International Journal of Innovative
Computing, Information and Control
Volume 12, Number 5, October 2016

ICIC International ©2016 ISSN 1349-4198
pp. 1581–1596

IDENTIFICATION AND RECOGNITION OF THE PERSPECTIVE DYNAMIC VISION SYSTEM VIA DETERMINISTIC LEARNING

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Received February 2016; revised June 2016

Abstract. *In this paper, we present a new approach for identification and recognition of the perspective dynamic vision system via deterministic learning theory. The states of the system are periodic or recurrent and only the outputs of the perspective system are measurable. The approach consists of two phases: a training (learning) phase and a test (recognition) phase. In the training phase, a Luenberger-type observer is first used to estimate the unknown system states for the case that the motion parameters of the system are constant, and the estimation error converges exponentially to zero. Then, a radial basis function (RBF) neural network (NN) is used to approximate the unknown dynamics of the vision system when the inputs of the RBF NN are the system states, which makes the RBF network satisfy the localized partial persistence of excitation (PE) condition and the approximation error converges exponentially to a small neighborhood around zero. The obtained knowledge of the approximated vision system dynamics is stored in the constant RBF network. A bank of RBF NN-based Luenberger-type nonlinear observers is constructed and served as dynamic representation for the training patterns. In the test phase, when a test dynamical pattern is presented to the RBF NN-based observers, a set of recognition errors is generated and the average L_1 norms of the errors will be taken as the similarity measure between the test and training dynamical patterns. Therefore, the test dynamical patterns can be rapidly recognized from a set of training dynamical patterns according to the smallest error principle. Finally, simulation results are included to demonstrate the effectiveness of the proposed scheme.*
Keywords: Dynamic vision, Luenberger-type observer, Deterministic learning, Similarity definition, Dynamical pattern recognition

1. Introduction.

Dynamic vision is an important and difficult problem for the reason that the system states are dynamic rather than static. It includes determining the position of a moving rigid body and/or any unknown parameters characterizing the motion and shape of the body. The perspective dynamic system theory arises in dealing with such a problem in the framework of dynamic system theory [1-13]. Consequently, the basic problem of the perspective system theory refers to as the state estimation and parameter estimation, and a specific class of algorithms for estimation problem can be formulated as nonlinear observers design.

The observation problem has been studied at the aspects of perspective observability condition and the observer design. A necessary and sufficient condition for the observability has been given in [1] for which the motion parameters are constant. When the motion parameters are time-varying, the observability condition has been mentioned in [14]. For the observer design, some recently reported works are concerned with nonlinear observers for estimating the unknown states of the perspective systems [2,6,15-21]. The problem of estimating three-dimensional structure and motion from two-dimensional perspective observations can be solved with a nonlinear observer design. Dahl et al. [18] presented structure estimation results by showing how a perspective system can be transformed into two observer forms. These forms naturally lead to observers with simple error dynamics systems. The simplicity of the error dynamics leads to a straightforward stability analysis. Grave and Tang [21] presented a simple design of observers for the range identification problem in perspective vision systems based on nonlinear contraction theory and synchronization. In this methodology, intermediate variables, which may be not measurable, are first introduced giving a simple observer. Analysis is then taken over to guarantee exponential convergence of the observer states. In particular, a Luenberger-type observer without transforming the perspective systems into implicit ones has been proposed in [22] and the estimation error converges exponentially to zero under certain reasonable assumptions. It is also possible to extend the nonlinear observer to the unknown parameters estimation, which can be used for simultaneous estimation of motion and structure [14,23,24]. The parameter estimation algorithms provide insight into stability analysis and into how the motion affects the estimation performance by using the persistence of excitation (PE) condition [23,24]. However, in many cases, including the above mentioned two references [23,24], the PE condition is hard to be satisfied and the convergency of the parameter estimation error is difficult to be guaranteed. There still remains a problem as how to identify the perspective vision system dynamics and utilize the various vision system dynamics achieved before through state estimation and parameter estimation for rapid recognition of new and similar patterns of the perspective dynamic vision system. It can find a way to solve these problems in this paper. In vision problems, the dynamics of an object moving in three dimensions are described via its image projected in a plane by a perspective dynamical system. For applications such as robot control, surveillance and medical imaging, the unknown depth of the three dimension object must be estimated. For practical applications, such as image based visual servo, nonlinear observer techniques can be used for depth and focal length observation. An autonomous underwater vehicle can be implemented with a global exponential convergent reduced-order observer design, which provides an estimate even if the observability condition is violated for some time instance. The aforementioned observers assume the dynamics with all the parameters known. The above mentioned cases together with those where the depth and the structure are unknown can also be solved with the method proposed in this paper.

In this paper, we propose a new approach for identification and rapid recognition of the perspective dynamic vision system undergoing periodic or recurrent motion based on deterministic learning theory [25-27], which can be used to solve the problem of the satisfaction of the PE condition. The states of the system are periodic or recurrent and only the outputs of the perspective system are measurable. The approach consists of two phases: a training (learning) phase and a test (recognition) phase. In the training phase, a Luenberger-type observer is first used to estimate the states of the vision system for the case that the motion parameters of the system are constant, and the estimation error converges exponentially to zero. Then, a radial basis function (RBF) neural network (NN) is used to approximate the unknown dynamics of the vision system when the inputs of the NN are the system states, which makes the NN satisfy the localized PE condition and the approximation error converges exponentially to a small neighborhood around zero. The system dynamics will be learned and kept in constant RBF networks. A bank of RBF NN-based Luenberger-type nonlinear observers is constructed and served as dynamic representation of the training patterns. In the test phase, when a test dynamical pattern is presented to the RBF NN-based observers, a set of recognition errors is generated and the average L_1 norms of the errors will be taken as the similarity measure between the test and training dynamical patterns. Therefore, the test dynamical patterns can be rapidly recognized from a set of training dynamical patterns according to the smallest error principle.

The rest of the paper is organized as follows. In Section 2, some preliminary knowledge about the dynamic vision system in time-invariant perspective type and its observation is given. In Section 3, when the unknown system states are observed, the unknown vision system dynamics can be approximated by RBF neural networks along the trajectories of the estimated system states. The learned knowledge about the unknown dynamics of the perspective vision system is stored in constant RBF networks. In Section 4, by using the learned knowledge about the unknown dynamics of the perspective vision system which is obtained in Section 3, the similarity definition and rapid recognition of the patterns of the perspective dynamic vision system are presented. In Section 5, simulation study is given to demonstrate the effectiveness of the proposed scheme. The conclusion is included in Section 6.

