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ROBUST H_∞ CONTROL FOR FUZZY TIME-DELAY SYSTEMS WITH PARAMETER UNCERTAINTIES – DELAY DEPENDENT CASE

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Abstract. *This paper is concerned with the problem of delay-dependent robust H_∞ control for uncertain Takagi-Sugeno (T-S) fuzzy systems with time-delay. The methodology is based on the direct Lyapunov method allied with a new Lyapunov fuctional choice. A fuzzy time-delay feedback controller is used to ensure the required H_∞ performance of the system to be achieved. The proposed stability conditions are derived in terms of linear matrix inequalities (LMIs). Finally, numerical examples are provided to illustrate the effectiveness of the proposed method.*

Keywords: Delay-dependent, H_∞ control, Stability, Linear matrix inequality (LMI), State time-delay, Parameter uncertainty

1. **Introduction.** Time-delay systems, also called systems with after-effect, have been a popular and challenging research area for decades. These kinds of systems can be found in many real life systems, such as electric power systems, neural networks, rolling mill systems, economic systems, aerospace systems, different types of societal systems and ecological systems. The uncertainties which include modeling error, parameter perturbations, approximation errors and external disturbances may enter a nonlinear system in a much more complex way. Both time delay and uncertainty are often a source of instability and degradation in control performance in many control systems. Hence, the stability analysis and the robust H_∞ control problem of time-delay systems with uncertainties have been studied in much literature (see for instance, [1, 2, 5, 6, 7, 9, 10, 11, 12, 15, 16, 18, 21, 22, 23, 24, 25, 26] and the references therein).

Depending on whether the existence condition of H_∞ controller includes the information of delay or not, stability criteria can be classified into two types: delay-dependent ones [1, 9, 11, 17, 19, 21, 22, 24, 25] and delay-independent ones [2, 3, 5, 7, 8, 15, 18]. Both of them have their own advantages. The delay-independent results are particularly good to deal with the systems without any information on the time delays, or even time-varying time delay. As the time delay is considered during the stability analysis, the delay-dependent result is less conservative comparatively, especially when the value of time delay is small. However, it can be seen that the delay-independent and delay-dependent results cannot replace each other. For delay-dependent case, the stability conditions always require the upper bound of derivative of the time-varying delay less than 1. In this paper, our result can avoid this restriction.

Fuzzy system model and theory [13, 14] have attracted a great deal of interest for system analysis and synthesis. It is a useful method to represent complex nonlinear systems by some fuzzy sets and reasoning. When the nonlinear plant is represented by a so-called Takagi-Sugeno (T-S) type fuzzy model, local dynamics in different state-space regions is represented by linear model. Then the system has a convenient dynamic structure so that some well-established linear systems theory can be easily applied for theoretical analysis of the overall closed-loop controlled system. For example, the direct Lyapunov method is a powerful tool for studying the problems of stability and H_∞ control for the systems mentioned above.

In this paper, we will consider the problem of H_∞ control for uncertain time-delay systems. Based on Lyapunov functional approach, a delay-dependent condition for the existence of a state time-delay feedback controller, which ensures asymptotic stability and a prescribed H_∞ performance level of these systems is obtained. The major contribution of our work is as follows. First, when the states of systems are measurable, we present a design method of state time-delay feedback controller for uncertain fuzzy systems with time-delay. Second, the design method of H_∞ controller is delay-dependent which can be used to study the stabilization of systems and to determine the maximal allowed value of time-delay. Third, the delay-dependent results can be used to determine the upper bound of time-delay to guarantee the robust H_∞ fuzzy stablizable of systems. These results are less conservative than those for the delay-dependent cases mentioned before. Fourth, all the results are given by LMIs, and they can be directly calculated by MATLAB LMI Toolbox. Fifth, our results can also be used to analyze the stability conditions of fuzzy time-delay systems without uncertainties.

The paper is organized as follows. In Section 2, a T-S fuzzy model is used to describe a time-delay systems with parameter uncertainties. In Section 3, based on Lyapunov functional approach, the existence conditions of a robust state time-delay feedback H_∞ controller are obtained in LMI form. All the results are delay-dependent. In Section 4, numerical examples are given to show the effectiveness of the obtained results. Section 5 concludes the paper.

Notation. For a symmetric matrix X, the notation $X > 0$ means that the matrix X is positive definite. I is an identity matrix of appropriate dimension. X^T denotes the transpose of matrix X. For any nonsingular matrix X, X^{-1} denotes the inverse of matrix X. R^n denotes the n-dimensional Euclidean space. $R^{m\times n}$ is the set of all $m \times n$ matrices. $*$ denotes the transposed element in the symmetric position of a matrix.

2. **System Description.**

In this section, we will introduce some related concepts. Consider the following parameter uncertain system with time-delay described by Takagi-Sugeno fuzzy model [13]:

Plant Rule i: If $z_1(t)$ is λ_{i1}, $z_2(t)$ is λ_{i2}, $\cdots$, $z_g(t)$ is λ_{ig}, then

$$\begin{cases} \dot{x}(t) = \tilde{A}_{i1}x(t) + \tilde{A}_{i2}x(t-d) + \tilde{B}_i u(t) + B_{\omega i}\omega(t), \\ \tilde{z}(t) = \tilde{C}_{i1}x(t) + \tilde{C}_{i2}x(t-d) + \tilde{D}_i u(t), \\ x(t) = \varphi(t), \qquad t \in [-d, 0], \end{cases} \tag{1}$$

where $i = 1, 2, \cdots, n$, n is the number of rules; $z_1(t), z_2(t), \cdots, z_g(t)$ are the premise variables; λ_{ij} $(i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, g)$ is the fuzzy set; $x(t) \in R^q$ is the state vector; $u(t) \in R^m$ is the input vector; $\omega(t)$ is the disturbance which belongs to $L_2[0, \infty)$; $\tilde{z}(t) \in R^l$ is the controlled output; $d > 0$ is the upper bound of time-delay; $\varphi(t)$ is the initial condition of system (1); $\tilde{A}_{i1} = A_{i1} + \Delta A_{i1}(t)$, $\tilde{A}_{i2} = A_{i2} + \Delta A_{i2}(t)$, $\tilde{B}_i = B_i + \Delta B_i(t)$, $\tilde{C}_{i1} = C_{i1} + \Delta C_{i1}(t)$, $\tilde{C}_{i2} = C_{i2} + \Delta C_{i2}(t)$, $\tilde{D}_i = D_i + \Delta D_i(t)$; A_{i1}, A_{i2}, B_i, C_{i1}, C_{i2} and D_i $(i = 1, 2, \cdots, n)$ are constant matrices of appropriate dimensions; $\Delta A_{i1}(t)$, $\Delta A_{i2}(t)$, $\Delta B_i(t)$, $\Delta C_{i1}(t)$, $\Delta C_{i2}(t)$, $\Delta D_i(t)$ $(i = 1, 2, \cdots, n)$ are realvalued unknown matrices representing time-varying parameter uncertainties of (1) and satisfying the following assumption.

