. In [13], it provides a low order robust stabilizer for the tracking rejection problem based on the internal model principle in the time varying setting. In [14], considering the robust output regulation and the internal model principle for infinite-dimensional linear systems, reduced order internal models in robust output regulation are presented. In [15], D. Ha et al. proposed an internal model principle (IMP)-based robust optimal nonlinear control with observer for position tracking of surface-mounted permanent magnet synchronous motor servo systems. In [16], a new control method based on a combination of robust control and internal model control has been proposed. However, in [12-15], the control structure and design are complicated and it restricts the application of the controller in real system. In [16], the robust controller and internal model controller are applied in position loop and velocity loop respectively. In this paper, the robust controller and internal model controller are applied to position loop simultaneously to improve the control performance of position loop.

Permanent magnet synchronous motor has been widely used in high-precision servo system [17,18]. For high-precision position servo system, many control methods are applied in PMSM system; in [19,20], a robust controller based on mixed sensitivity is presented to improve the speed control performance of the PMSM servo system. In [21,22], a sliding mode controller for PMSM servo system is introduced. In [23,24], based on PMSM dynamic and nonlinear load characteristics, a neural network controller for PMSM servo system is presented. The methods discussed above enrich the PMSM control theory and improve the performance of PMSM system from different aspects, but the robustness of sliding mode controller for PMSM servo system is not satisfied and the neural network controller is based on pattern recognition, which needs mass of data to practice self-learning. The internal model controller has the simple structure and convenient parameter tuning, but relies on the accurate model of the plant. Whether we could use robust control method to design the H_∞ feedback controller, which could compensate the influence of modeling error between the accurate model and the nominal model of the plant for PMSM servo system, motivates us to investigate this study.

In this paper, based on H_∞ control theory and internal model theory, an H_∞ feedback controller is combined with the internal model controller, which can improve the tracking performance of the PMSM system. The H_∞ feedback controller can compensate the modeling error between nominal model and the practical model of PMSM; thus the internal model controller which is designed based on the nominal model of PMSM can have better control performance. And the stability analysis proves that the internal model control system is stable. Simulation results and the analysis of mechanism on robustness and disturbance rejection indicate that the designed controller has satisfactory disturbance cancellation capability and robust stability.

The paper is organized into six sections including the introduction. Section 2 presents the model of a PMSM. Section 3 presents the design of the new internal model control system. Section 4 presents the application of the new controller in PMSM drive. Simulations are presented in Section 5. Section 6 summarizes the paper.

2. **Mathematics Model of a PMSM.** In *d*-*q* coordinates, the analytical model of PMSM is showed in Figure 1. The three-phase winding at stator is transformed to *d*-axis and *q*-axis two-phase winding which is rotating with rotor synchronously.

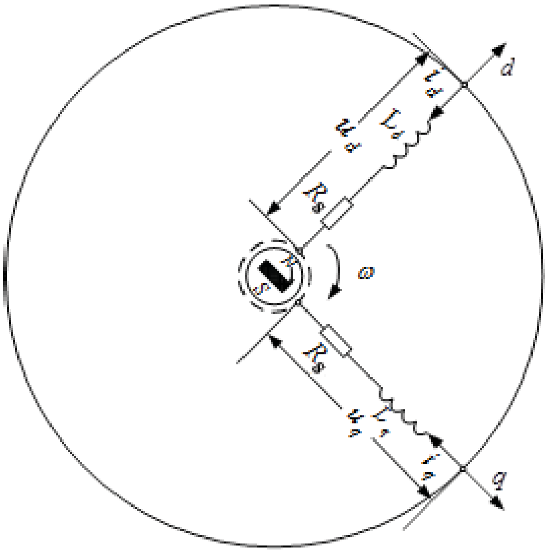

FIGURE 1. The analytical model of PMSM

From Figure 1, the model of the surface mounted PMSM can be described as [25],

$$\begin{cases} u_d = L_d \frac{di_d}{dt} + R_s i_d - \omega L_q i_q \\ u_q = L_q \frac{di_q}{dt} + R_s i_q + \omega L_d i_d + \omega \Psi_f \\ T_e = p_n \Psi_f i_q = K_T i_q \end{cases} \tag{1}$$

where u_d, u_q are the d-axis and q-axis voltage respectively. R_s is the stator resistance. i_d, i_q are the d-axis and q-axis current respectively. ω is the electrical rotor speed. L_d, L_q are the equivalent inductance at d-axis and q-axis respectively. Ψ_f is the effective flux of the magnet. p_n is the number of pole pairs. K_T is the thrust coefficient.

The mechanical equation of the PMSM drive is

$$\begin{cases} T_e - T_l = J \frac{d\omega_m(t)}{dt} + B\omega_m(t) \\ \theta_m = \int \omega_m(t)dt \end{cases} \tag{2}$$

where T_e is the electro-mechanical torque, and T_l is the external load. J is the inertia, B is the viscous coefficient, $\omega_m(t)$ is mechanical rotor speed, and θ_m is mechanical rotor position of the PMSM.

In Section 3, we will discuss the design approach of the new IMC system. In Section 3.1, the system structure of the new internal model control system is proposed. In Section 3.2, the design of the controller for the control system is presented.

