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OBSERVER-BASED CONSENSUS CONTROL FOR NETWORKED MULTI-AGENT SYSTEMS WITH DELAYS AND PACKET-DROPOUTS

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Abstract. *This paper is concerned with the issues of observer-based consensus control for a class of networked multi-agent systems with time-varying delays and random packet-dropouts. The distributed multi-agent system whose consensus control protocol is accomplished through a communication network, via which each agent can be remotely controlled, is referred to as networked multi-agent systems (NMAS). Firstly, under a directed graph, a new model of NMAS is established with delays and random packet-dropouts simultaneously considered. Then, a novel Lyapunov-Krasovskii function based on time-varying character of delay is constructed to derive a criterion of exponential stability in mean square for NMAS, from which a quantitative relationship between packet-dropouts probability and consensus condition of multi-agent system is established. By employing this stability criterion, a new observer-based consensus controller in terms of linear inequality matrix (LMI) is proposed to urge each following-agent's states to reach an agreement on the leader's state. Finally, simulations are included to demonstrate the theoretical results.*
Keywords: Multi-agent systems, Delays, Packet-dropouts, Observer-based consensus controller

1. **Introduction.** Distributed multi-agent systems have received increasing attention during the last decades due to its effective applications in coordination and control of distributed sensor networks, attitude of spacecraft alignment unmanned-air-vehicle formations, etc. [1-4]. The consensus problem referring to that a group of autonomous agents reach an agreement, as a fundamention for the study of distributed coordination, has been extensively studied in the past few years, and some seminal studies are established in [5-8]. A dramatic advance of various fruits on consensus, over these years, have been achieved, such as [9-22]. In [10,11], the finite-time consensus problem has been formulated. Two normalized and signed gradient flows of a differential function is proposed in [10], which is used to solve the finite-time consensus problem. A notion of finite-time semistability is used to develop the finite time rendezvous problem in [11]. Based on the frequency-domain analysis, Tian and Liu [12] shows that the first-order consensus condition is dependent on input delays but independent of communication delays. Considering transmission delays which may be non-uniform, some conditions on asymptotically reaching consensus under a leader following configuration were obtained and the convergence rate was estimated in [13]. Consensus in the first-order and second-order multi-agent systems has been studied by Cao et al. in [14], where the information of both current and historical position states was available. In [17], the discrete-time double-integrator consensus problem is addressed for multi-agent systems with directed switching proximity topologies and input constraints.

In [19-22], the consensus problem was formulated over networks for the first-order discrete-time multi-agent systems with random packet losses. It is concluded that consensus can be almost surely achieved if the expected interaction topology has spanning trees. Zhang and Tian [22] have considered a consensus problem for a team of second-order multi-agent systems via variable delays and occasional packet losses simultaneously, in which the multi-agent system is modeled as a discrete-time system. However, the results for discrete-time multi-agent systems could be not directly applicable to the continuous-time ones. Up to now, no relevant work addresses the quantitative relationship between packet-dropouts probability and consensus for continuous-time multi-agent systems with time-varying delays. This is one motivation of this literature.

It is worthy pointing out that, in the aforementioned literature, a point-to-point connected mean is the main method used to control the multi-agent systems. However, in many modern applications, multi-agent systems are required to be remotely operated and controlled, which shall cause the point-to-point control structure to be no longer applicable. Consequentially, it is a tendency that each agent in a distributed multi-agent system is controlled via a communication network. The communication network has the advantages of less wired, lower cost, being easier to maintain, more suitable and flexible structure, etc. However, it inevitably produces delays and packet-dropouts during information transmission in the channel because of limited network bandwidth, which may cause negative impact on the performance of the system, even leading instability to system. By introducing a communication network, a network-based consensus control protocol for distributed multi-agent system is proposed with network-induced delay considered by Ding et al. in [23], not refering to packet-dropouts, in which the real information of each agent is fully available. However, due to the introduction of communication network, the distributed multi-agent system remotely controlled possesses a rather large scale, and measuring all state information of each agent is so energy-consuming of nodes that it would be almost impossible.

In recent years, consensus for a multi-agent system with some states information of agents unavailable has been investigated. When the velocity states of agents are unavailable, the consensus for multi-agent systems is studied in [24,25]. However, the effect of communication network shall outdate the results in [24,25]. Therefore, new networked multi-agent systems model and new results need be timely established to cope with network-induced delays and packet-dropouts. So far, to the best of authors' knowledge, the consensus of multi-agent system controlled through a communication network when the information of each agent's state is not fully available, has not been studied. This is another motivation of this literature.

This paper aims at modeling and designing observer-based consensus controller for a networked multi-agent system with time-varying delays and random packet-dropouts. The main contributions of this paper are summarized as follows.

- A new continuous-time model for distributed multi-agent system that is the so-called NMAS will be established with delays and packet-dropouts simultaneously considered, where the consensus control protocol is accomplished through a communication network and the agents's information cannot be fully available.
- Combining with an F-function, the existence of exponentially stable in mean square (ESMS) in NMAS will be proved. And a new Lyapunov-Krasovskii function sufficiently considering time-varying character of delay will be constructed to derive a criterion of ESMS for NMAS.
- By employing this stability criterion, a new observer-based consensus controller in terms of linear inequality matrix (LMI) shall be proposed to urge each following-agent's states

to reach an agreement on the leader's state. And this design method is suitable for multi-agent systems of any order.

- Some simulations will be included to demonstrate the theoretical results.

Notation: R^n denotes the n-dimensional Euclidean space. The superscript 'T' stands for matrix transposition. The notation $X > 0$ means that the matrix X is a real positive definite matrix. I is the identity matrix of appropriate dimensions. $\begin{bmatrix} X & Z \\ * & Y \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry. The sign $\otimes$ represents the matrix Kronecker product.

The paper is organized in 5 sections including Introduction. Section 2 presents basic graph theory and model of networked multi-agent system. Section 3 presents some main results. There are some simulations to illustrate the results in Section 4. Section 5 summarizes this paper.

2. Preliminaries.

2.1. Graph theory.

Due to the fact that the network formed by multi-agent systems can always be represented by a graph, the graph theory is a significant tool to analyze the consensus problem for multi-agent systems. Some basic graph theory notions are first introduced in this subsection.

Let $\phi = (\chi, \lambda, \aleph)$ denote a directed weighted graph of n order with a vertex set $\chi = \{g_1, g_2, \cdots, g_n\}$, the edge set $\lambda \subseteq \chi \times \chi$, and a weighted adjacency matrix $\aleph = [a_{ij}]_{n\times n}$ with non-negative adjacency elements a_{ij}. It is assumed that $a_{ii} = 0$ for any i ($i = 1, 2, \cdots, n$). An edge is defined by the ordered pair of vertices $\upsilon = (g_i, g_j)$, where node g_j is a neighbor of node g_i (which means node g_i can receive the information of node g_j), and $a_{ij} \in \upsilon$ if and only if $a_{ij} > 0$. The degree matrix of graph ϕ is denoted by a diagonal matrix $\Lambda = diag\{g_1, g_2, \cdots, g_n\}$ with the diagonal element $g_i = \sum_{j=1}^{n} a_{ij}$, which is also called as the in-degree of node. The Laplacian matrix is defined as $L = \Lambda - \aleph$. Moreover, we denote the graph that contains one leader node and n follower nodes by $\hat{\phi}$. Similarly, a diagonal matrix $S = diag\{s_1, s_2, \cdots, s_n\} \in R^{n\times n}$ is referred to as the leader adjacency matrix with $s_i \geq 0$ for any i ($i = 1, 2, \cdots, n$). If and only if the leader is a neighbor of node g_i, it satisfies $s_i > 0$; otherwise, $s_i = 0$.