Assumption 2.1.

$$\left[\Delta A_{i1}(t), \Delta A_{i2}(t), \Delta B_i(t)\right] = U_i F_i(t) \left[E_{i1}, E_{i2}, E_i\right], \tag{2}$$

$$\left[\Delta C_{i1}(t), \Delta C_{i2}(t), \Delta D_i(t)\right] = H_i V_i(t) \left[G_{i1}, G_{i2}, G_i\right], \tag{3}$$

where U_i, E_{i1}, E_{i2}, E_i, H_i, G_{i1}, G_{i2} and G_i ($i = 1, 2, \cdots, n$) are known real constant matrices of appropriate dimensions. $F_i(t)$ and $V_i(t)$ ($i = 1, 2, \cdots, n$) are unknown real time-varying matrices with Lebesgue measurable elements satisfying

$$F_i^T(t) F_i(t) \leq I, \quad V_i^T(t) V_i(t) \leq I, \quad i = 1, 2, \cdots, n. \tag{4}$$

Let $\mu_i(z(t))$ be the normalized membership function of the inferred fuzzy set $\rho_i(z(t))$, i.e.,

$$\mu_i(z(t)) = \frac{\rho_i(z(t))}{\sum_{i=1}^{n} \rho_i(z(t))},$$

where $z(t) = [z_1(t), z_2(t), \cdots, z_g(t)]$, $\rho_i(z(t)) = \prod_{j=1}^{g} \lambda_{ij}(z_j(t))$. $\lambda_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in λ_{ij}. It is assumed that

$$\rho_i(z(t)) \geq 0, \quad i = 1, 2, \cdots, n, \quad \sum_{i=1}^{n} \rho_i(z(t)) > 0, \quad \forall t \geq 0.$$

Then, it can be seen that

$$\mu_i(z(t)) \geq 0, \quad i = 1, 2, \cdots, n, \quad \sum_{i=1}^{n} \mu_i(z(t)) = 1, \quad \forall t \geq 0.$$

By using the center-average defuzzifier, product inference and singleton fuzzifier, the T-S fuzzy model (1) can be expressed by the following model:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{n} \mu_i(z(t)) \Big[\tilde{A}_{i1} x(t) + \tilde{A}_{i2} x(t-d) + \tilde{B}_i u(t) + B_{\omega i} \omega(t) \Big], \\ \tilde{z}(t) = \sum\limits_{i=1}^{n} \mu_i(z(t)) \Big[\tilde{C}_{i1} x(t) + \tilde{C}_{i2} x(t-d) + \tilde{D}_i u(t) \Big], \\ x(t) = \varphi(t), \qquad t \in [-d, 0], \end{cases} \tag{5}$$

In this paper, state time-delay feedback T-S fuzzy-model-based H_∞ controller will be designed for the robust stabilization of system (5). The ith controller rule is

Plant Rule i: If $z_1(t)$ is λ_{i1}, $z_2(t)$ is λ_{i2}, $\cdots$, $z_g(t)$ is λ_{ig}, then

$$u(t) = K_{i1} x(t) + K_{i2} x(t-d), \tag{6}$$

where K_{i1} and K_{i2} ($i = 1, 2, \cdots, n$) are the controller gains of (6) to be determined. The defuzzified output of the controller rules is given by

$$u(t) = \sum_{i=1}^{n} \mu_i(z(t)) \Big[K_{i1} x(t) + K_{i2} x(t-d) \Big]. \tag{7}$$

Combining (5) and (7), the closed-loop fuzzy system can be obtained as

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{n} \mu_i \mu_j \Big[\left(\tilde{A}_{i1} + \tilde{B}_i K_{j1} \right) x(t) + \left(\tilde{A}_{i2} + \tilde{B}_i K_{j2} \right) x(t-d) + B_{\omega i} \omega(t) \Big], \\ \tilde{z}(t) = \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{n} \mu_i \mu_j \Big[\left(\tilde{C}_{i1} + \tilde{D}_i K_{j1} \right) x(t) + \left(\tilde{C}_{i2} + \tilde{D}_i K_{j2} \right) x(t-d) \Big], \\ x(t) = \varphi(t), \qquad t \in [-d, 0], \end{cases} \tag{8}$$

where $\mu_i = \mu_i(z(t))$ for short.

In order to study the design method of state time-delay feedback H_∞ controller, we always consider the following performance index.

Definition 2.1. *For a prescribed scalar $\gamma > 0$, define the performance index as*

$$J(\omega) = \int_0^\infty \left[\tilde{z}^T(\tau)\tilde{z}(\tau) - \gamma^2\omega^T(\tau)\omega(\tau)\right] d\tau. \tag{9}$$

Remark 2.1. *The purpose of this paper is to design a robust H_∞ controller (7) for the T-S fuzzy system (5) such that for all admissible uncertainties satisfying (2), (3), (4) and for a prescribed scalar $\gamma > 0$,*
[a] the closed-loop fuzzy system (8) is asymptotically stable when $\omega(t) = 0$;
[b] for all nonzero $\omega(t) \in L_2[0,\infty)$ under the zero initial condition, the closed-loop fuzzy system (8) satisfies $\|\tilde{z}(t)\|_2 < \gamma\|\omega(t)\|_2$.