3. Design of the New Internal Model Control System.

3.1. **System structure.** The proposed structure of control system is shown in Figure 2. $G_p(s)$ is the practical process, and its nominal model is $\widetilde{G}_p(s)$ which will be used in internal model controller of position loop. $G_C(s)$ is the equivalent form of internal model controller which is shown as Figure 3. The practical process is compensated by H_∞ feedback controller, which is of great help to improve the control performance. The design of equivalent internal model controller $G_C(s)$ and H_∞ feedback controller will be discussed in Section 3.2 and Section 4.2 respectively.

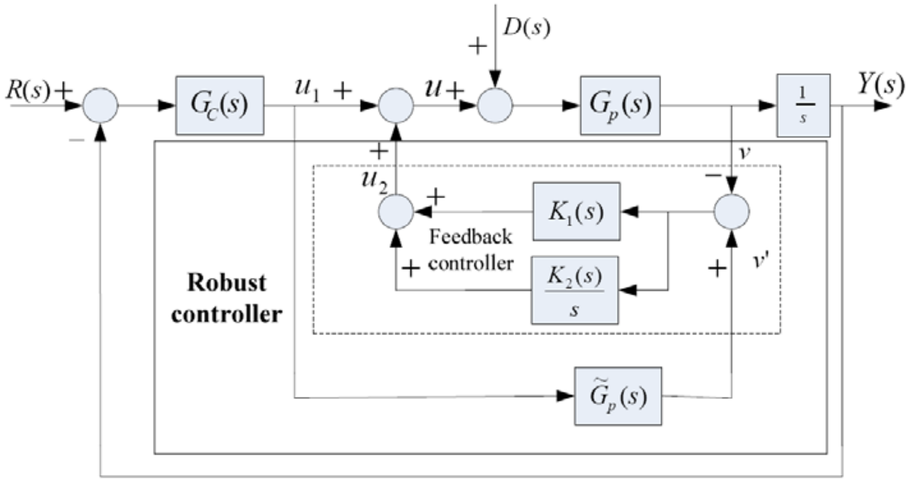

FIGURE 2. The structure of control system

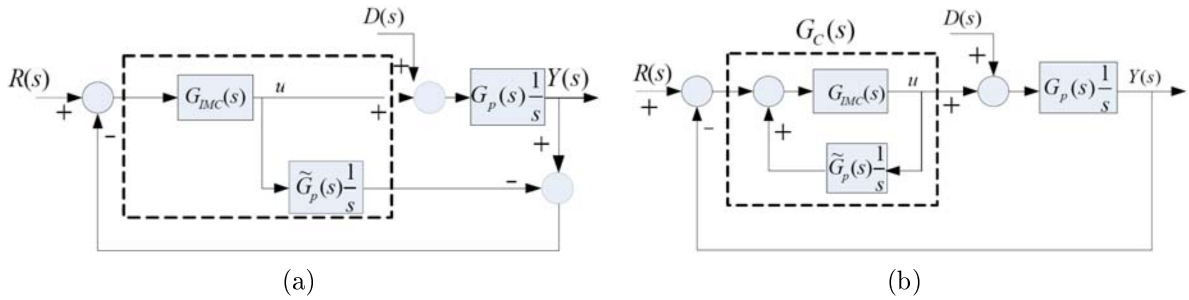

FIGURE 3. Block diagram of internal model control

3.2. **Design of internal model controller.** As shown in Figure 2, the internal model controller $G_C(s)$ is designed for position loop. The structure of internal model control is shown in Figure 3. The block diagram in Figure 3(b) is the equivalent form for Figure 3(a).

In Figure 3, $G_p(s)$ is the practical process, $\widetilde{G}_p(s)$ is the nominal model of $G_p(s)$, $G_{IMC}(s)$ is the internal model controller, $G_C(s)$ is the equivalent controller, $R(s)$ is the set point reference, u is the control variable, $Y(s)$ is the output of the process, and $D(s)$ is an unknown disturbance affecting the system.

From Figure 3(b), it follows that

$$G_C(s) = \frac{G_{IMC}(s)s}{s - G_{IMC}(s)\widetilde{G}_p(s)} \tag{3}$$

$$Y(s) = \frac{G_p(s)G_C(s)}{s + G_p(s)G_C(s)}R(s) + \frac{G_p(s)}{s + G_p(s)G_C(s)}D(s) \tag{4}$$

Substituting (3) to (4), we can obtain

$$Y(s) = \frac{G_p(s)G_{IMC}(s)}{s + G_{IMC}(s)\left(G_p(s) - \widetilde{G}_p(s)\right)}R(s) + \frac{\left(s - G_{IMC}(s)\widetilde{G}_p(s)\right)G_p(s)}{s^2 + G_{IMC}(s)\left(G_p(s) - \widetilde{G}_p(s)\right)s}D(s) \tag{5}$$

If the model is precise and the inverse model exists, i.e., $\widetilde{G}_p(s) = G_p(s)$ and $G_{IMC}(s) = G_p^{-1}(s)s$, the output of system is equivalent to the input, i.e., $Y(s) = R(s)$ and free from any interference.

