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EDGE RANKING IN GRAPH USING DISCRETE CHOICE

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Abstract. *In recent years there has been great interest in the automobile society in finding the impact on the transportation network by new construction, the collapse of roads and as well as thinking on long-term urban planning problem which implies to be necessary to estimate the traffic volume by mathematical modeling as a graph network. In this paper, assuming the probability of several people's path selection, we associate several physical quantities with the weight of the edge by using the concept of Discrete Choice and introduce a new graph network model to estimate the traffic volume by using the Entropy Maximization theory. Finally, we define the importance of each edge as the Edge Ranking and illustrate numerically its derivation using a simple graph network.*
Keywords: Urban planning problem, Transportation network, Entropy Maximization theory, Discrete Choice, Edge Ranking

1. Introduction.

Recently the automobile society is very interested in finding the impact on the transportation network by new construction, the collapse of roads and thinking on the long-term urban planning problem. In order to solve this typical problem, we thought that it is necessary to estimate the traffic volume by mathematical modeling as a graph network defining the edges as roads and the nodes as intersections, and present the importance of edges or nodes as something of theoretical value.

In the paper [1], Geographical Advantages (GAs) have been proposed as the index of node-importance. It is one of theoretical values for the advantage of position. The value of GA depends on the shape of the network structure and is proportional to the total sum of the number of neighbor nodes; therefore, it becomes large if the number of connected nodes increases. It is corresponding to the maximum eigenvalue of the adjacency matrix representing the structure of graph.

In the urban planning problem based on the concept that spatial distance affects human behavior, a spatial interaction model where an effect to certain point from certain city is assumed to be proportional to the city proportion size and be inversely proportional to the distance from point to city, has been proposed in [2, 3]. When the attenuation function with respect to distance is expressed by a negative exponential function, such a spatial interaction model is called a gravity model. In the paper [4], Maximization Entropy model [5] whose attenuation function is given by a negative exponential function has been proposed. It is shown that network flow, that is traffic volume, can be estimated from inflow and outflow of each node.

In the paper [6], for the basic spatial interaction model without considering path selection from each original-destination (OD) pair, a traffic assignment model has been formulated. Furthermore, extended model which permits consideration of several path selections has been developed in the paper [7]. This paper clarifies the relationship between the estimated OD trip and the GA. It derives the new GA which depends on not only network structure but also OD information, in other words, which is parametrized by traffic volume.

In these models, edges have weights corresponding to only one physical quantity, indicating that people evaluate paths by the physical quantity. However, for example, when several paths are presented on the car navigation systems, it is supposed to be natural for us to decide a path by evaluating various physical quantities comprehensively, such as distance, time, and toll.

The concept of Discrete Choice (DC) model has been proposed in [8, 9, 10]. DC model can describe mathematically human behavior that a person selects a choice from the set of several ones. Namely, it is assumed that for each choice, there exists utility, the value is proportional to obtain the degree of satisfaction when a parson selects a choice, and the selection probability depends on utility. In the paper [11], the method of estimating utility has been proposed on the condition that selection probability of people is given. Noteworthy of this paper is that the new model that people give each choice utility evaluated from several value factors comprehensively is constructed.

In this paper, by introducing the concept of DC in [11], we have represented how much influence various factors have on the people's path selection by values called the utility, and then we associated weight in the graph network model in [7] with utility in the DC. As a result, we constructed a new model with several physical quantities in a weight. Furthermore, introducing the idea of edge-importance in [12], we have formulated Edge Ranking parametrized by estimated traffic volume and shown numerical examples.

The paper is organized as follows. In Section 2, in order to model the road network, we formulate Entropy Maximization model of graph network to be used in the present study [7], and construct a new theorem to estimate the OD traffic volume by iteration of the eigenvalue problem. Then, we define the importance of edges depending on traffic volume as the Edge Ranking from the estimated OD matrix. In Section 3, based on [11], we introduce the mechanism to estimate utility of the people by observing several selection probabilities and describe the method of estimation of these utilities and link weight to utility in order to apply graph network. Section 4 presents the results through two numerical examples of the Edge Ranking using a simple graph network, and concluding remarks are offered in Section 5.

