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# NETWORK-BASED CONTAINMENT CONTROL PROTOCOL OF MULTI-AGENT SYSTEMS WITH TIME-VARYING DELAYS

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Abstract. *This paper is concerned with the problem of Network-based containment control for a class of multi-agent systems. Under a directed graph, the containment control protocol of distributed multi-agent system is accomplished through a communication network, in which way all followers' states can be driven asymptotically converge to the convex hull spanned by the leaders. By combining Lyapunov stability theory and linear matrix inequality (LMI) method, a sufficient condition guaranteeing containment control is derived, and the network-based containment control gain is achieved by solving the feasibility of linear matrix inequality. The results show the network-based containment control protocol effectively helps compensate the time-varying delays produced in the communication network. Finally, the effectiveness and feasibility of the control method are verified with a simulation example.*
**Keywords:** Multi-agent systems, Containment control, Communication network, Delays

1. **Introduction.** Distributed multi-agent systems have received increasing attention during the last decades due to the rapid developments of computer science and sensing technologies, distributed cooperative control of multi-agent systems has made great progress. Applications of cooperative control include UAV formation control [1], transportation systems [2], autonomous vehicle systems [3]. Therefore, investigations into fundamental aspects of cooperative control of multi-agent behavior have been widely reported, e.g., consensus [4-6], formation control [7,8], flocking [9,10] and coverage control [11].

As a kind of cooperative behavior, containment control of multi-agent systems has been investigated a lot in recent years [12-18,25-28], which can be found in many application scenarios such as when a collection of autonomous robots are to secure and then remove hazardous materials. Containment control problems have been investigated for first-order multiagent systems under undirected and directed network topologies [12]. The containment control was also studied for second-order dynamical systems in the presence of multiple leaders [13]. The authors of [14] proposed a hybrid control scheme based on stop-go rules for the leader-agents to guarantee that the followers remain in the convex ploy-tope spanned by the leader agents during their transportation. In [15], the containment control of first-order and second-order integral multi-agent systems with communication noises is investigated, and the results are extended to linear multi-agent systems in [16]. In [17], a hybrid model predictive control scheme for containment and distributed sensing in multi-agent systems was proposed. In [18], periodic sampled-data based containment control of multi-agent systems with single-integrator and double-integrator dynamics was studied. Some necessary and sufficient conditions on sampling period were given to guarantee the achievement of containment control. In [25], the containment control problem for multi-agent systems with general linear dynamics and multiple leaders whose control inputs are possibly nonzero and time varying has been studied. The containment problem of both first-order and second-order integral multi-agent systems with communication noises was investigated in [26]. [27] investigated the distributed containment tracking control problem for first-order agents with multiple dynamic leaders under directed Markovian switching network topologies. The event-based broadcasting containment control problem for both first-order and second-order multi-agent systems under directed topology has been investigated by K. Liu et al. in [28]. However, all literature [25-28] has not referred to the affection of time delay which usually cannot be avoided in multi-agent systems.

It is worthy pointing out, in the aforementioned literature, a point-to-point connected mean is the main method used to control the multi-agent systems. However, in many modern applications, multi-agent systems are required to be remotely operated and controlled, which shall cause the point-to-point control structure is no longer applicable. Consequentially, it is a tendency that each agent in a distributed multi-agent system is controlled via a communication network. The communication network has the advantages of less wired, lower cost, easier to maintain, more suitable and flexible structure, etc. However, it inevitably produces delays and packet-dropouts during information transmission in the channel because of limited network bandwidth, which may cause negative impact on the performance of the system, even leading instability to system. By introducing a communication network, a network-based consensus control protocol for distributed multi-agent system is proposed with network-induced delay considered by Ding et al. in [19]. However, the result in [19] can only be suitable for the cases that only one leader exists in MAS, not referring to the containment control problem, which is a significant but challenging work.

In this paper, the problem of network-based containment control for a class of multi-agent systems with multiple leaders is concerned with. Firstly, the containment control protocol of distributed multi-agent system is accomplished through a communication network, in which way all followers' states can be driven asymptotically converge to the convex hull spanned by the leaders. Then, by combining Lyapunov stability theory and linear matrix inequality (LMI) method, a sufficient condition guaranteeing containment control is derived, and the network-based containment control gain is achieved by solving the feasibility of linear matrix inequality. Finally, the effectiveness and feasibility of the control method are verified with a simulation example.

