International Journal of Innovative Computing, Information and Control
Volume **10**, Number **3**, June **2014**

ICIC International ©2014 ISSN 1349-4198
pp. 1095-1109

ROBUST TRACKING CONTROL OF PNEUMATIC MUSCLE USING PASSIVITY BASED CONTROL

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Received April 2013; revised October 2013

Abstract. *Presently artificial pneumatic muscles are used in various applications due to their simple construction, lightweight and high force to weight ratio. However, controls of different mechanical systems actuated by pneumatic muscles face various problems. Current models are nonlinear and time-varying. This paper deals with a passivity-based control (PBC) for Pneumatic Muscle (PM) with a model pulling a mass against gravity. The objective of this study is to design a robust controller for PM under modeling uncertainties and perturbations associated with Passivity Based Control (PBC) design methodology. In order to improve the effectiveness and robustness of our control system, we designed a PBC in combination with a non-linear disturbance observer (PBCNDO) that provides robust performance in tracking a desired trajectory with guaranteed accuracy regardless of the disturbances. We use the NDO for estimation of friction as an exogenous disturbance in our system. Stability and performance analysis of the composite closed-loop system is provided using Lyapunov theory. Simulation results are provided to validate our theoretical results.*

Keywords: Pneumatic muscle, Passivity based control, Non-linear disturbance observer, Lyapunov theory, Robust control

1. **Introduction.** In order to be suitable for task-oriented rehabilitation therapy, robots must comply with and meet safety measures for patients: a challenge for current rehabilitation robot design. Traditional robots are driven by electric motors that are deficient, in exoskeleton applications, from the standpoint of necessary compliance between the actuator and the limb being moved. To satisfy the needs of a compliant actuator, a pneumatic muscle (PM) actuator similar to human skeletal muscles in size, weight, and power output is extensively used in rehabilitation robots. The PM system is inherently a passive device and is classified as a "soft actuator" (due to its compliance) [1]. In contrast to traditional motor actuators, a PM actuator possesses many unique advantages (i.e., lower cost, light weight, compliance). Concurrently, the PM has high power/weight and power/volume ratios [2]. However, compared with electric motors, PM has a slower response time in force-generating, time-varying parameters depending on the load position and speed. A shortcoming in PM technology for use in precision and/or force applications is the inherent difficulty of controlling them accurately, due to their complex nonlinear dynamics. Another limitation of PMs is the fact that they only can be operated in the contractile direction. Hence, PMs have to be used in antagonistic pairs to achieve bi-directional motion

for a simple flexion and extension joint movement [3]. To overcome these complexities and hindrances, in recent years researchers have presented several novel approaches to alleviate shortcomings in current controller design and to allow for easier practical application of PMA [4, 5, 6, 7]. For the design and control of nonlinear systems, many effective methods have been proposed, e.g., Lyapunov function method, sliding mode control method, co-prime factorization method, and passivity based control method. Among these methods, passivity based control has been proven to be a promising method of analysis, design, stabilization, and control for nonlinear feedback systems. Passive systems constitute an important class of dynamical system whose energy is exchanged with the environment. In passive systems, the amount of the stored energy cannot exceed that of the supplied energy from the outside with the difference being the dissipated energy. Given this property, passivity characteristics have been regarded as a building block for stabilization of nonlinear systems by an increasing number of researchers [8, 9, 10, 11, 12, 13, 14]. If a nonlinear system is passive, it can be stabilized by any negative linear feedback even in the lack of a detailed description of its mathematical model, a property that is very attractive for use in different physical applications [15]. Although there are a lot of control methods that have been used for Pneumatic Muscle, to our knowledge none of them uses Passivity Based Control combined with nonlinear disturbance observer to control the Pneumatic Muscle with a model pulling a mass against gravity which is a new application domain. This study proposes a Nonlinear Disturbance Observer Based Control (NDOBC) approach for the PM-System. Passivity concepts are the fundamental tools invoked to analyze the closed-loop PM-System behavior which leads to the utilization of asymptotic position tracking performance. A major advantage of this framework allows us to develop in a separate way the control law from the observer design provided that each part satisfies some passivity properties. Within this framework, instead of considering the control problem for the PM-System under disturbance as a single entity, we approach it from the standpoint of two separate subproblems, each with its own design objectives. The first subproblem is how to find a dynamic output feedback controller which makes the system dissipative with respect to a specific supply rate while simultaneously capable of maintaining internal stability. The second subproblem is formulated with respect to attenuating disturbances. A nonlinear disturbance observer is designed to deduce external disturbances and then to compensate for the influence of the disturbances using proper feedback. This paper can be summarized as follows. In Section 2, passivity theory and mathematical preliminaries are reviewed. A pneumatic muscle model description and its features are outlined in Section 3. The main results consisting of: a design of the proposed robust control scheme, verification of the passivity of the nonlinear feedback system, and the design of a nonlinear disturbance observer are demonstrated in Section 4. Simulation results are provided to show the validity of the proposed methods in Section 5, the final part is the conclusion of the paper.