2. **Preliminaries.**

Consider a rigid object described by a set of feature points in the three-dimentional vision system, and select $[x_1, x_2, x_3]^T \in R^3$ to express the characteristic point coordinate. The input and output model of the uniform rotation system [22] (ignoring the mapping fault) can be described by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + v(t), \quad x(0) = x_0 \\ y(t) &= h(Cx) \\ h(\xi) &= \left[\frac{\xi_1}{\xi_{m+1}} \ \cdots \ \frac{\xi_m}{\xi_{m+1}} \right]^T \\ \xi &= [\xi_1 \ \cdots \ \xi_m \ \xi_{m+1}]^T = Cx \in \mathbf{R}^{m+1} \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the system state which is not measurable, x_0 is the initial state, $v(t) \in R^n$ is the external input, $y(t) \in R^m$ is the perspective observation of the state $x(t)$, $A \in R^{n\times n}$, $C \in R^{(m+1)\times n}$ are matrices with $m < n$, $h : R^{m+1} \to R^m$ is a nonlinear function that produces the perspective observation with ξ being the system state. The vision system can be parameterized by the angular velocity vector $w = [w_1 \ w_2 \ w_3]^T$, and the rotation part is

$$A = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix}$$

It is assumed that the system trajectory generated from the above dynamical vision system, denoted as $\varphi_\varsigma(t, x_0)$, is a periodic or recurrent trajectory. It is also assumed that the camera is calibrated and the output $y(t)$ can be measured, thus C is known. To simplify the system, we choose matrix $C = \mathrm{I}_3$ (the identity matrix) and $n = 3$, $m = 2$, and then the output can be turned into:

$$y(t) = h(Cx) = [y_1, y_2]^T = \left[\frac{x_1}{x_3}, \frac{x_2}{x_3} \right]^T \tag{2}$$

Assumption 2.1. *System (1) with assumptions as follows can be observed by a Luenberger-type nonlinear observer [22].*

(i) The system for observation is Lyapunov stable with periodic orbits.

(ii) The observation vector y(t) is a continuous and bounded function of t, that is

$$y(\cdot) \in C^m[0, \infty) \cap L_\infty^m[0, \infty).$$

(iii) Express the set $\sigma(A)$ of all eigenvalues of A as $\sigma(A) = \sigma_-(A) \cup \sigma_0(A)$ where $\sigma_-(A)$ and $\sigma_0(A)$ indicate the sets of eigenvalues with strictly negative real part and zero real

part, respectively. Let W_-, $W_0 \subset C^n$ denote the generalized eigenspace corresponding to $\sigma_-(A)$ and $\sigma_0(A)$, respectively, and choose a basis matrix $E_0 = [\xi_1 \ \cdots \ \xi_r]$ for W_0 where $r := dim(W_0)$. Then, there exist $T > 0$ and $\varepsilon > 0$ such that

$$\int_0^T E_0^* e^{A^*\tau} C^* B^*(y(t+\tau)) \times B(y(t+\tau)) C e^{A\tau} E_0 d\tau \geq \iota \mathrm{I_r}, \ \ \forall \mathrm{t} \geq 0 \tag{3}$$

(iv) (C, A) is a detectable pair, the external input $v(t)$ is never identical to zero.

(v) Select any positive-definite diagonal matrix P that can make the equation below come into existence.

$$A^*P + PA = 0 \tag{4}$$

Then a Luenberger-type nonlinear observer can be designed for the above system as follows:

$$\begin{aligned} \frac{d}{dt}\hat{x}(t) &= A\hat{x}(t) + v(t) + K(y(t), \hat{x}(t))[y(t) - h(C\hat{x}(t))], \\ \hat{x}(0) &= \hat{x}_0 \in R^n \end{aligned} \tag{5}$$

where $\hat{x}(t)$ is the estimated state. Using System (1), one can easily obtain

$$\begin{aligned} y - h(C\hat{x}) &= h(Cx) - h(C\hat{x}) \\ &= \left[\frac{\xi_1}{\xi_{m+1}} - \frac{\hat{\xi}_1}{\hat{\xi}_{m+1}} \ \cdots \ \frac{\xi_m}{\xi_{m+1}} - \frac{\hat{\xi}_m}{\hat{\xi}_{m+1}} \right] \\ &= \frac{1}{C_{m+1}\hat{x}} \begin{bmatrix} 1 & 0 & \cdots & 0 & -y_1 \\ 0 & 1 & \cdots & 0 & -y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -y_m \end{bmatrix} \begin{bmatrix} C_1(x - \hat{x}) \\ C_2(x - \hat{x}) \\ \vdots \\ C_{m+1}(x - \hat{x}) \end{bmatrix} \\ &= \frac{1}{C_{m+1}\hat{x}} B(y) C \rho \end{aligned} \tag{6}$$

where $B(y)$ is the matrix-valued function given by

$$B(y) := [I_m \ \ -y] \in R^{m \times (m+1)}, \ \ m = 2 \tag{7}$$

The estimation error of the system states $\rho(t) := x(t) - \hat{x}(t)$ satisfies the differential equation:

$$\frac{d}{dt}\rho(t) = \left\{ A - \frac{1}{C_{m+1}\hat{x}(t)} K(y(t), \hat{x}(t)) B(y(t)C) \right\} \times \rho(t) \tag{8}$$

where $\rho(t_0) = x(0) - \hat{x}(0) \in R^n$. To eliminate the singularity in (8), one can choose a gain matrix $K(y, \hat{x})$ of the form

$$K(y, \hat{x}) := C_{m+1}\hat{x} P^{-1} C^* B^*(y) \tag{9}$$

The estimation error ρ has been proven to converge exponentially to zero in [22].

3. **Learning from System Dynamics.**

When the system states are observed, it lacks a universal way to learn the unknown system dynamics and reuse the learned knowledge for another similar process. In this paper, we choose to learn the system dynamics by using the RBF networks. When the above observation system satisfies the PE condition, the learned knowledge will be kept in constant RBF networks.

The employed RBF network is with the following form:

$$f_{nn} = \sum_{i=1}^{N} W_i S_i(Z) = W^T S(Z)$$

where $Z \in \Omega_Z \subset \mathbf{R}^q$ is the input vector, $W = [W_1, \cdots, W_N]^T \in \mathbf{R}^N$ is the weight vector, $S(Z) = [S_1(||Z - \mu_1||), \cdots, S_N(||Z - \mu_N||)]^T$ with $S_i(\cdot)$ being the RBFs. The Gaussian RBF is

$$S_i(||Z - \mu_i||) = \exp\left[\frac{-(Z - \mu_i)^T(Z - \mu_i)}{\eta_i^2}\right], \quad i = 1, 2, \cdots, N$$

where μ_i is the center of the receptive field, η_i is the width of the receptive field, and N is the NN node number.