Notation: $\mathcal{R}^n$ denotes the $n$-dimensional Euclidean space. The superscript '$T$' stands for matrix transposition. The notation $X > 0$ means that the matrix $X$ is a real positive definite matrix. $I$ is the identity matrix of appropriate dimensions. $\begin{bmatrix} X & Z \\ * & Y \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry. The sign $\otimes$ represents the matrix Kronecker product.

The paper is organized in 5 sections including the introduction. Section 2 presents the preliminaries for networked multi-agent system. Section 3 presents some main results on containment control for networked multi-agent system. There are some simulations to illustrate the results in Section 4. Section 5 summarized this paper.

## 2. **Preliminaries.**

### 2.1. **Graph theory.**

Some basic graph theory notions are first introduced in this subsection.

Let $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$ be a weighted directed graph with a set of vertices $\mathcal{V}=\{v_1, v_2, \cdots, v_n\}$, a set of edges $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}]_{n\times n}$ of graph $\mathcal{G}$. If there exists an edge from vertex $v_j$ to vertex $v_i$ denoted by $\varepsilon_{ji} = (v_i, v_j)$, the adjacency elements associated with the edges are positive, i.e., $a_{ij} > 0$, it means agent $v_i$ can receive the information of agent $v_j$. For any vertex $v_i \in \mathcal{V}$, it holds $a_{ii} = 0$. The set of neighbors of vertex $v_i$ is denoted by $N_i = \{v_j \in \mathcal{V} | (j, i) \in \varepsilon\}$. The Laplacian matrix of directed graph $\mathcal{G}$ is denoted by $\boldsymbol{L} = [l_{ij}]_{n\times n}$, where $l_{ii} = \sum\limits_{j=1, j\neq i}^{n} a_{ij}$; $l_{ij} = -a_{ij}$, $i \neq j$. A directed path in a directed graph $\mathcal{G}$ is a sequence consisting of a group of orderly edges, and it can be denoted by $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \cdots$, where $i_k \in \{1, 2, \cdots, n\}$ and $v_{i_k} \in \mathcal{V}$. A directed tree is a directed graph, where every vertex, except one special vertex without any neighbor, which is called the root vertex, has exactly one neighbor, and the root vertex can be connected to any other vertices through paths. A directed forest is a directed graph consisting of one or more directed trees, not two of which have a vertex in common. A directed spanning tree (directed spanning forest) is a directed tree (directed forest), which consists of all the vertices and some edges in $\mathcal{G}$.

If an agent has no neighbor, it is called as a leader; otherwise, it is called as a follower. Assume a multi-agent system consists of $m$ $(m < n)$ leaders and $n-m$ followers. Without loss of generality, it assumes agents $1, 2, \cdots, m$ are leaders, and agents $m+1, m+2, \cdots, n$ are followers. It defines $\mathcal{L} = \{1, 2, \cdots, m\}$ and $\mathcal{F} = \{m + 1, m + 2, \cdots, n\}$ respectively representing leaders set and follower set. Because leaders have no neighbor, Laplacian matrix of directed graph $\mathcal{G}$ can be decomposed as

$$L = \begin{bmatrix} \mathbf{0}_{m\times m} & \mathbf{0}_{m\times(n-m)} \\ L_1 & L_2 \end{bmatrix}$$

where $L_1 \in \mathcal{R}^{(n-m)\times m}$ and $L_2 \in \mathcal{R}^{(n-m)\times(n-m)}$.

For the sake of further discussion, now we give the following assumption.

**Assumption 2.1.** *The communication digraph $\mathcal{G}$ has a directed spanning forest.*

**Definition 2.1.** ([20]). *A set $F \subset \mathcal{R}^m$ is said to be convex if $(1-\lambda)x + \lambda y \in F$ for any $x, y \in F$ and $\lambda \in [0, 1]$. The convex hull of a finite set of points $\chi_1, \cdots, \chi_n \in \mathcal{R}^m$, denoted by $\mathbf{Co}\{\chi_1, \cdots, \chi_n\}$, is the minimal convex set containing all points $\chi_i$, $i = 1, \cdots, n$. More specifically,* $\mathbf{Co}\{\chi_1, \cdots, \chi_n\} = \left\{ \sum\limits_{i=1}^{n} \alpha_i \chi_i | \alpha_i \geq 0 \text{ and } \sum\limits_{i=1}^{n} \alpha_i = 1 \right\}$.

