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DESIGN OF A THREE-PHASE SWITCHED-CAPACITOR AC-AC CONVERTER WITH SYMMETRICAL TOPOLOGY

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Abstract. *In this paper a novel three-phase direct ac-ac converter is proposed in order to offer sinusoidal three-phase voltages. To achieve light weight and small size, the proposed ac-ac converter is designed by switched-capacitor (SC) techniques. While keeping the frequencies of the input and output voltages equal, the proposed converter provides stepped-up or stepped-down voltages to output loads. Unlike conventional SC ac-ac converters, the proposed converter has symmetrical topology without flying capacitors. By reducing the number of capacitors, the proposed ac-ac converter can improve power efficiency as well as input power factor from conventional converters. In the case of delta-wye, delta-delta, wye-delta and wye-wye connections, the operation principle, qualitative analysis and simulation evaluation are described in order to clarify the characteristics of the proposed converter. The results of the simulation program with integrated circuit emphasis (SPICE) simulation demonstrate the effectiveness of the proposed ac-ac converter.*

Keywords: ac-ac converters, Switched-capacitor circuits, Switching converters, Symmetrical topology, Three-phase

1. Introduction.

In power distribution systems, a three-phase transformer is commonly used to offer stepped-up or stepped-down three-phase voltages [1]. However, the three-phase transformer is difficult to achieve high power efficiency due to core losses and copper losses. Furthermore, due to the magnetic core and winding, the transformer is heavy and bulky. To overcome these problems, several types of an inductor-less ac-ac converter have been designed by using switched-capacitor (SC) techniques [2-11]. Due to the inductor-less topology, the SC ac-ac converter can realize light weight, small size, no flux of magnetic induction, and so on.

To the best of our knowledge, the first SC ac-ac converter was suggested by Ueno et al. [2,3]. To light an electroluminescent lamp, the first SC ac-ac converter offers a stepped-up modified sinusoidal waveform [2,3] by changing the connection of N $(= 2, 3, \ldots)$ charge-transfer capacitors by N-phase clock pulses. Following this study, in order to enhance the flexibility of conversion ratios, the ring-type SC ac-ac converter was proposed by Terada et al. and Eguchi et al. [4-6]. Unlike the first SC ac-ac converter, the ring-type SC ac-ac converter can provide stepped-up or stepped-down voltages by controlling transistor switches by N-phase clock pulses. However, the circuit topology of the ring-type SC ac-ac converter is complex. To simplify circuit topology, Lazzarin et al. and Andersen et al designed a direct ac-ac converter [7-9] by using SC techniques. By connecting a flying capacitor to charge-transfer capacitors alternately, the direct ac-ac converter can achieve not only step-down conversion but also step-up conversion [7-9]. Following this study, You and Hui expanded Lazzarin's ac-ac converter to realize the conversion ratio of 1/4 [10], because the direct ac-ac converter reported in [7-9] achieves only 1/2x step-down or 2x step-up conversion. However, the conventional ac-ac converters reported in [2-10] are the single-phase ac-ac converter. For this reason, the direct ac-ac converter was expanded to a three-phase converter topology by Lazzarin et al. [11]. However, due to the existence of flying capacitors, the conventional three-phase ac-ac converter [11] is difficult to achieve high input power factor and high power efficiency, because the instantaneous equivalent circuits of the conventional three-phase ac-ac converter have asymmetrical converter topology. In other words, there is still room for improvement in the converter topology of the conventional three-phase ac-ac converter.

In this paper, an SC three-phase direct ac-ac converter is proposed in order to offer sinusoidal three-phase voltages. While keeping the frequencies of the input and output voltages equal, the proposed converter provides stepped-up or stepped-down voltages to output loads. Unlike conventional SC ac-ac converters, the proposed converter has symmetrical topology without flying capacitors. By reducing the number of capacitors, the proposed ac-ac converter can achieve not only high power efficiency but also high input power factor. To demonstrate the effectiveness of the proposed ac-ac converter, simulation program with integrated circuit emphasis (SPICE) simulations and theoretical analysis are performed.

The rest of this paper is organized as follows. In Section 2, the circuit configuration of the SC ac-ac converters is presented. In Section 3, the characteristics of the SC ac-ac converter are clarified by assuming a four-terminal equivalent circuit. To confirm validity of circuit design, simulation results are demonstrated in Section 4. Finally, conclusion and future work are drawn in Section 5.