In this paper, for simplicity, let

$$\tilde{S}_{ij} = \tilde{A}_{i1} + \tilde{B}_iK_{j1},\ \tilde{T}_{ij} = \tilde{A}_{i2} + \tilde{B}_iK_{j2},\ \tilde{M}_{ij} = \tilde{C}_{i1} + \tilde{D}_iK_{j1},\ \tilde{N}_{ij} = \tilde{C}_{i2} + \tilde{D}_iK_{j2},$$
$$S_{ij} = A_{i1} + B_iK_{j1},\ T_{ij} = A_{i2} + B_iK_{j2},\ M_{ij} = C_{i1} + D_iK_{j1},\ N_{ij} = C_{i2} + D_iK_{j2},$$
$$\tilde{W}_{ij} = \tilde{A}_{i1} + \tilde{A}_{i2} + \tilde{B}_iK_{j1},\ W_{ij} = A_{i1} + A_{i2} + B_iK_{j1}.$$

3. **Main Results.** In this section, based on the Lyapunov approach, we will present a new method to design the robust H_∞ controller for uncertain time delay systems. First, three important lemmas are presented as follows because they are the key to proving the main theorems.

Lemma 3.1 ([4])**.** *For any two vectors $x(t), y(t) \in R^n$, we have*

$$2x^T(t)y(t) \le x^T(t)G^{-1}x(t) + y^T(t)Gy(t),$$

where $G \in R^{n\times n}$ and $G > 0$.

Lemma 3.2 ([20])**.** *Y, U and E are the matrices of appropriate dimensions, and $Y = Y^T$, then for any matrix F satisfying $F^TF \le I$, we have the following equivalent condition*

$$Y + UFE + E^TF^TU^T < 0$$

if and only if there exists a constant $\varepsilon > 0$ satisfying

$$Y + \varepsilon UU^T + \varepsilon^{-1}E^TE < 0.$$

Lemma 3.3 ([20])**.** *(Schur complements) For a symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, the following conditions are equivalent:*
[i] $S < 0$;
[ii] $S_{11} < 0,\ S_{22} - S_{12}^TS_{11}^{-1}S_{12} < 0$;
[iii] $S_{22} < 0,\ S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.

Firstly, we consider the following closed-loop fuzzy system.

$$\dot{x}(t) = \sum_{i=1}^n\sum_{j=1}^n \mu_i\mu_j\left[\tilde{S}_{ij}x(t) + \tilde{T}_{ij}x(t-d) + B_{\omega i}\omega(t)\right]. \tag{10}$$

Using the Newton-Leibniz formula $\int_{-d}^0 \dot{x}(t+\theta)\mathrm{d}\theta = x(t) - x(t-d)$, we can have an equivalent form of fuzzy systems (10) as follows:

$$\dot{x}(t) = \sum_{i=1}^n\sum_{j=1}^n \mu_i\mu_j\left[\tilde{W}_{ij}x(t) + \tilde{B}_iK_{j2}x(t-d) - \tilde{A}_{i2}\int_{-d}^0 \dot{x}(t+\theta)\mathrm{d}\theta + B_{\omega i}\omega(t)\right]. \tag{11}$$

When the states are measurable, based on the Lyapunov functional approach, the stabilization results of system (10) while $\omega(t) = 0$ are summarized in the following theorem.

Theorem 3.1. *Suppose $\omega(t) = 0$. For a given positive scalar d_M such that $d \in [0, d_M]$, if there exist matrices $P > 0$, $Q > 0$ and $R > 0$ of appropriate dimensions such that*

$$\begin{bmatrix} P\tilde{W}_{ii} + \tilde{W}_{ii}^T P & * & * & * & * \\ K_{i2}^T \tilde{B}_i^T P & -Q & * & * & * \\ \tilde{A}_{i2}^T P & 0 & -d_M^{-1} R & * & * \\ \tilde{S}_{ii} & \tilde{T}_{ii} & 0 & -d_M^{-1} R^{-1} & * \\ I & 0 & 0 & 0 & -Q^{-1} \end{bmatrix} < 0, \quad 1 \le i \le n \tag{12}$$

$$\begin{bmatrix} P\left(\tilde{W}_{ij} + \tilde{W}_{ji}\right) + \left(\tilde{W}_{ij} + \tilde{W}_{ji}\right)^T P & * & * & * & * & * \\ K_{j2}^T \tilde{B}_i^T P + K_{i2}^T \tilde{B}_j^T P & -2Q & * & * & * & * \\ \tilde{A}_{i2}^T P & 0 & -d_M^{-1} R & * & * & * \\ \tilde{A}_{j2}^T P & 0 & 0 & -d_M^{-1} R & * & * \\ \tilde{S}_{ij} + \tilde{S}_{ji} & \tilde{T}_{ij} + \tilde{T}_{ji} & 0 & 0 & -2d_M^{-1} R^{-1} & * \\ I & 0 & 0 & 0 & 0 & -0.5Q^{-1} \end{bmatrix} < 0,$$

$$1 \le i < j \le n \tag{13}$$

then system (10) is asymptotically stable.

Proof: Choose the Lyapunov function as

$$V(x(t)) = x^T(t)Px(t) + \int_{t-d}^{t} x^T(s)Qx(s)\mathrm{d}s + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$

by (11), then the derivative of $V(x(t))$ is

$$\begin{aligned} \dot{V}(x(t)) &= 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-d)Qx(t-d) + d\dot{x}^T(t)R\dot{x}(t) \\ &\quad - \int_{-d}^{0} \dot{x}^T(t+\theta)Q\dot{x}(t+\theta)\mathrm{d}\theta \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i\mu_j \Big[2x^T(t)P\tilde{W}_{ij}x(t) + x^T(t)Qx(t) - x^T(t-d)Qx(t-d) \\ &\quad + d\dot{x}^T(t)R\dot{x}(t) + 2x^T(t)P\tilde{B}_iK_{j2}x(t-d) - 2x^T(t)P\tilde{A}_{i2}\int_{-d}^{0}\dot{x}(t+\theta)\mathrm{d}\theta \\ &\quad - \int_{-d}^{0} \dot{x}^T(t+\theta)R\dot{x}(t+\theta)\mathrm{d}\theta \Big]. \end{aligned} \tag{14}$$