2.2. Modeling of networked multi-agent systems.

Here we consider an l-order distributed multi-agent system consisting of n follower agents and a leader agent. It assumes that the movement of leader agent is independent on that of each follower agent, and each follower agent's movement could be affected by others'. The continuous-time dynamics of each follower agent is described as follows.

$$\begin{cases} \dot{\delta}_i^0(t) = \delta_i^1(t) \\ \quad\vdots \\ \dot{\delta}_i^{l-2}(t) = \delta_i^{l-1}(t) \\ \dot{\delta}_i^{l-1}(t) = u_i(t) \\ y_i(t) = \begin{bmatrix} \delta_i^0(t) & \delta_i^1(t) & \cdots & \delta_i^{l-1} \end{bmatrix}^T \end{cases} \qquad i = 1, 2, \cdots, n \tag{1}$$

where $\left(\delta_i^0, \delta_i^1, \cdots, \delta_i^{l-1}\right) \in R^l$, $u_i(t)$ and $y_i(t)$ represent the ith follower agent's states, control input and output separately. And the dynamics of leader agent is described as follows.

$$\begin{cases} \dot{\delta}_0^0(t) = \delta_0^1(t) \\ \quad \vdots \\ \dot{\delta}_0^{l-1}(t) = d \\ y_i(t) = \left[\begin{array}{cccc} \delta_i^0(t) & \delta_i^1(t) & \cdots & \delta_i^{l-1} \end{array} \right]^T \end{cases} \tag{2}$$

where d is a constant, and for definiteness and without loss of generality, we take $d = 0$. With a definition of a vector $\delta_i(t) = \left[\delta_i^0(t), \delta_i^1(t), \cdots, \delta_i^{l-1}(t)\right]^T$, systems (1) and (2) can be equivalently expressed in terms of the following matrix equations.

$$\begin{cases} \dot{\delta}_i(t) = A\delta_i(t) + Bu_i(t), \quad (i = 1, 2, \cdots, n) \\ \dot{\delta}_0(t) = A\delta_0(t) \\ y(t) = C\delta_i(t) \\ y_0(t) = C\delta_i(t) \end{cases} \tag{3}$$

where

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \in R^{l\times l}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^{l\times 1}, \quad C = I \in R^{l\times l}.$$

It notes that the pair (A, B) is stabilizable and the pair (A, C) is observable from system (3).

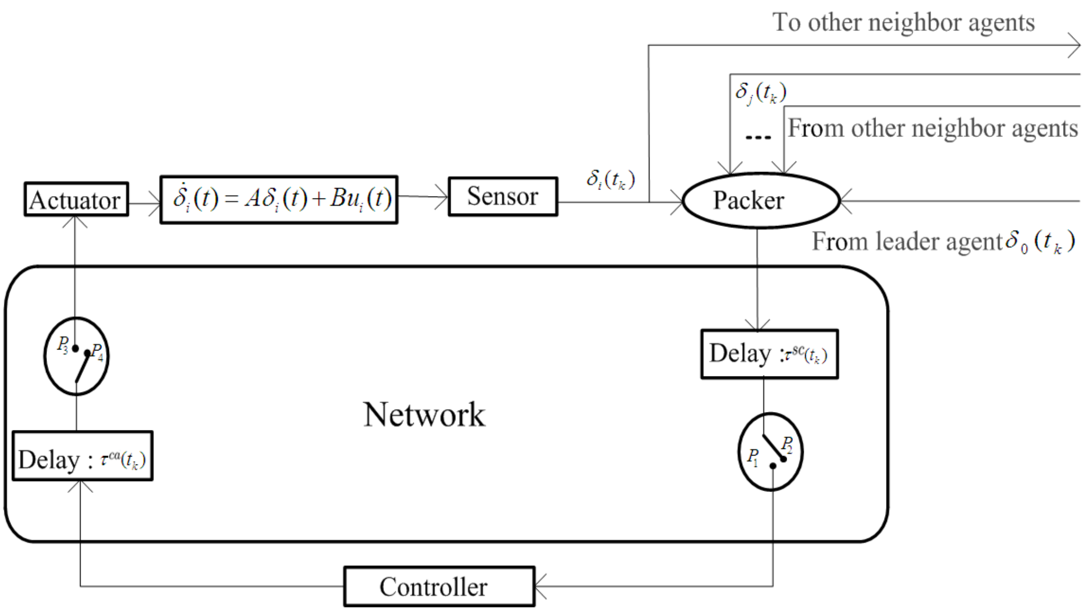

FIGURE 1. The basic structure of networked multi-agent systems

Here each agent of distributed multi-agent system is controlled by a feedback controller connected through a communication network, and we call it as NMAS, whose structure is shown in Figure 1. It is assumed that the sensor is clock-driven, and both the controller and the actuator are event-driven. During the information transmission in the communication network, a network delay is inevitably induced, which usually consists of two kinds of delays: one is sensor-to-controller delay τ^{sc} which is produced in forward channel, and the other is controller-to-actuator delay τ^{ca} which is produced in backward channel. A "packer" is employed to package the sampled data of agent i and its neighbor agents into one single packet before they get to the network channel. Namely, the data contained in one packet are transmitted in feedback channel, which suggests $\tau^{sc}(t) = \tau_i^{sc}(t) = \tau_j^{sc}(t)$, $\tau^{ca}(t) = \tau_i^{ca}(t) = \tau_j^{ca}(t)$, where $j \in$ {neighbor agents of agent i}. So the time delay can be lumped together as one $\tau(t) = \tau_i^{sc}(t) + \tau^{ca}(t)$. If agent i's states, as well as its neighbors', and the leader's can be fully measured, the consensus control protocol for system (3) is given as

$$\begin{cases} u_i(t) = -K \sum\limits_{j=1}^{n} a_{ij} \left[\delta_i(t_k - \tau(t)) - \delta_j(t_k - \tau(t))\right] \\ \qquad\qquad -K s_i \left[\delta_i(t_k - \tau(t)) - \delta_0(t_k - \tau(t))\right] \\ t \in [t_k, t_{k+1}], \quad i = 1, 2, \cdots, n \end{cases} \tag{4}$$

where K is the consensus control gain that remains to be designed. t_k is the sampling instant of the kth packet, and the sensor's sampling period is denoted by $\mathrm{T} = t_{k+1} - t_k$, $k \in Z^+$ is the sequence number of current sampled-packet.

Because of limited bandwidth in communication network, besides network-induced delay, packet-dropouts also inevitably exist. Considering that packet-dropouts may occur, we model the communication network as two switches. When the switch in forward channel is located in position of P_1, the packet containing currently sampled data is transmitted, and the controller utilizes the updated data; while when it is located in position P_2, the packet dropouts occur, and the controller uses the old data. Let a binary-valued variable β_{sc} denote the packet dropouts occur or not in forward channel, i.e., β_{sc}: $R \to \{0, 1\}$, where "1" implies packet is transmitted to controller, and "0" implies packet dropouts occur. Correspondingly, when the switch in backward channel is located in position P_3, the packet containing control inputs is transmitted; while the packet dropouts occur when it is located in position P_4. Similarly, let another binary-valued variable β_{ca} denote the packet dropouts occur or not in backward channel, i.e., β_{ca}: $R \to \{0, 1\}$. Let a variable $\beta(t)$ denote the product of variable β_{sc} and β_{ca}, and it is obvious $\beta(t)$ is also a binary-valued, i.e., $\beta(t)$: $R \to \{0, 1\}$. Taking packet dropouts into consideration, the consensus control protocol is transferred to

$$\begin{cases} u_i(t) = -K\beta(t) \sum\limits_{j=1}^{n} a_{ij} \left[\delta_i(t_k - \tau(t)) - \delta_j(t_k - \tau(t))\right] \\ \qquad\qquad -K\beta(t) s_i \left[\delta_i(t_k - \tau(t)) - \delta_0(t_k - \tau(t))\right] \\ t \in [t_k, t_{k+1}], \quad i = 1, 2, \cdots, n \end{cases} \tag{5}$$

Remark 2.1. *It notes that, different from [26], where the packet-dropouts were lumped together with communication delay and the maximal number of successive packet-dropouts does not exceed a setting bound, we adopt the "0/1" strategy in this paper to deal with the problem of packet-dropouts in NMAS.*

Defining a "synthetical delay" as $\eta(t) = t - t_k + \tau(t)$. To facilitate further discussion, the following assumption is made.