2. **Circuit Configuration.**

2.1. **Conventional three-phase ac-ac converter.**

Figure 1 illustrates the circuit configuration of the conventional three-phase ac-ac converter in the case of delta-wye connection [11]. As you can see from Figure 1, the conventional ac-ac converter has three

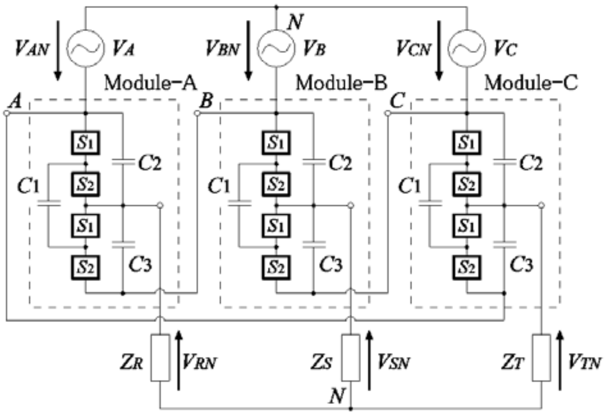


FIGURE 1. Conventional three-phase ac-ac converter in the case of delta-wye connection

modules: Modules-A, B and C. In this case, the voltage gains of the conventional three-phase ac-ac converter of Figure 1 are expressed as

$$\frac{V_{RN}}{V_{AN}} = \frac{1}{2} \text{ and } \frac{V_{RN}}{V_{AB}} = \frac{1}{2\sqrt{3}}, \tag{1}$$

where the phase-shift between V_{RN} and V_{AN} is –60° and the phase-shift between V_{RN} and V_{AB} is –90°. Of course, other connections, such as delta-delta, wye-delta and wye-wye connections, can be realized by changing the connections between converter modules. Table 1 shows the summary of the voltage gain and phase-shift.

Table 1. Voltage gain and phase-shift of the three-phase ac-ac converter

Connection	Voltage gain	Phase shift
Delta-Wye	$\frac{V_{RN}}{V_{AN}} = \frac{1}{2}$ and $\frac{V_{RN}}{V_{AB}} = \frac{1}{2\sqrt{3}}$	Between V_{RN} and V_{AN}: –60° Between V_{RN} and V_{AB}: –90°
Wye-Wye	$\frac{V_{RN}}{V_{AN}} = \frac{1}{2}$ and $\frac{V_{RN}}{V_{AB}} = \frac{1}{2\sqrt{3}}$	Between V_{RN} and V_{AN}: 0° Between V_{RN} and V_{AB}: –30°
Delta-Delta	$\frac{V_{RS}}{V_{AN}} = \frac{\sqrt{3}}{2}$ and $\frac{V_{RS}}{V_{AB}} = \frac{1}{2}$	Between V_{RS} and V_{AN}: –30° Between V_{RS} and V_{AB}: –60°
Wye-Delta	$\frac{V_{RS}}{V_{AN}} = \frac{\sqrt{3}}{2}$ and $\frac{V_{RS}}{V_{AB}} = \frac{1}{2}$	Between V_{RS} and V_{AN}: –30° Between V_{RS} and V_{AB}: 0°

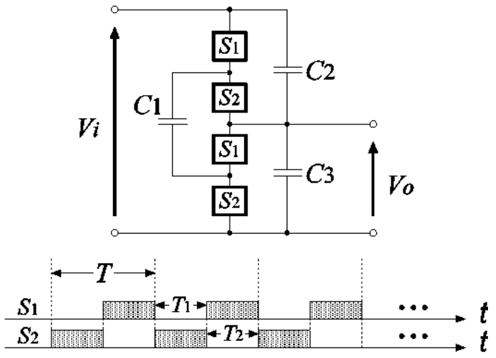
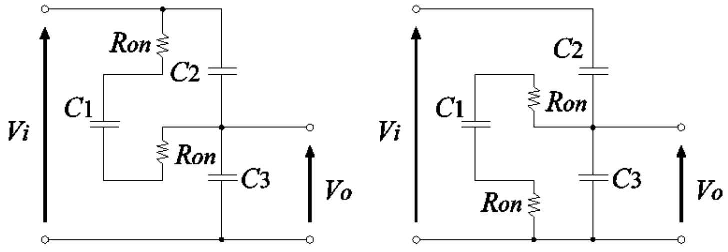