2. **Passivity Review and Mathematical Preliminaries.**

In this section the concepts which will serve as the basis for the further developments are introduced. To put our discussion in perspective and to introduce some notations, let us briefly recall some aspects and results from the theory of passive systems, which can be found in [14, 16, 17, 18, 19, 20] and references therein. Consider an affine nonlinear system described by equations of the form

$$\begin{aligned} \dot{x} &= f(x) + g(x)\,u \\ y &= h(x) \end{aligned} \tag{1}$$

Definition 2.1. *The system (1) is said to be passive if there exists a C^0 nonnegative function $V : R^n \to R$ with $V(0) = 0$, called storage function, such that for all $u \in R^P$ and all $x_0 \in R^n$*

$$V(x) - V(x_0) \le \int_0^t y^T(\tau)\, u(\tau) d\tau \tag{2}$$

where $x(t) = \varphi(t, x_0, u)$ denotes a solution of (1) starting from $x(0) = x_0$. If, in addition, there is a positive definite function $S : R^n \to R$ such that for all $u \in R^P$ and all $x_0 \in R^n$

$$V(x) - V(x_0) \le \int_0^t y^T(\tau)\, u(\tau) d\tau - \int_0^t S(x(\tau)) d\tau \tag{3}$$

then the system (1) is said to be strictly passive. The inequality (2) is called the passive inequality which is equivalent to

$$\dot{V}(x) \le y^T(t)\, u(t) \quad \forall u \in \mathrm{R}^p \tag{4}$$

if V is C^r $(r \ge 1)$ function. Similarly, (3) can be expressed as

$$\dot{V}(x) < y^T(t)\, u(t) \quad \forall u \in R^p,\ \forall x \ne 0 \tag{5}$$

if V is differentiable.

2.1. **Kalman-Yacubovitch-Popov Lemma (KYP for short).** A fundamental property of passive systems is characterized by the nonlinear version of the KYP Lemma, which can be summarized in the following statement.

Lemma 2.1. *The system (1) is passive with a C^1 storage function if and only if there exists a C^1 nonnegative function $V : R^n \to R$ with $V(0) = 0$ such that*

$$L_f V(x) \le 0, \quad L_g V(x) = h^T(x) \tag{6}$$

The system (1) is strictly passive if and only if there exists a C^1 positive definite function $V(x)$ such that

$$L_f V(x) < 0 \ \ \forall x \ne 0, \quad L_g V(x) = h^T(x) \tag{7}$$

2.2. **Robust KYP Lemma.** The KYP Lemma described in a previous section provides a necessary and sufficient condition for a nonlinear system of the form (1) to be passive or strictly passive. The purpose of this section is to show how this result can be extended to nonlinear systems with structural uncertainty, and how robust passivity can be tested using the robust KYP Lemma [19]. The class of uncertain nonlinear systems under consideration is described by

$$\begin{aligned} \dot{x} &= f(x) + \Delta f(x) + g(x)\, u \\ y &= h(x) \end{aligned} \tag{8}$$

where $\Delta f : R^n \to R^n$ represents structural uncertainty or uncertain perturbation which is characterized by $\Delta f = e(x)\, \delta(x)$ where $e : R^n \to R^{n\times m}$ is a known smooth function and $\delta : R^n \to R^m$ is an unknown function, which belongs to the following compact set

$$\Omega = \{\delta(x) \mid \|\delta(x)\| \le \|n(x)\|\} \tag{9}$$

The smooth mapping $n : R^n \to R^m$ is given, with $n(0) = 0$. $\Delta f(x)$ or $\delta(x)$ is said to be admissible if $\delta(x) \in \Omega$. In view of Definition 2.1 and Lemma 2.1, it is quite natural to introduce the following concept for uncertain systems.