The objective is to identify the unknown vision system dynamics $f(x) = Ax$ along the estimated system trajectory $\varphi_\zeta(t, \hat{x}_0)$, which can be implemented in two steps. Firstly, we employ the Luenberger-type observer (5) to obtain the unknown system states. After observation, the observation error will satisfy $||\hat{x}(t) - x(t)|| \leq d$, $\forall t \geq t'$, we denote it as $\hat{x} \to x$ which means that the estimate $\hat{x}(t)$ converges to a sufficiently small neighborhood of the state $x(t)$ in finite time. Secondly, we employ the following dynamical RBF network for identification of the system dynamics $f(x)$:

$$\dot{\chi} = -a(\chi - \hat{x}) + \hat{W}^T S(\hat{x}) \tag{10}$$

with $\chi = [\chi_1, \ldots, \chi_3]^T$ being the state vector of the dynamical RBF network, $a = \mathrm{diag}[a_1, \ldots, a_n]$ being a diagonal matrix with $a_i > 0$ being design constants, $Z = \hat{x}$ being the estimate of system states obtained from (5), and $\hat{W}^T S(\hat{x}) = \left[\hat{W}_1^T S_1(\hat{x}), \hat{W}_2^T S_2(\hat{x}), \hat{W}_3^T S_3(\hat{x})\right]^T$ being localized RBF NNs used to identify (learn) the system dynamics. The NN weight updating law is given by

$$\dot{\hat{W}} = -\Gamma S(\hat{x})\bar{x} - \Gamma\sigma\hat{W} \tag{11}$$

where $\Gamma = \Gamma^T > 0$, $\sigma > 0$ is a small value, and $\bar{x} := \chi - \hat{x}$.

Thus the neural weights for each state can be expressed as follows:

$$\dot{\hat{W}}_i = -\Gamma_i S_i(\hat{x})\bar{x}_i - \Gamma_i\sigma\hat{W}_i, \quad i = 1, 2, 3 \tag{12}$$

Remark 3.1. *Consider the adaptive system consisting of the nonlinear dynamical system (1), the RBF network (10), and the NN weight updating law (11). For almost every recurrent trajectory $\varphi_\zeta(t, x_0)$, with initial values $\hat{W}(0) = 0$, we have: (i) all signals in the adaptive system remain uniformly ultimately bounded; and (ii) locally accurate approximation for the unknown $f(x)$ to the error level ϵ^* is obtained along the trajectory $\varphi_\zeta(t, \hat{x}_0)$ when $\hat{x} \to x$. These results can be found in [28].*

After learning, the entire RBF network $\hat{W}^T S(\hat{x})$ can approximate the unknown system dynamics along the estimated trajectory $\hat{x}(t)$ as:

$$f(\hat{x}) = A\hat{x} = \hat{W}^T S(\hat{x}) + \epsilon_1 \tag{13}$$

where $|\epsilon_1| = |d + \varepsilon|$ is small, and $f(\hat{x})$ is the practical system dynamics. Moreover, choose

$$\bar{W} = mean_{t\in[t_a, t_b]}\hat{W}(t) \tag{14}$$

where "mean" is the arithmetic mean, $t_b > t_a > 0$ represent a time segment after the transient process, the system dynamics

$$f(\hat{x}) = \bar{W}^T S(\hat{x}) + \epsilon_2 \tag{15}$$

and

$$\left|\bar{W}^T S(\hat{x}) - f(x)\right| < \epsilon^* \tag{16}$$

In the following, we show that the knowledge obtained through deterministic learning [25-27] can be reused in another state observation process and the estimation error can still converge exponentially to zero.

For state observation of the same nonlinear system (1), an RBF NN-based Luenberger-type nonlinear observer is constructed as follows:

$$\begin{aligned}\frac{d}{dt}\hat{x}(t) &= \bar{W}^T S(\hat{x}) + v(t) + K(y(t), \hat{x}(t))[y(t) - h(C\hat{x}(t))], \\ \hat{x}(0) &= \hat{x}_0 \in R^n\end{aligned} \tag{17}$$

where $K(y(t), \hat{x}(t))$ is defined in (9). The vector of the estimation error for the system states is defined as $\rho(t) := x(t) - \hat{x}(t)$. The following theorem indicates that identification of the unknown $f(x)$ can be achieved along the trajectory $\varphi_\zeta(t, \hat{x}_0)$ when $\hat{x} \to x$.

Theorem 3.1. *Consider the adaptive system consisting of the nonlinear dynamical system (1), the Luenberger-type nonlinear observer (17), the gain matrix (9) and $B(y)$ given by (7). The following statements hold:*

(i) The estimation error $\rho(t) := x(t) - \hat{x}(t)$ satisfies the differential equation

$$\begin{aligned}\frac{d}{dt}\rho(t) &= Ax - \bar{W}^T S(\hat{x}) - P^{-1}C^*B^*BC\rho \\ &= -\epsilon^* - P^{-1}C^*B^*BC\rho \\ \rho(0) &\in R^n\end{aligned} \tag{18}$$

(ii) $\rho(t)$ converges exponentially to zero, that is, there exist $\alpha > 0$, $\beta > 0$ such that

$$\begin{aligned}||\rho(t)|| &:= ||x(t) - \hat{x}(t)|| \\ &\leq \beta e^{-\alpha t}||\rho(0)||^2, \ \ \forall t \geq 0\end{aligned} \tag{19}$$

Proof: The statement (i) in Theorem 3.1 directly follows (1), (9), (16) and (17).

