A NOVEL STRATEGY FOR BERTH AND QUAY CRANE ALLOCATION UNDER DISRUPTION IN CONTAINER TERMINAL

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Abstract. The port operational plan which is optimized before the vessel arrives tends to be disrupted by some uncertainties. And the pre-arranged scheduling plan will be not optimal or even infeasible. In this paper, we focus on the integrated berth and quay crane allocation under disruption. First, an initial allocation model is formulated without taking account of the disruption. Second, a simulation-based strategy is proposed to deal with disruptions. Vessels’ berthing times and berthing positions are constrained within stable modes which are extracted through multiple disruption simulations. Under these constraints, a mixed integer linear programming model is presented to solve the integrated berth and quay crane allocation problem. Furthermore, an efficient method is presented to determine the specific quay cranes which serve a vessel. Additionally, four experimental scenarios are designed and conducted on several test instances to validate the performance of the proposed strategy. This strategy proves to be viable. It not only better maintains the stability of the initial scheduling plan but also prevents excessive use of port resources.

Keywords: Berth and quay crane allocation, Disruption, Container terminal, Stable mode

1. Introduction. With the development of the global economy, the logistics industry has rapidly developed. As a major component of logistics links, port plays an important role. Port is not only a hinge of different transport modes, but more importantly also a gathering place for technology, economy and information. In order to increase the port profit and improve the port competitiveness, port operators attempt to optimally allocate port scarce resources (i.e., berths and quay cranes). Due to the opening of liner routes in the world’s container traffic, the modern container terminal gets a revolution in the way of scheduling management. Terminal operators can make an optimal plan of the seaside operations by taking account of shipping schedules and loading and unloading information provided by the shipping company. This helps to increase utilization of port resources and enhance satisfaction of ship owners [1]. However, the coordination of components of the ship loading and unloading system, such as vessels, berths, quay cranes, and trucks, is extremely complicated.

In practice, when a vessel calls a port, some information of the vessel including ship type, estimated arrival time, stowage plan, and so on, should be sent to the port. Based on the information and the status of port resources (e.g., berths, quay cranes, and yard trucks), port operator will make the optimal berthing plan and equipment scheduling

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for upcoming vessels within a planning horizon. However, because of many uncertainties (e.g., awful weather, arrival delay, and equipment failure), the initial berthing plan and equipment scheduling will not be optimal or even infeasible. These uncertainties will disturb the normal operation plan of the terminal. For example, changes of berthing time and berthing position of a certain vessel will directly affect the normal handling of other vessels and cause a series of chain reactions.

When disruptions happen, the initial plan has to be adjusted accordingly so as to minimize the negative impacts, which is called disruption management. Its main idea is to rapidly generate an adjustment plan which has the minimum deviation from the original scheduling after the disruption happens. To the best of our knowledge, the application of this method in berth allocation has received more attention recently [2, 3]. Only a few studies applied this method to the integrated berth and quay crane allocation [4]. The port operator deals with the disruption by formulating a disruption recovery model, in which the deviation cost caused by disruption is the commonly used evaluation criterion. However, it will be difficult to balance the deviation cost and the original objective. And solving the multi-objective programming is very complicated.

In this paper, we present a novel strategy for berth and quay crane allocation under disruption, which differs from the disruption management method. The remainder of this paper is organized as follows. Following this introduction, Section 2 reviews recent research on berth and quay crane allocation. Section 3 presents in detail the novel strategy for berth and quay crane allocation under disruption and the method for determination of specific quay cranes. In Section 4, four experimental scenarios are designed and carried out on several instances to validate the proposed strategy. Section 5 summaries the conclusions and possible future research.

2. Literature Review. Many researches are very concerned with allocating berths and quay cranes to vessels. This research includes three problems: berth allocation problem (BAP), quay crane assignment problem (CAP) and integrated berth and quay crane allocation problem (BCAP). The BAP is to make a berthing plan, including berthing time and berthing position for each vessel, aimed to optimize the objective function. The CAP is to assign quay cranes, which are lined up on rail tracks alongside the quay and responsible for loading and unloading containers, to vessels. The two problems are basically interrelated. Vessels’ handling times in the BAP are impacted to a large extent by the CAP solution. Therefore, the BCAP, which determines berths and quay cranes for vessels simultaneously, is extensively researched by many scholars. When formulating the problem, the spatial attribute of berth layout (i.e., discrete, continuous or hybrid layout) and the temporal attribute of vessel arrival (i.e., static arrival or dynamic arrival) should always be considered [5].

2.1. BAP. The BAP has been widely studied in different combinations of the spatial attribute and the temporal attribute [6, 7, 8]. Most models aim at minimizing the total of waiting and handling times of all vessels [9, 10], and some also consider the effect of earliness or tardiness departures on the port and the ship company [11]. Hansen et al. [11] added ship-dependent earliness premiums and lateness penalties into the objective function, apart from waiting costs and handling costs. They acknowledged the correlation between handling cost and handling time, but did not assume that handling cost was proportional to handling time.