Assumption 2.1. *The "synthetical delay" varies between η_m and η_M, i.e., $\eta(t) \in [\eta_m, \eta_M]$. And its derivative is upper bounded by h, i.e., $\dot{\eta}(t) \leq h$, where h is a constant satisfying $h < 1$.*

Further, control protocol (5) can be rewritten as

$$\begin{cases} u_i(t) = -K\beta(t) \sum\limits_{j=1}^{n} a_{ij} \left[\delta_i(t - \eta(t)) - \delta_j(t - \eta(t))\right] \\ \qquad\qquad -K\beta(t) s_i \left[\delta_i(t - \eta(t)) - \delta_0(t - \eta(t))\right] \\ t \in [t_k, t_{k+1}], \quad i = 1, 2, \cdots, n \end{cases} \tag{6}$$

We suppose that packet-dropouts are nonuniformly distributed, and the probability of packet-dropouts occurring is $\bar{\beta}$, where $0 \leq \bar{\beta} \leq 1$. Then the probability of packets successfully reaching the actuator is $1 - \bar{\beta}$. The above given statistic characteristic can be described by a Bernoulli distributed white sequence as

$$\begin{cases} Prob\{\beta(t) = 0\} = E\{\beta(t)\} = \bar{\beta} \\ Prob\{\beta(t) = 1\} = 1 - E\{\beta(t)\} = 1 - \bar{\beta} \end{cases} \tag{7}$$

In a real-world application, the distributed multi-agent system which is remotely controlled possesses a rather large scale, and it would usually be impossible to measure all state information of each agent. Under this condition, an observer is designed to observe the states of each follower agent and the leader agent, whose dynamic model, corresponding to multi-agent system (1)-(3), is given as follows.

$$\begin{cases} \dot{\hat{\delta}}_i(t) = A\hat{\delta}_i(t) + Bu_i(t) + G\left(\delta_i(t) - \hat{\delta}_i(t)\right), \quad (i = 1, 2, \cdots, n) \\ \dot{\hat{\delta}}_0(t) = A\hat{\delta}_0(t) + G\left(\delta_0(t) - \hat{\delta}_0(t)\right) \\ t \in [t_k, t_{k+1}], \quad i = 1, 2, \cdots, n \end{cases} \tag{8}$$

where G is observer gain that remains to be designed. Because the real state information of the agents cannot be fully used, the control inputs are calculated by using the estimating value $\hat{\delta}_i(t)$ instead of $\delta_i(t)$. Based on this, the consensus control protocol (6) can be rebuilt as

$$\begin{cases} u_i(t) = -K\beta(t) \sum\limits_{j=1}^{n} a_{ij} \left[\hat{\delta}_i(t - \eta(t)) - \hat{\delta}_j(t - \eta(t))\right] \\ \qquad\qquad -K\beta(t) s_i \left[\hat{\delta}_i(t - \eta(t)) - \hat{\delta}_0(t - \eta(t))\right] \\ t \in [t_k, t_{k+1}], \quad i = 1, 2, \cdots, n \end{cases} \tag{9}$$

Definition 2.1. *Consensus in high-order multi-agents system (3) under the control protocol (9), which means each following-agent's states reach an agreement on the leader's state, is said to be achieved if, for any initial conditions,*

$$\lim_{t \to \infty} \|\delta_i(t) - \delta_0(t)\| = 0, \quad i = 1, 2, \cdots, n$$

Next, defining relative error of states as $x_i(t) = \delta_i(t) - \delta_0(t)$ and that of observations $\hat{x}_i(t) = \hat{\delta}_i(t) - \hat{\delta}_0(t)$, from (3) and (8), it produces

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) \\ \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + G\left(x_i(t) - \hat{x}_i(t)\right) \\ t \in [t_k, t_{k+1}], \quad i = 1, 2, \cdots, n \end{cases} \tag{10}$$

Also, let the variable $e_i(t)$ denote the predictive error produced by observers as $e_i(t) = \delta_i(t) - \hat{\delta}_i(t)$, from (3) and (8), the predictive error system can be obtained

$$\begin{cases} \dot{e}_i(t) = (A - G)e_i(t) \\ t \in [t_k, t_{k+1}], \quad i = 0, 1, 2, \cdots, n \end{cases} \tag{11}$$

Then, augmenting $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$, $\hat{x}(t) = [\hat{x}_1(t), \hat{x}_2(t), \cdots, \hat{x}_n(t)]$, $e(t) = [e_0(t), e_1(t), \cdots, e_n(t)]$ and $\tilde{x}(t) = [x(t), \hat{x}(t), e(t)]$, based on control input (9) and dynamical systems (10) and (11), the model of NMAS can be established as follows

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \beta(t)\tilde{B}\tilde{x}(t - \eta(t)) \\ t \in [t_k, t_{k+1}] \end{cases} \tag{12}$$

where $\tilde{A} = \begin{bmatrix} I_n \otimes A & 0 & 0 \\ I_n \otimes G & I_n \otimes (A-G) & 0 \\ 0 & 0 & I_{n+1} \otimes (A-G) \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} 0 & (L+S) \otimes BK & 0 \\ 0 & (L+S) \otimes BK & 0 \\ 0 & 0 & 0 \end{bmatrix}$.
And its initial state is supplemented as

$$\tilde{x}(\ell) = \psi(\ell), \quad \ell \in [-\eta_M, 0] \tag{13}$$

where $\psi(\ell) = \tilde{x}(\ell) = [x(\ell), \hat{x}(\ell), e(\ell)]$, which is smooth and its derivative $\dot{\psi}(\ell)$ exists.

Remark 2.2. *The NMAS is modeled as system (12) with the effects of time-varying delay and random packet-dropouts, and the problem of studying the consensus of distributed multi-agent systems (1)-(3) is transformed to stabilizing NMAS (12).*

3. Main Results.

3.1. Stability analysis.

In this subsection, the relationship between the stability and the possibility of packet-dropouts for NMAS (12) will be derived. First of all, the following necessary definition and lemma are introduced.

Definition 3.1. *NMAS (12) is said to be exponentially stable in mean square (ESMS) if, there exist two constants $a > 0$ and $\varepsilon > 0$ such that for all $t \geq 0$, the following inequality holds*

$$E\left\{\|x(t)\|^2\right\} \leq ae^{-\varepsilon t} \sup_{-\eta_M \leq s,v \leq 0} E\left\{\|\psi(s)\|^2 + \left\|\dot{\psi}(v)\right\|^2\right\} \tag{14}$$

Lemma 3.1. *If there exist symmetric matrices Z_{11}, Z_{22}, Z_{33}, and arbitrary matrices Z_{12}, Z_{13}, Z_{23}, such that*

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ * & Z_{22} & Z_{23} \\ * & * & Z_{33} \end{bmatrix} \geq 0 \tag{15}$$

then the following inequality holds.

$$\begin{aligned} & -\int_{\eta(t)}^{t} \dot{x}^T(v) Z_{33} x(v) dv \\ & \leq \int_{\eta(t)}^{t} \begin{bmatrix} x^T(t) & x^T(t-\eta(t)) & \dot{x}^T(v) \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ * & Z_{22} & Z_{23} \\ * & * & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta(t)) \\ \dot{x}(v) \end{bmatrix} dv \end{aligned} \tag{16}$$

Proof: This is proved in [20], and thus is omitted here.