**Lemma 2.1.** (Cao et al. [13]). *Under Assumption 2.1, all eigenvalues of $L_2$ have positive real parts, $-L_2^{-1}L_1$ is nonnegative and each row of $-L_2^{-1}L_1$ has a sum equal to one.*

2.2. **Modeling of networked multi-agent systems.** Here consider a second-order distributed multi-agent system, which consists of $m$ leader agents labelled as $1, 2, \cdots, m$, whose dynamics are described as follows.

$$\begin{cases} \dot{s}_i(t) = v_i(t) \\ \dot{v}_i(t) = 0 \end{cases} \quad i \in \mathcal{L} \tag{1}$$

And $n - m$ follower agents are labelled as $m + 1, m + 2, \cdots, n$, whose dynamics are described as follows.

$$\begin{cases} \dot{s}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases} \quad i \in \mathcal{F} \tag{2}$$

where $s_i(t) \in \mathcal{R}^N$, $v_i(t) \in \mathcal{R}^N$ and $u_i(t) \in \mathcal{R}^N$ represent the position, velocity and the control input of the $i$th agent. Denote the leader set by $\mathcal{L}$ and the follower set by $\mathcal{F}$, respectively.

**Definition 2.2.** ([21]). *The containment control is achieved for system (2) under a certain control input if the position and velocity states of the followers asymptotically converge to the convex hull formed by those of the leaders as system (1).*

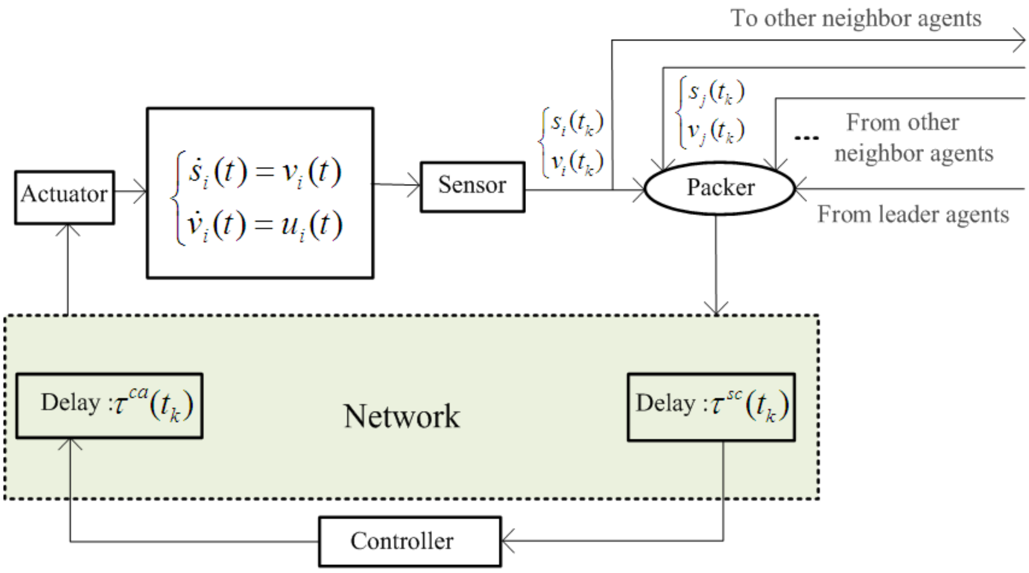



FIGURE 1. The basic structure of multi-agent systems through a network

Here each agent of distributed multi-agent system is controlled by a feedback controller connected through a communication network, which is helpful to control the multi-agent system remotely. And its structure is shown in Figure 1. It is assumed that the sensor is clock driven, and both the controller and the actuator are event-driven. During the information transmission in the communication network, a network delay is inevitably induced, which usually consists of two kinds of delays: one is sensor-to-controller delay $\tau^{sc}$ which is produced in forward channel, and the other is controller-to-actuator delay $\tau^{ca}$ which is produced in backward channel. A "packer" is employed to package the sampled data of agent $i$ and its neighbor agents into one single packet before they get to the network channel. Namely, the data contained in one packet is transmitted in feedback channel, which suggests $\tau^{sc}(t) = \tau_i^{sc}(t) = \tau_j^{sc}(t)$, $\tau^{ca}(t) = \tau_i^{ca}(t) = \tau_j^{ca}(t)$, where $j \in$ {neighbor agents of agent $i$}. So the time delay can be lumped together as one $\tau(t) = \tau_i^{sc}(t) + \tau^{ca}(t)$. The networked containment control protocol for system (2) is given as