The assumption about the handling time is very important. In the paper of Li et al. [12], the vessel handling time was assumed to be fixed. They proposed the continuous static BAP for the first time and formulated a model with the objective of minimizing
the makespan of the scheduling. Compared with continuous static BAP, there are more researches on continuous dynamic BAP. In the majority of them, the vessel handling time is assumed to be fixed [8, 13]. There are still a few researches in which the vessel handling time depends on the number of quay cranes assigned to the vessel and the handling location of the vessel [14, 15]. The model of Umang et al. [15] assumed that the vessel handling time was position-dependent. The authors formulated the dynamic hybrid BAP in bulk ports taking account of the type of vessel cargo. And this is the first time that the BAP is studied in the context of bulk ports.

In the real world, the pre-arranged berthing plan will be disrupted by some uncertainties, such as awful weather, and arrival delay. Zhen et al. [16] optimized the baseline schedule and recovery schedule simultaneously with a two-stage strategy. The objective is to minimize the baseline schedule cost and the recovery cost simultaneously. Zeng et al. [17] formulated a disruption recovery model for berth allocation and designed a simulation-based optimization approach to deal with disruptions and generate a new berthing schedule. Umang and Bierlaire [3] studied the problem of real-time recovery and formulated a dynamic hybrid BAP model in the context of bulk ports. When disruptions happen, the berthing schedule is recovered with minimal realized cost of the modified scheduling.

2.2. BCAP. The interplay between BAP and CAP makes it necessary to study the BCAP. A deep integration of the discrete BAP and the CAP is studied [18, 19]. Imai et al. [18] assumed that vessel's handling time depends on its berth position but they did not consider the relationship between the handling time and the number of cranes. Liang et al. [19] explicitly took account of handling time as a function of the number of cranes assigned to the vessel. More BCAPs are researched under the continuous berth layout. Hu et al. [20] considered fuel consumption by and emissions from vessels. They proposed a model for BCAP that minimized the port operation cost and vessel’s fuel consumption simultaneously. The authors regarded the vessel arrival time as a decision variable so that the shipping company can adjust the vessel’s sailing speed to reduce the fuel consumption and emissions and maximize the utilization of port resources. However, vessel handling time was assumed fixed in this model, which lacks rationality in real operation. In some researches, the vessel handling time is assumed to be impacted by the crane resources assigned to the vessel. Meisel and Bierwirth [21] devised a BCAP model in which productivity losses caused by interference among quay cranes were considered. Squeaky wheel optimization was proposed to solve the model, and the solutions were compared with those of Lagrangian heuristics algorithm. Afterward, the authors provided a framework to align all seaside operational decisions (berth allocation and quay crane scheduling) in an integrative manner [22].

In some CAPs, the specific quay cranes assigned to a vessel also need to be determined. These CAPs are referred to as CASPs. Türkoğulları et al. [23] formulated BCAP model and established a necessary and sufficient condition for generating the optimal solution of BCASP (integration of BAP and CASP) from that of BCAP. If this condition is not satisfied, an exact solution algorithm will be used to add constraints to the BCAP model so that the condition is satisfied. However, the assumption that the number of cranes assigned to a vessel cannot change during its stay at the berth may not hold true to some extent in practice. Zhang et al. [24] allowed for limited adjustments of quay cranes which are assigned to a vessel. Both the number of assigned quay cranes and the specific quay cranes can be changeable. This makes their integrated allocation model more adaptable to the real situation. They considered the problem as a two-dimensional
cutting stock problem and solved it by using Lagrangian relaxation and sub-gradient optimization algorithm.

Only a tiny minority of researchers study the integration of the hybrid BAP and the CAP. Lokuge and Alahakoon [25] handled this problem by using a multi-agent system (MAS). The architecture of the MAS constructs a feedback loop integration of the BAP and the CAP.

The BCAP under uncertainty has also been researched. Han et al. [26] proposed a robust integrated scheduling generation model in the context of discrete berth layout. The vessel arrival time and the container handling time are assumed to be stochastic in this model. Li et al. [4] assumed that the quay cranes assigned to a vessel can be changeable. A real-time recovery model was presented to handle disruptions caused by service interruption and arrival delay. The pursued objective is to minimize the weighted sum of time cost (from service time and tardiness of vessels) and recovery cost, and the vessels which arrive late are allowed to be early dispatched.

Actually, the strategy presented by Li et al. [4] is a reactive recovery strategy. It was conducted to minimize the negative impacts of the specific disruption. However, the recovery result was not a robust solution. While the strategy presented by Han et al. [26] is a proactive strategy. The authors considered that vessel arrival time and the container handling time were normal distributed. They tried to obtain a robust solution by minimizing the expected value plus standard deviation of total service time and weighted tardiness time for all vessels in a planning time horizon. No matter what disruption might happen, this solution would have a statistically good performance. However, it was not the optimal result and depended on stochastic values’ distributions which remain to be proved.