By (10), we have

$$\begin{aligned} d\dot{x}^T(t)R\dot{x}(t) &= d\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} \mu_i\mu_j\mu_k\mu_l \Big[\tilde{S}_{ij}x(t) + \tilde{T}_{ij}x(t-d) + B_{\omega i}\omega(t)\Big]^T R \Big[\tilde{S}_{kl}x(t) \\ &\quad + \tilde{T}_{kl}x(t-d) + B_{\omega i}\omega(t)\Big] \\ &\le d\sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i\mu_j \Big[\tilde{S}_{ij}x(t) + \tilde{T}_{ij}x(t-d) + B_{\omega i}\omega(t)\Big]^T R \Big[\tilde{S}_{ij}x(t) + \tilde{T}_{ij}x(t-d) \\ &\quad + B_{\omega i}\omega(t)\Big]. \end{aligned} \tag{15}$$

By Lemma 3.1, we can obtain

$$\begin{aligned}
&-\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j 2x^T(t)P\tilde{A}_{i2}\int_{-d}^{0}\dot{x}(t+\theta)\mathrm{d}\theta \\
&= -\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j\int_{-d}^{0}2x^T(t)P\tilde{A}_{i2}\dot{x}(t+\theta)\mathrm{d}\theta \\
&\le \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j\left[dx^T(t)P\tilde{A}_{i2}R^{-1}\tilde{A}_{i2}^TPx(t)+\int_{-d}^{0}\dot{x}^T(t+\theta)R\dot{x}(t+\theta)\mathrm{d}\theta\right].
\end{aligned} \tag{16}$$

Substituting (15) and (16) into (14), we have

$$\begin{aligned}
\dot{V}(x(t)) &\le \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j\xi^T(t)\tilde{\Omega}_{ij}\xi(t) \\
&= \sum_{i=1}^{n}\mu_i^2\xi^T(t)\tilde{\Omega}_{ii}\xi(t)+\sum_{i=1}^{n-1}\sum_{j>i}^{n}\mu_i\mu_j\xi^T(t)\left(\tilde{\Omega}_{ij}+\tilde{\Omega}_{ji}\right)\xi(t)
\end{aligned} \tag{17}$$

where $\xi^T(t)=\begin{bmatrix}x^T(t) & x^T(t-d) & \omega^T(t)\end{bmatrix}$,

$$\tilde{\Omega}_{ij}=\begin{bmatrix}P\tilde{W}_{ij}+\tilde{W}_{ij}^TP+Q+d\tilde{S}_{ij}^TR\tilde{S}_{ij}+dP\tilde{A}_{i2}R^{-1}\tilde{A}_{i2}^TP & * & * \\ K_{j2}^T\tilde{B}_i^TP+d\tilde{T}_{ij}^TR\tilde{S}_{ij} & -Q+d\tilde{T}_{ij}^TR\tilde{T}_{ij} & * \\ B_{\omega i}^TP+dB_{\omega i}^TR\tilde{S}_{ij} & dB_{\omega i}^TR\tilde{T}_{ij} & dB_{\omega i}^TRB_{\omega i}\end{bmatrix}.$$

By setting $\omega(t)=0$ and using Schur complements, we can easily obtain (18) and (19) such that $\dot{V}(x(t))<0$.

$$\begin{bmatrix}P\tilde{W}_{ii}+\tilde{W}_{ii}^TP & * & * & * & * \\ K_{i2}^T\tilde{B}_i^TP & -Q & * & * & * \\ \tilde{A}_{i2}^TP & 0 & -d^{-1}R & * & * \\ \tilde{S}_{ii} & \tilde{T}_{ii} & 0 & -d^{-1}R^{-1} & * \\ I & 0 & 0 & 0 & -Q^{-1}\end{bmatrix}<0, \quad 1\le i\le n \tag{18}$$

$$\begin{bmatrix}P\left(\tilde{W}_{ij}+\tilde{W}_{ji}\right)+\left(\tilde{W}_{ij}+\tilde{W}_{ji}\right)^TP & * & * & * & * & * \\ K_{j2}^T\tilde{B}_i^TP+K_{i2}^T\tilde{B}_j^TP & -2Q & * & * & * & * \\ \tilde{A}_{i2}^TP & 0 & -d^{-1}R & * & * & * \\ \tilde{A}_{j2}^TP & 0 & 0 & -d^{-1}R & * & * \\ \tilde{S}_{ij}+\tilde{S}_{ji} & \tilde{T}_{ij}+\tilde{T}_{ji} & 0 & 0 & -2d^{-1}R^{-1} & * \\ I & 0 & 0 & 0 & 0 & -0.5Q^{-1}\end{bmatrix}<0,$$

$$1\le i<j\le n. \tag{19}$$

Note that the matrices in (18) and (19) are monotonic increasing with respect to $d>0$. If there exist a positive scalar d_M such that $d\in[0,d_M]$, then $\dot{V}(x(t))<0$ still holds when '$-d^{-1}$' in (18) and (19) are replaced by '$-d_M^{-1}$'. Then we can finish the proof.

Then we study the design method of state time-delay feedback H_∞ controller of system (1). For H_∞ control, we always consider the performance index $J(\omega)$ (see Equation (9)) under zero initial condition.

Theorem 3.2. *For a prescribed constant $\gamma > 0$ and a positive scalar d_M such that $d \in [0, d_M]$, if there exist $P > 0$, $Q > 0$ and $Q > 0$ satisfying the following matrix inequalities, then $J(\omega) < 0$.*

$$\begin{bmatrix} P\tilde{W}_{ii} + \tilde{W}_{ii}^T P & * & * & * & * & * & * \\ K_{i2}^T \tilde{B}_i^T P & -Q & * & * & * & * & * \\ B_{\omega i}^T P & 0 & -\gamma^2 I & * & * & * & * \\ \tilde{A}_{i2}^T P & 0 & 0 & -d_M^{-1} R & * & * & * \\ \tilde{S}_{ii} & \tilde{T}_{ii} & B_{\omega i} & 0 & -d_M^{-1} R^{-1} & * & * \\ \tilde{M}_{ii} & \tilde{N}_{ii} & 0 & 0 & 0 & -I & * \\ I & 0 & 0 & 0 & 0 & 0 & -Q^{-1} \end{bmatrix} < 0, \quad 1 \le i \le n \tag{20}$$