Figure 2. Converter module of the conventional three-phase ac-ac converter

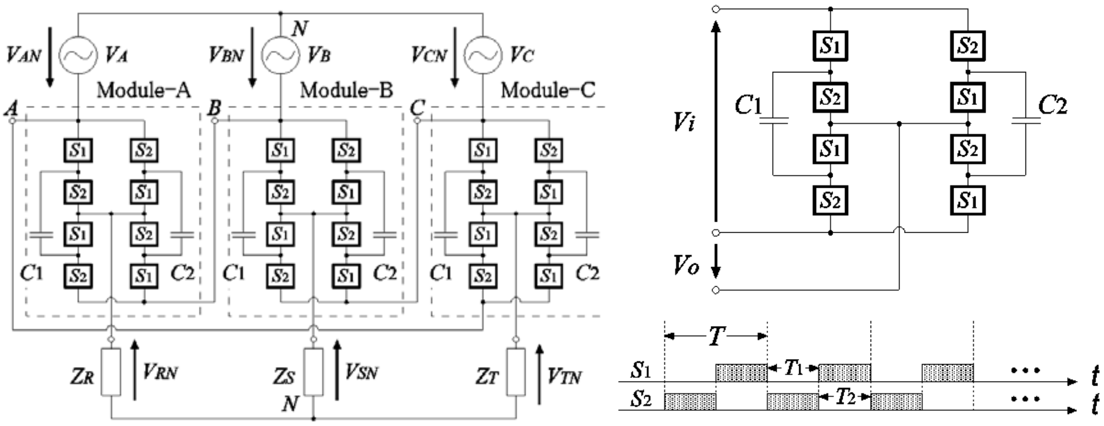
As an example of the operation principle of the module, Figure 2 illustrates the circuit configuration in the case of 1/2x step-down conversion. The topology of the converter module is based on the SC dc-dc converter suggested by Anderson et al. and Hara et al. [9,12]. As Figure 2 shows, each module consists of four bidirectional switches and three capacitors, where switches S_1 and S_2 are driven by non-overlapped two-phase clock pulses with constant switching frequency and duty cycle. Of course, the converter module can offer not only the 1/2x stepped-down voltage but also the 2x stepped-up voltage by swapping the input terminal and the output terminal.

Figure 3 illustrates the instantaneous equivalent circuits of the converter module in the case of the 1/2x step-down conversion, where the bidirectional switch is modeled by an ideal lossless switch and an on-resistance R_{on}. As you can see from Figure 3, the voltage conversion is performed by connecting the flying capacitor C_1 to the main capacitors C_2 and C_3 alternately.



(a) (b)

Figure 3. Instantaneous equivalent circuit of the conventional module: (a) State-T_1 and (b) State-T_2



(a) (b)

Figure 4. Proposed three-phase ac-ac converter in the case of delta-wye connection: (a) circuit topology and (b) converter module

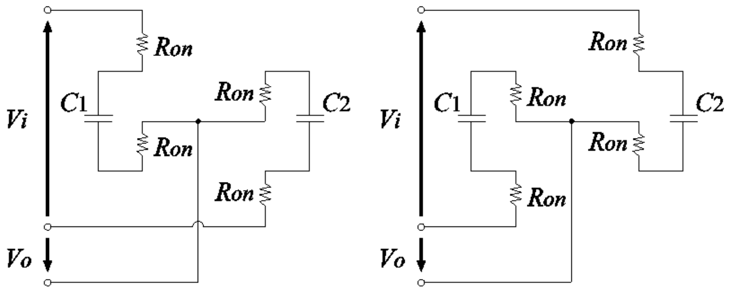
Table 2. Number of circuit components

	Number of capacitors	Number of switches
Proposed module	2	8
Conventional module	3	4

2.2. **Proposed converter.** Figure 4 illustrates the proposed three-phase ac-ac converter in the case of delta-wye connection. As you can see from Figure 4(a), the proposed ac-ac converter has three module blocks, where each module consists of eight bidirectional switches and two capacitors. Unlike the converter module of Figure 2, the proposed module block of Figure 4(b) has symmetrical converter topology.

Table 2 shows the comparison of the number of circuit components between the proposed module and the conventional module. As you can see from Table 2, the number of switches for the proposed module is larger than that for the conventional module. However, the number of capacitors for the proposed module is less than that for the conventional module block, because the proposed module has no flying capacitor.

Figure 5 illustrates the instantaneous equivalent circuits of the proposed module in the case of the 1/2x step-down conversion. By changing the connection order of C_1 and C_2, the proposed module achieves voltage conversion. Therefore, unlike the conventional module block, the number of capacitors connected to the I/O terminals is constant. Of course, the proposed module can offer not only the 1/2x stepped-down voltage but also the 2x stepped-up voltage by swapping the input and output terminals.