In this paper, we solve this problem with a novel approach. Our approach combines the advantages of the two strategies well. It extracts stable modes of scheduling through multiple simulations. These modes are robust to different disruptions. When a disruption happens, the stable modes are added into the rescheduling model as constraints. This ensures that the scheduling result will be the optimal solution under a specific disruption.

More extensive reviews on the BAP and the BCAP can be found in related literature [5, 27].

3. A Novel Strategy for Berth and Quay Crane Allocation under Disruption.
In the actual terminal operations, some disruptions may happen. These uncertainties will disrupt the initial scheduling plan. Two types of disruptions are fairly representative: disruptions caused by vessel arrival delay (referred to as first-class disruptions) and disruptions caused by some unexpected events, such as awful weather, and equipment failure, during terminal operations (referred to as second-class disruptions). There are two main reasons behind the first-class disruption: delayed departure from the last service port and time-extended sailing. This will disturb some vessels’ pre-scheduled berthing plans. This paper focuses on the first-class disruption. The disruption mentioned below is the first-class disruption.

In this section, we attempt to determine the optimal berthing time, berthing position and quay crane scheduling of each vessel under disruption on the basis of continuous berth layout and dynamic vessel arrivals. First, a non-disruption BCAP model is formulated to determine the initial berthing schedule. Second, computer simulation method is used to extract stable modes of the initial scheduling. We construct a new berth and quay crane allocation model based on the stable modes. When a disruption happens, this model can be used to determine the optimal berthing plan for upcoming vessels within a planning horizon and the number of quay cranes which serve each vessel. At last, an efficient method is presented to determine the specific quay cranes assigned to each vessel.
Before describing our strategy for BCAP under disruption, we state the underlying assumptions as follows.

- **(a)** The planning horizon and the berth are divided into equal-sized time periods and equal-sized berth sections, respectively.
- **(b)** Each vessel has a preferred berthing position.
- **(c)** Vessels are handled continuously by quay cranes.
- **(d)** Each vessel has a minimum and maximum number of quay cranes that can be assigned to it, and the quay cranes assigned to a vessel cannot be changed during the service period.
- **(e)** Each quay crane can serve at most one vessel in each time period.
- **(f)** The interference between quay cranes is not considered.

Related parameters and decision variables of our models are defined as shown in Table 1 and Table 2.

Here, we display the solution of the BCAP in a two-dimensional time-berth space. The horizontal axis corresponds to the time within the planning horizon, while the vertical axis corresponds to the berth space within the wharf boundary. All vessels are depicted in the space as rectangles. The width of the rectangle represents the vessel handling time which depends on the number of assigned quay cranes, and the height represents the length of the vessel (including the safety margin). Figure 1 shows an optimal BCAP solution.

**Table 1. Definition of the parameter involved in mathematical models**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>Number of vessels</td>
</tr>
<tr>
<td>$i$</td>
<td>Index of the vessel in set $VS = 1, 2, \cdots, V$</td>
</tr>
<tr>
<td>$T$</td>
<td>Number of time periods</td>
</tr>
<tr>
<td>$t$</td>
<td>Index of the time period in set $TS = 1, 2, \cdots, T$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Number of quay cranes</td>
</tr>
<tr>
<td>$q$</td>
<td>Index of the number of quay cranes in set $NS = 0, 1, 2, \cdots, Q$</td>
</tr>
<tr>
<td>$p$</td>
<td>Index of the quay crane number in set $CS = 1, 2, \cdots, Q$</td>
</tr>
<tr>
<td>$a_i$</td>
<td>Arrival time of vessel $i$</td>
</tr>
<tr>
<td>$b_i$</td>
<td>Preferred berthing position of vessel $i$ (measured in the number of berth sections)</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Estimated departure time of vessel $i$</td>
</tr>
<tr>
<td>$l_i$</td>
<td>Length of vessel $i$, including the safety margin (measured in the number of berth sections)</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of the wharf (measured in the number of berth sections)</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>The quay crane capacity demand of vessel $i$, which is the number of crane-time sections required to load and unload all containers for vessel $i$ (measured in QC · hours)</td>
</tr>
<tr>
<td>$q_i$</td>
<td>The minimum number of quay cranes that can be assigned to vessel $i$</td>
</tr>
<tr>
<td>$\bar{q}_i$</td>
<td>The maximum number of quay cranes that can be assigned to vessel $i$</td>
</tr>
<tr>
<td>$q_i$</td>
<td>The number of quay cranes which are assigned to vessel $i$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Cost of waiting for one time period for each vessel</td>
</tr>
<tr>
<td>$c_2$</td>
<td>Cost of one-unit deviation from the preferred berth position for each vessel</td>
</tr>
<tr>
<td>$c_3$</td>
<td>Cost of delaying one time period for departure for each vessel</td>
</tr>
<tr>
<td>$c_4$</td>
<td>Cost of loading and discharging one time period for each quay crane</td>
</tr>
<tr>
<td>$M$</td>
<td>A large positive number</td>
</tr>
</tbody>
</table>
Table 2. Definition of the decision variable involved in mathematical models