For convenience of further formulation, we define

$$\varepsilon(t) = \left[\tilde{x}^T(t), \tilde{x}^T(t-\eta_m), \tilde{x}^T(t-\eta_M), \tilde{x}^T(t-\eta(t))\right]^T.$$

Next, a new Lyapunov-Krasovskii function which plays an important role in stability analysis of system (13) is constructed as follows.

$$V\left(\tilde{x}(t), t\right) = \sum_{k=1}^{3} V_k\left(\tilde{x}(t), t\right) \tag{17}$$

where

$$\begin{aligned} V_1(\tilde{x}(t), t) &= \tilde{x}^T(t) P \tilde{x}(t) \\ V_2(\tilde{x}(t), t) &= \int_{t-\eta_m}^{t} \tilde{x}^T(\tau) Q_1 \tilde{x}(\tau) d_\tau + \int_{-\eta_m}^{0} \int_{t+s}^{t} \dot{\tilde{x}}^T(\tau) R_1 \dot{\tilde{x}}(\tau) d_s d_\tau \\ V_3(\tilde{x}(t), t) &= \int_{t-\eta_M}^{t-\eta_m} \tilde{x}^T(\tau) Q_2 \tilde{x}(\tau) d_\tau + \int_{-\eta_M}^{-\eta_m} \int_{t+s}^{t} \dot{\tilde{x}}^T(\tau) R_2 \dot{\tilde{x}}(\tau) d_s d_\tau \end{aligned}$$

$$V_4(\tilde{x}(t),t) = \int_{t-\eta(t)}^{t} \tilde{x}^T(\tau)Q_3\tilde{x}(\tau)d_\tau + \int_{-\eta(t)}^{0}\int_{t+s}^{t} \dot{\tilde{x}}^T(\tau)R_3\dot{\tilde{x}}(\tau)d_s d_\tau$$
$$(t \in [t_k, t_{k+1}])$$

Let $P = diag\{I_n \otimes \bar{P}, I_n \otimes \bar{P}, I_{n+1} \otimes \bar{P}\}$ and $R_i = diag\{I_n \otimes R_{i1}, I_n \otimes R_{i2}, I_{n+1} \otimes R_{i3}\}$, $Q_i \in R^{(3n+1)l\times(3n+1)l}$ is a symmetric positive definite matrix for any $i = 1, 2, 3$. Based on the properties of Kronecker product, we know P, $R_i \in R^{(3n+1)l\times(3n+1)l}$ are also a symmetric positive definite matrices if and only if $\bar{P}$, $R_{ij} \in R^{l\times l}$, $(i, j = 1, 2, 3)$ are symmetric positive definite.

Before we give the consensus criteria, we use ∇V to denote the infinitesimal operator of V, which is defined as (see [21]).

$$\dot{V}(\tilde{x}(t),t) = \lim_{\Delta t\to 0^+} \frac{E\{V(\tilde{x}(t+\Delta t), t+\Delta t) \mid (\tilde{x}(t),t)\} - V(\tilde{x}(t),t)}{\Delta t} \tag{18}$$

Now, based on the given Lyapunov function V (17), and its infinitesimal operator (18), we state and prove the existence of ESMS criteria for NMAS.

Theorem 3.1. *For NMAS (12) is ESMS if there exists a scalar $\lambda < 0$, such that the following inequality holds*

$$E\{\nabla V(\tilde{x}(t),t)\} \le \lambda E\left\{\|\tilde{x}(t)\|^2 + \left\|\dot{\tilde{x}}(t)\right\|^2\right\} \tag{19}$$

for any $t \ge 0$.

Proof: A function is defined as

$$F(\tilde{x}(t),t) = e^{\varepsilon t}V(\tilde{x}(t),t) \tag{20}$$

Its infinitesimal operator goes

$$\nabla F(\tilde{x}(t),t) = \varepsilon e^{\varepsilon t}V(\tilde{x}(t),t) + e^{\varepsilon t}\nabla V(\tilde{x}(t),t) \tag{21}$$

Based on the constructed Lyapunov-Krasovskii Function (17) and (19), we have

$$\begin{aligned} &E\left\{\varepsilon e^{\varepsilon t}V(\tilde{x}(t),t) + e^{\varepsilon t}\nabla V(\tilde{x}(t),t)\right\} \\ &\le E\left\{e^{\varepsilon t}\left[(\varepsilon\zeta+\lambda)\sup_{t>0}\|\tilde{x}(t)\|^2 + (\varepsilon\sigma+\lambda)\sup_{t>0}\left\|\dot{\tilde{x}}(t)\right\|^2\right]\right\} \end{aligned} \tag{22}$$

where $\zeta = \lambda_{\max}(P)+\varepsilon\eta_m\lambda_{\max}(Q_1)+\varepsilon(\eta_M-\eta_m)\lambda_{\max}(Q_2)+\varepsilon\eta_M\lambda_{\max}(Q_3)$, $\sigma = \eta_m^2\lambda_{\max}(R_1)$ $+ (\eta_M-\eta_m)^2\lambda_{\max}(R_2) + \eta_M^2\lambda_{\max}(R_3)$.

A sufficiently small scalar $\varepsilon > 0$ being chosen to simultaneously guarantee $\varepsilon\zeta + \lambda < 0$ and $\varepsilon\sigma + \lambda < 0$ for any $t \ge 0$, it follows that

$$\begin{aligned} &E\{F(\tilde{x}(t),t)\} - E\{F(\tilde{x}(0),0)\} \\ &= \int_0^t [\varepsilon e^{\varepsilon v}E\{V(\tilde{x}(v),v)\} + e^{\varepsilon v}E\{\nabla V(\tilde{x}(v),v)\}]\, d_v \le 0 \end{aligned} \tag{23}$$

From (23) and Lyapunov-Krasovskii Function (17), we have

$$\begin{aligned} &e^{\varepsilon t}E\{V(\tilde{x}(t),t)\} \\ &\le E\{V(\tilde{x}(0),0)\} \le \zeta \sup_{-\eta_M<s<0} E\left\{\|\psi(s)\|^2\right\} + \sigma \sup_{-\eta_M<s<0} E\left\{\left\|\dot{\psi}(s)\right\|^2\right\}, \\ &t \ge 0 \end{aligned} \tag{24}$$

Therefore,

$$E\{V(\tilde{x}(t),t)\} \le \bar{\alpha}e^{-\varepsilon t}\zeta \sup_{-\eta_M<s,v<0} E\left\{\|\psi(s)\|^2 + \left\|\dot{\psi}(v)\right\|^2\right\}, \quad t \ge 0 \tag{25}$$

where $\bar{\alpha} = \max\{\zeta, \sigma\}$. Undoubtedly, $V(\tilde{x}(t), t) \geq \lambda_{\min}(P) \|\tilde{x}(t)\|^2$ holds. After replacing the item $V(\tilde{x}(t), t)$ with the item $\|\tilde{x}(t)\|^2$, we have

$$E\{V(\tilde{x}(t), t)\} \leq \alpha e^{-\varepsilon t} \zeta \sup_{-\eta_M < s, v < 0} E\left\{\|\psi(s)\|^2 + \left\|\dot{\psi}(v)\right\|^2\right\}, \quad t \geq 0 \tag{26}$$

where $\alpha = \frac{\bar{\alpha}}{\lambda_{\max}(P)}$. So far, the property described in (14) is satisfied. From Definition 3.1, we know Theorem 3.1 holds. This can complete the proof.

Next, on the basis of Theorem 3.1, by employing Lemma 3.1, following sufficient ESMS condition for NMAS (12) is established.