To prove statement (ii), first of all, we deduce several important facts from Assumption 2.1. Let $\pi_- : C^n \to W_-$, $\pi_0 : C^n \to W_0$ denote the matrix representations of the projection operators along W_0, W_-. Using the notation in Theorem 3.1, we define

$$\rho_-(t) := \pi_-\rho(t), \quad \rho_0(t) := \pi_0\rho(t), \ \ \forall t \geq 0 \tag{20}$$

Then since $C^n = W_- \oplus W_0$ one has

$$\rho(t) = \rho_-(t) + \rho_0(t) \tag{21}$$

which leads to the inequality

$$\begin{aligned}\int_0^\infty ||\rho(t)||^2 dt &= \int_0^\infty ||\rho_-(t) + \rho_0(t)||^2 dt \\ &\leq 2\int_0^\infty ||\rho_-(t)||^2 dt + 2\int_0^\infty ||\rho_0(t)||^2 dt\end{aligned} \tag{22}$$

Using (18) and (20), we can obtain

$$\begin{aligned}\frac{d}{dt}(\rho^*(t)P\rho(t)) &= -\epsilon^* P(\rho^*(t) + \rho(t)) - 2\rho^*(t)C^*B^*(y(t))B(y(t))C\rho(t) \\ &\leq -2\epsilon^*||P\rho_-(t)|| - 2||B(y(t))C\rho(t)||^2\end{aligned} \tag{23}$$

Then, for any $t \geq 0$,

$$0 \leq \rho^*(t)P\rho(t) \leq \rho^*(0)P\rho(0) - 2\epsilon^* \int_0^t ||P\rho_-(s)||ds - 2\int_0^t ||B(y(s))C\rho(s)||^2 ds$$

Hence, one can get that

$$2\epsilon^* \int_0^t ||P\rho_-(s)||ds \leq \rho^*(0)P\rho(0), \ \ \forall t \geq 0$$

$$2\int_0^t ||B(y(s))C\rho(s)||^2 ds \le \rho^*(0)P\rho(0), \quad \forall t \ge 0 \tag{24}$$

Since $P^* = P > 0$, then $\rho^*(0)P\rho(0) \le ||P||\ ||\rho(0)||^2$, one can obtain the inequality from (24) that

$$\int_0^\infty ||\rho_-(t)||dt \le \frac{||\rho(0)||^2}{2\epsilon^*} \Rightarrow \int_0^\infty ||\rho_-(t)||^2 dt \le \frac{||\rho(0)||^4}{4\epsilon^{*2}} =: \gamma_-||\rho(0)||^4 \tag{25}$$

where $\gamma_- > 0$ is a constant independent of $\rho(0)$.

From [Lemma 3.2, [22]], one can obtain that

$$\int_0^\infty ||\rho_0(t)||^2 dt \le \gamma_0||\rho(0)||^2 \tag{26}$$

where $\gamma_0 > 0$ is a constant independent of $\rho(0)$.

It follows from (22), (25) and (26) that

$$\begin{aligned}\int_0^\infty ||\rho(t)||^2 dt &\le 2\int_0^\infty ||\rho_-(t)||^2 dt + 2\int_0^\infty ||\rho_0(t)||^2 dt \\ &\le 2\gamma_-||\rho(0)||^4 + 2\gamma_0||\rho(0)||^2 \\ &= 2\gamma_-\left(||\rho(0)||^2 + \frac{\gamma_0}{2\gamma_-}\right)^2 - \frac{\gamma_0^2}{2\gamma_-}\end{aligned} \tag{27}$$

where $\frac{\gamma_0}{2\gamma_-} > 0$, $\frac{\gamma_0^2}{2\gamma_-} > 0$ are constants independent of $\rho(0)$. This completes the proof of Theorem 3.1. □

4. Recognition Scheme of the Perspective Dynamic Vision System.

In this section, we consider the problem of representation, similarity definition and rapid recognition of the patterns of the perspective dynamic vision system.

4.1. Representation using estimated states.

Consider the dynamical pattern φ_ζ generated from the following dynamic vision system:

$$\begin{aligned}\dot{x}_\zeta(t) &= Ax_\zeta(t) + v(t), \quad x_\zeta(0) = x_{\zeta 0} \\ y_\zeta(t) &= h(Cx_\zeta)\end{aligned} \tag{28}$$

where $x_\zeta \in R^n$ is the system state, and $f_\zeta(x_\zeta) = Ax_\zeta$ is a smooth but unknown function. For representation of a dynamical pattern, complete information on both its estimated pattern states and its underlying system dynamics is used. The pattern φ_ζ can be represented via deterministic learning by using the constant RBF network $\bar{W}^T S(Z)$, which provides a locally accurate NN approximation of the underlying system dynamics $f_\zeta(x_\zeta)$. The knowledge represented in RBF network $\bar{W}^T S(Z)$ is valid in a local region Ω_{φ_ζ}, which can be described as: for the pattern state trajectory φ_ζ, there exist constants d_ζ, $\xi^* > 0$ such that

$$dist(Z, \varphi_\zeta) < d_\zeta \Rightarrow \left|\bar{W}^T S(\hat{x}_\zeta) - f_\zeta(x_\zeta)\right| < \xi^* \tag{29}$$

where $\hat{x}_\zeta(t)$ is the estimate of $x_\zeta(t)$, ξ^* is the approximation error within Ω_{φ_ζ}, which is of small value.

Thus, a dynamical pattern is represented in a time-invariant and spatially distributed manner by using information regarding both its estimated pattern states $\hat{x}_\zeta$ and its underlying system dynamics $f_\zeta(x_\zeta)$ along the estimated state trajectory $\hat{x}_\zeta(t)$.

4.2. **Similarity definition.** Consider the dynamical pattern φ_ζ (as given by Equation (28)), and another dynamical pattern φ_ς generated from the following dynamical system:

$$\begin{aligned} \dot{x}_\varsigma(t) &= Ax_\varsigma(t) + v(t), \quad x_\varsigma(0) = x_{\varsigma 0} \\ y_\varsigma(t) &= h(Cx_\varsigma) \end{aligned} \tag{30}$$

where $x_\varsigma \in R^n$ is the system state, and $f_\varsigma(x_\varsigma) = Ax_\varsigma$ is a smooth and unknown function.

Since the state variables are mostly unknown, we rely on the difference between corresponding system dynamics within a local region Ω_ς:

$$\Omega_\varsigma := \{x | dist(x, \varphi_\varsigma) < d_\varsigma\}$$

where $d_\varsigma > 0$ is a constant.

We have the following definitions for similarity of dynamical patterns.

Definition 4.1. *Dynamical pattern φ_ς is said to be similar to dynamical pattern φ_ζ, if the state of pattern φ_ς stays within a neighborhood region of the state of pattern φ_ζ, and the difference between the corresponding system dynamics within a local region Ω_ς, that is*

$$\Delta f = |f_\zeta(x) - f_\varsigma(x)|_{\forall x \in \Omega_\varsigma} \le \varepsilon^* \tag{31}$$

where $\varepsilon^* > 0$, which is the similarity measure, is small.