2. Estimation of Traffic Volume and Derivation of Edge Ranking.

2.1. Entropy Maximization model.

With reference to [7], consider a graph network based on the Entropy Maximization model. V_i $(V_i = V_1, V_2, \cdots, V_m)$ represents the traffic inflow of node i and U_j $(U_j = U_1, U_2, \cdots, U_n)$ represents the traffic outflow of node j. It is assumed that V_i and U_j can be obtained. A set of paths to move from i to j is represented by $\{ij\}$ $(\{ij\} = \{ij^1, ij^2, \cdots, ij^{K_{ij}}\} \in \Omega^{K_{ij}})$. Each ij dose not contain backtrack paths and $\{ij\} = \{ji\}$ is not always satisfied.

Let r_{ij^k} be the traffic volume of the path ij^k. By the condition that V_i and U_j are constants, these constraints equations

$$V_i = \sum_{j=1}^{m} \sum_{k=1}^{K_{ij}} r_{ij^k} \qquad (\forall i) \tag{1}$$

$$U_j = \sum_{i=1}^{n} \sum_{k=1}^{K_{ij}} r_{ij^k} \qquad (\forall j) \tag{2}$$

are satisfied. The cost of ij^k is represented by c_{ij^k}. In graph network problem, cost means that the weight given to the edge, that is, its quantified information is proportional to the loss of physical quantity by passing through the edge, for example, time, money, and fuel.

Assuming that the total cost C, which is the sum of value payed by all drivers, is constant, this constraint equation

$$TC = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{K_{ij}} c_{ij^k} r_{ij^k} \tag{3}$$

is satisfied similarly. Here T is the total traffic volume. By considering the combination of T cars, we can obtain the following equation

$$\begin{aligned} \omega(r_{ij^k}) &= \frac{T!}{r_{12^1}!(T - r_{12^1})!} \cdot \frac{(T - r_{12^1})!}{r_{12^2}!(T - r_{12^1} - r_{12^2})!} \cdot \frac{(T - r_{12^1} - r_{12^2})!}{r_{12^3}!(T - r_{12^1} - r_{12^2} - r_{12^3})!} \cdots \\ &= \frac{T!}{\prod_{i=1}^{n} \prod_{j=1}^{m} \prod_{k=1}^{K_{ij}} r_{ij^k}!}. \end{aligned} \tag{4}$$

Equation (4) is called Entropy Maximization model estimating the traffic volume which makes the most of $\omega(r_{ij^k})$ under constraints Equations (1), (2) and (3). Consider an application of Stirling's approximation $\ln T! \approx T \ln T - T$ with the Lagrange undetermined multipliers γ_i $(\gamma_i = \gamma_1, \gamma_2, \cdots, \gamma_m)$, μ_j $(\mu_j = \mu_1, \mu_2, \cdots, \mu_n)$, ρ for each constraint and the condition of total sum of r_{ij^k} being equivalent to T, the interaction can be derived as

$$r_{ij^k} = \varphi \exp\left(-\gamma_i - \mu_j - \rho c_{ij^k}\right) \tag{5}$$

$$\varphi = \frac{T}{\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{K_{ij}} \exp\left(-\gamma_i - \mu_j - \rho c_{ij^k}\right)}. \tag{6}$$

Entropy Maximization model [5] coincides with gravity model and moreover, Geographical Advantage is connected with accessibility in the paper [1]. By obtaining the GA, that is, obtaining the eigenvector $\mathbf{g}$ of the maximum eigenvalue of the matrix $\mathbf{M}$, it attempts to estimate the Lagrange undetermined multipliers. In our study we analyze the property of matrix $\mathbf{M}$ through the new theorem as follows.

Theorem 2.1. *Assuming* $\gamma_i = -\log g_i^{(\gamma)}$, *then the vector* $\mathbf{g}^{(\gamma)} = \left[g_1^{(\gamma)}, g_2^{(\gamma)}, g_3^{(\gamma)}, \cdots, g_n^{(\gamma)}\right]^{\mathrm{T}} \in \Re^{n\times 1}$ *is the eigenvector of the matrix* $\mathbf{M}^{(\mu,\rho)} \in \Re^{n\times n}$ *which has a unique eigenvector* 1. *Subscript* (γ) *or* (μ, ρ) *means this vector or matrix is constituted by* γ_i *or* μ_j, ρ.