Theorem 3.2. *Given the packet-dropouts possibility $\bar{\beta}$, as well as scalars η_m, η_M, $h < 1$, and consensus control gain matrix $K \in R^{1\times l}$ and observer gain $G \in R^{l\times l}$, NMAS (12) is ESMS, correspondingly, the consensus of multi-agent system described by (3) is exponentially achieved in mean square under the consensus protocol described by (9) if there exist symmetric positive definite matrices $\bar{P} \in R^{l\times l}$, $T \in R^{(3n+1)l\times(3n+1)l}$, $Q_i \in R^{(3n+1)l\times(3n+1)l}$ $(i = 1, 2, 3)$ and $R_{ij} \in R^{l\times l}$ $(i, j = 1, 2, 3)$, symmetric definite matrices Z_{11}, Z_{22}, and matrices Z_{12}, Z_{13}, Z_{23} of appropriate dimensions such that*

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ * & Z_{22} & Z_{23} \\ * & * & R_3 \end{bmatrix} \geq 0 \tag{27}$$

and

$$\begin{bmatrix} \Pi & \Xi_1 & \Xi_2 \\ * & \Lambda_1 & 0 \\ * & * & \Lambda_2 \end{bmatrix} < 0 \tag{28}$$

where

$$\Pi = \begin{bmatrix} \Pi_1 & \frac{1}{\eta_m}R_1 & 0 & \Pi_2 \\ * & -Q_1 + Q_2 - \frac{1}{\eta_m}R_1 - \frac{1}{\eta_M-\eta_m}R_2 & \frac{1}{\eta_M-\eta_m}R_2 & 0 \\ * & * & -Q_2 - \frac{1}{\eta_M-\eta_m}R_2 & 0 \\ * & * & * & \Pi_3 \end{bmatrix},$$

$$\Pi_1 = \Phi + \Phi^T + Q_1 + Q_3 - \frac{1}{\eta_m}R_1 + (1-h)Z_{11} + (1-h)Z_{13} + (1-h)Z_{13}^T,$$

$$\Pi_2 = \Psi + (1-h)\eta_M Z_{12} - (1-h)Z_{13} + (1-h)Z_{23}^T,$$

$$\Pi_3 = -(1-h)Q_3 + (1-h)\eta_M Z_{22} - (1-h)Z_{23} - (1-h)Z_{23}^T,$$

$$\Xi_1 = \begin{bmatrix} \sqrt{\bar{\beta}\eta_m}\Gamma & \sqrt{\bar{\beta}(\eta_M - \eta_m)}\Gamma & \sqrt{\bar{\beta}\eta_M}\Gamma & \sqrt{\bar{\beta}}\Gamma \end{bmatrix},$$

$$\Xi_2 = \begin{bmatrix} \sqrt{\eta_m}\Sigma & \sqrt{\eta_M - \eta_m}\Sigma & \sqrt{\eta_M}\Sigma \end{bmatrix},$$

$$\Lambda_1 = diag\{-R_1^{-1}, -R_2^{-1}, -R_3^{-1}, -T^{-1}\}, \quad \Lambda_2 = diag\{-R_1^{-1}, -R_2^{-1}, -R_3^{-1}\},$$

$$\Phi = \begin{bmatrix} I_n \otimes \bar{P}^T A & 0 & 0 \\ I_n \otimes \bar{P}^T G & I_n \otimes \bar{P}^T (A - G) & 0 \\ 0 & 0 & I_{n+1} \otimes \bar{P}^T (A - G) \end{bmatrix},$$

$$\Psi = \begin{bmatrix} 0 & \bar{\beta}(L+S) \otimes \bar{P}^T BK & 0 \\ 0 & \bar{\beta}(L+S) \otimes \bar{P}^T BK & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \tilde{A}^T \\ 0 \\ 0 \\ \tilde{B}^T \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \tilde{A}^T \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Proof: Inspired by Theorem 3.1, $\nabla V\left(\tilde{x}(t),t\right)$ described by (18) for the evolution of Lyapunov function $V\left(\tilde{x}(t),t\right)$ described by (17) with respect to $t\in[t_k,t_{k+1}]$ is solved as follows.

$$\begin{aligned}&\nabla V\left(\tilde{x}(t),t\right)\\&=\tilde{x}^T(t)P^T\dot{\tilde{x}}(t)+\dot{\tilde{x}}^T(t)P\tilde{x}(t)+\tilde{x}^T(t)Q_1\tilde{x}(t)-\tilde{x}^T(t-\eta_m)Q_1\tilde{x}(t-\eta_m)\\&\quad+\tilde{x}^T(t-\eta_m)Q_2\tilde{x}(t-\eta_m)-\tilde{x}^T(t-\eta_M)Q_2\tilde{x}(t-\eta_M)+\tilde{x}^T(t)Q_3\tilde{x}(t)\\&\quad-(1-\dot{\eta}(t))\tilde{x}^T(t-\eta(t))Q_3\tilde{x}(t-\eta(t))+\eta_m\dot{\tilde{x}}^T(t)R_1\dot{\tilde{x}}(t)\\&\quad-\int_{t-\eta_m}^{t}\dot{\tilde{x}}^T(\tau)R_1\dot{\tilde{x}}(\tau)d_\tau+(\eta_M-\eta_m)\dot{\tilde{x}}^T(t)R_2\dot{\tilde{x}}(t)-\int_{t-\eta_M}^{t-\eta_m}\dot{\tilde{x}}^T(\tau)R_2\dot{\tilde{x}}(\tau)d_\tau\\&\quad+\eta(t)\dot{\tilde{x}}^T(t)R_3\dot{\tilde{x}}(t)-(1-\dot{\eta}(t))\int_{t-\eta(t)}^{t}\dot{\tilde{x}}^T(\tau)R_3\dot{\tilde{x}}(\tau)d_\tau\end{aligned}\tag{29}$$

From Jessen's inequality, the following two inequalities hold.

$$\begin{aligned}&-\int_{t-\eta_m}^{t}\dot{\tilde{x}}^T(s)R_1\dot{\tilde{x}}(s)d_s\\&\le\frac{1}{\eta_m}\left[\tilde{x}^T(t),\tilde{x}^T(t-\eta_m)\right]\begin{bmatrix}-R_1&R_1\\R_1&-R_1\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\\tilde{x}(t-\eta_m)\end{bmatrix}\end{aligned}\tag{30}$$

$$\begin{aligned}&-\int_{t-\eta_M}^{t-\eta_m}\dot{\tilde{x}}^T(s)R_2\dot{\tilde{x}}(s)d_s\\&\le\frac{1}{\eta_M-\eta_m}\left[\tilde{x}^T(t-\eta_m),\tilde{x}^T(t-\eta_M)\right]\begin{bmatrix}-R_2&R_2\\R_2&-R_2\end{bmatrix}\begin{bmatrix}\tilde{x}(t-\eta_m)\\\tilde{x}(t-\eta_M)\end{bmatrix}\end{aligned}\tag{31}$$

Because Inequality (27) holds, applying the Leibniz-Newton formula $\left(\int_{t-\eta(t)}^{t}\dot{\tilde{x}}(v)dv=\tilde{x}(t)-\tilde{x}(t-\eta(t))\right)$ and Lemma 3.1 to the last item $-(1-\dot{\eta}(t))\int_{t-\eta(t)}^{t}\dot{\tilde{x}}^T(\tau)R_3\dot{\tilde{x}}(\tau)d_\tau$ in Equation (29), we have

$$\begin{aligned}&-(1-\dot{\eta}(t))\int_{t-\eta(t)}^{t}\dot{\tilde{x}}^T(\tau)R_3\dot{\tilde{x}}(\tau)d_\tau\le(1-h)\int_{t-\eta(t)}^{t}\dot{\tilde{x}}^T(\tau)R_3\dot{\tilde{x}}(\tau)d_\tau\\&\le(1-h)\Big[\eta_M\tilde{x}^T(t)Z_{11}\tilde{x}(t)+\eta_M\tilde{x}^T(t)Z_{12}\tilde{x}(t-\eta(t))+\tilde{x}^T(t)Z_{13}\tilde{x}(t)\\&\quad-\tilde{x}^T(t-\eta(t))Z_{13}^T\tilde{x}(t)+\eta_M\tilde{x}^T(t-\eta(t))Z_{12}^T\tilde{x}(t)+\eta_M\tilde{x}^T(t-\eta(t))Z_{22}\tilde{x}(t-\eta(t))\\&\quad+\tilde{x}^T(t-\eta(t))Z_{23}\tilde{x}(t)-\tilde{x}^T(t-\eta(t))Z_{23}\tilde{x}(t-\eta(t))+\tilde{x}^T(t)Z_{13}^T\tilde{x}(t)\\&\quad-\tilde{x}^T(t)Z_{13}\tilde{x}(t-\eta(t))+\tilde{x}^T(t)Z_{23}^T\tilde{x}(t-\eta(t))-\tilde{x}^T(t-\eta(t))Z_{23}^T\tilde{x}(t-\eta(t))\Big]\end{aligned}\tag{32}$$