(a) (b)

Figure 5. Instantaneous equivalent circuit of the proposed module: (a) State-T_1 and (b) State-T_2

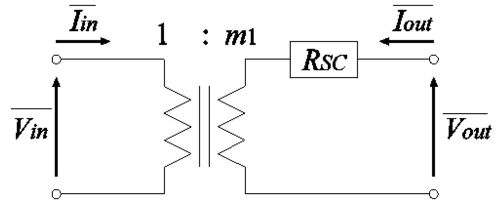


Figure 6. Four-terminal equivalent circuit

3. **Theoretical Analysis.** In a conversion ratio of 1/2, the characteristics of the proposed module are analyzed theoretically, where the ac input is assumed as a pulse ac waveform in order to estimate the maximum power efficiency and the maximum output voltage. For the pulse input, the proposed module behaves like a dc-dc converter. Therefore, the theoretical analysis is performed by using a four-terminal equivalent circuit shown in Figure 6, because it is known that an SC dc-dc converter can be expressed by a K-matrix [13,14]. In Figure 6, R_{SC} is called the SC resistance and m_1 is the conversion ratio of an ideal transformer. These parameters are derived by utilizing instantaneous equivalent circuits of Figure 5.

In steady state, the differential value of electric charges $\Delta q_{T_i}^k$ in C_k $(k = 1, 2)$ satisfies the following equation:

$$\Delta q_{T_1}^k + \Delta q_{T_2}^k = 0, \tag{2}$$

where $\Delta q_{T_i}^k$ $((i = 1, 2)$ and $(k = 1, 2))$ denotes the electric charge of the k-th capacitor in State-T_i. The interval of State-T_i satisfies

$$T = T_1 + T_2 \text{ and } T_1 = T_2 = \frac{T}{2}, \tag{3}$$

where T is a period of the clock pulse and T_i $(i = 1, 2)$ is the interval of State-T_i.

In State-T_1, the differential values of electric charges in the input and the output, $\Delta q_{T_1,v_i}$ and $\Delta q_{T_1,v_o}$, are expressed as

$$\Delta q_{T_1,v_i} = \Delta q_{T_1}^1 \text{ and } \Delta q_{T_1,v_o} = -\Delta q_{T_1}^1 + \Delta q_{T_1}^2. \tag{4}$$

On the other hand, in State-T_2, the differential values of electric charges in the input and the output, $\Delta q_{T_2,v_i}$ and $\Delta q_{T_2,v_o}$, are expressed as

$$\Delta q_{T_2,v_i} = \Delta q_{T_2}^2 \text{ and } \Delta q_{T_2,v_o} = \Delta q_{T_2}^1 - \Delta q_{T_2}^2. \tag{5}$$

In (4) and (5), the following conditions are satisfied:

$$\Delta q_{T_1}^1 = \Delta q_{T_2}^2 \text{ and } \Delta q_{T_1}^2 = \Delta q_{T_2}^1, \tag{6}$$

because the proposed module has symmetrical topology. Using (4) and (5), the I/O currents, i_i and i_o, can be expressed as

$$i_i = \frac{\Delta q_{v_i}}{T} = \frac{\Delta q_{T_1,v_i} + \Delta q_{T_2,v_i}}{T} \text{ and } i_o = \frac{\Delta q_{v_o}}{T} = \frac{\Delta q_{T_1,v_o} + \Delta q_{T_2,v_o}}{T}, \tag{7}$$

because the overall change of I/O currents is zero in steady state. In (7), Δq_{v_i} and Δq_{v_o} are electric charges in v_i and v_o, respectively. Substituting (2)-(6) into (7), we have the relation between the input current and the output current as follows:

$$i_i = -\frac{1}{2} i_o. \tag{8}$$

Therefore, the conversion ratio in Figure 6 can be obtained as $m_1 = 1/2$.