From (1) and (5), the equation

$$g_i^{(\gamma)} = \frac{V_i}{T} \sum_{j=1}^{n} \frac{\sum_{s=1}^{m} \sum_{k=1}^{K_{js}} \exp(-\mu_s - \rho c_{js^k})}{\sum_{s'=1}^{m} \sum_{k'=1}^{K_{j's'}} \exp(-\mu_s - \rho c_{j's'^{k'}})} g_{j'}^{(\gamma)}. \tag{7}$$

can be obtained. We define

$$a_{ij}^{(\gamma)} = V_i W_j, \tag{8}$$

where $W_j = \sum_{s=1}^{m} \sum_{k=1}^{K_{js}} \exp(-\mu_s - \rho c_{j s^k})$. (7) can be written as

$$g_i^{(\gamma)} = \sum_{j=1}^{n} \frac{a_{ij}^{(\gamma)}}{\sum_{t'=1}^{n} a_{t'i}^{(\gamma)}} g_j^{(\gamma)}. \tag{9}$$

The array of the matrix $\mathbf{M}^{(\mu,\rho)}$ is defined such that

$$m_{ij}^{(\mu,\rho)} = \frac{a_{ij}^{(\gamma)}}{\sum_{t'=1}^{n} a_{t'i}^{(\gamma)}}. \tag{10}$$

Hence, (7) can be rewritten as

$$\mathbf{M}^{(\mu,\rho)} \mathbf{g}^{(\gamma)} = \mathbf{g}^{(\gamma)} . \tag{11}$$

This means that (7) is converted to the problem of finding the eigenvector corresponding to the eigenvalue 1 of the matrix $\mathbf{M}^{(\mu,\rho)}$. We define $a_*^{(\gamma)} = \sum_{i=1}^{n} V_i$; then

$$\mathbf{M}^{(\mu,\rho)} = \frac{1}{a_*^{(\gamma)}} \begin{bmatrix} V_1 & V_1 & \dots & V_1 \\ V_2 & V_2 & \dots & V_2 \\ \vdots & \vdots & \ddots & \vdots \\ V_n & V_n & \dots & V_n \end{bmatrix}. \tag{12}$$

We find that the rank of the matrix $\mathbf{M}^{(\mu,\rho)}$ is 1. Let Jordan normal form of the matrix $\mathbf{M}^{(\mu,\rho)}$ be $\mathbf{P}\mathbf{M}^{(\mu,\rho)}\mathbf{P}^{-1}$ by using a regular matrix $\mathbf{P}^{(\gamma)}$; the equation

$$\mathrm{tr}\left(\mathbf{P}\mathbf{M}^{(\mu,\rho)}\mathbf{P}^{-1}\right) = \mathrm{tr}\left(\mathbf{M}^{(\mu,\rho)}\mathbf{P}^{-1}\mathbf{P}\right) = \mathrm{tr}\left(\mathbf{M}^{(\mu,\rho)}\right) = 1 \tag{13}$$

indicates that the sum of the eigenvalues is 1. Thus, it is shown that matrix $\mathbf{M}^{(\mu,\rho)}$ can have a unique eigenvalue 1 and $\mathbf{g}^{(\gamma)}$ is its eigenvector. Similarly, in the case of the μ_j, the vector $\mathbf{g}^{(\mu)}$ satisfies the equation

$$\mathbf{M}^{(\gamma,\rho)} \mathbf{g}^{(\mu)} = \mathbf{g}^{(\mu)}, \tag{14}$$

where

$$m_{ij}^{(\gamma,\rho)} = \frac{a_{ij}^{(\mu)}}{\sum_{t'=1}^{n} a_{t'i}^{(\mu)}}. \tag{15}$$

By using Theorem 2.1, the Lagrange undetermined multipliers γ_i, μ_j, ρ can be estimated from iteration algorithm in [12] and then the OD matrix of traffic volume is derived. For example, consider the OD matrix in Table 1 with $k = 1$.

TABLE 1. OD matrix with $k = 1$

r_{11}^1	r_{12}^1	r_{13}^1	$\dots$	r_{1m}^1	V_1
r_{21}^1	r_{22}^1	r_{23}^1	$\dots$	r_{2m}^1	V_2
r_{31}^1	r_{32}^1	r_{33}^1	$\dots$	r_{3m}^1	V_3
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
r_{n1}^1	r_{n2}^1	r_{n3}^1	$\dots$	r_{nm}^1	V_n
U_1	U_2	U_3	$\dots$	U_m	T

2.2. **Edge Ranking.** In light of the below definition, we show the formulation of the edge importance, that is the Edge Ranking. Consider a graph network with more than one path to the OD as shown in Section 2.1. It contains n number of starting point i, m number of point of arrival j and o number of the edge l. In order to derive the probability $p(l)$ passing through the edge l, we derive the probability that contains edge l in ij^k, one path of the set $\{ij\}$ of the path from i to j.