Then, by combining (30)-(32), and taking mathematical expectation at both sides of Equation (29) with the factor $E\{\beta(t)\}=E\{\beta^2(t)\}=\bar{\beta}$, the inequality follows

$$E\{\nabla V(\tilde{x}(t),t)\}\le\varepsilon^T(t)\Omega\varepsilon(t)\tag{33}$$

where

$$\Omega=\begin{bmatrix}\Omega_1&\frac{1}{\eta_m}R_1&0&\Omega_2\\ *&-Q_1+Q_2-\frac{1}{\eta_m}R_1-\frac{1}{\eta_M-\eta_m}R_2&\frac{1}{\eta_M-\eta_m}R_2&0\\ *&*&-Q_2-\frac{1}{\eta_M-\eta_m}R_2&0\\ *&*&*&\Omega_3\end{bmatrix},$$

$$\Omega_1 = P^T\tilde{A} + \tilde{A}^T P + Q_1 + Q_3 - \frac{1}{\eta_m}R_1 + \tilde{A}^T\left(\eta_m R_1 + (\eta_M - \eta_m)R_2 + \eta_M R_3\right)\tilde{A} + (1-h)Z_{11} + (1-h)Z_{13} + (1-h)Z_{13}^T,$$

$$\Omega_2 = \bar{\beta}P^T\tilde{B} + \bar{\beta}\tilde{A}^T(\eta_m R_1 + (\eta_M - \eta_m)R_2 + \eta_M R_3)\tilde{B} + (1-h)\eta_M Z_{12} - (1-h)Z_{13} + (1-h)Z_{23}^T,$$

$$\Omega_3 = -(1-h)Q_3 + \bar{\beta}\tilde{B}^T\left(\eta_m R_1 + (\eta_M - \eta_m)R_2 + \eta_M R_3\right)\tilde{B} + (1-h)\eta_M Z_{22} - (1-h)Z_{23} - (1-h)Z_{23}^T.$$

Using the properties of Kronecker product, it holds

$$(L+S)\otimes\bar{P}^T BK = \left(I_n\otimes\bar{P}^T\right)[(L+S)\otimes BK]$$

and $\Phi = P^T\tilde{A}$, and applying Schur complement theory to Inequality (28) yields

$$\varepsilon^T(t)\Omega\varepsilon(t) \le -\bar{\beta}\tilde{x}^T(t)\tilde{A}^T\left(\eta_m R_1 + (\eta_M - \eta_m)R_2 + \eta_M R_3\right)\tilde{A}\tilde{x}(t) - \bar{\beta}\dot{\tilde{x}}^T(t)T\dot{\tilde{x}}(t) < 0 \quad (34)$$

We define $\lambda = -\max\left\{\bar{\beta}\tilde{A}^T\left(\eta_m R_1 + (\eta_M - \eta_m)R_2 + \eta_M R_3\right)\tilde{A}, \bar{\beta}T\right\}$. It is obvious that $E\{\nabla V(\tilde{x}(t),t)\} \le \lambda\left(\tilde{x}^T(t)\tilde{x}(t) + \dot{\tilde{x}}^T(t)\dot{\tilde{x}}(t)\right) = \lambda\left(\|\tilde{x}(t)\|^2 + \|\dot{\tilde{x}}(t)\|^2\right)$ with $\lambda < 0$.

From Theorem 3.1, we know NMAS (10) is ESMS. This can complete the proof.

Remark 3.1. *So far, from Theorem 3.2, we know the quantitative relationship between packet-dropouts probability and consensus condition of multi-agent system (3) has been established through Inequality (28). When constructing the Lyapunov-Krasovskii Function (17), we have considered the delay's time-varying character with the introduction of* $V_4(\tilde{x}(t),t)$*, and it sufficiently allows us to use more delay information to develop the stable criteria for the NMAS, which has not been considered in [27]. Due to this, unfortunately, the item* $-(1-\dot{\eta}(t))\int_{t-\eta(t)}^{t}\dot{\tilde{x}}^T(\tau)R_3\dot{\tilde{x}}(\tau)d_\tau$ *is inevitably produced in the infinitesimal operator* $\Delta V(\tilde{x}(t),t)$*, which makes system's stability difficult to analyze. To overcome this problem, we have considered a constraint* $\dot{\eta}(t) \le h < 1$.

3.2. **Observer-based consensus controller design.** By employing stable criteria expressed by Theorem 3.2, a sufficient condition on the existence of the observer-based consensus controller will in this subsection be given for the NMAS.

Theorem 3.3. *Given the packet-dropouts possibility* $\bar{\beta}$*, as well as scalars* η_m*,* η_M*,* $h < 1$*,* $\mu_i > 0$ $(i = 1,2,3)$*, the consensus of multi-agent system described by (3) is exponentially achieved in mean square under the consensus protocol described by (9), if there exist symmetric positive definite matrices* $X \in R^{l\times l}$*,* $U \in R^{(3n+1)l\times(3n+1)l}$ *and* $\tilde{Q}_i \in R^{(3n+1)l\times(3n+1)l}$ $(i = 1,2,3)$*, symmetric matrices* $\tilde{Z}_{11}$*,* $\tilde{Z}_{22} \in R^{(3n+1)l\times(3n+1)l}$*, and matrices* $W \in R^{1\times l}$*,* $Y \in R^{l\times l}$*,* $\tilde{Z}_{12}$*,* $\tilde{Z}_{13}$*,* $\tilde{Z}_{23}$ *of appropriate dimensions such that LMIs*

$$\begin{bmatrix} \tilde{Z}_{11} & \tilde{Z}_{12} & \tilde{Z}_{13} \\ * & \tilde{Z}_{22} & \tilde{Z}_{23} \\ * & * & \mu_3^{-1}\tilde{X}^T \end{bmatrix} \ge 0 \quad (35)$$

and

$$\begin{bmatrix} \tilde{\Pi} & \tilde{\Xi}_1 & \tilde{\Xi}_2 \\ * & \tilde{\Lambda}_1 & 0 \\ * & * & \tilde{\Lambda}_2 \end{bmatrix} < 0 \quad (36)$$

where

$$\tilde{\Pi} = \begin{bmatrix} \tilde{\Pi}_1 & \frac{\mu_1}{\eta_m}\tilde{X}^T & 0 & \tilde{\Pi}_2 \\ * & -\tilde{Q}_1 + \tilde{Q}_2 - \frac{\mu_1^{-1}}{\eta_m}\tilde{X}^T - \frac{\mu_2^{-1}}{\eta_M - \eta_m}\tilde{X}^T & \frac{\mu_2^{-1}}{\eta_M - \eta_m}\tilde{X}^T & 0 \\ * & * & -Q_2 - \frac{\mu_2^{-1}}{\eta_M - \eta_m}\tilde{X}^T & 0 \\ * & * & * & \tilde{\Pi}_3 \end{bmatrix},$$

$$\tilde{\Pi}_1 = \tilde{\Phi} + \tilde{\Phi}^T + \tilde{Q}_1 + \tilde{Q}_3 - \frac{\mu_1^{-1}}{\eta_m}\tilde{X} + (1-h)\tilde{Z}_{11} + (1-h)\tilde{Z}_{13} + (1-h)\tilde{Z}_{13}^T,$$

$$\tilde{\Pi}_2 = \tilde{\Psi} + (1-h)\eta_M\tilde{Z}_{12} - (1-h)\tilde{Z}_{13} + (1-h)\tilde{Z}_{23}^T,$$

$$\tilde{\Pi}_3 = -(1-h)\tilde{Q}_3 + (1-h)\eta_M\tilde{Z}_{22} - (1-h)\tilde{Z}_{23} - (1-h)\tilde{Z}_{23}^T,$$