Definition 4.2. *Dynamical pattern φ_ς is said to be similar to dynamical pattern φ_ζ, if the state of pattern φ_ς stays within a neighborhood region of the state of pattern φ_ζ, and the difference between the corresponding system dynamics within a local region Ω_ς, that is*

$$\Delta f_N = \left|\bar{W}^T S(\hat{x}_\varsigma) - f_\varsigma(x)\right|_{\forall x \in \Omega_\varsigma} \le \varepsilon^* + \xi^* \tag{32}$$

where $\hat{x}_\varsigma(t)$ is the estimate of $x_\varsigma(t)$, ε^* is the similarity measure and ξ^* is the approximation error given in Equation (29), and is small.

4.3. **Rapid recognition via Luenberger-type state observation.** In this subsection, we present how to achieve rapid recognition of dynamical patterns via Luenberger-type state observation.

Consider a dynamical pattern φ_ς (as given by Equation (30)) as a test dynamical pattern. Consider again a set of training patterns φ_ζ^k, $k = 1, \ldots, M$, with the kth training pattern φ_ζ^k generated from

$$\begin{aligned} \dot{x}_\zeta^k(t) &= Ax_\zeta^k(t) + v(t), \quad x_\zeta^k(0) = x_{\zeta 0}^k \\ y_\zeta^k(t) &= h(Cx_\zeta^k) \end{aligned} \tag{33}$$

where x_ζ^k are the state variables of the kth training pattern φ_ζ^k, $f_\zeta^k\left(x_\zeta^k\right) = Ax_\zeta^k$ are unknown smooth functions.

For rapid recognition of a test dynamical pattern φ_ς from a set of training dynamical patterns, a set of RBF NN-based Luenberger-type nonlinear observers is constructed as follows:

$$\frac{d}{dt}\hat{x}^k(t) = \bar{W^k}^T S(\hat{x}_\varsigma) + v(t) + K^k(y_\varsigma(t), \hat{x}(t))\left[y_\varsigma(t) - h\left(C\hat{x}^k(t)\right)\right] \tag{34}$$

where $k = 1, \ldots, M$, the superscript $(\cdot)^k$ denotes the component for the kth training pattern, $\hat{x}^k$ is the state of the set of Luenberger-type nonlinear observers, $\hat{x}_\varsigma$ is the estimate of x_ς, the constant RBF network $\bar{W^k}^T S(\hat{x}_\varsigma)$ is embedded to provide a locally accurate approximation of system dynamics $f_\zeta^k\left(x_\zeta^k\right) = Ax_\zeta^k$ of the training dynamical pattern φ_ζ^k. These observers are taken as dynamic representations of the corresponding training dynamical patterns.

When a test dynamical pattern φ_ς is presented to one RBF network-based observer (i.e., the dynamical model for training pattern φ_ζ), a state observation error system (i.e., recognition error system) is yielded as follows:

$$\begin{aligned}\dot{e}^k &= Ax_\varsigma - \bar{W^k}^T S(\hat{x}_\varsigma) - K^k(y_\varsigma(t), \hat{x}(t)) \left[y_\varsigma(t) - h\left(C\hat{x}^k(t)\right)\right] \\ &= Ax_\varsigma - \bar{W^k}^T S(\hat{x}_\varsigma) - P^{-1}C^*B^*BCe^k \end{aligned} \tag{35}$$

where $e^k := x_\varsigma - \hat{x}^k \in R^n$ is the recognition error, and $K^k(y_\varsigma(t), \hat{x}(t))$ has the same definition in (9). The following theorem describes how to achieve rapid recognition of a test dynamical pattern.

Theorem 4.1. *Consider the recognition error system (35) corresponding to the test pattern φ_ς and the RBF NN-based Luenberger-type observer for the training pattern φ_ζ^k. If the estimated state $\hat{x}^k$ stays within a local region Ω_ς along the orbit of the test pattern φ_ς, then the recognition error $||e^k||$ will be approximately proportional to the difference between system dynamics of test pattern φ_ς and training pattern φ_ζ^k.*

Proof: From Equations (30) and (34), we have

$$\dot{e}^k = Ax_\varsigma - Ax_\zeta^k + Ax_\zeta^k - \bar{W^k}^T S\left(\hat{x}^k\right) - P^{-1}C^*B^*BC\rho \tag{36}$$

Choose Lyapunov function $V^k = \frac{1}{2}{e^k}^T Pe^k$. Thus, its derivative satisfies:

$$\begin{aligned}\dot{V}^k &= {e^k}^T P\left[Ax_\varsigma - \bar{W^k}^T S\left(\hat{x}^k\right) - P^{-1}C^*B^*BCe^k\right] \\ &= -{e^k}^T C^*B^*BCe^k + {e^k}^T P\left[Ax_\varsigma - \bar{W^k}^T S\left(\hat{x}^k\right)\right] \\ &\le -||B||^2\ \|e^k\|^2 + \|e^k\|\ ||P||\ \left\|Ax_\varsigma - Ax_\zeta^k + Ax_\zeta^k - \bar{W^k}^T S\left(\hat{x}^k\right)\right\| \\ &\le -\frac{1}{2}||B||^2\ \|e^k\|^2 + \frac{||P||^2\left(\varepsilon^{k*} + \xi^{k*}\right)^2}{2||B||^2} \\ &\le -\frac{||B||^2}{2\lambda_{\max}(P)}\lambda_{\max}(P)\|e^k\|^2 + \frac{||P||^2\left(\varepsilon^{k*} + \xi^{k*}\right)^2}{2||B||^2} \\ &\le -\frac{||B||^2}{\lambda_{\max}(P)}V^k + \frac{||P||^2\left(\varepsilon^{k*} + \xi^{k*}\right)^2}{2||B||^2} \le -\alpha V^k + \delta \end{aligned} \tag{37}$$

where

$$\begin{aligned}\alpha &:= \frac{||B||^2}{\lambda_{\max}(P)} \\ \delta &:= \frac{||P||^2\left(\varepsilon^{k*} + \xi^{k*}\right)^2}{2||B||^2} \\ \eta &:= \frac{\delta}{\alpha} = \lambda_{\max}(P)\frac{||P||^2\left(\varepsilon^{k*} + \xi^{k*}\right)^2}{2||B||^4}\end{aligned}$$

Then, Equation (37) deduces

$$\lambda_{\min}(P)||e^k||^2 \le V^k(t) < \eta + \left(V^k(0) - \eta\right)\exp(-\alpha t) \tag{38}$$