<table>
<thead>
<tr>
<th>Decision variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_i$</td>
<td>Berthing time of vessel $i$</td>
</tr>
<tr>
<td>$B_i$</td>
<td>Berthing position of vessel $i$ (measured in the number of berth sections)</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Departure time of vessel $i$</td>
</tr>
<tr>
<td>$U_{it}$</td>
<td>1 if vessel $i$ is served in time period $t$, 0 otherwise</td>
</tr>
<tr>
<td>$q_{it}$</td>
<td>$q$ quay cranes are assigned to vessel $i$ in time period $t$</td>
</tr>
<tr>
<td>$\sigma_{im}$</td>
<td>1 if vessel $i$ is berthed to the left of vessel $m$ along the wharf, 0 otherwise</td>
</tr>
<tr>
<td>$\delta_{im}$</td>
<td>1 if vessel $i$ is berthed earlier than vessel $m$, 0 otherwise</td>
</tr>
<tr>
<td>$p_i$</td>
<td>The $p$-th quay crane is the leftmost quay crane which is assigned to vessel $i$</td>
</tr>
<tr>
<td>$\varphi_{im}$</td>
<td>1 if $p_i$ is smaller than $p_m$, 0 otherwise</td>
</tr>
</tbody>
</table>

Figure 1. An optimal BCAP solution of an instance with 25 vessels

of an instance with 25 vessels. In Figure 1, take the rectangle representing vessel $i$ for example. Its lower left corner and top right corner are points $(E_i, B_i)$ and $(D_i, B_i + l_i)$, respectively. The digit in parentheses is the number of quay cranes assigned to the vessel. For a feasible solution of the BCAP, all rectangles are non-overlapping and vessels and quay cranes should satisfy the spatial and temporal constraints.

3.1. Initial allocation model. In order to clarify the effect of disruptions on berth and quay crane allocation, we should first study the BCAP without considering the disruption. This problem is formulated as a mixed integer linear programming model [M1]. The model is used to determine the optimal berthing time and berthing position of each vessel. The mathematical formulation for [M1] is as follows.

$$\text{[M1]} \quad \min f = \sum_i \left[ c_1 (E_i - a_i) + c_2 |B_i - b_i| + c_3 (D_i - d_i)^+ + c_4 \sum_t q_{it} \right] \quad (*)$$

s.t. \quad $E_i \geq a_i$, \quad \forall i \quad (1)$

$B_i + l_i \leq L$, \quad \forall i \quad (2)$
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The objective function (∗) is to minimize the total cost of waiting for berthing, deviation from the preferred berthing position, tardiness of vessels and the use of quay cranes. Constraint (1) ensures that a vessel cannot berth before its arrival time. Constraint (2) guarantees that all vessels must be berthed within the wharf length. Constraints (3)-(5) ensure that no overlap exists between any two rectangles representing vessels in the time-berth space. Constraint (6) limits the amount of quay cranes available at a time period. Constraint (7) states that the quay crane capacity demand of each vessel must be satisfied. Constraint (8) relates variable \( q_{it} \) to \( U_{it} \). Constraint (9) restricts the number of quay cranes which are assigned to serve a vessel. Constraints (10) and (11) ensure that the number of quay cranes assigned to a vessel is unchanged during the service period. Constraint (12) means that a vessel must be served between its berthing and departure times. Constraint (13) states that a vessel will depart from the port the moment its loading and unloading operations have been completed. Constraints (14)-(17) specify the range of the variables. Constraint (18) defines the tardiness of vessels.

3.2. The strategy for integrated berth and quay crane allocation under disruption. In the actual world, some disruptions may happen on a vessel’s voyage, and this will disrupt the pre-arranged terminal operational plan. Two approaches are generally used to deal with the disruption. One is to reschedule the BCAP. Rescheduling focuses on the generation of a new schedule for the upcoming vessels within the planning horizon. The new schedule is based on the updated information, and it has no connection with the initial scheduling. The other is to adjust the initial scheduling using the disruption management method. Its main idea is to rapidly generate an adjustment plan which has the minimum deviation from the initial scheduling after the disruption happens. This method will be more efficient than rescheduling method when the number of considered vessels is large. In this section, we present a novel strategy for integrated berth and quay
of the optimal scheduling. The mathematical formulation for [M2] is as follows.

(1) Simulate the disruption.

In practice, when a vessel calls a port, some information of the vessel including its ship type, estimated arrival time, stowage plan, estimated departure time, and so on, should be sent to the port. Port operators will make an optimal berthing plan and quay crane scheduling for upcoming vessels within a planning horizon based on vessels’ information and the status of port resources. When disruption happens, the estimated arrival time of a vessel may be changed. This will make the pre-arranged scheduling plan not optimal or even infeasible.