It is natural to think that this probability is obtained by dividing the traffic volume of path ij^k flowing through the edge l by the base volumes of traffic flowing to the whole edge. This leads to the following definition.

Definition 2.1. *The probability that contains the edge l in the path ij^k is given as follows*

$$p(l|kij) = \frac{(\text{The traffic volume of path } ij^k \text{ flowing through the edge } l)}{(\text{The base volumes of traffic flowing to the whole edge})}.$$

In order to formulate this definition, we designate the arrays $b_{kl}(i,j)$ of matrix $\mathbf{B}_{kl}$ as:

$$b_{kl}(i,j) = \begin{cases} 1 & (\text{the root } ij^k \text{ contains edge } l) \\ 0 & (\text{otherwise}) \end{cases}, \tag{16}$$

then, the probability $p(l|kij)$ is

$$p(l|kij) = \frac{b_{kl}(i,j) r_{ij^k}}{\sum_{l'=1}^{o} \sum_{k'=1}^{K_{i'j'}} \sum_{i'=1}^{n} \sum_{j'=1}^{m} b_{k'l'}(i',j') r_{i'j'^{k'}}}. \tag{17}$$

Finally, the probability $p(l)$ to pass through the edge l is

$$p(l) = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{K_{ij}} p(l|kij). \tag{18}$$

In this paper, this probability $p(l)$ defines the Edge Ranking.

3. **Discrete Choice.** The conventional network model defines the cost as a value corresponding to one physical quantity. To match a plurality of physical quantities to cost, apply the DC models that are introduced in the paper [11]. Now, it is assumed that the path set $\{ij\}$ is presented towards from i to j to hth driver. It is assumed that the hth driver evaluates the path based on his/her own value factor, for example, "I want to arrive early at the destination", "I want to drive a little way of signals", or "I want to drive a road that is not crowded".

It is assumed that the path set $\{ij\}$ is presented to hth driver and he/she has personal value factors q $(q = 1, 2, \cdots, Q)$. He/She will evaluate the plan based on individual values w_{hq} $(q = 1, 2, \cdots, Q)$ whose vector is denoted by

$$\mathbf{w}_h = [w_{h1}, w_{h2}, \cdots, w_{hQ}]^{\mathrm{T}} \in \Re^Q, \tag{19}$$

where $\sum_{q=1}^{Q} w_{hq} = 1$ $(\forall h)$. We can collect facility vector

$$\mathbf{x}^{ij^k} = \left[x^{ij^k1}, x^{ij^k2}, \cdots, x^{ij^kS}\right]^{\mathrm{T}} \in \Re^S, \tag{20}$$

where x^{ij^ks} is information about facility s $(s = 1, 2, 3, \cdots, S)$ for path ij^k, furthermore, measure selection results as vector $y_h^{ij^k*}$ $(= \{0, 1\})$ which takes 1 if the hth driver chooses the path ij^k and otherwise 0, or the choice probability vector

$$\mathbf{p}(\mathbf{y}_h^*) = \left[p\left(y_h^{ij^1*} = 1\right), p\left(y_h^{ij^2*} = 1\right), \cdots, p\left(y_h^{ij^{K_{ij}}*} = 1\right)\right]^{\mathrm{T}} \in \Re^{K_{ij}}. \tag{21}$$