That is:

$$\begin{aligned}\lambda_{\min}(P)||e^k||^2 &< \eta + \left(V^k(0) - \eta\right)\exp(-\alpha t) \\ &< \eta + V^k(0))\exp(-\alpha t)\end{aligned} \tag{39}$$

and

$$||e^k||^2 < \left[\eta + V^k(0)\exp(-\alpha t)\right]/\lambda_{\min}(P) \tag{40}$$

which implies that given $\nu > \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \cdot \frac{||P||\left(\varepsilon^{k^*}+\xi^{k^*}\right)}{\sqrt{2}||B||^2}$, there exists a finite time T, such that for all $t \geq T$, the recognition error $||e^k||$ will exponentially converge to a small neighborhood around zero; that is, $||e^k|| \leq \nu$, with the size of the neighborhood ν approximately proportional to $\varepsilon^{k^*} + \xi^{k^*}$, and inversely proportional to $||B||$. Thus, the recognition error $||e^k||$ will be approximately proportional to the difference between system dynamics of test pattern φ_ς and training pattern φ_ζ^k. We compute the average L_1 norm of the error $e^k(t)$

$$\|e^k(t)\|_1 = \frac{1}{\mathrm{T}} \int_{t-\mathrm{T}}^{t} \left|e^k(\tau)\right| d\tau, \quad k = 1, \ldots, n, \quad t \geq \mathrm{T} \tag{41}$$

□

5. **Simulation Study.** In order to show the effectiveness of the Luenberger-type nonlinear observer for perspective linear system (1) and the scheme of the rapid dynamical pattern recognition, some simulations are given in this section. Consider a simple vision system described in a perspective linear form with the following data:

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}; \quad V(t) = 2\pi \begin{bmatrix} -\sin(2\pi t) \\ \cos(2\pi t) \\ 0 \end{bmatrix}$$

The matrix $C = \mathrm{I}_3$, A is Lyapunov stable. (C, A) is observable, hence it is detectable, and all the assumptions in Assumption 2.1 are satisfied. We choose the form of the observer as in [22] and the initial conditions are:

$$\hat{x}_0 = [1 \;\; 1 \;\; 2]^T, \quad P^{-1} = \mathrm{diag}(8.2, 8.2, 8.2) = 8.2\mathrm{I}_3$$

where the free parameter P is an identity matrix with suitable gain, such that the Lyapunov equation $A^*P + PA = 0$ is satisfied.

In the training phase, we consider three training dynamical patterns generated from system (1) which are denoted as φ_ζ^1, φ_ζ^2 and φ_ζ^3. They are started from initial states: $x_\zeta^1(0) = [1\ 1\ 2]$, $x_\zeta^2(0) = [1\ 1\ 1]$ and $x_\zeta^3(0) = [1\ 1\ 3]$, respectively.

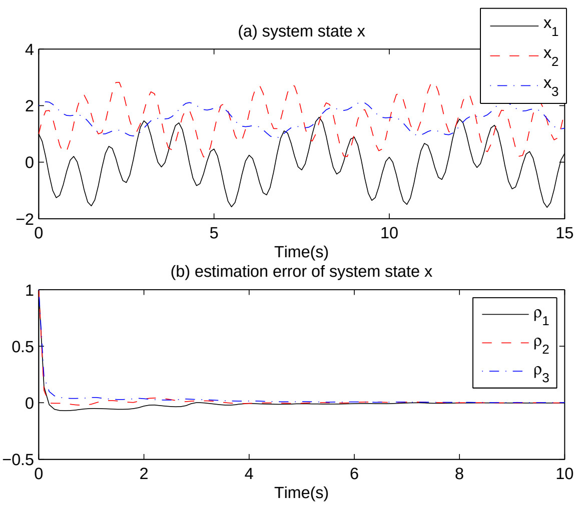

FIGURE 1. System state $x = [x_1, x_2, x_3]^T$ and its observation error $\rho = [\rho_1, \rho_2, \rho_3]^T$

To identify the unknown dynamics $f(x) = Ax$ of the three training patterns, the dynamical RBF network is employed. The RBF network $\hat{W}^T S(x)$ is constructed in a regular lattice, with nodes $N = 240$, the center μ_i evenly spaced on $[-0.61\ 2] \times [0.18\ 2.9] \times [0.8\ 3]$, and the width $\eta_i = 0.5$. The design parameters for Equations (10) and (11) are $a = [10\ 2\ 10; 2\ 10\ 10; 2\ 2\ 10]$, $\Gamma = 10$, $\sigma = 0.001$. The initial weights $\hat{W}(0) = 0$.

Figure 1 shows the estimation of the system state $x = [x_1\ x_2\ x_3]^T$ and its estimation error using Luenberger-type nonlinear observer. Figure 2 shows the three-dimensional trajectory tracking of system states between (x_1, x_2, x_3) and the estimated $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$. As can be seen from Figure 3, the RBF network has learned the unknown system dynamics along the estimated state trajectory $\hat{x}$ with a good effect. It can be seen from Figure 4 that

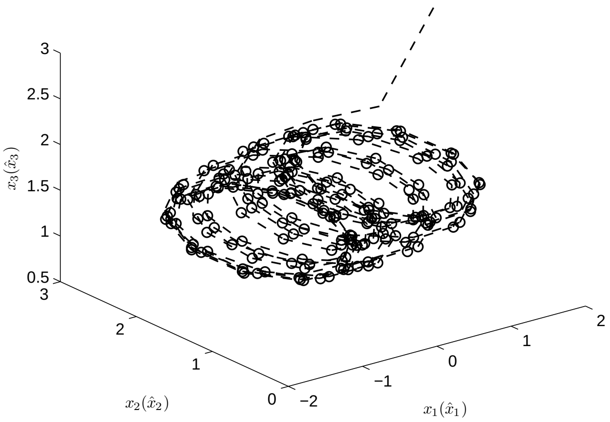

FIGURE 2. The three-dimensional trajectory tracking of system states between (x_1, x_2, x_3) ("o") and the estimated $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$("− −")