Definition 2.2. *The system (8) is said to be robust passive if there exists a C^1 nonnegative function $V : R^n \to R$ with $V(0) = 0$, such that for all admissible $\Delta f(x)$*

$$L_{f+\Delta f} V(x) \leq 0, \quad L_g V(x) = h^T(x) \tag{10}$$

If the inequality in (10) becomes a strict inequality that holds for a C^1 positive definite $V(x)$ and for all $x \neq 0$, then (8) is said to be strictly robust passive.

Lemma 2.2. *The system (8) is robust passive with a C^1 $V(x)$, which is nonnegative, with $V(0) = 0$, if and only if*

$$\begin{cases} L_f V(x) + \|L_e V(x)\| \, \|n\| \leq 0 \\ L_g V(x) = h^T(x) \end{cases} \tag{11}$$

3. **Pneumatic Muscle Model Description.** PMAs represent the main force control operator in many applications, where their static and dynamic characteristics play an important role in the overall behavior of the control system. Therefore, improving the dynamic behavior of the pneumatic actuator is of prime interest to control system designers. Two categories for the mathematical models of a pneumatic muscle actuator are prevalent: the theoretical and the phenomenological models [21, 22, 23, 24]. In this paper, we adopt the phenomenological model method as a combination of effects from non-linear friction, spring and contraction components as shown in Figure 1, to describe the dynamic behavior of a pneumatic muscle (PM) pulling a mass against gravity [5]. The coefficients related to these three elements depend on the input pressure of the PM [2]. The equations describing approximately the dynamics of a PM are given by

$$M\ddot{x} + B(P)\dot{x} + K(P)x = F(P) - Mg \tag{12}$$

$$K(P) = K_0 + K_1 P \tag{13}$$

$$B(P) = B_{0i} + B_{1i} P(\text{inflation}) \tag{14}$$

$$B(P) = B_{0d} + B_{1d} P(\text{deflation}) \tag{15}$$

$$F(P) = F_0 + F_1 P \tag{16}$$

where M is the mass, g is the acceleration of gravity, $x = 0$ corresponds to the fully deflated position, and P is the input pressure. The coefficients $K(P)$ and $B(P)$ are pressure dependent for the spring and the damping, respectively. The contractile element presents the effective force $F(P)$. The damping coefficient depends on whether the PM

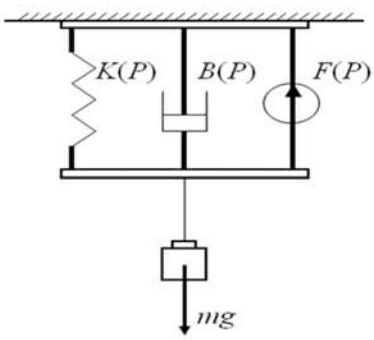

FIGURE 1. Pneumatic muscle (PM) components

is inflated and deflated. From the dynamic Equations (12)-(16), we can write for the PM the following dynamic model:

$$\begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{M}\left[(F_0 - Mg - B_0x_2 - K_0x_1) + (F_1 - B_1x_2 - K_1x_1)\,P\right] \end{array} \tag{17}$$

From the dynamic Equations (12)-(16), with $x_1 = x$ the system state (actual position), $x_2 = \dot{x}$ is the speed of the system and $X = \left[\begin{array}{cc} x_1 & x_2 \end{array}\right]^T$ we can obtain

$$\dot{X} = f\,(X) + b\,(X)\,P \tag{18}$$

where $f\,(X) = \left[\begin{array}{cc} x_2 & \frac{1}{M}\,(F_0 - Mg - B_0x_2 - K_0x_1) \end{array}\right]^T$ and $b\,(X) = \left[\begin{array}{cc} 0 & \frac{1}{M}(F_1 - B_1x_2 - K_1x_1) \end{array}\right]^T$. In summary, the dynamic model of PM can be concretely represented as

$$M\ddot{x} + B_0\dot{x} + K_0x + (Mg - F_0) = (F_1 - B_1\dot{x} - K_1x)\,P \tag{19}$$

4. **Design of the Proposed Robust Control.**

Robust control stabilizes an uncertain system by assuming that its uncertainties are bounded in size by a known function [7]. Typically implemented in a computer, the action of a controller may be understood in energy terms as another dynamical system interconnected with the nonlinear system to modify its behavior. The control problem can then be recast as finding a dynamical system and an interconnection pattern such that the overall energy function takes the desired form. This "energy-shaping" approach is the essence of passivity-based control (PBC), a controller design technique that is very well-known in mechanical systems [1].