Next, in order to derive the SC resistance R_{SC}, the consumed energy in one period is discussed. The consumed energy W_T in one period can be expressed as

$$W_T = W_{T_1} + W_{T_2} = 2W_{T_1}, \tag{9}$$

where

$$W_{T_1} = \frac{\left(\Delta q_{T_1}^1\right)^2}{T_1} 2R_{on} + \frac{\left(\Delta q_{T_1}^2\right)^2}{T_1} 2R_{on}.$$

In (9), dielectric loss is not considered to estimate the maximum power efficiency and the maximum output voltage. Using (2)-(6), the consumed energy (9) can be rewritten as

$$W_T = \frac{\left(\Delta q_{v_o}\right)^2}{T} R_{on}. \tag{10}$$

Here, the consumed energy W_T of the four-terminal equivalent circuit shown in Figure 6 is expressed as

$$W_T = R_{SC} \frac{\left(\Delta q_{v_{out}}\right)^2}{T}. \tag{11}$$

Therefore, from (10) and (11), the SC resistance R_{SC} in Figure 6 can be obtained as

$$R_{SC} = R_{on}. \tag{12}$$

Finally, by combining (8) and (12), the equivalent circuit is given by

$$\begin{bmatrix} v_i \\ i_i \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & R_{on} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_o \\ -i_o \end{bmatrix}, \tag{13}$$

because the four-terminal equivalent circuit of Figure 6 can be expressed by the K-matrix. From (13), the maximum efficiency and the maximum output voltage can be derived as

$$\eta_{\max} = \frac{R_L}{R_{SC} + R_L} \text{ and } v_{o\_\max} = \left(\frac{R_L}{R_{SC} + R_L}\right)\left(\frac{v_i}{2}\right). \tag{14}$$

Of course, other conversion modes can be analyzed by the same method. Table 3 shows the comparison between the proposed module and the conventional module in conversion ratios of 1/2 and 2. The theoretical analysis of other conversion modes will be described in Appendix. As you can see from Table 3, the proposed module can achieve not only fewer number of capacitors but also smaller SC resistance than the conventional module. In other words, the proposed ac-ac converter can achieve higher power efficiency than the conventional ac-ac converter.

TABLE 3. SC resistance obtained by theoretical analysis

	2x Step-up	1/2x Step-down
Proposed module block	$4R_{on}$	R_{on}
Conventional module block	$8R_{on}$	$2R_{on}$

4. **Simulations.** To clarify the characteristic of the proposed converter shown in Figure 7, SPICE simulations are performed under conditions that input voltage $V_A = V_B = V_C = 200\text{V@}50\text{Hz}$, $C_1 = C_2 = 33\mu\text{F}$, $C_{out} = 100\text{nF}$, $R_{on} = 0.83\Omega$, $T = 10\mu\text{s}$, and $D(= T_1/T) = 0.45$ (5% of dead time), where C_{out} is a filter capacitor.

Figure 8 demonstrates the simulated voltages of the proposed three-phase ac-ac converter with a balanced three-phase resistive loads, where $Z_R = Z_S = Z_T = 100\Omega$. As you can see from Figure 8, the proposed converter can offer the output voltages shown in Table 1. Figure 9 demonstrates the simulated power efficiency in the case of delta-wye connection. To save space, the simulation result of other connections is omitted in this paper, because the power efficiency does not depend on the connections. As you can see from Figure 9, the power efficiency of the proposed three-phase ac-ac converter is higher