In order to simulate the disruption, we first suppose the rate of disrupted vessels $\rho$ is obtained according to port operators’ experiences and the upcoming vessels within the planning horizon are disrupted with uniform probability. Then we randomly select some vessels according to the rate $\rho$ and add a disturbance term $\delta$ to their arrival times. Each vessel has a latest arrival time $la_{i}$ ($i = 1, 2, \cdots, V$). If a vessel’s estimated arrival time is later than its latest arrival time, it will be rejected by the port. So we should update all vessels’ arrival times and eliminate the vessels which are rejected by the port.

(2) Extract stable modes of the initial scheduling.

Based on the updated information of vessels, we use the initial allocation model [M1] to solve the BCAP and obtain the optimal berthing time ($IE_{i}$) and berthing position ($IB_{i}$) of vessel $i$ (not including eliminated vessels). For eliminated vessels, their optimal berthing times and berthing positions are set to NaN (Not-a-Number). By carrying out the process mentioned above $N$ times, we can get a sequence, $\{(IE_{in}, IB_{in})\}$, where $i = 1, 2, \cdots, V$, $n = 1, 2, \cdots, N$, $IE_{in}$ and $IB_{in}$ are the optimal berthing time and berthing position of the $i$-th vessel under the $n$-th simulated disruption. Then, for each vessel, we search for intervals which cover most of its optimal berthing times or berthing positions. These intervals are referred to as stable modes of the initial scheduling. For some vessels, this kind of interval may not exist. The detailed steps are as follows (take the $i$-th vessel for example).

**Step 1.** Plot points $(IE_{in}, IB_{in})$, $n = 1, 2, \cdots, N$, in the time-berth space.

**Step 2.** Use a $2\sigma$-wide vertical strip to scan these scatter points. If there is a strip which intersects the horizontal axis at $t_{i} - \sigma, 0$, $t_{i} + \sigma, 0$ covers at least $R$ points, output the intersection points and record the interval $st_{i} = [t_{i} - \sigma, t_{i} + \sigma]$. All the recorded intervals are designated $\bigcup st_{i}$.

**Step 3.** Similar to Step 2, use a $2\mu$-wide horizontal strip to scan the scatter points. If there is a strip which intersects the vertical axis at $0, b_{i} - \mu$, $0, b_{i} + \mu$ covers at least $R$ points, output the intersection points and record the interval $sp_{i} = [b_{i} - \mu, b_{i} + \mu]$. All the recorded intervals are designated $\bigcup sp_{i}$.

The sets $\bigcup st_{i}$ and $\bigcup sp_{i}$, $i = 1, 2, \cdots, V$, are the stable modes of the initial scheduling. Here, values for $\sigma$, $\mu$ and $R$ should be set appropriately. If $\sigma$ (or $\mu$) is too large or $R$ is too small, the entire search space will be the stable mode of scheduling, while if $\sigma$ (or $\mu$) is too small or $R$ is too large, no stable mode of scheduling will be obtained.

(3) Generate the optimal scheduling.

The model [M2] is used to solve the BCAP under a specific disruption. The stable modes of scheduling are added to this model as constraints which narrow the search space of the optimal scheduling. The mathematical formulation for [M2] is as follows.

$$[M2] \quad \min \quad f = \sum_{i} \left[ c_{1}(E_{i} - a_{i}) + c_{2}|B_{i} - b_{i}| + c_{3}(D_{i} - d_{i})^{+} + c_{4}\sum_{t} q_{it} \right] \quad (*)$$
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s.t. Equations (1)-(18)

\[ E_i \in \bigcup_{s_t}, \forall i \]  \hspace{1cm} (19)

\[ B_i \in \bigcup_{s_p}, \forall i \]  \hspace{1cm} (20)

The objective function is the same as that of [M1]. Constraints (19) and (20) specify the stable modes of scheduling.

By solving the model [M2], we can obtain the optimal scheduling (i.e., the optimal berthing time, berthing position, departure time and number of assigned quay cranes of each vessel). And this scheduling plan has a satisfactory deviation from the initial scheduling.

The flowchart of the proposed Stable-BCAP strategy is shown in Figure 2.