The condition that ideal controller can be designed is that $G_p(s)$ has the inverse model, but inertial element and integration element always exist in the process, so the inverse model will hardly be achieved in practical application. To solve this problem, the internal model controller is usually designed as the inverse of process normal model in series with a low pass filter, which can be described as [16],

$$G_{IMC} = \widetilde{G}_p^{-1}(s)F(s)s \tag{6}$$

where $F(s)$ is a low pass filter. Substituting (6) to (3), we can get equivalent internal model controller,

$$G_C(s) = \frac{\widetilde{G}_p^{-1}(s)F(s)s}{1 - F(s)} \tag{7}$$

From Equation (7) and Figure 3(b), we can get the closed loop transfer function of the internal control system

$$\Phi(s) = F(s)$$

In Section 4, we will discuss the application of the new IMC controller in PMSM drive. In Section 4.1, the problem of conventional IMC for PMSM drive is presented. In Section 4.2, the design of H_∞ feedback controller is proposed to compensate the conventional IMC for PMSM drive system. In Section 4.3, mechanism on robustness and disturbance rejection of control system using H_∞ feedback controller will be discussed. In Section 4.4, the stability of the internal model control system using H_∞ feedback controller is discussed.

4. The Application of the New IMC Controller in PMSM Drive.

4.1. The problem of conventional IMC for PMSM drive. It is assumed that the Laplace transform of $\omega_m(t)$ is $\omega_m(s)$ and the Laplace transform of $i_q(t)$ is $i_q(s)$, from (1) and (2), ignoring the impact of load disturbance, we can get the normal model of plant is as follows,

$$\widetilde{G}_p(s) = \frac{\omega_m(s)}{i_q(s)} = \frac{K_T}{Js + B} \tag{8}$$

Then we can obtain,

$$\widetilde{G}_p^{-1}(s) = \frac{Js + B}{K_T} \tag{9}$$

Based on (9), we set a low pass filter [4],

$$F(s) = \frac{1}{(\lambda s + 1)^2} \tag{10}$$

Substituting (9) and (10) to (7), we can get equivalent internal model controller,

$$G_c(s) = \frac{Js + B}{K_T\left(\lambda^2 s + 2\lambda\right)} \tag{11}$$

where λ is the only parameter needed to be set.

Letting $R(s) = 0$, $G_p(s) = \widetilde{G}_p(s)$, adding a step disturbance, i.e., $D(s) = \frac{1}{s}$, substituting (8) and (11) to (4), we get impact of disturbance to the output,

$$Y(s) = \frac{K_T\lambda(\lambda s + 2)}{(\lambda s + 1)^2(Js + B)}\frac{1}{s} \tag{12}$$

Through the 'Final value theorem', we will get the steady state of the disturbance to the output,

$$y(t) = \lim_{s\to 0} Y(s)s = \lim_{s\to 0}\frac{K_T\lambda(\lambda s + 2)}{(\lambda s + 1)^2(Js + B)} = \frac{2K_T\lambda}{B} \tag{13}$$

From Equation (13), we can see the problem that the impact of the step disturbance cannot be ignored. The H_∞ feedback controller which will be discussed in 4.2 can solve this problem.

4.2. **The design of H_∞ feedback controller.** The design of H_∞ feedback controller can be transferred as H_∞ standard robust control problem which can be described as Figure 4. Figure 4 is the part of Figure 2, where $D(s) = \omega$ is the external disturbance input signal, u is the control input signal, u_1 is the output signal of internal model controller $G_C(s)$, u_2 is the output signal of H_∞ feedback controller $K(s)$, v is mechanical rotor speed from $G_p(s)$, v' is mechanical rotor speed from nominal model of $G_p(s)$ and z is the evaluation function.

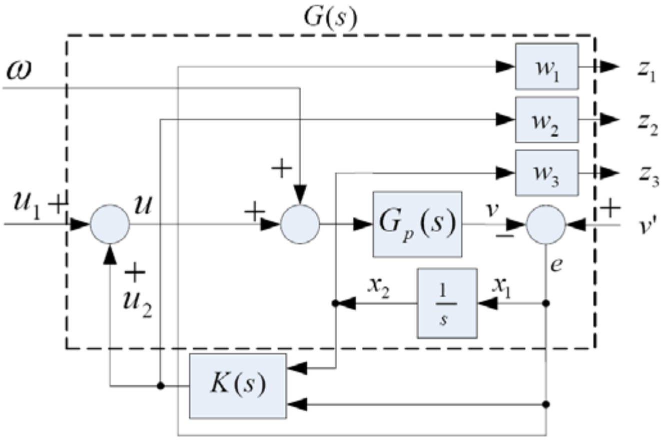

FIGURE 4. The block diagram of standard robust controller

Letting $G_p(s) = \widetilde{G}_p(s)$, from Figure 2 and Figure 4, based on Equation (2), the mechanical equations of the PMSM servo system are described as

$$\dot{v} = -\frac{B}{J}v + \frac{K_T}{J}\omega + \frac{K_T}{J}u_1 + \frac{K_T}{J}u_2 \tag{14}$$

$$\dot{v'} = -\frac{B}{J}v' + \frac{K_T}{J}u_1 \tag{15}$$

Let $w_1 = r_1$, $w_3 = r_2$, $w_2 = 1$, where r_1 and r_2 are the weighting factors selected by the frequency-domain requirement of w_1 and w_3. As in [26,27], selecting state variables as shown in Figure 4, subtracting (14) from (15), the state-space realization of the generalized plant $G(s)$ can be written as

$$\begin{cases} \dot{x} = Ax + B_1\omega + B_2u_2 \\ z = C_1x + D_{12}u_2 \end{cases} \tag{16}$$

where $x = [x_1 \;\; x_2]^T$, $x_1 = v' - v$, $x_2 = \int(v' - v)dt$, $z = [z_1 \;\; z_2 \;\; z_3]$.