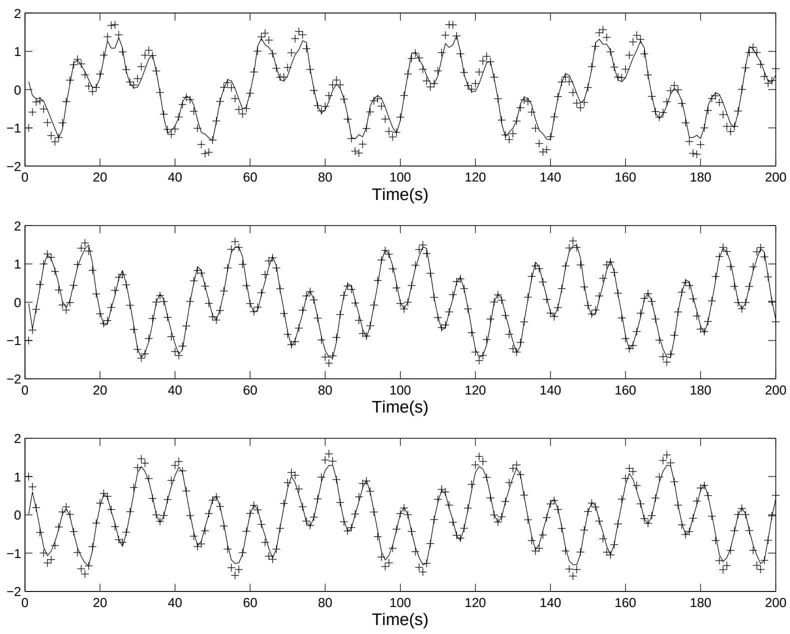

FIGURE 3. Learning of system dynamics: $A\hat{x}$ ("—"), $\hat{W}^T S(\hat{x})$("+")

function approximation errors are small. The convergence of the neural weights is shown in Figures 5 to 8. Especially, Figures 6 to 8 demonstrate partial parameter convergence of the neural weights. It is seen from these figures that weights of the neurons of RBF networks whose centers are closed to the state $\hat{x}$ converge to constant values, while some other weights (of whose neurons centered far away from the orbit) remain almost zero. Then, the learned knowledge is kept in the constant RBF networks for the following rapid recognition of system dynamical patterns.

In the test phase, consider a test pattern generated from (30) which is denoted as φ_ς, with initial state: $x_\varsigma(0) = [1\ 1\ 2]$. For rapid recognition of a test dynamical pattern from training dynamical patterns, we construct a set of RBF network-based nonlinear observers with suitably designed gain matrix. By taking the test pattern as input to the set of RBF network-based observers, we choose the suitable gain matrix to find out which observer yields the smallest recognition error. The corresponding training dynamical pattern is considered to be most similar to the test pattern.

The recognition errors $||e^k(t)||$ $(k = 1, 2, 3)$ are shown in Figures 9 to 11. The average L_1 norms of the recognition errors are shown in Figure 12. Hence the result of the rapid

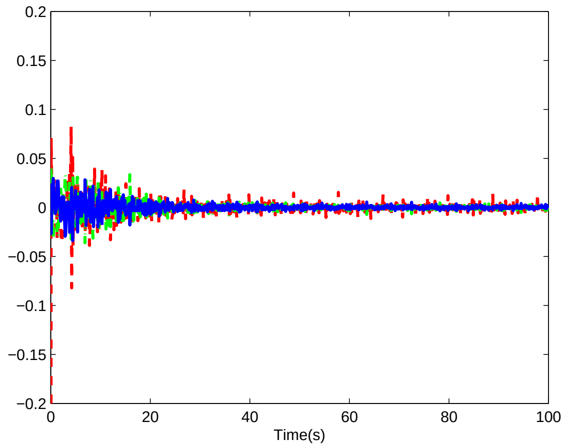

Figure 4. Function approximation errors $A\hat{x} - \hat{W}^T S(\hat{x})$

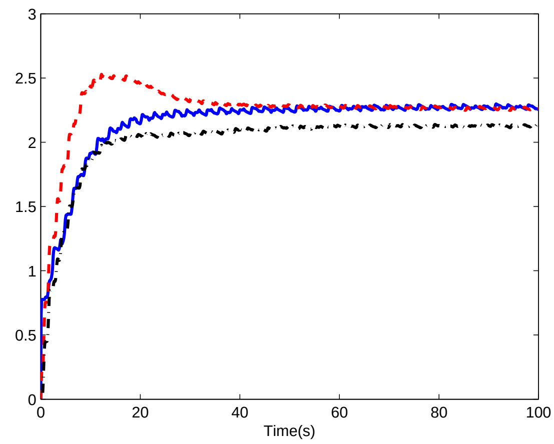

Figure 5. Parameter convergence: $||\hat{W}_1||$ ("—"), $||\hat{W}_2||$ ("–"), $||\hat{W}_3||$ ("$- \cdot - \cdot$")