Under the conditions that facility vector $\mathbf{x}^k$ and choice probability $p\left(y_h^{ij^k*}=1\right)$ can be observed, we have

$$\mathbf{z}_h^{ij^k} = \left[\mathbf{x}^{ij^k\mathrm{T}}, p\left(y_h^{ij^k*}=1\right)\right]^{\mathrm{T}} \in \Re^d \,, \tag{22}$$

where $d = S+1$. Vector $\mathbf{m}_{hq}$ which is the mean vector of $\mathbf{z}_h^{ij^k}$ consists of a part $\mathbf{m}_{hq}^x \in \Re^S$ about $\mathbf{x}^{ij^k}$ facility vector and a part $m_{hq}^y \in \Re^1$ about evaluation $y_h^{ij^k*}$ as follows

$$\mathbf{m}_{hq} = \left[\mathbf{m}_{hq}^{x\mathrm{T}}, m_{hq}^y\right]^{\mathrm{T}} \tag{23}$$

and matrix $\mathbf{\Sigma}_{hq}$ which is the variance-covariance matrix of $\mathbf{z}_h^{ij^k}$ consists of $\mathbf{G}_{hq}^x \in \Re^{S\times S}$ about $\mathbf{x}^{ij^k}$, $G_{hq}^y \in \Re^{1\times 1}$ about $y_h^{ij^k*}$ and $\mathbf{G}_{hq}^{xy} \in \Re^{S\times 1}$, $\mathbf{G}_{hq}^{yx} \in \Re^{1\times S}$ about both $\mathbf{x}^{ij^k}$ and $y_h^{ij^k*}$ respectively such as

$$\mathbf{\Sigma}_{hq} = \begin{bmatrix} \mathbf{G}_{hq}^x & \mathbf{G}_{hq}^{xy} \\ \mathbf{G}_{hq}^{yx} & G_{hq}^y \end{bmatrix}. \tag{24}$$

It is known that the vector $\mathbf{z}_h^{ij^k}$ obeys mixture normal distribution; thus, parameters $\mathbf{w}_h$, $\mathbf{m}_{hq}$, and $\mathbf{\Sigma}_{hq}$ can be derived by applying Expectation Maximization algorithm [11, 13]. The hth driver evaluates path ij^k by utility of each q value factor with respect to faculty vector $\mathbf{x}^{ij^k} \in \Re^S$ $(s = 1, 2, \cdots, S)$

$$u_{hq}^{ij^k} = m_{hq}^y + \mathbf{D}_{hq}^y\left(\mathbf{x}^{ij^k} - \mathbf{m}_{hq}^x\right), \tag{25}$$

where $\mathbf{D}_{hq}^y \in \Re^{1\times S}$ is given by $\mathbf{D}_{hq}^y = \mathbf{G}_{hq}^{yx}\mathbf{G}_{hq}^{x^{-1}}$.

We show the method of deriving utility as reference [11]. The utility is a value which is to quantify benefit obtained when drivers have selected, by evaluating each option from several physical quantities. We calculate each of the utility from the path selection probability of several people and derive the utility average $\overline{u}^{ij^k}$. The average of H driver's utility is calculated by

$$\overline{u}^{ij^k} = \frac{1}{H}\sum_{h=1}^{H}\sum_{q=1}^{Q} u_{hq}^{ij^k}. \tag{26}$$

In this paper, consider that it is possible to correspond several physical quantities to only one cost by linking the negative value of utility average to cost, and define the conversion function below.

Definition 3.1. *Define a set of S-number of cost c_s $(c_s = c_1, c_2, \cdots, c_S)$ obtained a set of S-number of utility average $\overline{u}_s$ $(\overline{u}_s = \overline{u}_1, \overline{u}_2, \cdots, \overline{u}_S)$. Then, when $\overline{u}_{\max} = \max\{\overline{u}_1, \overline{u}_2, \cdots, \overline{u}_S\}$ and $\overline{u}_{\min} = \min\{\overline{u}_1, \overline{u}_2, \cdots, \overline{u}_S\}$, the following relationship is satisfied in cost and utility averages,*

$$c_s = \frac{9}{\overline{u}_{\min} - \overline{u}_{\max}}\overline{u}_s + \frac{\overline{u}_{\min} - 10\overline{u}_{\max}}{\overline{u}_{\min} - \overline{u}_{\max}}. \tag{27}$$

It means that cost is set to a value of 1 to 10 and the path having lower/higher utility is assigned higher/lower cost. Incidentally, when the actual traffic volume data were obtained, it is possible to suppress the error between the estimated and obtained value by adjusting the conversion function.