$$\begin{bmatrix} P\left(\tilde{W}_{ij} + \tilde{W}_{ji}\right) + \left(\tilde{W}_{ij} + \tilde{W}_{ji}\right)^T P & * & * & * & * & * & * & * \\ \left(\tilde{B}_i K_{j2} + \tilde{B}_j K_{i2}\right)^T P & -2Q & * & * & * & * & * & * \\ (B_{\omega i} + B_{\omega j})^T P & 0 & -2\gamma^2 I & * & * & * & * & * \\ \tilde{A}_{i2}^T P & 0 & 0 & -d_M^{-1} R & * & * & * & * \\ \tilde{A}_{j2}^T P & 0 & 0 & 0 & -d_M^{-1} R & * & * & * \\ \tilde{S}_{ij} + \tilde{S}_{ji} & \tilde{T}_{ij} + \tilde{T}_{ji} & B_{\omega i} + B_{\omega j} & 0 & 0 & -2d_M^{-1} R^{-1} & * & * \\ \tilde{M}_{ij} + \tilde{M}_{ji} & \tilde{N}_{ij} + \tilde{N}_{ji} & 0 & 0 & 0 & 0 & -2I & * \\ I & 0 & 0 & 0 & 0 & 0 & 0 & -0.5Q^{-1} \end{bmatrix} < 0,$$

$$1 \le i < j \le n \tag{21}$$

Proof: By (8), we have

$$\begin{aligned} &\tilde{z}^T(t)\tilde{z}(t) - \gamma^2 \omega^T(t)\omega(t) \\ \le &\sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i \mu_j \Big[x^T(t) \tilde{M}_{ij}^T \tilde{M}_{ij} x(t) + 2x^T(t) \tilde{M}_{ij}^T \tilde{N}_{ij} x(t-d) \\ &+ x^T(t-d) \tilde{N}_{ij}^T \tilde{N}_{ij} x(t-d) - \gamma^2 \omega^T(t)\omega(t) \Big] \\ = &\sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i \mu_j \left\{ \xi^T(t) \begin{bmatrix} \tilde{M}_{ij}^T \\ \tilde{N}_{ij}^T \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{M}_{ij} & \tilde{N}_{ij} & 0 \end{bmatrix} \xi(t) - \gamma^2 \omega^T(t)\omega(t) \right\}. \end{aligned} \tag{22}$$

On the other hand, under zero initial condition, we can obtain that

$$\begin{aligned} J(\omega) &= \int_0^\infty \left[\tilde{z}^T(\tau)\tilde{z}(\tau) - \gamma^2 \omega^T(\tau)\omega(\tau) \right] \mathrm{d}\tau \\ &= \int_0^\infty \left[\tilde{z}^T(\tau)\tilde{z}(\tau) - \gamma^2 \omega^T(\tau)\omega(\tau) + \dot{V}(x(\tau)) \right] \mathrm{d}\tau - V(x(\infty)) \\ &\le \int_0^\infty \left[\tilde{z}^T(\tau)\tilde{z}(\tau) - \gamma^2 \omega^T(\tau)\omega(\tau) + \dot{V}(x(\tau)) \right] \mathrm{d}\tau \end{aligned} \tag{23}$$

Substituting (17), (22) into (23), by schur complements, we can complete the proof.

Remark 3.1. *It is easy to see that (20) implies (12), and (21) implies (13).*

Noting that the parameter uncertainties are contained in (20) and (21). So Theorem 3.2 cannot be directly used to determine whether $J(\omega) < 0$. By combining Remark 2.1, Remark 3.1 and Theorem 3.2, we propose a new design method of state time-delay robust H_∞ controller in the following theorem, and the results are delay-dependent.

Theorem 3.3. *For a prescribed scalar $\gamma > 0$ and a scalar $d_M > 0$ such that $d \in [0, d_M]$, T-S fuzzy system (8) is stable and satisfies $\|\tilde{z}(t)\|_2 < \gamma\|\omega(t)\|_2$ for all nonzero $\omega(t) \in L_2[0,\infty)$ under the zero initial condition, if there exist matrices $X > 0$, $Y > 0$, $Z > 0$, L_{j1} and L_{j2} $(j = 1, 2, \cdots, n)$ of appropriate dimensions and positive constants ε_{ij} $(i, j = 1, 2, \cdots, n)$ such that the following LMIs simultaneously hold:*

$$\begin{bmatrix} \Phi_{11}^{ii} & * & * \\ \Phi_{21}^{ii} & \Phi_{22}^{ii} & * \\ \Phi_{31}^{ii} & 0 & \Phi_{33}^{ii} \end{bmatrix} < 0, \quad 1 \le i \le n, \tag{24}$$

$$\begin{bmatrix} \Psi_{11}^{ij} & * & * \\ \Psi_{21}^{ij} & \Psi_{22}^{ij} & * \\ \Psi_{31}^{ij} & 0 & \Psi_{33}^{ij} \end{bmatrix} < 0, \quad 1 \le i < j \le n, \tag{25}$$

$$\Phi_{11}^{ii} = \begin{bmatrix} A_{i1}X + A_{i2}X + B_iL_{i1} + (A_{i1}X + A_{i2}X + B_iL_{i1})^T + \varepsilon_{ii}U_iU_i^T & * & * & * \\ L_{i2}^TB_i^T & -Y & * & * \\ B_{\omega i}^T & 0 & -\gamma^2 I & * \\ ZA_{i2}^T & 0 & 0 & -d_M^{-1}Z \end{bmatrix},$$

$$\Phi_{21}^{ii} = \begin{bmatrix} A_{i1}X + B_iL_{i1} & A_{i2}Y + B_iL_{i2} & B_{\omega i} & 0 \\ C_{i1}X + D_iL_{i1} & C_{i2}Y + D_iL_{i2} & 0 & 0 \\ X & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_{22}^{ii} = \begin{bmatrix} -d_M^{-1}Z + \varepsilon_{ii}U_iU_i^T & * & * \\ 0 & -I + \varepsilon_{ii}H_iH_i^T & * \\ 0 & 0 & -Y \end{bmatrix},$$

$$\Phi_{31}^{ii} = \begin{bmatrix} E_{i1}X + E_{i2}X + E_iL_{i1} & E_iL_{i2} & 0 & E_{i2}Z \\ E_{i1}X + E_iL_{i1} & E_{i2}Y + E_iL_{i2} & 0 & 0 \\ G_{i1}X + G_iL_{i1} & G_{i2}Y + G_iL_{i2} & 0 & 0 \end{bmatrix},$$

$$\Phi_{33}^{ii} = diag\Big\{ -\varepsilon_{ii}I, -\varepsilon_{ii}I, -\varepsilon_{ii}I \Big\},$$