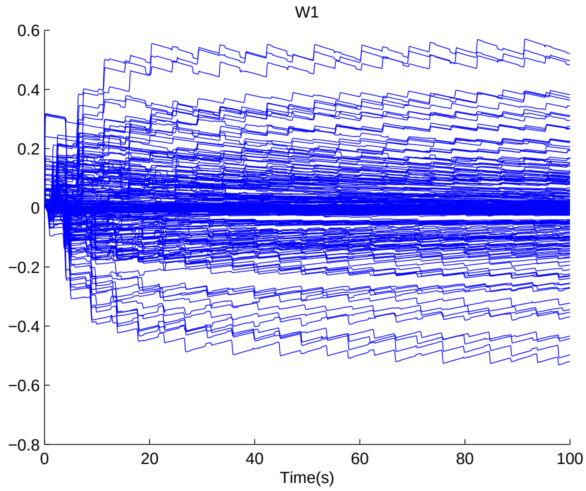

Figure 6. Partial parameter convergence W_1

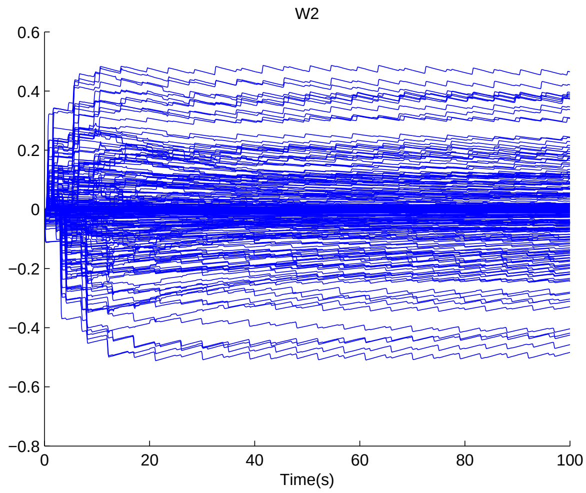

Figure 7. Partial parameter convergence W_2

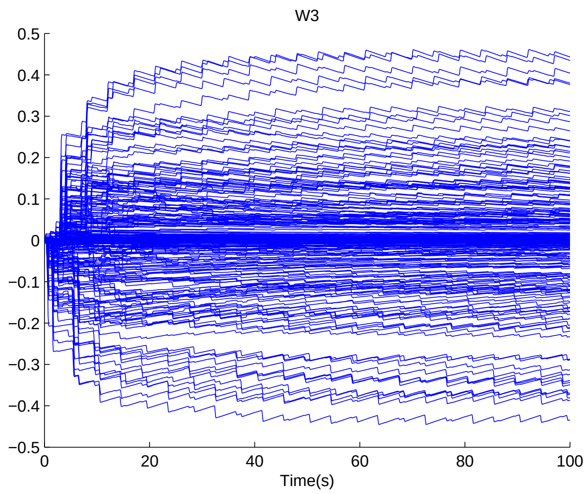

Figure 8. Partial parameter convergence W_3

recognition can easily be achieved. It is obvious from Figure 12 that only training pattern φ_ζ^1 is similar to the test pattern φ_ς.

Remark 5.1. *In comparison with other methods mentioned in [10,18,21-23], the proposed scheme can not only estimate the unknown states of the perspective vision system, but also identify and approximate the unknown dynamics of the vision system by using RBF networks. The obtained knowledge of the various vision system dynamics can be used for rapid recognition of new and similar patterns of the perspective dynamic vision system, which cannot be achieved in other methods. Moreover, in other methods the persistence of excitation (PE) condition was adopted for the parameter estimation and stability analysis. However, in many cases the PE condition is hard to be satisfied and the convergency of the parameter estimation error is difficult to be guaranteed. By using the proposed scheme, the PE condition can be satisfied and the approximation error converges exponentially to a small neighborhood around zero.*

6. **Conclusion.** This paper shows that the deterministic learning theory can be used to learn the vision system dynamics, and has an excellent effect. For the perspective dynamic vision systems with constant parameters and unknown system states, a Luenberger-type

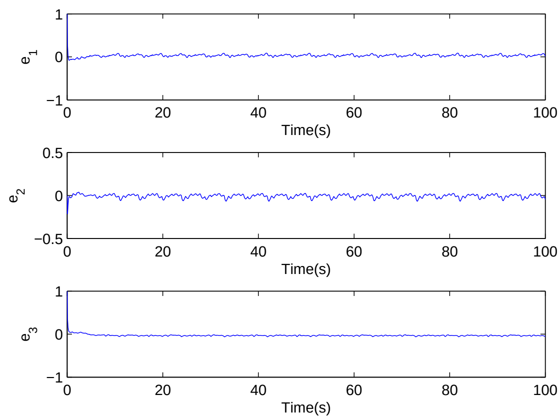

FIGURE 9. Recognition error corresponding to training pattern φ_ζ^1

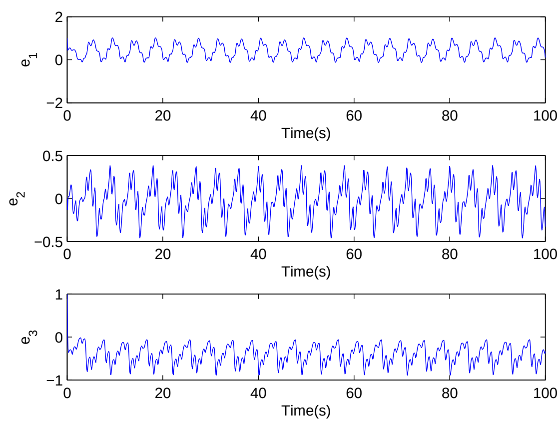

FIGURE 10. Recognition error corresponding to training pattern φ_ζ^2

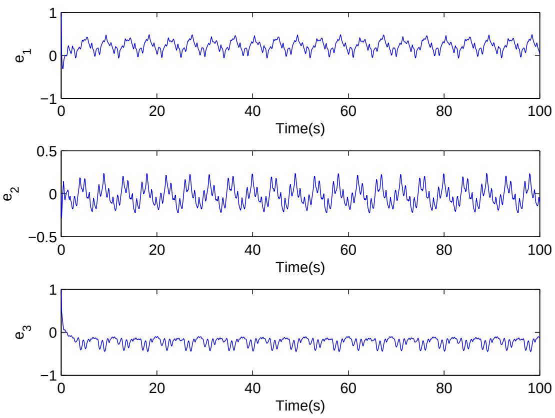

FIGURE 11. Recognition error corresponding to training pattern φ_ζ^3

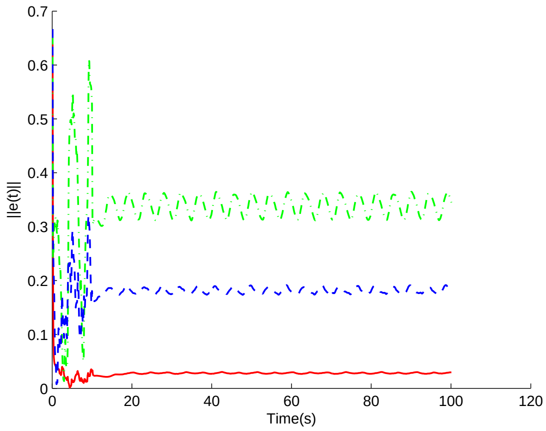

FIGURE 12. Average L_1 norm of $||e(t)||$ for training pattern φ_ζ^1 ("—"), φ_ζ^2 ("$- \cdot - \cdot$") and φ_ζ^3 ("–")

observer can be used to estimate the states. Through learning the underlying system dynamics from observation, we can keep the learned knowledge in the form of constant RBF network to construct a set of RBF NN-based nonlinear observers as dynamic representation for the training patterns. When a test dynamical pattern is presented to the RBF NN-based observers, a set of recognition errors are generated and taken as the similarity measure between the test and training dynamical patterns. Then, rapid recognition of the test patterns from corresponding training patterns can be achieved.

Acknowledgments. This work was supported by the National Natural Science Foundation of China (Grant no. 61304084), by the Program for New Century Excellent Talents in Fujian Province University, by the Educational and Scientific Research Project for Middle-aged and Young Teachers of Fujian Province of China (Grant nos. JA14298, JA15498), and by the Science and Technology Project of Longyan University (Grant nos. LC2014005, LC2015008, LC2015009, LQ2015027).

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