$$\begin{cases} u_i(t) = K_{c1} \sum\limits_{j\in N_i} a_{ij}[s_i(t_k - \tau(t)) - s_j(t_k - \tau(t))] \\ \qquad\qquad + K_{c2} \sum\limits_{j\in N_i} a_{ij}[v_i(t_k - \tau(t)) - v_j(t_k - \tau(t))] \\ t \in [t_k, t_{k+1}], \quad i \in \mathcal{F} \end{cases} \tag{3}$$

where $K_{c1}$, $K_{c2} \in \mathcal{R}^{N\times N}$ are network-based containment control gains that remain to be designed later. $t_k$ is the sampling instant of the $k$th packet, and the sensor's sampling period is denoted by $\mathrm{T} = t_{k+1} - t_k$, and $k \in Z^+$ is the sequence number of current sampled-packet.

Defining a "synthetical delay" as $\eta(t) = t - t_k + \tau(t)$, the control protocol (5) can be rewritten as

$$\begin{cases} u_i(t) = K_{c1} \sum\limits_{j\in N_i} a_{ij}[s_i(t - \eta(t)) - s_j(t - \eta(t))] \\ \qquad\qquad + K_{c2} \sum\limits_{j\in N_i} a_{ij}[v_i(t - \eta(t)) - v_j(t - \eta(t))] \\ i \in \mathcal{F} \end{cases} \tag{4}$$

For the sake of further discussion, about the synthetical delay $\eta(t)$, we give the following assumption.

**Assumption 2.2.** *$\eta(t)$ is upper bounded by $\eta$, and it is smooth and its derivative is upper bounded by $\beta$, i.e., $\eta(t) \le \eta$ and $\dot{\eta}(t) \le \beta$.*

Let $x_i(t) = \left[ \begin{array}{cc} s_i(t) & v_i(t) \end{array} \right]^T$ $(i = 1, \cdots, n)$, $x_{\mathcal{L}} = \left[ \begin{array}{ccc} x_1^T & \cdots & x_m^T \end{array} \right]^T$, and $x_{\mathcal{F}} = \left[ x_{m+1}^T \cdots x_n^T \right]^T$. It notes that vector $x_i$ of the leaders is a linear function with respect to $t$: $x_i(t) = s_i(t_0) + v_i(t_0)t$. Then, based on (4), the dynamics of a closed-loop systems resulting from (1) and (2) can be written as

$$\dot{x}_{\mathcal{L}}(t) = (I_m \otimes A)x_{\mathcal{L}}(t) \tag{5}$$

and

$$\dot{x}_{\mathcal{F}}(t) = (I_{n-m} \otimes A)\, x_{\mathcal{F}}(t) + (L_1 \otimes BK)x_{\mathcal{L}}(t - \eta(t)) + (L_2 \otimes BK)x_{\mathcal{F}}(t - \eta(t)) \tag{6}$$

where $A = \left[ \begin{array}{cc} 0 & I_2 \\ 0 & 0 \end{array} \right]$, $B = \left[ \begin{array}{c} 0 \\ I_2 \end{array} \right]$, $K = \left[ \begin{array}{cc} K_{c1} & K_{c2} \end{array} \right]$.

Letting $\tilde{x}(t) = x_{\mathcal{F}}(t) + \left(L_2^{-1}L_1 \otimes I_4\right) x_{\mathcal{L}}(t)$, it follows from (5) and (6).

$$\dot{\tilde{x}}(t) = (I_{n-m} \otimes A)\tilde{x}(t) + (L_2 \otimes BK)\tilde{x}(t - \eta(t)) \tag{7}$$

**Remark 2.1.** *By Lemma 2.1, we know each entry of $-L_2^{-1}L_1$ is nonnegative and all row sums of $-L_2^{-1}L_1$ equal one, so the convex hull spanned by the leader agents is calculated as $(-L_2^{-1}L_1 \otimes I_{2N})x_{\mathcal{L}}$. The containment control problem is transformed into stabilizing system (7).*

3. **Main Results.** Before the analysis of consensus, some necessary lemmas in the following sections are introduced.