$$\Psi_{11}^{ij} = \begin{bmatrix} \Lambda_{ij} & * & * & * & * \\ (B_iL_{j2} + B_jL_{i2})^T & -2Y & * & * & * \\ (B_{\omega i} + B_{\omega j})^T & 0 & -2\gamma^2 I & * & * \\ ZA_{i2}^T & 0 & 0 & -d_M^{-1}Z & * \\ ZA_{j2}^T & 0 & 0 & 0 & -d_M^{-1}Z \end{bmatrix},$$

$$\begin{aligned} \Lambda_{ij} = {} & (A_{i1} + A_{j1} + A_{i2} + A_{j2})X + B_iL_{j1} + B_jL_{i1} + ((A_{i1} + A_{j1} + A_{i2} + A_{j2})X \\ & + B_iL_{j1} + B_jL_{i1})^T + \varepsilon_{ij}U_iU_i^T + \varepsilon_{ji}U_jU_j^T, \end{aligned}$$

$$\Psi_{21}^{ij} = \begin{bmatrix} (A_{i1} + A_{j1})X + B_iL_{j1} + B_jL_{i1} & (A_{i2} + A_{j2})Y + B_iL_{j2} + B_jL_{i2} & B_{\omega i} + B_{\omega j} & 0 & 0 \\ (C_{i1} + C_{j1})X + D_iL_{j1} + D_jL_{i1} & (C_{i2} + C_{j2})Y + D_iL_{j2} + D_jL_{i2} & 0 & 0 & 0 \\ X & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{22}^{ij} = \begin{bmatrix} -2d_M^{-1}Z + \varepsilon_{ij}U_iU_i^T + \varepsilon_{ji}U_jU_j^T & * & * \\ 0 & -2I + \varepsilon_{ij}H_iH_i^T + \varepsilon_{ji}H_jH_j^T & * \\ 0 & 0 & -0.5Y \end{bmatrix},$$

$$\Psi_{31}^{ij} = \begin{bmatrix} E_{i1}X + E_{i2}X + E_iL_{j1} & E_iL_{j2} & 0 & E_{i2}Z & 0 \\ E_{i1}X + E_iL_{j1} & E_{i2}Y + E_iL_{j2} & 0 & 0 & 0 \\ G_{i1}X + G_iL_{j1} & G_{i2}Y + G_iL_{j2} & 0 & 0 & 0 \\ E_{j1}X + E_{j2}X + E_jL_{i1} & E_jL_{i2} & 0 & 0 & E_{j2}Z \\ E_{j1}X + E_jL_{i1} & E_{j2}Y + E_jL_{i2} & 0 & 0 & 0 \\ G_{j1}X + G_jL_{i1} & G_{j2}Y + G_jL_{i2} & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{33}^{ij} = diag\left\{-\varepsilon_{ij}I, -\varepsilon_{ij}I, -\varepsilon_{ij}I, -\varepsilon_{ji}I, -\varepsilon_{ji}I, -\varepsilon_{ji}I\right\}.$$

Moreover, the state time-delay feedback H_∞ controller gains of (7) are given by

$$K_{j1} = L_{j1}X^{-1}, \quad K_{j2} = L_{j2}Y^{-1}, \quad j = 1, 2, \cdots, n. \tag{26}$$

Proof: Consider the parameter uncertainties satisfying (2), (3) and (4) in Assumption 2.1. Then replace $\tilde{A}_{i1}$, $\tilde{A}_{i2}$, $\tilde{B}_i$, $\tilde{C}_{i1}$, $\tilde{C}_{i2}$ and $\tilde{D}_i$ with $A_{i1} + U_iF_i(t)E_{i1}$, $A_{i2} + U_iF_i(t)E_{i2}$, $B_i + U_iF_i(t)E_i$, $C_{i1} + H_iV_i(t)G_{i1}$, $C_{i2} + H_iV_i(t)G_{i2}$ and $D_i + H_iV_i(t)G_i$ in (20) and (21), respectively. By Lemma 3.1, we can obtain

$$\begin{aligned} (20) \Leftrightarrow \Theta_{ii} + \Gamma_i \begin{bmatrix} F_i(t) & * & * \\ 0 & F_i(t) & * \\ 0 & 0 & V_i(t) \end{bmatrix} \Xi_{ii} + \Xi_{ii}^T \begin{bmatrix} F_i^T(t) & * & * \\ 0 & F_i^T(t) & * \\ 0 & 0 & V_i^T(t) \end{bmatrix} \Gamma_i^T < 0 \\ \Leftrightarrow \Theta_{ii} + \varepsilon_{ii}\Gamma_i\Gamma_i^T + \varepsilon_{ii}^{-1}\Xi_{ii}^T\Xi_{ii} < 0, \end{aligned} \tag{27}$$

where

$$\Theta_{ii} = \begin{bmatrix} PW_{ii} + W_{ii}^TP & * & * & * & * & * & * \\ K_{i2}^TB_i^TP & -Q & * & * & * & * & * \\ B_{\omega i}^TP & 0 & -\gamma^2I & * & * & * & * \\ A_{i2}^TP & 0 & 0 & -d_M^{-1}R & * & * & * \\ S_{ii} & T_{ii} & B_{\omega i} & 0 & -d_M^{-1}R^{-1} & * & * \\ M_{ii} & N_{ii} & 0 & 0 & 0 & -I & * \\ I & 0 & 0 & 0 & 0 & 0 & -Q^{-1} \end{bmatrix},$$

$$\Gamma_i = \begin{bmatrix} PU_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & U_i & 0 \\ 0 & 0 & H_i \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_{ii} = \begin{bmatrix} E_{i1} + E_{i2} + E_iK_{i1} & E_iK_{i2} & 0 & E_{i2} & 0 & 0 & 0 \\ E_{i1} + E_iK_{i1} & E_{i2} + E_iK_{i2} & 0 & 0 & 0 & 0 & 0 \\ G_{i1} + G_iK_{i1} & G_{i2} + G_iK_{i2} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We can transfer (21) in the same way. By Remark 3.1, we know that when (20) and (21) hold, system (8) satisfies condition [a] and [b] in Remark 2.1. Define $X = P^{-1}$, $Y = Q^{-1}$, $Z = R^{-1}$, $L_{j1} = K_{j1}X$ and $L_{j2} = K_{j2}Y$. Using Schur complements in (27), then pre- and post-multiplying both sides of the obtained matrix with diag$\{X, Y, I, Z, I, I, I, I, I, I\}$, we can get (24). Analogously we can prove that (21) $\Leftrightarrow$ (25).