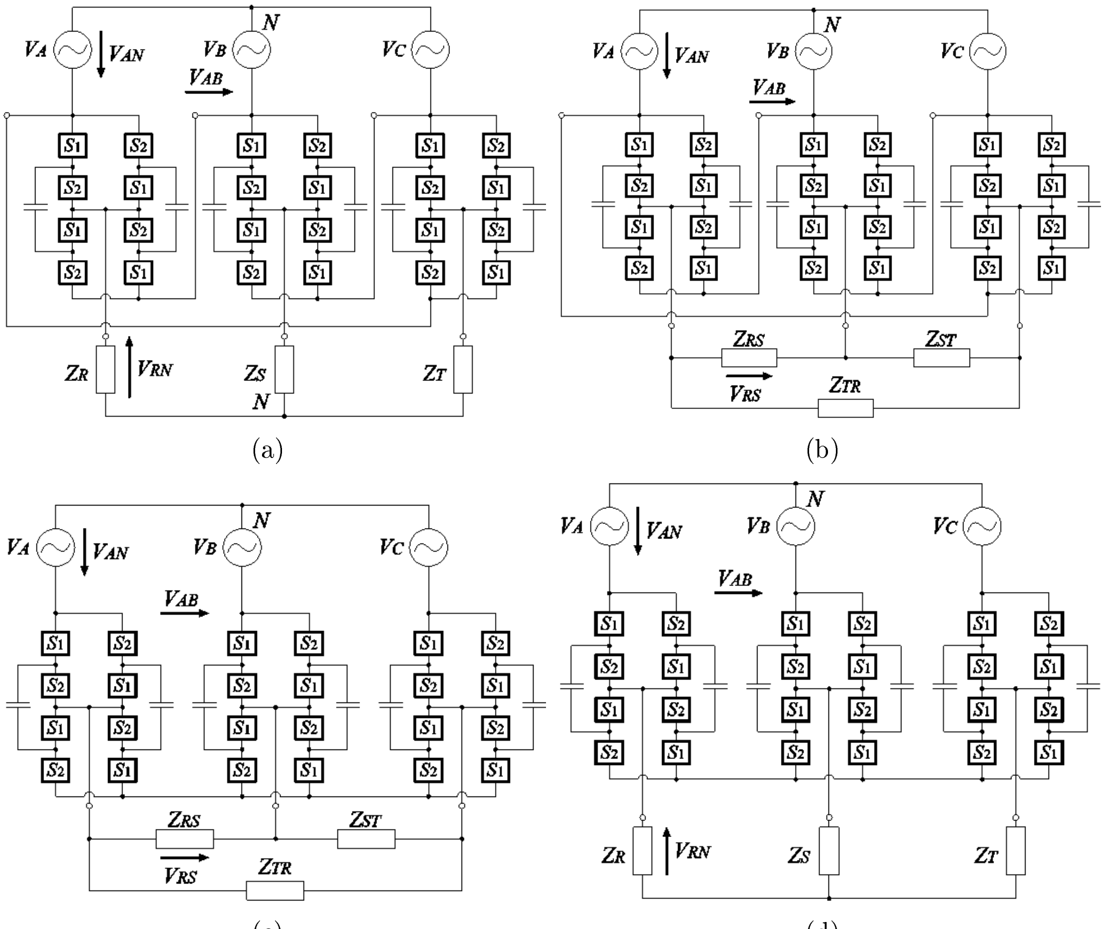
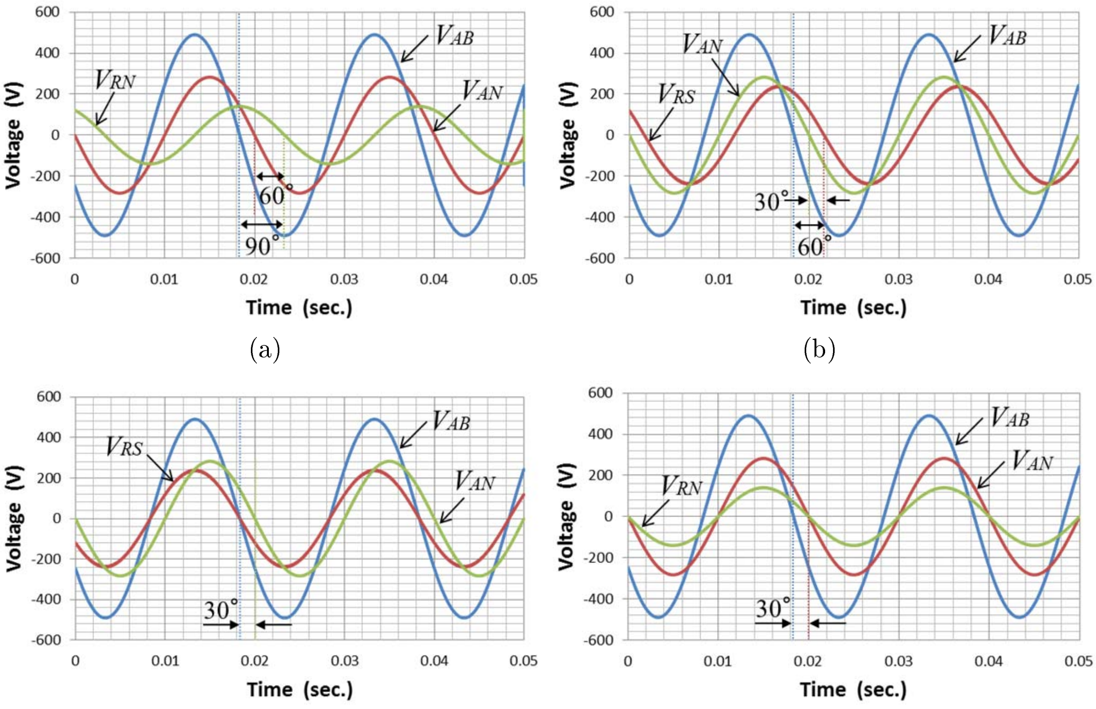


FIGURE 7. Circuit configuration of the proposed three-phase ac-ac converter with balanced resistive loads: (a) delta-wye configuration, (b) delta-delta configuration, (c) wye-delta configuration, and (d) wye-wye configuration