And $A = \begin{bmatrix} -\frac{B}{J} & 0 \\ 1 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} -\frac{K_T}{J} \\ 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} -\frac{K_T}{J} \\ 0 \end{bmatrix}$, $C_1 = \begin{bmatrix} r_1 & 0 \\ 0 & 0 \\ 0 & r_2 \end{bmatrix}$, $D_{12} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

The control law is given by

$$u_2 = Kx \tag{17}$$

Then, the H_∞ control problem is to find the state-feedback controller $K(s)$ which satisfies

$$\|T_{z\omega}(s)\|_\infty < \gamma \tag{18}$$

where $T_{z\omega}(s)$ is the transfer function from single-input ω to multiple-output z.

The controller $K(s)$ can be obtained by the following lemma.

Lemma 4.1. [19]: *For the presented control plant (16), the sufficient and necessary condition for the existence of the state feedback controller which can make the closed-loop system internally stable and satisfy (18) is the existence of the positive definite matrix X which satisfies the Riccati inequality,*

$$\begin{aligned} &A^T X + XA + \frac{1}{\gamma^2} X B_1 B_1^T X + C_1^T C_1 \\ &- \left(XB_2 + C_1^T D_{12}\right)\left(D_{12}^T D_{12}\right)^{-1}\left(B_2^T X + D_{12}^T C_1\right) < 0 \end{aligned} \tag{19}$$

In order to calculate controller parameters more conveniently, Riccati inequality is transformed into Riccati equation,

$$\begin{aligned} &A^T X + XA + \frac{1}{\gamma^2} X B_1 B_1^T X + C_1^T C_1 \\ &- \left(XB_2 + C_1^T D_{12}\right)\left(D_{12}^T D_{12}\right)^{-1}\left(B_2^T X + D_{12}^T C_1\right) + \varepsilon I = 0 \end{aligned} \tag{20}$$

where $\gamma = 1$ and ε is an arbitrary small finite positive real number and the state-feedback controller $K(s)$ can be calculated as

$$K = -\left(D_{12}^T D_{12}\right)^{-1}\left(B_2^T X + D_{12}^T C_1\right) \tag{21}$$

Equation (21) can be solved by using the MATLAB robust control toolbox. Then the required controller K can be obtained. With the H_∞ feedback controller, the impact of external disturbance will be limited to ignorable range, which can deal with the problem of conventional IMC for PMSM drive.

4.3. **Analysis of mechanism on robustness and disturbance rejection.** For convenience of statement for the mechanism on robustness, adding modeling error $\Delta G_p(s)$, Figure 4 can be transformed to Figure 5.

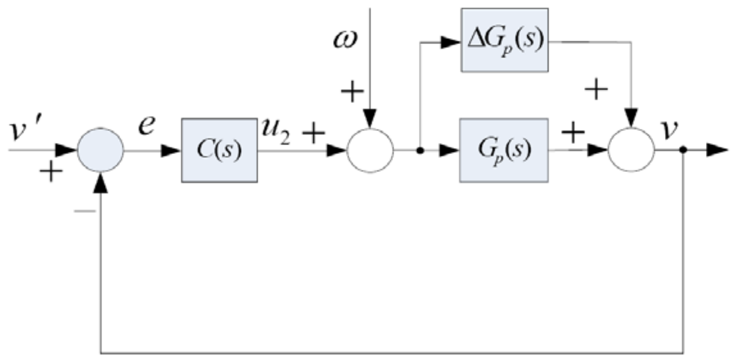

FIGURE 5. The block diagram of control system

In Figure 5, $C(s) = k_1 + \frac{k_2}{s}$, $\Delta G_p(s)$ is the modeling error which can also be regarded as parameters variation equivalently.

Letting $\omega = 0$, Figure 5 can be transformed to Figure 6 equivalently.

In Figure 6, $T(s) = \frac{-C(s)}{1+C(s)G_p(s)}$.

If $T(s)$ is stable, based on Nyqusit Criterion, the sufficient condition of stability for the closed system showed as Figure 6 is as follows [28],

$$|T(j\omega)\Delta G_p(jw)| < 1, \quad \forall \omega \in [0, \infty) \tag{22}$$

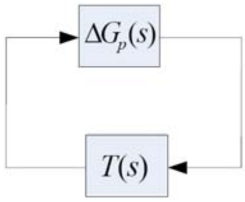

FIGURE 6. Equivalent system of Figure 5

Inequality (22) can be proved as follows.

If $|\Delta G_p(s)| < |G_p(s)|$, the left side of Inequality (22) can be transformed as

$$\begin{aligned} |T(j\omega)\Delta G_p(j\omega)| &= |T(j\omega)||\Delta G_p(j\omega)| \leq |T(j\omega)||G_p(j\omega)| \\ &= |T(j\omega)G_p(j\omega)| = |T'(j\omega)| \end{aligned} \tag{23}$$

where $T'(s) = T(s)G_p(s)$ is the transfer function from ω to u_2 in Figure 4.