4. **Numerical Examples.** In this section, two examples are presented to illustrate the proposed methods.

Example 4.1. *In the following, we consider a time-delay T-S fuzzy system with uncertainties:*

Plant Rule 1: *If $x_2(t)$ is small, then*

$$\dot{x}(t) = (A_{11} + \Delta A_{11}(t))\, x(t) + A_{12}x(t-d) + B_1u(t).$$

Plant Rule 2: *If $x_2(t)$ is big, then*

$$\dot{x}(t) = (A_{21} + \Delta A_{21}(t))\, x(t) + A_{22}x(t-d) + B_2u(t).$$

The model parameters are given as follows:

$$A_{11} = \begin{bmatrix} 0 & -0.9 \\ -0.3 & -1.5 \end{bmatrix},\ A_{12} = \begin{bmatrix} 0 & 0.01 \\ -0.018 & 0.2 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 0 & -0.8 \\ -0.4 & -1.7 \end{bmatrix},\ A_{22} = \begin{bmatrix} 0 & 0.01 \\ -0.012 & 0.19 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$U_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\ E_{11} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},\ F_1(t) = \sin(t),$$

$$U_2 = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},\ E_{21} = \begin{bmatrix} -0.2 & 0.2 \end{bmatrix},\ F_2(t) = -\sin(t).$$

The membership functions for x_2 are as follows:

$$small(x_2) = \begin{cases} 1, & x_2 \in (-\infty, -1], \\ 0.5(1-x), & x_2 \in [-1, 1], \\ 0, & x_2 \in [1, +\infty), \end{cases} \quad , \quad big(x_2) = \begin{cases} 0, & x_2 \in (-\infty, -1], \\ 0.5(1+x), & x_2 \in [-1, 1], \\ 1, & x_2 \in [1, +\infty). \end{cases}$$

The system with $u(t) = 0$, $d = 0.5$ has unstable response as shown in Figure 1 for the initial condition $x(0) = [2\ \ -5]^T$.

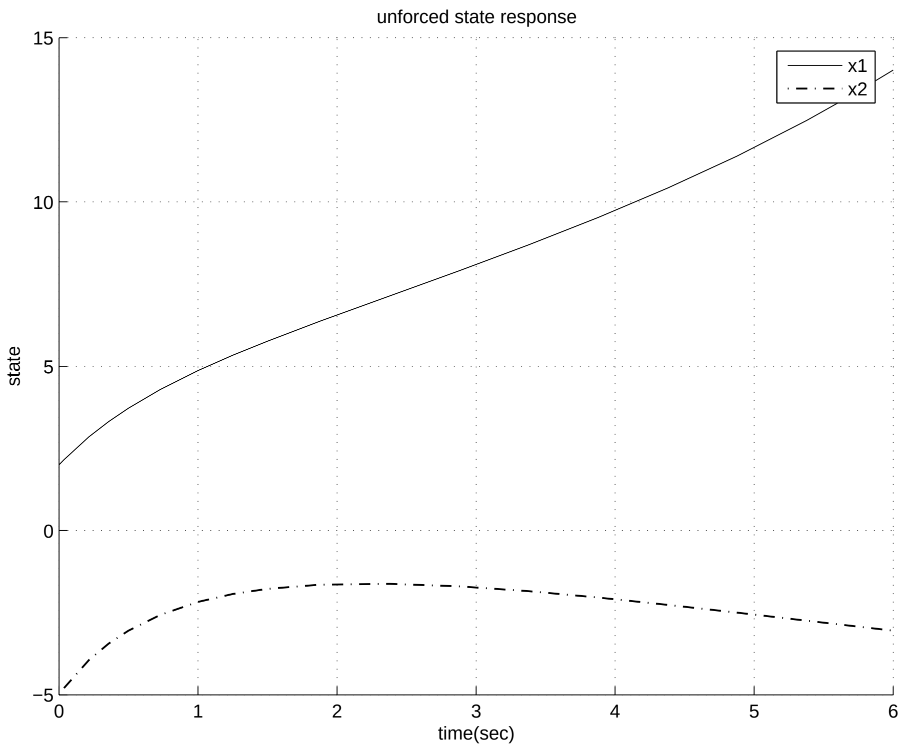

FIGURE 1. Unforced state response

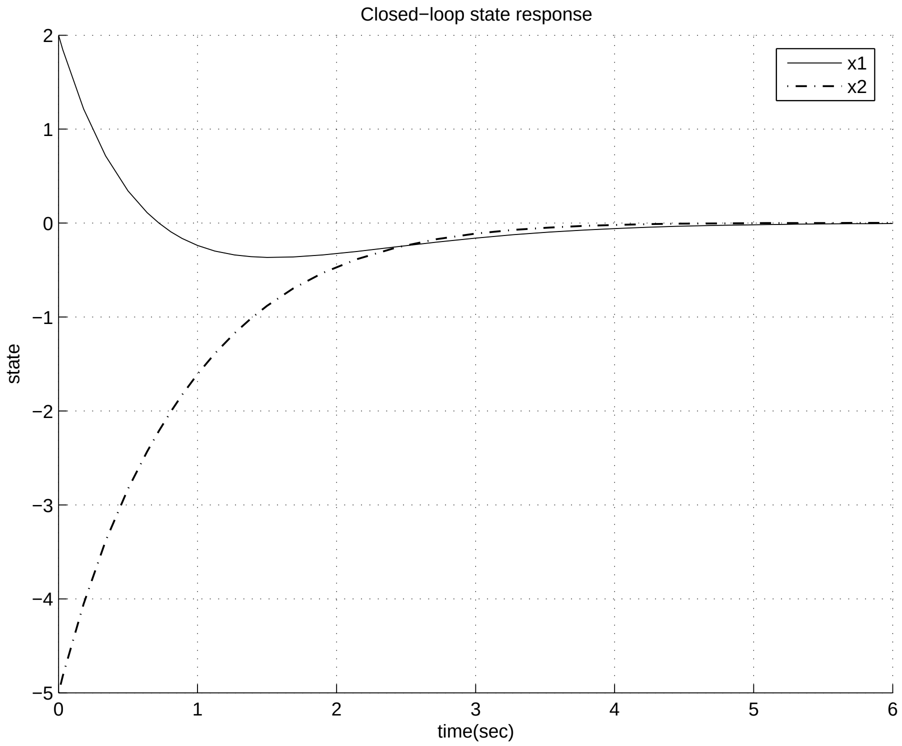

FIGURE 2. Closed-loop state response

Then we consider the simple state feedback controller as $u(t) = \sum_{i=1}^{n} \mu_i K_i x(t)$. *Finally, we can have the state feedback gain* K_i *as* $K_1 = [-1.2665\ 0.6977]$, $K_2 = [-1.2445\ 0.7281]$, *and the simulation result is shown in Figure 2.*