**Lemma 3.1.** ([22]). *Given a positive definite matrix $\mathcal{Q} \in \mathcal{R}^{m\times m}$, two constants $a$ and $b$ satisfying $a < b$, and a vector function $\nu$: $[a, b] \to \mathcal{R}^m$ such that the integrations concerned are well defined, the following inequality holds*

$$\left(\int_a^b \nu(s)d_s\right)^T Q \left(\int_a^b \nu(s)d_s\right) \le (b-a)\left(\int_a^b \nu^T(s)Q\nu(s)d_s\right) \tag{8}$$

**Lemma 3.2.** (Jia [23]). *For a given symmetric matrix $S$ with the form $S = [S_{ij}]$, $S_{11} \in \mathcal{R}^{r\times r}$, $S_{12} \in \mathcal{R}^{r\times(n-r)}$, $S_{22} \in \mathcal{R}^{(n-r)\times(n-r)}$. Then, $S < 0$ if and only if the following inequalities hold.*

$$\left\{ \begin{array}{l} S_{11} < 0 \\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0 \end{array} \right. \tag{9a}$$

*or*

$$\left\{ \begin{array}{l} S_{22} < 0 \\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0 \end{array} \right. \tag{9b}$$

**Theorem 3.1.** *Under Assumption 2.1 and Assumption 2.2, given constants $\beta$, $\eta$, $\sigma < 1$ satisfying $\sigma\beta < 1$, as well as network-based containment control gain $K$, if there exists symmetric positive definite matrix $P$, positive definite matrices $R$, $Q$ of appropriate dimensions such that*

$$\left[ \begin{array}{cccc} -(I \otimes T)^{-1} & I \otimes A & \sqrt{\sigma\eta}L_2 \otimes BKP & 0 \\ * & I \otimes (A^T P + PA) & L_2 \otimes BKP & 0 \\ * & * & -I \otimes Q & 0 \\ * & * & * & \Xi \end{array} \right] < 0 \tag{10}$$

*where* $\Xi = -\frac{1}{\sigma\eta}(1-\sigma\beta)(I\otimes T) + I\otimes R$, *then multi-agent systems (1) and (2) solve the containment problem.*

**Proof:** First of all, construct the following Lyapunov-Krasovskii function.

$$\begin{aligned} V(\tilde{x}(t),t) &= \tilde{x}^T(t)(I\otimes P)\tilde{x}(t) + \int_{t-\eta(\boldsymbol{t})}^{t} \tilde{x}^T(\tau)(I\otimes Q)\tilde{x}(\tau)d\tau \\ &\quad + \int_{t-\sigma\eta(t)}^{t} \tilde{x}^T(\tau)(I\otimes R)\tilde{x}(\tau)d\tau + \int_{-\sigma\eta(t)}^{0}\int_{t+s}^{t} \dot{\tilde{x}}^T(\tau)(I\otimes T)\dot{\tilde{x}}(\tau)dsd\tau \end{aligned} \tag{11}$$

Obviously, because $P>0$, $Q>0$, $R>0$, $T>0$, $V(\bar{\boldsymbol{x}}(t),t)$ is positive definite. Taking the derivative of $V(\tilde{x}(t),t)$ with respect to $t$ along the trajectory (7) yields:

$$\begin{aligned} \dot{V}(\tilde{x}(t),t) &= 2\dot{\tilde{x}}^T(t)(I\otimes P)\tilde{x}(t) + \tilde{x}^T(t)(I\otimes Q)\tilde{x}(t) + \tilde{x}^T(t)(I\otimes R)\tilde{x}(t) \\ &\quad - \tilde{x}^T(t-\sigma\eta(t))(I\otimes R)\tilde{x}(t-\sigma\eta(t)) \\ &\quad - \tilde{x}^T(t-\eta(t))(I\otimes Q)\tilde{x}(t-\eta(t)) + \sigma\eta(t)\dot{\tilde{x}}^T(t)(I\otimes T)\dot{\tilde{x}}(t) \\ &\quad - (1-\sigma\dot{\eta}(t))\int_{t-\sigma\eta(t)}^{t} \dot{\tilde{x}}^T(\tau)(I\otimes T)\dot{\tilde{x}}(\tau)d\tau \end{aligned} \tag{12}$$