From (18), letting $w_2 = 1$, the transfer functions from ω to z_2 satisfy the inequality

$$|T'(s)w_2|_\infty = |T'(s)| < 1, \quad s = j\omega \quad \omega \in [0, \infty) \tag{24}$$

where w_2 is the weighting function of control system in Figure 4.

Combining (23) and (24), Inequality (22) is proved to be right.

So if $\Delta G_p(s)$ satisfies the condition of $|\Delta G_p(s)| < |G_p(s)|$, the closed loop system which is showed as Figure 5 is stable; thus the output signal v of $G_p(s)$ can follow v' of $\widetilde{G}_p(s)$. Therefore, the nominal model $\widetilde{G}_p(s)$ can be used for internal model controller precisely and the impact of parameters variation can be eliminated.

Let $\Delta G_p(s) = 0$. From Inequality (18), letting $w_1 = r_1$, the transfer function from ω to z_1 satisfies inequality:

$$|S(s)w_1|_\infty = |S(s)r_1| < 1 \tag{25}$$

where $S(s) = \frac{-G_p(s)}{1+G_p(s)C(s)}$ is the transfer function from ω to e in Figure 4, and w_1 is the weighting function of control system in Figure 4.

The necessary and sufficient condition of Inequality (25) is as follows [28],

$$|S(j\omega)| < |r_1^{-1}|, \quad \forall\omega \in [0, \infty) \tag{26}$$

From (26), the impact of the disturbance can be limited to a small range by selecting an appropriate value of r_1.

In conclusion, it indicates that the proposed control system has better robustness which the parameters variation can be eliminated and problem of the disturbance rejection can be dealt with by the H_∞ feedback controller in proposed system. So the proposed control system has the advantages over the conventional IMC system.

4.4. **Stability analysis of system.** For the discussion of 4.3 when there exists the disturbance and parameters variation, the practical velocity v can follow the observation velocity v' precisely. So $\widetilde{G}_p(s)$ can be used as the nominal model of the practical model $G_p(s)$, which almost has no modeling error, that is to say, the modeling error is very small.

From Figure 3(a), the characteristic equation of control system can be described as:

$$1 + G_{IMC}(s)\Delta\widetilde{G}_p(s)\frac{1}{s} = 0 \tag{27}$$

where $\Delta\widetilde{G}_p(s) = G_p(s) - \widetilde{G}_p(s)$ is the modeling error.

Because $\Delta\widetilde{G}_p(s)$ is very small, $\Delta\widetilde{G}_p(s)$ can be transformed to $d\widetilde{G}_p(s)$, which is the differential of $\widetilde{G}_p(s)$. From (8), in PMSM servo system, it can be described as:

$$\begin{aligned} d\widetilde{G}_p(s) = d\frac{K_T}{Js+B} &= \frac{\partial \widetilde{G}_p}{\partial K_T}dK_T + \frac{\partial \widetilde{G}_p}{\partial J}dJ + \frac{\partial \widetilde{G}_p}{\partial B}dB \\ &= \frac{1}{Js+B}dK_T - \frac{K_T s}{(Js+B)^2}dJ - \frac{K_T}{(Js+B)^2}dB \end{aligned} \tag{28}$$

From (6), (9) and (10), $G_{IMC}(s)$ can be described as:

$$G_{IMC}(s) = \frac{(Js+B)s}{K_T(\lambda s+1)^2} \tag{29}$$

Substituting (28) and (29) to Equation (27), it can be described as:

$$N_3 s^3 + N_2 s^2 + N_1 s + N_0 = 0 \tag{30}$$

where $N_3 = K_T\lambda^2 J$, $N_2 = K_T(2\lambda J + B\lambda^2)$, $N_1 = K_T J + 2\lambda B K_T + J dK_T - K_T dJ$, $N_0 = K_T B + B dK_T - K_T dB$.

The stable conditions of the system can be obtained by Roth Criterion of stability, which can be described as:

$$\Delta_3 = N_3 > 0, \quad \Delta_2 = N_2 > 0, \quad \Delta_1 = \frac{N_1 N_2 - N_0 N_3}{N_2} > 0, \quad \Delta_0 = N_0 > 0 \tag{31}$$

In PMSM servo system, the parameters satisfy

$$K_T > 0, \quad J > 0, \quad B > 0, \quad \lambda > 0 \tag{32}$$

So conditions $\Delta_3 = N_3 > 0$, $\Delta_2 = N_2 > 0$ are obviously satisfied.

In order to prove the other conditions of Inequality (31), the parameters variation can be described as:

$$dK_T = g_1 K_T, \quad dJ = g_2 J, \quad dB = g_3 B \ (|g_{1,2,3}| \ll 1) \tag{33}$$

Then, we have (34) and (35):

$$N_0 = (g_1 - g_3 + 1)K_T B \tag{34}$$

$$N_1 N_2 - N_0 N_3 = M + (3 - g_1 + g_3)\lambda^2 K_T^2 B J \tag{35}$$

where $M = (g_1 - g_2 + 1)\left(2\lambda K_T^2 J^2 + \lambda^2 K_T^2 B J\right) + 2\lambda^3 K_T^2 B^2$.