![Flowchart of the Stable-BCAP strategy](Image)

**FIGURE 2.** The flowchart of the Stable-BCAP strategy

3.3. **An efficient method for determination of specific quay cranes.** To the best of our knowledge, vessels should be handled continuously by quay cranes and the quay cranes which are assigned to serve a vessel have consecutive numbers. In order to give the status of a quay crane for vessels more intuitively, we display vessels in a two-dimensional time-crane space as shown in Figure 3. The horizontal axis corresponds to the time within the planning horizon, while the vertical axis corresponds to the quay crane number. All vessels are depicted in the space as rectangles. The width of the rectangle represents the vessel handling time, and the height represents the number of quay cranes which are assigned to the vessel. For the \( i \)-th rectangle representing vessel, its lower left corner and top right corner are points \((E_i, p_i)\) and \((D_i, p_i + q_i)\), respectively. And \( p_i \) is the leftmost quay crane among \( q_i \) quay cranes which are assigned to vessel \( i \). Therefore, the quay cranes which are numbered from \( p_i \) to \( p_i + q_i - 1 \) are responsible for serving vessel \( i \). To ensure that each quay crane can serve at most one vessel in each time period, all rectangles must be non-overlapping.
Figure 3. The quay crane allocation result for an instance with 25 vessels

So, the specific quay cranes can be determined by solving a simple integer linear programming problem [M3].

\[
\text{min } f = \sum_i p_i + \sum_{i<m} \sum_t U_{it} \times U_{mt} \times |\varphi_{im} - \sigma_{im}|
\]

\(\text{s.t.}\)

\[
p_i + q_i \leq p_m + M (1 - \varphi_{im}), \forall i \neq m
\] (21)

\[
D_i \leq E_m + M (1 - \delta_{im}), \forall i \neq m
\] (22)

\[
1 \leq \varphi_{im} + \varphi_{mi} + \delta_{im} + \delta_{mi} \leq 2, \forall i < m
\] (23)

\[
p_i \in \{1, 2, \ldots, Q\}, \forall i
\] (24)

\[
\varphi_{im}, \delta_{im} \in \{0, 1\}, \forall i \neq m
\] (25)

In this programming problem, values of \(U_{it}, U_{mt}, \sigma_{im}, D_i\) and \(E_m\) are obtained by solving model [M2]. The objective function (2) is to minimize the frequency of cross movements of quay cranes. Constraints (21)-(23) ensure that no overlap exists between any two rectangles representing vessels in the time-crane space. Constraints (24) and (25) specify the range of the variables.

This model provides an efficient way to determine the specific quay cranes for each vessel. Combined with the Stable-BCAP strategy mentioned in Section 3.2, we provide an integrated berth and quay crane allocation under disruption.

Considering the optimal BCAP solution given in Figure 1, the specific allocation of quay cranes is shown in Figure 3. For example, quay cranes numbered from 3 to 6 are assigned to serve vessel 7.

4. Computational Experiments. In this section, computational experiments are performed on a set of instances to investigate the performance of the proposed Stable-BCAP strategy. We firstly introduce the generation of the test instances and the parameter settings of the strategy. Then four experimental scenarios are used to assess the deviation from the initial scheduling, utilization degree of berths and quay cranes and parameter sensitivity, respectively. The BCAP models [M1], [M2] and [M3] are formulated in MATLAB R2009a and solved by CPLEX 12.6. All experiments are carried out on a computer with 3.1 GHz CPU and 4 GB RAM.
4.1. Generation of the test instance. In the experiments, the length of the wharf is 1200 m and the planning horizon is 72 h. And there are 12 quay cranes available. We first divide the wharf into 24 equal berth sections and set the unit of time of the planning horizon to 1 h. Then test instances are randomly generated according to parameter settings of instance and vessel shown in Table 3. 20 test instances are generated. For these instances, set the number of vessels from 6 to 25 with increments of one. The rate of disrupted vessels $\rho$ is obtained from port operators’ experiences. In each instance, three vessel classes (i.e., feeder, medium and jumbo) are considered. Their proportions are 30%, 50% and 20%, respectively. Because the specific objective function value is not a focus, we set cost parameters $c_1 = 150$, $c_2 = 100$, $c_3 = 200$ and $c_4 = 150$. Related parameters of the Stable-BCAP strategy are also shown in Table 3. In Table 3, $U[a,b]$ denotes the uniform distribution from $a$ to $b$.

Table 3. Parameter settings for test instances, vessels and Stable-BCAP strategy

<table>
<thead>
<tr>
<th>Instance</th>
<th>V</th>
<th>$\rho$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[6, 25]</td>
<td>20%</td>
<td>$U[1,4]$</td>
</tr>
<tr>
<td>Vessel</td>
<td>Class</td>
<td>$l_i$</td>
<td>$a_i$</td>
</tr>
<tr>
<td>Feeder</td>
<td>$U[1,4]$</td>
<td>$a_i$</td>
<td>$U[1,60]$</td>
</tr>
<tr>
<td>Medium</td>
<td>$U[4,6]$</td>
<td>$a_i + U[2,3]$</td>
<td>$U[15,36]$</td>
</tr>
<tr>
<td>Jumbo</td>
<td>$U[6,8]$</td>
<td>$U[36,48]$</td>
<td>$\frac{a_i}{R} \cdot U[1,2] + a_i$</td>
</tr>
<tr>
<td>Strategy</td>
<td>$N$</td>
<td>$\sigma$</td>
<td>$\mu$</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>$\sum \frac{\omega_i}{R}$</td>
<td>$\sum l_i$</td>
</tr>
</tbody>
</table>

4.2. Experimental scenarios. In order to investigate the performance of the proposed Stable-BCAP strategy, the deviation from the initial scheduling, utilization degree of berths and quay cranes, time complexity and parameter sensitivity should be considered. Table 4 shows the information of scenario settings and experimental steps. In the first three scenarios, the Stable-BCAP strategy is compared with the rescheduling method.