(a) (b)

(c) (d)

Figure 8. Simulated voltages with balanced resistive loads: (a) delta-wye configuration, (b) delta-delta configuration, (c) wye-delta configuration, and (d) wye-wye configuration

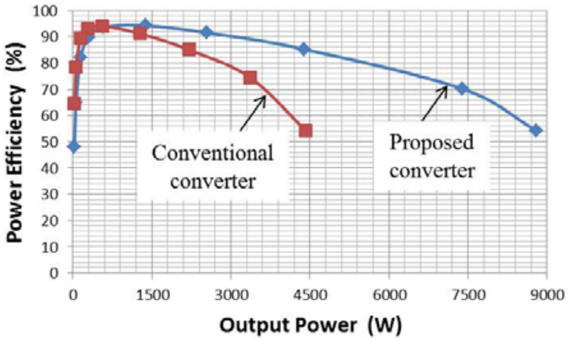


Figure 9. Simulated power efficiency with balanced resistive loads in the case of delta-wye configuration

than that of the conventional three-phase ac-ac converter. Concretely, the proposed converter can improve power efficiency more than 11% when the output power is 3kW. In the range of 300W to 4.5kW, the proposed ac-ac converter can achieve more than 85% power efficiency. Figure 10 demonstrates the simulated input power factor in the case of delta-wye connection. As Figure 10 shows, the input power factor of the proposed ac-ac converter is higher than that of the conventional ac-ac converter. Concretely, the proposed converter can improve input power factor about 0.4 when the output power is 3kW. The input power factor of the proposed ac-ac converter is more than 0.85 when the output power is higher than 3kW.

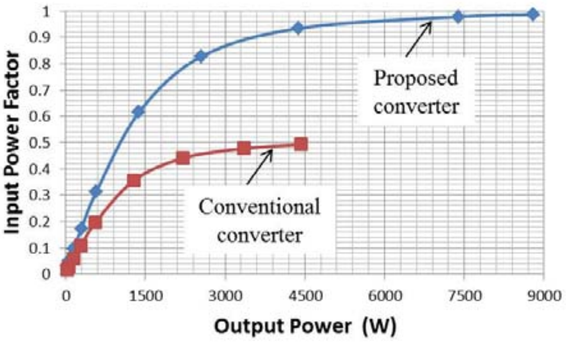


FIGURE 10. Simulated input power factor with balanced resistive loads in the case of delta-wye configuration

5. **Conclusions.** A novel three-phase SC ac-ac converter has been proposed in this paper. Unlike conventional SC ac-ac converters, the proposed converter has symmetrical topology without flying capacitors. Therefore, the proposed converter can reduce the number of capacitors from the conventional converter, though the circuit topology is complex. Concretely, the proposed three-phase ac-ac converter can reduce three capacitors from the conventional three-phase ac-ac converter.

The validity of circuit design was confirmed by SPICE simulations. The SPICE simulation demonstrated that the proposed three-phase ac-ac converter can offer the converted three-phase voltages in the case of delta-wye, delta-delta, wye-delta and wye-wye connections. Furthermore, the proposed ac-ac converter improved 11% power efficiency and 0.4 input power factor from the conventional ac-ac converter. As these results show, the proposed three-phase SC ac-ac converter can achieve not only high power efficiency but also input power factor.

The circuit evaluation through experiments is left to a future study.

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Appendix.

A.1. **Proposed converter in a conversion ratio of 2.** In a conversion ratio of 2, the characteristics of the proposed module are analyzed theoretically, where the ac input is assumed as a pulse ac waveform.

In the steady state, the differential value of electric charges in C_k $(k = 1, 2)$ satisfies (2). In State-T_1, $\Delta q_{T_1,v_i}$ and $\Delta q_{T_1,v_o}$ are obtained as

$$\Delta q_{T_1,v_i} = -\Delta q_{T_1}^1 + \Delta q_{T_1}^2 \text{ and } \Delta q_{T_1,v_o} = \Delta q_{T_1}^1. \tag{15}$$

On the other hand, in State-T_2, $\Delta q_{T_2,v_i}$ and $\Delta q_{T_2,v_o}$ are obtained as

$$\Delta q_{T_2,v_i} = \Delta q_{T_2}^1 - \Delta q_{T_2}^2 \text{ and } \Delta q_{T_2,v_o} = \Delta q_{T_2}^2. \tag{16}$$

Substituting (2), (6), (15) and (16) into (7), we have

$$i_i = -2i_o. \tag{17}$$

From (17), the conversion ratio is obtained as $m_1 = 2$.