For $|g_{1,2,3}| \ll 1$, then $g_1 - g_2 + 1 > 0$, $3 - g_1 + g_3 > 0$, considering Inequality (32), we have

$$N_0 > 0, \quad N_1 N_2 - N_0 N_3 > 0.$$

So conditions $\Delta_1 > 0$, $\Delta_0 > 0$ are obviously satisfied.

From what has been discussed above, based on Roth Criterion, the PMSM control system is proved to be stable.

In next section, the simulation research is given to demonstrate the effectiveness of the proposed scheme.

5. **Simulations Research.**

The control system is simulated based on Matlab/Simulink which is shown in Figure 7. The simulation system uses a current control strategy of '$i_d = 0$' and two-loop structure including the position loop and the current loop. In simulations, the parameters of the PMSM are as follows $R = 2.875\Omega$, $L_d = L_q = 8.5$mH, $\Psi_f = 0.375$Wb, $J = 0.0008$kg·m^2, $p_n = 4$, $B = 0.016$N.m.s, $K_T = p_n\Psi_f = 1.5$. Based on the method discussed in 3.2, using controller designed by Equation (7), the closed loop transfer function of internal model control system can be $F(s)$. Then from Equation (10), we know that the less the λ is, the faster the system responds, but considering the saturation of the control system, we select $\lambda = 0.05$, and then we get the equivalent IMC controller,

$$G_C(s) = 0.213 + \frac{-0.027}{0.00375s + 0.2}$$

Based on the method discussed in 4.2, from Inequality (26), the larger the r_1 is, the impact of the disturbance can be smaller. Because $S(s)+G_p(s)T'(s) = -G_p(s)$, the larger r_1 will have bad impact on the following performance of observation velocity v' to practical velocity v. Thus, the value of r_1 should be in an appropriate range, so as the value of r_2. ε is an arbitrary small finite positive real number to get the state-feedback controller K. After several choices we select $r_1 = 50$, $r_2 = 1000$, $\varepsilon = 0.001$. Using the Matlab robust software package, the H_∞ feedback controller is calculated as $K = [57.8 \;\; 1154.7]$.

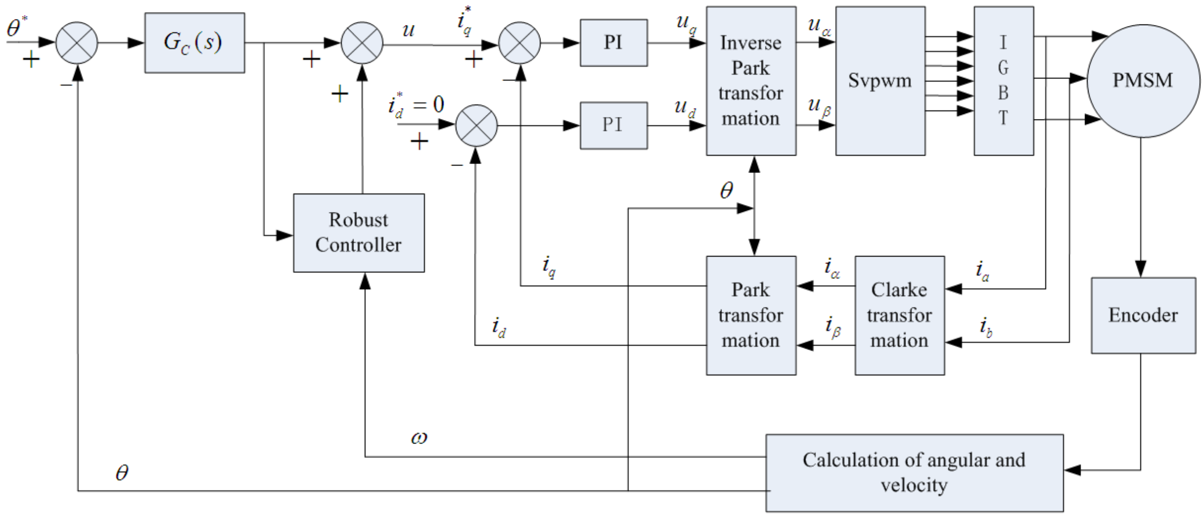

FIGURE 7. Simulation structure of the control system

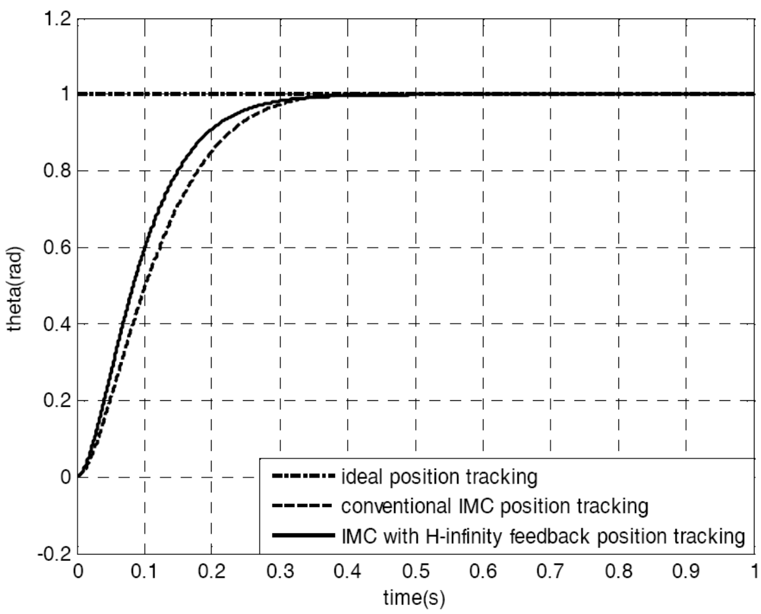

FIGURE 8. Step position response of the system without disturbance and model perturbation