4.3. Analysis.

4.3.1. Deviation analysis. Vessels may arrive late because of many disruptions on the voyage. This will make the initial berthing plan and quay crane scheduling not optimal or even infeasible. However, some port equipment may have been pre-positioned at the specified locations according to the initial berthing plan. In order to reduce the operational complexity of the port, the deviation between the new and the initial scheduling should be as small as possible. Here, we compare the scheduling deviation generated by our Stable-BCAP strategy with that generated by rescheduling method. Figure 4 provides a more intuitive representation of the deviations. By analyzing Figure 4(a), we find that the proposed Stable-BCAP strategy is superior to the rescheduling method in maintaining the stability of the initial scheduling. And for most instances, the port operational cost obtained by our strategy is less than that obtained by rescheduling method (as shown in Figure 4(b)).

4.3.2. Resource utilization analysis. The utilization of port resources greatly affects the profit of a port. However, it is impossible to achieve the reasonable utilization of berths and quay cranes in practice. This is due to the unreasonable allocation of resources during a planning horizon. Therefore, port operators strive to optimize the integrated berth and
Table 4. The experimental scenario settings

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Experimental steps</th>
</tr>
</thead>
</table>
| 1   | Deviation analysis           | (1) For each generated instance, solve the initial allocation model [M1] and record the values $IE_i$ and $IB_i$, $i = 1, 2, \cdots, V$;  
(2) Implement the Stable-BCAP strategy, record the objective value $f_1$ and record the values $E_i$ and $B_i$, $i = 1, 2, \cdots, V$;  
(3) Reschedule the BCAP. Record the objective value $f_2$ and record berthing time $(RE_i)$ and berthing position $(RB_i)$ of each vessel $i \in V$;  
(4) Calculate deviations from the initial scheduling $\Delta E_1 = \sum_i |E_i - IE_i|$, $\Delta B_1 = \sum_i |B_i - IB_i|$, $\Delta_1 = \sqrt{(\Delta E_1)^2 + (\Delta B_1)^2}$,  
$\Delta E_2 = \sum_i |RE_i - IE_i|$, $\Delta B_2 = \sum_i |RB_i - IB_i|$ and $\Delta_2 = \sqrt{(\Delta E_2)^2 + (\Delta B_2)^2}$, and compare the results of the Stable-BCAP strategy with those of rescheduling method. |
| 2   | Resource utilization analysis| (1) For two instances, solve the BCAP with the Stable-BCAP strategy and rescheduling method respectively;  
(2) Calculate and compare utilization rates of berths and quay cranes under the two methods. |
| 3   | Time complexity analysis     | (1) For ten instances, solve the BCAP with the Stable-BCAP strategy and rescheduling method respectively;  
(2) Record the time consumed by each step of the Stable-BCAP and compare the result with that by rescheduling method. |
| 4   | Parameter sensitivity analysis| (1) For a instance, set the instance parameter $\rho = 20\%, 25\%, 30\%, 35\%, 40\%$ and the Stable-BCAP strategy parameter $N = 5, 10, 15, 20, 25, 30, 35, 40, 45$;  
(2) For the adjustment of parameters $\rho$ and $N$, solve the BCAP with our Stable-BCAP strategy and analyze deviations from the initial scheduling plan and port operational costs. |

![Figure 4](image_url)  
Figure 4. Comparison between Stable-BCAP strategy and rescheduling method: (a) deviations from initial scheduling and (b) objective values
quay crane allocation. The utilization rate of berths (or quay cranes) is calculated by Equation (26).

\[
\omega = \frac{\theta}{\psi} \times 100\% \tag{26}
\]

where \(\omega\) is the utilization rate of berths (or quay cranes); \(\theta\) is the number of used berth length (or used quay cranes) in a time period; \(\psi\) is the wharf length (or the total number of quay cranes).

Figure 5 and Figure 6 show a comparison of utilization rate for berths and quay cranes when \(V = 22\) and \(V = 25\). We find that port resources (both berths and quay cranes) under the rescheduling method are overused in a planning horizon. In contrast, by using the Stable-BCAP strategy, the port will get smaller operational cost (Figure 4(b)) with a lower rate of resource utilization.