Because $\sigma\beta<1$, based on Lemma 3.2, the following inequality holds.

$$\begin{aligned} &- (1-\sigma\dot{\eta}(t))\int_{t-\sigma\eta(t)}^{t} \dot{\tilde{x}}^T(\tau)(I\otimes T)\dot{\tilde{x}}(\tau)d\tau \\ \le &- (1-\sigma\beta)\int_{t-\sigma\eta(t)}^{t} \dot{\tilde{x}}^T(\tau)(I\otimes T)\dot{\tilde{x}}(\tau)d\tau \\ \le &- \frac{1}{\sigma\eta(t)}(1-\sigma\beta)\left(\int_{t-\sigma\eta(t)}^{t}\dot{\tilde{x}}(\tau)d\tau\right)^T (I\otimes T)\left(\int_{t-\sigma\eta(t)}^{t}\dot{\tilde{x}}(\tau)d\tau\right) \\ = &- \frac{1}{\sigma\eta(t)}(1-\sigma\beta)\left[\tilde{x}(t)-\tilde{x}(t-\sigma\eta(t))\right]^T(I\otimes T)[\tilde{x}(t)-\tilde{x}(t-\sigma\eta(t))] \end{aligned} \tag{13}$$

Substituting (7) and (13) into (12), it follows

$$\begin{aligned} \dot{V}(\tilde{x}(t),t) \le\, & 2\left[(I\otimes A)\tilde{x}(t) + (L_2\otimes BK)\tilde{x}(t-\eta(t))\right]^T(I\otimes P)\tilde{x}(t) \\ & + \tilde{x}^T(t)(I\otimes Q)\tilde{x}(t) + \tilde{x}^T(t)(I\otimes R)\tilde{x}(t) \\ & - \tilde{x}^T(t-\sigma\eta(t))(I\otimes R)\tilde{x}(t-\sigma\eta(t)) - \tilde{x}^T(t-\eta(t))(I\otimes Q)\tilde{x}(t-\eta(t)) \\ & + \sigma\eta\left[(I\otimes A)\tilde{x}(t) + (L_2\otimes BK)\tilde{x}(t-\eta(t))\right]^T(t)(I\otimes T)\big[(I\otimes A)\tilde{x}(t) \\ & + (L_2\otimes BK)\tilde{x}(t-\eta(t))\big] - \frac{1}{\sigma\eta}(1-\sigma\beta)\big[\tilde{x}(t) \\ & - \tilde{x}(t-\sigma\eta(t))\big]^T(I\otimes T)\left[\tilde{x}(t)-\tilde{x}(t-\sigma\eta(t))\right] \end{aligned} \tag{14}$$

Let $\xi = \tilde{x}(t)-\tilde{x}(t-\sigma\eta(t))$, $\varphi = \left[\tilde{x}^T(t), \tilde{x}^T(t-\eta(t)), \xi\right]^T$, inequality (14) can be equivalently written as

$$\dot{V}(\tilde{x}(t),t) \le \varphi^T\Pi\varphi \tag{15}$$

where

$$\Pi = \begin{bmatrix} I\otimes(A^TP+PA)+\Pi_1 & L_2\otimes BKP+\Pi_2 & 0 \\ * & -I\otimes Q+\Pi_3 & 0 \\ * & * & \Xi \end{bmatrix}$$

$$\Pi_1 = I\otimes Q + \sigma\eta(I\otimes A^T)(I\otimes T)(I\otimes A)$$

$$\Pi_2 = \sigma\eta(I\otimes A^T)(I\otimes T)(L_2\otimes BKP)$$

$$\Pi_3 = \sigma\eta (L_2 \otimes BKP)^T (I \otimes T)(L_2 \otimes BKP)$$

$$\Xi = -\frac{1}{\sigma\eta}(1 - \sigma\beta)(I \otimes T) + I \otimes R.$$

Applying Schur complement theory described as Lemma 3.2 to inequality (10), it follows $\Pi < 0$. From (15), it holds $\dot{V}(\tilde{x}(t), t) < 0$, so system (6) is asymptotically stable. Therefore, as $t \to \infty$, $x_F(t) \to -\left(L_2^{-1}L_1 \otimes I_{2N}\right) x_F(t)$. This completes the proof.