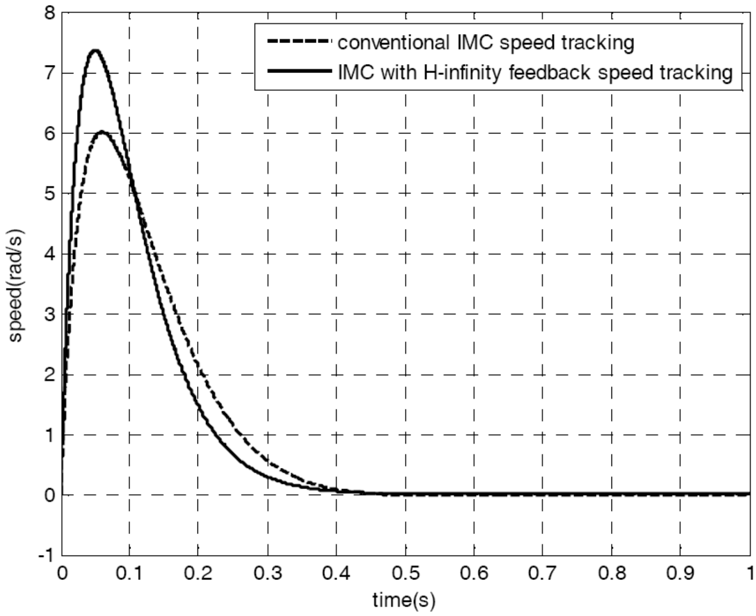

FIGURE 9. Speed response of the system without disturbance and model perturbation

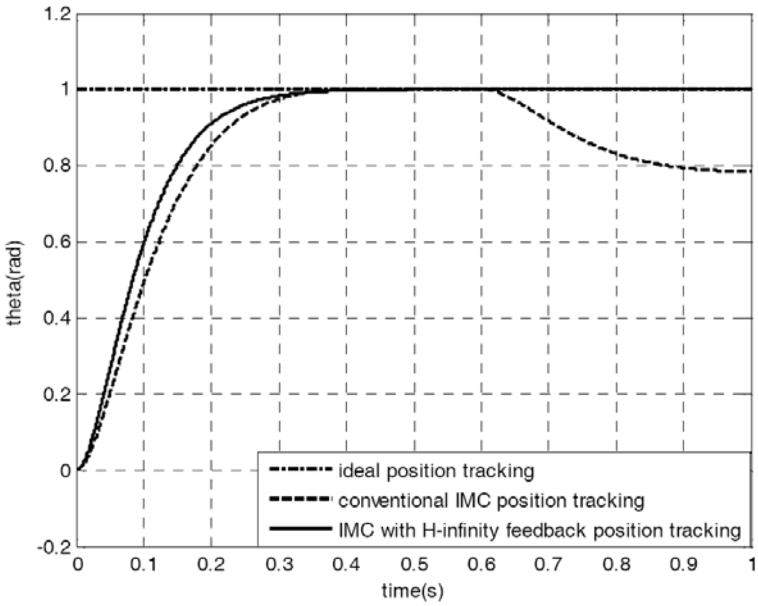

FIGURE 10. Step position response of the system with disturbance

In simulations, the control performance of the new internal model controller is compared with that of the conventional internal model controller. Figure 8 and Figure 9 show the dynamic response of the system without disturbance and model perturbation. The simulation is carried out for a PMSM with step position reference changing from 0rad to 1rad at 0s. From Figure 8 and Figure 9, we can see the control performances of conventional IMC and IMC with H_∞ feedback controller are almost the same when there is no disturbance and model perturbation. However, in practical process, the disturbance and model perturbation always exist. The position response of the system with the disturbance is shown in Figure 10 and Figure 11. The step disturbance changing from 0N.m to 0.03N.m is added to the system at 0.6s. From Figure 10 and Figure 11 we can find that the conventional IMC is sensitive to the disturbance while the IMC with H_∞