Example 4.2. *Consider an uncertain nonlinear system with time-delay as follows:*

$$\begin{cases} \dot{x}_1(t) = -\left(6 - \cos^2(x_2(t))\right) + x_2(t) - 0.1\sin^2(x_2(t))x_1(t-d) \\ \qquad\quad - \left(2 + \sin^2(x_2(t))\right) x_2(t-d) + c(t)\sin^2(x_2(t))\left[x_2(t) + x_1(t-d)\right] \\ \qquad\quad + c(t)\cos^2(x_2(t))\left[x_1(t) + x_2(t-d)\right] + c(t)u(t) + \left(1 + \sin^2(x_2(t))\right)\omega(t), \\ \dot{x}_2(t) = -\left(0.2 - 0.3\cos^2(x_2(t))\right)x_1(t) - x_2(t) + \left(0.1 - 0.2\cos^2(x_2(t))\right)x_1(t-d) \\ \qquad\quad - 0.1x_2(t-d) + u(t), \end{cases} \tag{28}$$

where $c(t)$ *is an uncertain parameter satisfying* $c(t) \in [-0.2, 0.2]$. *If we select the membership function as* $M_1(x_2(t)) = \sin^2(x_2(t))$ *and* $M_2(x_2(t)) = \cos^2(x_2(t))$, *then the nonlinear time-delay system (28) can be represented by the following time-delay T-S fuzzy model with parameter uncertainties:*

Plant Rule 1: *If* $x_2(t)$ *is* M_1, *then*

$$\begin{cases} \dot{x}(t) = \left(A_{11} + \Delta A_{11}(t)\right)x(t) + \left(A_{12} + \Delta A_{12}(t)\right)x(t-d) + \left(B_1 + \Delta B_1(t)\right)u(t) \\ \qquad\quad + B_{\omega 1}\omega(t), \\ \tilde{z}(t) = \left(C_{11} + \Delta C_{11}(t)\right)x(t) + \left(C_{12} + \Delta C_{12}(t)\right)x(t-d) + D_1 u(t), \end{cases}$$

Plant Rule 2: *If* $x_2(t)$ *is* M_2, *then*

$$\begin{cases} \dot{x}(t) = \left(A_{21} + \Delta A_{21}(t)\right)x(t) + \left(A_{22} + \Delta A_{22}(t)\right)x(t-d) + \left(B_2 + \Delta B_2(t)\right)u(t) \\ \qquad\quad + B_{\omega 2}\omega(t), \\ \tilde{z}(t) = \left(C_{21} + \Delta C_{21}(t)\right)x(t) + \left(C_{22} + \Delta C_{22}(t)\right)x(t-d) + D_2 u(t), \end{cases}$$

where

$$A_{11} = \begin{bmatrix} -6 & 1 \\ -0.2 & -1 \end{bmatrix},\ A_{12} = \begin{bmatrix} -0.1 & -3 \\ 0.1 & -0.1 \end{bmatrix},\ A_{21} = \begin{bmatrix} -5 & 1 \\ 0.1 & -1 \end{bmatrix},\ A_{22} = \begin{bmatrix} 0 & -2 \\ -0.1 & -0.1 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ B_{\omega 1} = \begin{bmatrix} 2 \\ 0 \end{bmatrix},\ B_{\omega 2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ C_{11} = C_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C_{12} = C_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ D_1 = D_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ U_1 = U_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ E_{11} = E_{22} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix},$$

$$E_{12} = E_{21} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix},\ E_1 = E_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ H_1 = H_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$G_{11} = G_{22} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix},\ G_{12} = G_{21} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix},\ G_1 = G_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Choosing the H_∞ performance level $\gamma = 1$, designing the state time-delay feedback controller as (7), then for $d = 0.9$, according to Theorem 3.3, by solving LMIs (24) and (25), we can obtain

$$X = \begin{bmatrix} 1.5974 & -0.0155 \\ -0.0155 & 0.2703 \end{bmatrix},\ Y = \begin{bmatrix} 5.8110 & -0.2732 \\ -0.2732 & 0.5845 \end{bmatrix},\ Z = \begin{bmatrix} 15.5326 & -0.5140 \\ -0.5140 & 0.9229 \end{bmatrix},$$

$$L_{11} = \begin{bmatrix} -0.1273 & -0.1286 \end{bmatrix},\ L_{12} = \begin{bmatrix} -0.1908 & 0.0186 \end{bmatrix},$$

$$L_{21} = \begin{bmatrix} 0.0173 & -0.0806 \end{bmatrix},\ L_{22} = \begin{bmatrix} 0.0106 & 0.0085 \end{bmatrix}.$$

Finally, by (26), the responding controller gains are calculated as follows:

$$K_{11} = \begin{bmatrix} -0.0843 & -0.4806 \end{bmatrix},\quad K_{12} = \begin{bmatrix} -0.0320 & 0.0169 \end{bmatrix},$$
$$K_{21} = \begin{bmatrix} 0.0079 & -0.2979 \end{bmatrix},\quad K_{22} = \begin{bmatrix} 0.0026 & 0.0157 \end{bmatrix},$$

and the maximal delay allowed is $d_{Max} = 1.2230$.

5. **Conclusions.** In this paper, we considered a class of T-S fuzzy time-delay systems with parameter uncertainties. Based on Lyapunov functional approach, we obtained some new delay-dependent conditions of designing a stable state time-delay feedback controller. All the results are given in terms of LMIs. Finally, we gave two numerical examples to demonstrate the effectiveness of our methods.

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