![Figure 5. Comparison of utilization rate for berths and quay cranes (V = 22): (a) utilization rate of berths and (b) utilization rate of quay cranes](image1)

![Figure 6. Comparison of utilization rate for berths and quay cranes (V = 25): (a) utilization rate of berths and (b) utilization rate of quay cranes](image2)

### 4.3.3. Time complexity analysis

The time consumed by the Stable-BCAP strongly depends on the hardware, operating system and development environment. For ten selected instances, we analyze the time complexity of our strategy and compare with rescheduling method in the same operating environment. The following four time values for each instance are listed in Table 5.
time\textsubscript{1}: The average time consumed by the first step of the Stable-BCAP to simulate disruptions.

time\textsubscript{2}: The average time consumed by the second step of the Stable-BCAP to extract stable modes.

time\textsubscript{3}: The average time consumed by the third step of the Stable-BCAP to generate optimal scheduling.

time\textsubscript{4}: The average time consumed by the rescheduling method.

The Stable-BCAP and the rescheduling method are all reactive strategies. Apparently, the Stable-BCAP consumes more time than rescheduling method for each instance (as shown in Table 5). This depends on the number of simulations to a large extent, especially the values of time\textsubscript{1} and time\textsubscript{2}. The more the disruption is simulated, the longer the Stable-BCAP will take. However, considering the result of deviation analysis and resource utilization analysis, we think it is worth taking a longer time to obtain a better scheduling result. Also, if the second disruption happens, the Stable-BCAP will consume less time than rescheduling method, because we only need to implement the third step of the Stable-BCAP and the value of time\textsubscript{3} less than that of time\textsubscript{4} for each instance.

**Table 5.** The average time values for instances

<table>
<thead>
<tr>
<th>V</th>
<th>Stable-BCAP</th>
<th></th>
<th></th>
<th>Rescheduling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>time\textsubscript{1}(s)</td>
<td>time\textsubscript{2}(s)</td>
<td>time\textsubscript{3}(s)</td>
<td>time\textsubscript{4}(s)</td>
</tr>
<tr>
<td>6</td>
<td>3.58E-02</td>
<td>2.418E+01</td>
<td>1.318E+00</td>
<td>2.440E+00</td>
</tr>
<tr>
<td>8</td>
<td>3.426E-02</td>
<td>3.692E+01</td>
<td>4.585E+00</td>
<td>4.869E+00</td>
</tr>
<tr>
<td>10</td>
<td>2.685E-02</td>
<td>6.810E+01</td>
<td>2.723E+00</td>
<td>5.185E+00</td>
</tr>
<tr>
<td>12</td>
<td>3.406E-02</td>
<td>6.957E+02</td>
<td>7.674E+00</td>
<td>4.277E+01</td>
</tr>
<tr>
<td>14</td>
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<td>9.600E+01</td>
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<tr>
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<td>1.560E+01</td>
<td>9.940E+01</td>
</tr>
<tr>
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<td>2.138E+01</td>
<td>1.080E+02</td>
</tr>
</tbody>
</table>

4.3.4. *Parameter sensitivity analysis.* The rate of disrupted vessels $\rho$ and the strategy parameter $N$ also affect the port operational cost and deviation from the initial scheduling. Therefore, we investigate the sensitivity of the two parameters to reveal their influences on the BCAP solution. Take the 17th generated instance for example. In this instance, 22 vessels are considered in the planning horizon. Figure 7 shows the impact of $\rho$ and $N$. In Figure 7(a), the impact of $\rho$ and $N$ on deviation from the initial scheduling is shown, where $N$ varies from 5 to 45 with a step of 5 and $\rho$ is set to 20%, 25%, 30%, 35%, 40%. For a given value of $\rho$, deviation from the initial scheduling decreases with the increase of $N$ and flattens out gradually. Specially, when $\rho = 30\%$, there are large fluctuations in the deviation, and then the deviation tends to stabilize.

Figure 7(b) and Figure 7(c) show the impact of $\rho$ and $N$ on costs (cost of waiting for berthing, cost of deviation from the preferred berthing position and cost of tardiness). It can be observed from the two figures that the cost of waiting for berthing is most impacted by $\rho$ and $N$, the cost of tardiness comes second, and the cost of deviation from the preferred berthing position is less impacted. Also, the impact of $\rho$ on costs is larger than that of $N$.  

---

**Table:** The average time values for instances

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<th>Rescheduling</th>
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</tr>
</tbody>
</table>
5. **Conclusion.** This work mainly solved the integrated berth and quay crane allocation problem under disruption. First, a mixed integer programming model is formulated. Stable modes of the initial scheduling are extracted through multiple simulations and used for restricting the adjustment spaces of vessels’ berthing times and berthing positions. And
the model is more realistic in that vessels rejected by the port are considered. Second, we present an efficient way to determine the specific quay cranes for vessels. The objective is to minimize the frequency of cross movements of quay cranes. Also, numerical experiments show that the proposed Stable-BCAP strategy is superior in maintaining stability of the initial scheduling and preventing excessive use of port resources.

However, the operating environment for container ports is very complex in reality. In the future, more realistic situations should also be considered in the model, such as the traveling of quay cranes, more scenarios of disruptions. In addition, a more comprehensive and integrated scheduling, including berths, quay cranes and internal trucks, under disruption should be studied.

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REFERENCES