Next, the consumed energy W_T in one period can be expressed as (9). Using (2), (6), (15) and (16), the consumed energy (9) is rewritten as

$$W_T = \frac{(\Delta q_{v_o})^2}{T} 4R_{on}. \tag{18}$$

Therefore, from (11) and (18), we have the SC resistance:

$$R_{SC} = 4R_{on}. \tag{19}$$

By combining (17) and (19), the equivalent circuit of the conventional module block can be expressed as

$$\begin{bmatrix} v_i \\ i_i \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4R_{on} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_o \\ -i_o \end{bmatrix}. \tag{20}$$

A.2. **Conventional converter in a conversion ratio of 1/2.** In a conversion ratio of 1/2, the differential value of electric charges in C_k $(k = 1, 2, 3)$ satisfies (2). In State-T_1, $\Delta q_{T_1,v_i}$ and $\Delta q_{T_1,v_o}$ are obtained as

$$\Delta q_{T_1,v_i} = \Delta q_{T_1}^1 + \Delta q_{T_1}^2 \text{ and } \Delta q_{T_1,v_o} = -\Delta q_{T_1}^1 - \Delta q_{T_1}^2 + \Delta q_{T_1}^3. \tag{21}$$

On the other hand, in State-T_2, $\Delta q_{T_2,v_i}$ and $\Delta q_{T_2,v_o}$ are obtained as

$$\Delta q_{T_2,v_i} = \Delta q_{T_2}^2 \text{ and } \Delta q_{T_2,v_o} = \Delta q_{T_2}^1 - \Delta q_{T_2}^2 + \Delta q_{T_2}^3. \tag{22}$$

Substituting (2), (6), (21) and (22) into (7), we have (8). Therefore, the conversion ratio m_1 is 1/2.

Next, the consumed energy W_T in one period can be expressed as

$$W_T = W_{T_1} + W_{T_2}, \tag{23}$$

where

$$W_{T_1} = \frac{\left(\Delta q^1_{T_1}\right)^2}{T_1} 2R_{on} \text{ and } W_{T_2} = \frac{\left(\Delta q^1_{T_2}\right)^2}{T_2} 2R_{on}.$$

Using (2), (6), (21) and (22), the consumed energy (23) is rewritten as

$$W_T = \frac{(\Delta q_{v_o})^2}{T} 2R_{on}. \tag{24}$$

Therefore, from (11) and (24), we have the SC resistance:

$$R_{SC} = 2R_{on}. \tag{25}$$

By combining (8) and (25), the equivalent circuit of the conventional module can be expressed as

$$\begin{bmatrix} v_i \\ i_i \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 2R_{on} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_o \\ -i_o \end{bmatrix}. \tag{26}$$

A.3. **Conventional converter in a conversion ratio of 2.** In a conversion ratio of 2, the differential value of electric charges in C_k $(k = 1, 2, 3)$ satisfies (2). In State-T_1, $\Delta q_{T_1,v_i}$ and $\Delta q_{T_1,v_o}$ are obtained as

$$\Delta q_{T_1,v_i} = \Delta q^3_{T_1} - \Delta q^1_{T_1} - \Delta q^2_{T_1} \text{ and } \Delta q_{T_1,v_o} = \Delta q^1_{T_1} + \Delta q^2_{T_1}. \tag{27}$$

On the other hand, in State-T_2, $\Delta q_{T_2,v_i}$ and $\Delta q_{T_2,v_o}$ are obtained as

$$\Delta q_{T_2,v_i} = \Delta q^1_{T_2} - \Delta q^2_{T_2} + \Delta q^3_{T_2} \text{ and } \Delta q_{T_2,v_o} = \Delta q^2_{T_2}. \tag{28}$$

Substituting (2), (6), (27) and (28) into (7), we have (17). Therefore, the conversion ratio m_1 is 2.

Next, the consumed energy W_T in one period can be expressed as (23). Using (2), (6), (21) and (22), the consumed energy (23) is rewritten as

$$W_T = \frac{(\Delta q_{v_o})^2}{T} 8R_{on}. \tag{29}$$

Therefore, from (17) and (29), we have the SC resistance:

$$R_{SC} = 8R_{on}. \tag{30}$$

By combining (17) and (30), the equivalent circuit of the conventional module can be expressed as

$$\begin{bmatrix} v_i \\ i_i \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 8R_{on} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_o \\ -i_o \end{bmatrix}. \tag{31}$$