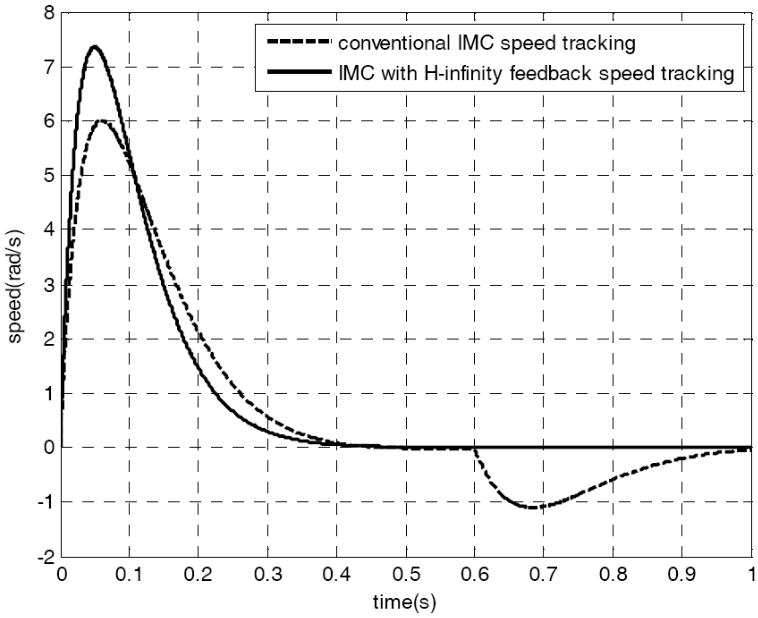

FIGURE 11. Speed response of the system with disturbance

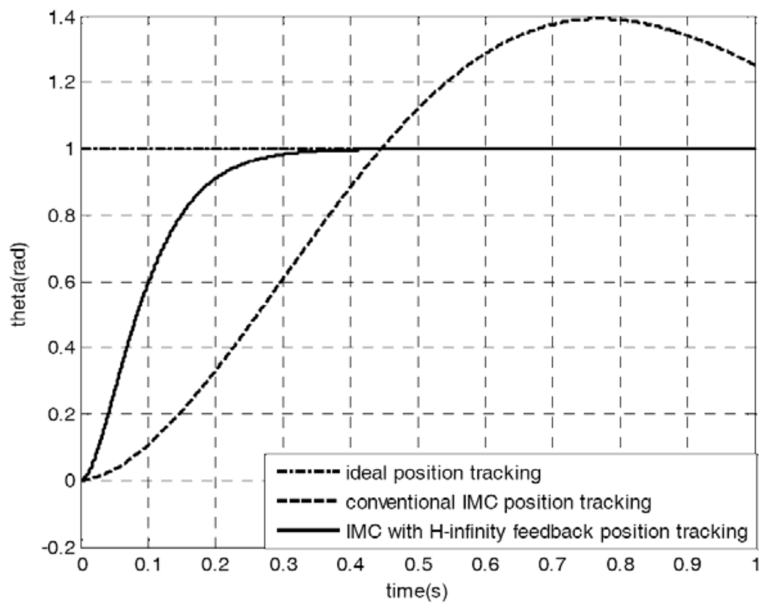

FIGURE 12. Step speed response of the system with model perturbation

feedback controller can overcome this problem and the disturbance nearly has no impact on the output of system due to H_∞ feedback controller. The position response of the system with the model perturbation is shown in Figure 12 and Figure 13. The parameter perturbation of inertia changing from 0.0008kg·m^2 to 0.008kg·m^2 occurs in the system. From Figure 12 and Figure 13, we can find that the IMC with H_∞ feedback controller has better performance to resist the parameter perturbation. So the designed H_∞ feedback controller can make conventional IMC have better robustness and disturbance attenuation.

6. **Conclusions.** This paper proposes an internal model control system in which the feedback controller is designed by the H_∞ control theory. The compensation of H_∞ feedback controller can make the process act as the nominal model. This approach can

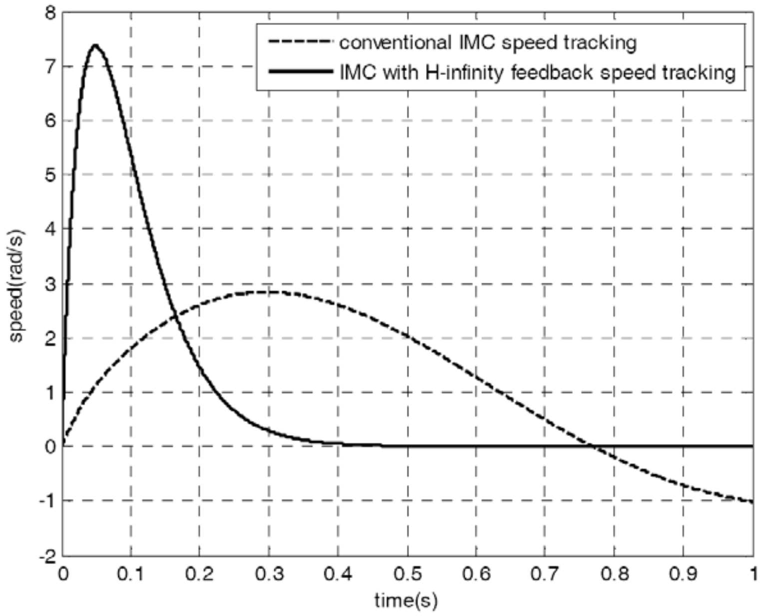

Figure 13. Speed response of the system with model perturbation

resolve the problems caused by the model perturbation and the disturbance. So the H_∞ feedback controller can promote the robust performance of IMC and make IMC more effective for PMSM control. The simulation results demonstrate the effectiveness of the proposed scheme. Our next work will consider the impact of time delay to PMSM servo system, which will be a challenge work.

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