4. **Numerical Example.** The graph network used in the numerical experiments is shown in Figure 1. As the basic information of this network, the total number of nodes is seven, the total number of edges is eight and the total number of paths is 98. Although edges l_1, l_3, l_4, l_5, l_6, l_7 and l_8 are possible to pass in both directions, edge l_2 is only one direction. We gave this graph network the inflow vector $V = [\ V_1, V_2, \cdots, V_7\ ]$, the outflow vector $U = [\ U_1, U_2, \cdots, U_7\ ]$, the total traffic volume T and the total cost constant C below, and each edge two pieces of information (distance and toll) as Figure 2.

$$V = [\ 195 \quad 250 \quad 160 \quad 120 \quad 160 \quad 180 \quad 215\ ]$$
$$U = [\ 260 \quad 130 \quad 320 \quad 150 \quad 200 \quad 70 \quad 110\ ]$$
$$T = 1240$$
$$C = 2.6.$$

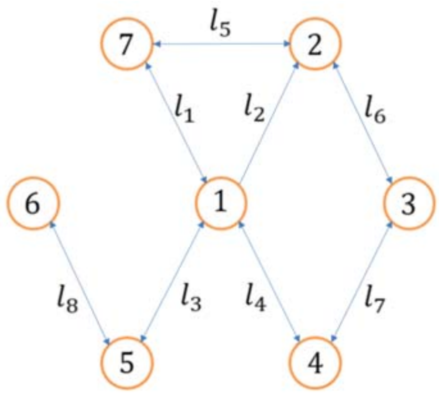

FIGURE 1. Graph network

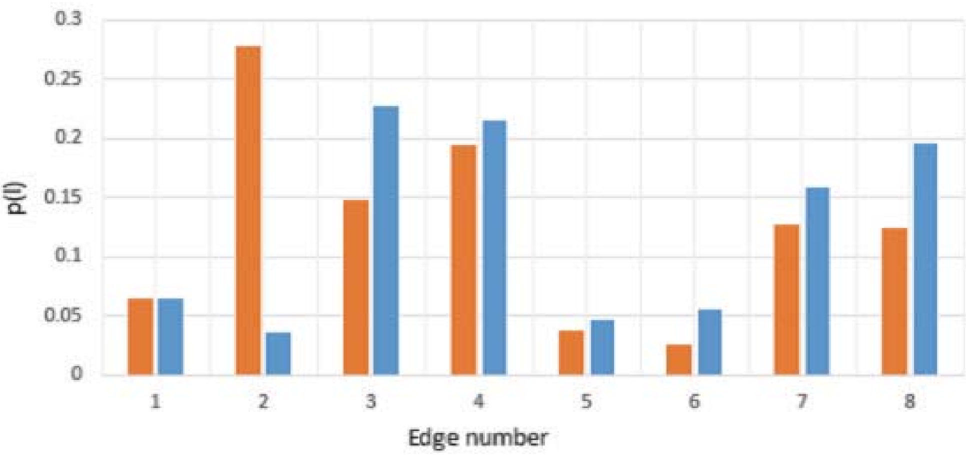

FIGURE 2. Results of the Edge Ranking

From two pieces of information, a feature vector of all the 98 paths is derived. We excerpt a feature vector of path {12}, {32}, {42}, {52}, {62} and {72} and show below. Those sets have just three paths, respectively,

$$\mathbf{x}^{12^1} = [\ 9.205882 \quad 5.655172\ ]^{\mathrm{T}} \qquad \mathbf{x}^{12^2} = [\ 7.088235 \quad 6.431034\ ]^{\mathrm{T}}$$
$$\mathbf{x}^{12^3} = [\ 6.823529 \quad 8.137931\ ]^{\mathrm{T}} \qquad \mathbf{x}^{32^1} = [\ 6.823529 \quad 4.103448\ ]^{\mathrm{T}}$$
$$\mathbf{x}^{32^2} = [\ 9.205882 \quad 9.689655\ ]^{\mathrm{T}} \qquad \mathbf{x}^{32^3} = [\ 4.705882 \quad 4.879310\ ]^{\mathrm{T}}$$