Based on the stability condition described as Theorem 3.1, next, the method for solving the network-based containment control gain K will be studied.

**Theorem 3.2.** *Under Assumption 2.1 and Assumption 2.2, given constants $\beta$, $\eta$, $\sigma < 1$ satisfying $\sigma\beta < 1$, if there exists symmetric positive definite matrix $X$, positive definite matrices $\tilde{R}$, $\tilde{Q}$, and matrix $W$ of appropriate dimensions such that*

$$\begin{bmatrix} -X \otimes I & I \otimes A & \lambda\sqrt{\sigma\eta}L_2 \otimes BW & 0 \\ * & \lambda I \otimes (XA^T + AX) & \lambda L_2 \otimes BW & 0 \\ * & * & -I \otimes \tilde{Q} & 0 \\ * & * & * & \tilde{\Xi} \end{bmatrix} < 0 \tag{16}$$

*where $\tilde{\Xi} = -\frac{1}{\sigma\eta}(1 - \sigma\beta)(I \otimes X) + I \otimes \tilde{R}$, then multi-agent systems (1) and (2) solve the containment problem with network-based containment control gain $K = WX^{-1}$.*

**Proof:** Defining $P = \lambda T$, pre- and post-multiplying both sides of inequality (10) with $diag\{I, I \otimes T^{-1}, I \otimes T^{-1}, I \otimes T^{-1}\}$ and with the introduction of other new variables $T^{-1} = X$, $W = KT^{-1}$, $\tilde{Q} = T^{-1}QT^{-\mathrm{T}}$, $\tilde{R} = T^{-1}RT^{-\mathrm{T}}$, it follows inequality (16). Then, Theorem 3.2 is proofed.

**Remark 3.1.** *It is obvious that inequality (16) is linear, so here the network-based containment control gain $K$ can be directly achieved by searching the feasibility to LMI (16).*

4. **Simulations.** Consider the second-order multi-agent system including three leader agents marked as 1, 2 and 3, and three follower agents marked as 4, 5 and 6. The topology formed between leaders and followers has a directed spanning forest, which is shown as Figure 2. For simplicity, here we suppose that all the weights of edges are set as 1. It assumes the "synthetical delay" $\eta(t)$ produced in the communication network is not less than 0.08 and not greater than 0.65, i.e., $0.08 \le \eta(t) \le 0.65$. And its derivative is not greater than 0.36, i.e., $\dot{\eta}(t) \le 0.36$. We choose parameter $\sigma = 0.45$. By solving the LMI in Theorem 3.2, the network-based containment control gain is calculated as follows.

$$K = \begin{bmatrix} -2.0343 & -5.2128 & -5.4245 & 2.5481 \\ -0.0178 & -2.7042 & 1.0012 & -0.6253 \end{bmatrix}$$

The agents' initial states are assumed to obey Gaussian distribution. Applying the designed network-based containment control gain to the multi-agent system, we depict the position and velocity trajectories of all the agents in Figure 3. From this figure, it is obvious that, after 20 seconds, even affected by the time-varying delay produced in the communication channel, all followers' states reach the convex hull spanned by the leaders which is represented by dotted line. This shows the effectiveness and feasibility of the proposed design method.

Moreover, the method proposed in [24] is used for the same problem. The trajectories of positions and velocities are shown in Figure 4, from which it is obvious to see that follower agents' states fail to reach the convex hull spanned by the leaders. After all, this further demonstrates the advantages of the method proposed in this paper.