$$\tilde{\Xi}_1 = \begin{bmatrix} \sqrt{\bar{\beta}\eta_m}\tilde{\Gamma} & \sqrt{\bar{\beta}(\eta_M - \eta_m)}\tilde{\Gamma} & \sqrt{\bar{\beta}\eta_M}\tilde{\Gamma} & \sqrt{\bar{\beta}}\tilde{\Gamma} \end{bmatrix},$$

$$\tilde{\Xi}_2 = \begin{bmatrix} \sqrt{\eta_m}\tilde{\Sigma} & \sqrt{\eta_M - \eta_m}\tilde{\Sigma} & \sqrt{\eta_M}\tilde{\Sigma} \end{bmatrix},$$

$$\tilde{\Lambda}_1 = diag\left\{-\mu_1\tilde{X}, -\mu_2\tilde{X}, -\mu_3\tilde{X}, -U\right\}, \quad \tilde{\Lambda}_2 = diag\left\{-\mu_1\tilde{X}, -\mu_2\tilde{X}, -\mu_3\tilde{X}\right\},$$

$$\tilde{X} = diag\left\{I_n \otimes X, I_n \otimes X, I_{n+1} \otimes X\right\}$$

$$\Phi = \begin{bmatrix} I_n \otimes AX & 0 & 0 \\ I_n \otimes Y & I_n \otimes (AX - Y) & 0 \\ 0 & 0 & I_{n+1} \otimes (AX - Y) \end{bmatrix},$$

$$\tilde{\Psi} = \begin{bmatrix} 0 & \bar{\beta}(L+S) \otimes BW & 0 \\ 0 & \bar{\beta}(L+S) \otimes BW & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{\Gamma} = \begin{bmatrix} \tilde{\Phi}^T \\ 0 \\ 0 \\ \tilde{\Psi}^T \end{bmatrix}, \quad \tilde{\Sigma} = \begin{bmatrix} \tilde{\Phi}^T \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The consensus controller gain is given by $K = WX^{-1}$; meanwhile, the observer gain is given by $G = YX^{-1}$.

Proof: The proof is based on a suitable transformation and a change of variables allowing us to transfer Inequalities (27) and (28) in Theorem 3.2. Firstly, one defines $R_{i1} = R_{i2} = R_{i3} = \mu_i^{-1}\bar{P}$ $(i = 1, 2, 3)$, $X = \bar{P}^{-1}$, $J = diag\{\tilde{X}, \tilde{X}, \tilde{X}, \tilde{X}\}$, and other new variables $W = KX$, $Y = GX$, $U = T^{-1}$, $\tilde{Q}_i = \tilde{X}^T Q_i \tilde{X}$ $(i, j = 1, 2, 3)$ are introduced, $\tilde{Z}_{pr} = \tilde{X}^T Z_{pr} \tilde{X}$ $(p = 1, 2;\ r = 1, 2, 3)$. Then, pre- and post-multiplying both sides of Inequality (27) with $diag\{\tilde{X}, \tilde{X}, \tilde{X}\}$, and pre- and post-multiplying both sides of Inequality (28) with $diag\{J^T, I, I\}$, Inequalities (35) and (36) can be obtained. Thus, one can say (35) and (36) respectively imply (27) and (28). Based on Theorem 3.2, this can complete the proof.

4. **Simulations.** Consider the third-order multi-agent system including one leader agent and four follower agents described as system (3) under a directed graph shown in Figure 2. For simplicity, here we suppose that all the weights are set as 1. The "synthetical delay" $\eta(t)$ produced in the communication network is assumed upper bounded by $\eta_M = 0.41$ and lower bounded by $\eta_m = 0.01$, respectively. And its derivative is upper bounded by $\dot{\eta}(t) \leq h = 0.40$. Packet-dropouts occurring in the channel shown in Figure 3 follow Bernoulli distribution with $prob\{\beta(t) = 0\} = E\{\beta(t)\} = 0.15$. We choose other scalars as: $\mu_1 = 0.17$, $\mu_2 = 0.29$, $\mu_3 = 1.10$. By solving the LMIs in Theorem 3.3, the parameters, which are used to determine the consensus controller gain and observer gain, are calculated

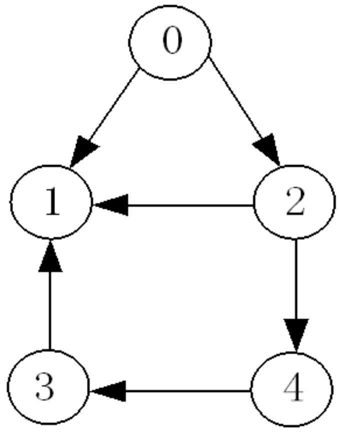

FIGURE 2. The topology graph for multi-agent system with a leader

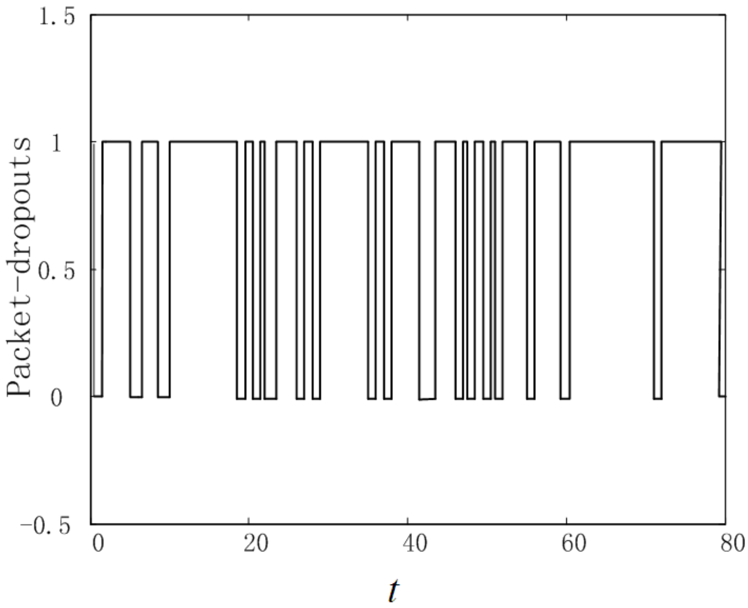

FIGURE 3. The distribution of packet-dropouts in NMAS

as follows.

$$W = [-0.0268, -0.5947, -0.1664], \quad X = \begin{bmatrix} 0.0301 & -0.0807 & 0.0211 \\ -0.0808 & 1.0600 & -0.1152 \\ 0.0203 & -0.1140 & 0.0518 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0.0001 & 0.6943 & 0.0354 \\ -0.0658 & 1.0196 & -0.1046 \\ -0.0114 & -0.1263 & -0.0038 \end{bmatrix}.$$

The agents' initial states and their observations are assumed to obey Gaussian distribution. Applying the designed observer-based consensus controller to the networked multi-agent system, the relative states' trajectories of NMAS are depicted in Figure 4. In Figure 4, all the relative states have arrived at zero equilibrium when $t = 60$. It implies that all follower agents' states can indeed reach consensus with the leader agent's before $t = 60$, even if the NMAS is simultaneously affected by time-varying delay and data dropouts. This shows the effectiveness and feasibility of the proposed design method.