$$\mathbf{x}^{42^1} = [\ 8.941176\ \ 5.189655\ ]^{\mathrm{T}} \qquad \mathbf{x}^{42^2} = [\ 7.088235\ \ 8.603448\ ]^{\mathrm{T}}$$
$$\mathbf{x}^{42^3} = [\ 6.823529\ \ 5.965517\ ]^{\mathrm{T}} \qquad \mathbf{x}^{52^1} = [\ 7.882352\ \ 4.413793\ ]^{\mathrm{T}}$$
$$\mathbf{x}^{52^2} = [\ 5.764705\ \ 5.189655\ ]^{\mathrm{T}} \qquad \mathbf{x}^{52^3} = [\ 5.5\ \ 6.896551\ ]^{\mathrm{T}}$$
$$\mathbf{x}^{62^1} = [\ 6.294117\ \ 4.103448\ ]^{\mathrm{T}} \qquad \mathbf{x}^{62^2} = [\ 4.176470\ \ 4.879310\ ]^{\mathrm{T}}$$
$$\mathbf{x}^{62^3} = [\ 3.911764\ \ 6.586206\ ]^{\mathrm{T}} \qquad \mathbf{x}^{72^1} = [\ 7.617647\ \ 7.206896\ ]^{\mathrm{T}}$$
$$\mathbf{x}^{72^2} = [\ 8.676470\ \ 4.879310\ ]^{\mathrm{T}} \qquad \mathbf{x}^{72^3} = [\ 6.294117\ \ 7.362068\ ]^{\mathrm{T}}.$$

Then, numerical results when the path selection probability of people is given are shown in two cases below.

Case 1: When we give a selection result in proportion to the distance

When we gave some paths {12}, {32}, {42}, {52}, {62} and {72} a selection result in proportion to the distance, each path selection probability is

$$\mathbf{p}\left(\mathbf{y}_h^*\right) = \frac{1}{x^{ij^1}(1,1) + x^{ij^2}(1,1) + x^{ij^3}(1,1)} \left[\ x^{ij^1}(1,1),\ x^{ij^2}(1,1),\ x^{ij^3}(1,1)\ \right]^{\mathrm{T}}. \tag{28}$$

The average utility $\overline{u}^{ij^k}$ is calculated from this probability and estimated parameter from iteration algorithm in [7], the vector $\gamma = [\ \gamma_1, \gamma_2, \cdots, \gamma_7\ ]$, the vector $\mu = [\ \mu_1, \mu_2, \cdots, \mu_7\ ]$, and ρ, are shown below. We defined the index $|E_c| = \left|TC - \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{K_{ij}} c_{ij^k} r_{ij^k}\right|$ to represent iteration error and continued iteration until $|E_c| < 0.6$. For the sake of simplicity, we set $q = 1$.

$$|E_c| = 0.26129$$
$$\rho = 0.796$$
$$\gamma = [\ 2.4453\ \ 2.8213\ \ 0.6784\ \ 3.0042\ \ 1.9245\ \ 2.2468\ \ 3.0994\ ]$$
$$\mu = [\ 3.0255\ \ 0.8998\ \ 2.7463\ \ 3.3972\ \ 2.6450\ \ 1.1950\ \ 2.6108\ ].$$

By using those results, the OD matrix is estimated. Finally, $p(l)$ is obtained from the resulting traffic volume, shown in Table 2.

Table 2. Information of each edge

	distance (km)	toll (yen)
@l_1	2.5	120
@l_2	2.0	220
@l_3	1.5	120
@l_4	3.0	140
@l_5	3.5	40
@l_6	1.8	20
@l_7	3.5	20
@l_8	2.0	160

Case 2: When we give a selection result in proportion to the toll

Similarly, we gave each path selection probability is

$$\mathbf{p}(\mathbf{y}_h^*) = \frac{1}{x^{ij^1}(2,1) + x^{ij^2}(2,1) + x^{ij^3}(2,1)} \left[\ x^{ij^1}(2,1),\ x^{ij^2}(2,1),\ x^{ij^3}(2,1)\ \right]^{\mathrm{T}}. \tag{29}$$

The average utility $\overline{u}^{ijk}$ is calculated from this probability and this result is shown below. For the sake of simplicity, we set $q = 1$.

$$\begin{aligned} |E_c| &= 0.52617 \\ \rho &= 0.574 \\ \gamma &= [\ 1.7950\ \ 2.3986\ \ 1.1211\ \ 2.4780\ \ 1.6084\ \ 2.9011\ \ 2.5508\ ] \\ \mu &= [\ 2.1949\ \ 1.3434\ \ 2.2696\ \ 2.6964\ \ 2.1332\ \ 1.7697\ \ 1.7836\ ]\ . \end{aligned}$$

By using those results, the OD matrix is estimated. Finally, $p(l)$ is obtained from the resulting traffic volume, shown in Table 2.