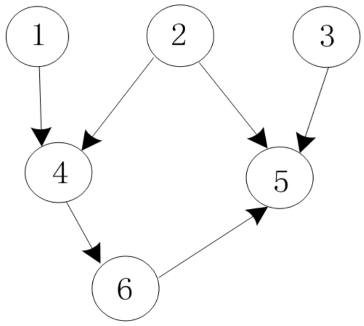
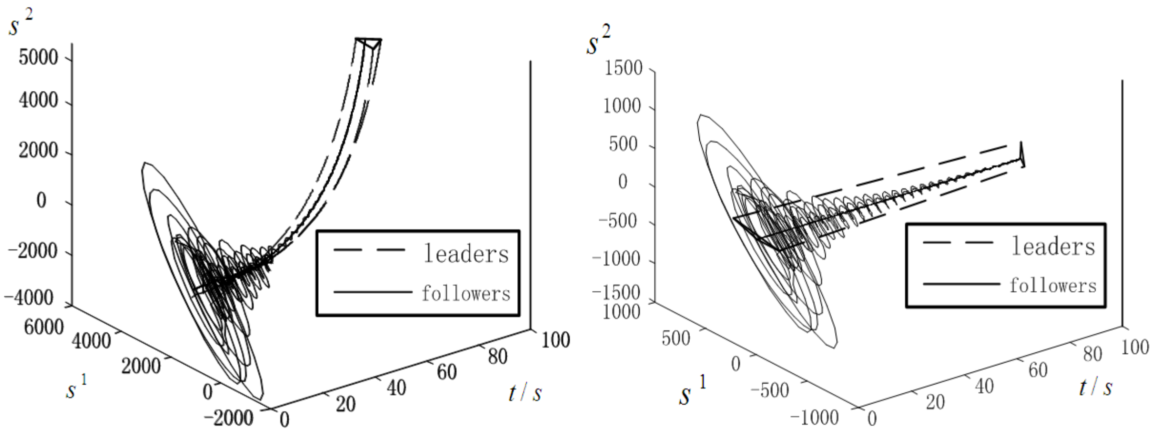
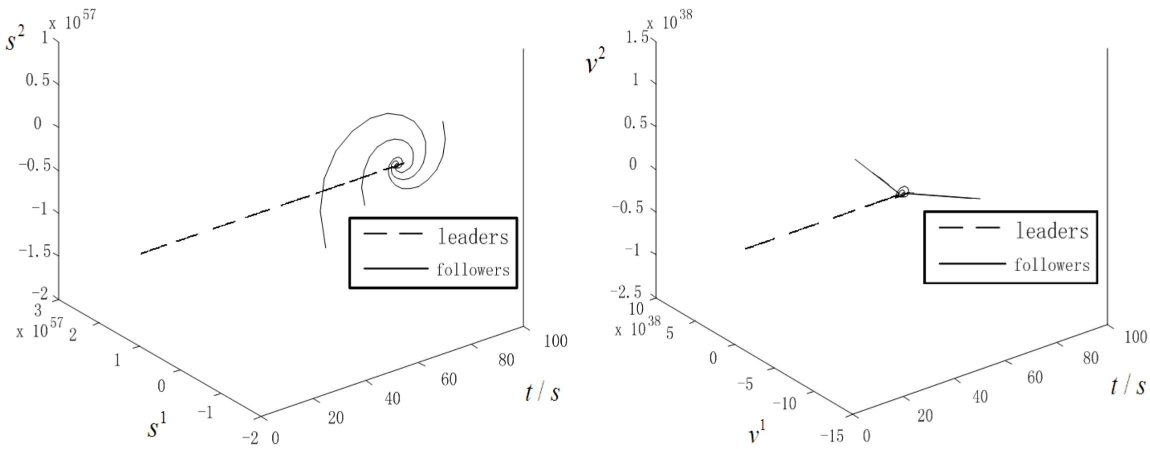



FIGURE 2. The topology graph for multi-agent system




(a) Position trajectories of all the agents (b) Velocity trajectories of all the agents

FIGURE 3. State trajectories of all the agents




(a) Position trajectories of all the agents (b) Velocity trajectories of all the agents

FIGURE 4. State trajectories of all the agents based on the method in [24]

5. **Conclusions.** The problem of network-based containment control for a class of multi-agent systems is studied in this paper. Under a directed graph, the containment control protocol of distributed multi-agent system is accomplished through a communication network, in which way all followers' states can be driven asymptotically converge to the convex hull spanned by the leaders. By combining Lyapunov stability theory and linear matrix inequality (LMI) method, a sufficient condition guaranteeing containment control

is derived, and the network-based containment control gain is achieved by solving the feasibility of linear matrix inequality. The results show the network-based containment control protocol effectively helps compensate the time-varying delays produced in the communication network.

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