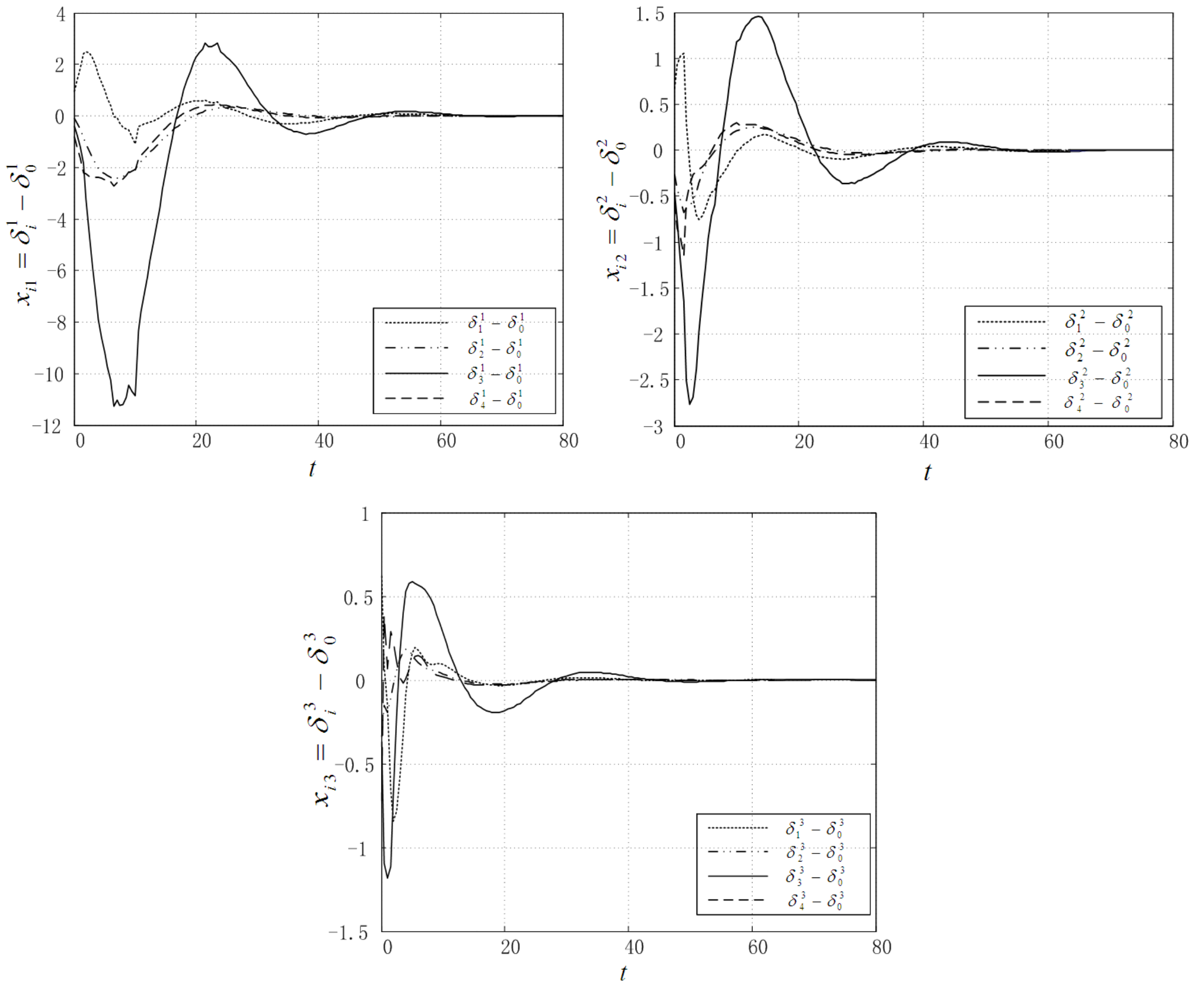

FIGURE 4. The trajectory of relative states $x_i = \delta_i - \delta_0$ of NMAS in the method proposed by this paper

In addition, the method proposed in [28] is applied into the same problem. The response of system's states are shown as Figure 5, from which it is obvious to see that follower agents' states fail to reach consensus with the leader's. By comparison, it further demonstrates the effectiveness and feasibility of the method proposed in this paper.

5. **Conclusions.** In this paper, we have studied the issues of modeling and consensus control for NMAS. Under a directed graph, the continuous-time NMAS is built as a closed-loop system with delays and random packet-dropouts simultaneously considered. An F-function is used to prove the existence of ESMS in NMAS. And a Lyapunov-Krasovskii function containing time-varying variable of delay is constructed to derive a criterion of exponential stability in mean square for NMAS, from which a quantitative relationship between packet-dropouts probability and consensus condition of multi-agent system has been established. By employing this stability criterion, an observer-based consensus controller in terms of LMI is proposed to urge each following-agent's states to reach an agreement on the leader's state. Finally, numerical examples are given to demonstrate the effectiveness of our method. The convergent rate of NMAS is not considered here, which is also a challenging problem remained to be studied in our future research work.

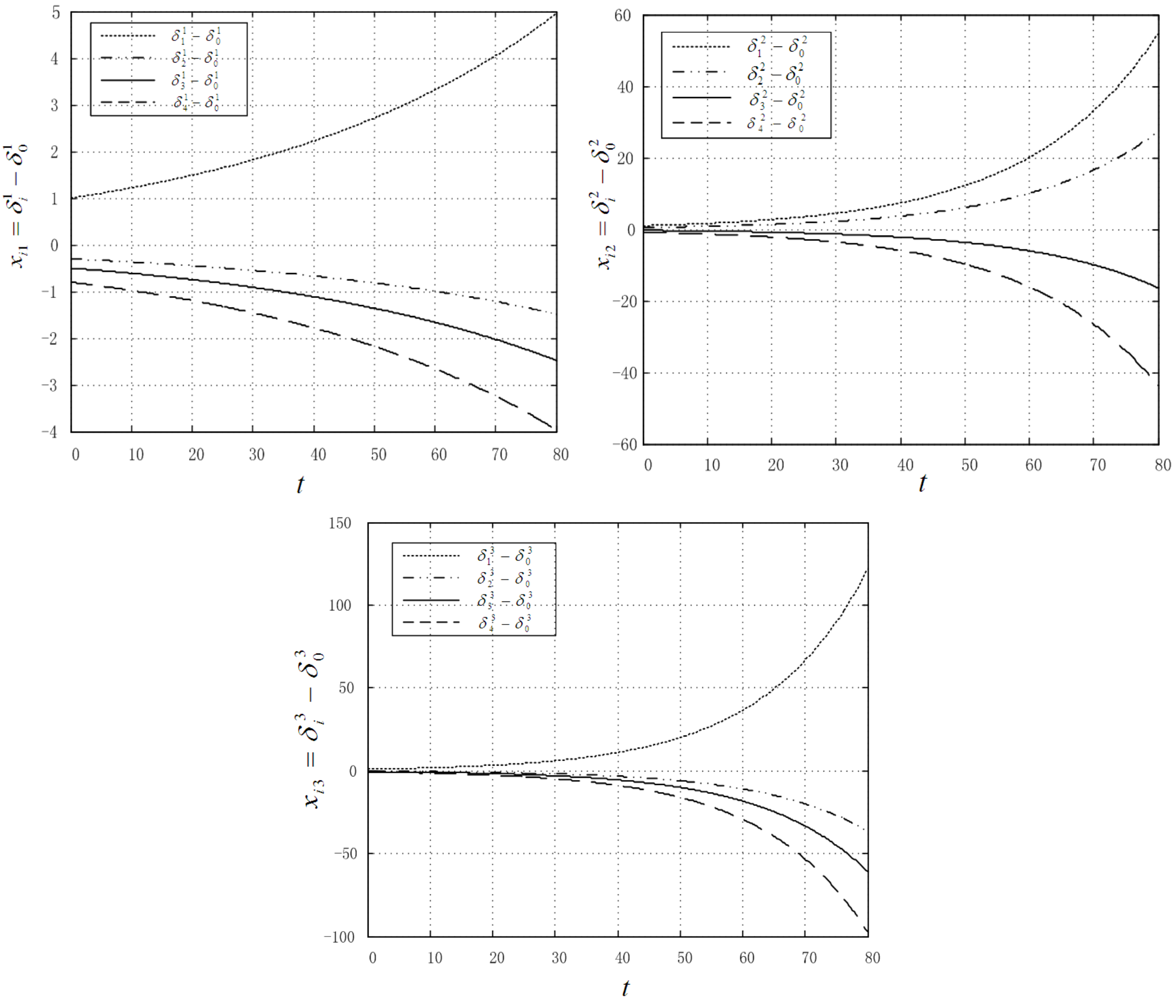

FIGURE 5. The trajectory of relative states $x_i = \delta_i - \delta_0$ of NMAS in the method proposed by [28]

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