The Edge Ranking of both **case 1** and **case 2** are shown in Figure 2. The vertical axis represents the value of $p(l)$ and the horizontal axis represents the edge number, and red bar graph (left side) indicates the results of **case 1** and blue bar graph (right side) indicates the result of **case 2**, respectively.

There are 3 paths in $\{12\}$, that is $1 \to 2$, $1 \to 7 \to 2$ and $1 \to 4 \to 3 \to 2$. The information shows 2.0km, 5.5km and 8.3km for distance and 220yen, 160yen and 180yen for toll, respectively. In **case 1**, it is considered that the Edge Ranking of l_2 becomes larger than l_4, l_5, l_6 and l_7 because the first path is the most shortest path in the path set $\{12\}$. In **case 2**, however, it is considered that the Edge Ranking of l_2 becomes smaller than l_4, l_5, l_6 and l_7 because the first path is the most highest path in the path set $\{12\}$ and traffic volume is dispersed in l_4, l_5, l_6 and l_7.

In the case of l_3 and l_8, from Figure 2 it is known that the ratio of **case 1** and **case 2** are almost the same (1.52309 and 1.584001), so it is considered that because of decreasing the Edge Ranking of l_2, traffic volume is dispersed in the direction node 6.

From these results, it was found that the importance of edge and the traffic volume flowing through each path depend on changes in the trend of the path selection of the people.

5. **Conclusions.** We considered network model in order to solve various problems in the road network. By using the Maximization Entropy theorem, OD matrix of traffic volume was obtained from traffic inflow and outflow of each node, but the cost did not correspond to only one physical quantity in the conventional model.

In this paper, by introducing the concept of DC, we represented how much influence various factors have on the people's path selection by value called utility, and then we associate weight in the graph network with utility.

As a result, we constructed a new model with several physical quantities in weight. From numerical examples, it was found that the importance of edge and the traffic volume flowing through each path depend on changes in the trend of the path selection of the people.

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REFERENCES

[1] H. Noda, The advantage of position as a quantitative criterion according to the characteristics of street network: Quantitative analysis of city planning using the characteristics: Of street network Part 1, *Journal of Architecture, Planning and Environmental Engineering, Transactions of AIJ*, vol.470, pp.139-148, 1995.

[2] D. L. Hull, A probabilistic analysis of shopping center trade areas, *Land Economics*, vol.39, pp.81-90, 1963.

[3] T. R. Laksshmanan and W. G. Hansen, A retail market potential model, *Journal of American Institute of Planners*, vol.33, pp.134-143, 1965.
[4] A. G. Wilson, *Entropy in Urban and Regional Modelling*, Pion, London, 1970.
[5] K. J. Narain and H. K. Kesavan, *Entropy Optimization Principles with Applications*, Academic Press, 1992.
[6] K. Okuhara, K.-Y. Yeh, J. Shibata, H. Ishii and H.-C. Hsia, Evaluation and assignment of traffic volume for urban planning based on planner and user stance, *International Journal of Innovative Computing, Information and Control*, vol.4, no.5, pp.1151-1160, 2008.
[7] K. Okuhara, K.-Y. Yeh, H.-C. Hsia and H. Ishii, Geographical advantage from accessibility based on spatial interaction model, *International Journal of Innovative Computing, Information and Control*, vol.6, no.9, pp.4193-4202, 2010.
[8] J. J. Louviere, D. A. Hensher and J. Swait, *Stated Choice Methods*, Cambridge University Press, 2000.
[9] K. Train, *Discrete Choice Methods with Simulation*, Cambridge University Press, 2003.
[10] D. A. Hensher, J. M. Rose and W. H. Greene, *Applied Choice Analysis*, Cambridge University Press, 2005.
[11] K. Okukara, R. Haruna and A. Ishii, Mechanism to improvement and modeling of creation of the value, *Proc. of Spring Meeting in Japan Operations Research Association (in Japanese)*, pp.204-205, 2010.
[12] K. Okuhara, A. O. N. Rene, Y. Ichihuji and N. Sonehara, Path ranking in graph based on structure and activity of network, *Information System for Social Innovation*, Technical Report, 2015.
[13] A. P. Dempster, N. M. Laird and D. B. Rubin, Maximum likelihood from incomplete data via the EM algorithm, *Journal of the Royal Statistical Society. Series B (Methodological)*, vol.39, no.1, pp.1-38, 1977.