4.1. **PBC controller design.**

For imprecisely known parameter values, the dynamic behavior of PM-System is actually described by

$$M\ddot{x} + B_0\dot{x} + K_0x + (Mg - F_0) = (F_1 - B_1\dot{x} - K_1x)\,P \tag{20}$$

Suppose that the desired position, speed and acceleration of the system are described by x_d, $\dot{x}_d$ and $\ddot{x}_d$, then the corresponding errors are defined as $e = x - x_d$, $\dot{e} = \dot{x} - \dot{x}_d$ and $\ddot{e} = \ddot{x} - \ddot{x}_d$. After we defined $H_0 = M\ddot{x}_d + B_0\dot{x}_d + K_0x_d + (Mg - F_0)$ and

$$H_1 = F_1 - B_1\dot{x} - K_1x$$

we can suppose for the right side of Equation (20) as the following

$$\begin{array}{l} (F_1 - B_1\dot{x} - K_1x)\,P = M\ddot{x}_d + B_0\dot{x}_d + K_0x_d + (Mg - F_0) + u \\ u = H_1P - H_0 \end{array} \tag{21}$$

where u is an auxiliary function to be determined. Substituting Equation (21) in (20), we obtain error dynamic equation

$$M\ddot{e} + B_0\dot{e} + K_0e = u \tag{22}$$

For Equation (22) we define $x_1 = e$ and $x_2 = \dot{e} + e$ as state variables. Obviously, we have $\lim\limits_{t\to\infty} e = 0$ and $\lim\limits_{t\to\infty} \dot{e} = 0$ if and only if $\lim\limits_{t\to\infty} x\,(t) = 0$. In the new coordinate we assume the nonlinear disturbance is ω. The error dynamic model in Equation (22) can then be translated into the following equation of state:

$$\begin{array}{l} \dot{x}_1 = -x_1 + x_2 \\ \dot{x}_2 = \frac{1}{M}\left((B_0 - K_0 - M)\,x_1 - (B_0 - M)\,x_2 - H_0\right) + \frac{H_1}{M}P + \frac{1}{M}\omega \\ y = x_2 \end{array} \tag{23}$$

This in general can be written as

$$\dot{X} = f\,(X) + g_1\,(X)\,P + g_2\,(X)\,\omega \tag{24}$$

where

$$\begin{aligned}
&\dot{X} = \begin{bmatrix} \dot{x}_1 & \dot{x}_2 \end{bmatrix}^T \\
&f(X) = \begin{bmatrix} -x_1 + x_2 & \frac{1}{M}\left((B_0 - K_0 - M)x_1 - (B_0 - M)x_2 - H_0\right) \end{bmatrix}^T \\
&g_1(X) = \begin{bmatrix} 0 & \frac{H_1}{M} \end{bmatrix}^T, \ g_2(X) = \begin{bmatrix} 0 & \frac{1}{M} \end{bmatrix}^T
\end{aligned} \tag{25}$$

Equation (24) can be written as

$$g_2(X)\,\omega = \dot{X} - f(X) - g_1(X)\,P \tag{26}$$

In this paper, a nonlinear disturbance observer to estimate the disturbance ω of system (23) is proposed as

$$\dot{\hat{\omega}} = \alpha\left(\dot{X} - f(X) - g_1(X)\,P - g_2(X)\,\hat{\omega}\right) \tag{27}$$

where α is the observer gain and $\hat{\omega}$ is the estimated disturbance. We choose the non-linear observer gain α as a constant matrix

$$\alpha = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \tag{28}$$

where $c_1 > 0$ and $c_2 > 0$. Substitution of Equations (25) and (28) into (27) yields

$$\dot{\hat{\omega}} = c_2\left[\dot{x}_2 - \frac{1}{M}\left((B_0 - K_0 - M)x_1 - (B_0 - M)x_2 - H_0\right) - \frac{H_1}{M}P - \frac{\hat{\omega}}{M}\right] \tag{29}$$

From Equation (23) we substitute $\dot{x}_2$ in (29) and then we can write for $\dot{\hat{\omega}}$ as the following

$$\dot{\hat{\omega}} = c_2\left[\frac{1}{M}(\omega - \hat{\omega})\right] = c_2\frac{\tilde{\omega}}{M} \tag{30}$$

We let $\tilde{\omega} = \omega - \hat{\omega}$ with the assumption that

$$|\tilde{\omega}| < \bar{\eta} < \eta \ \text{ then } \ 0 < \eta - \bar{\eta} < \eta - \tilde{\omega} < \eta + \bar{\eta} \tag{31}$$

where $\tilde{\omega}$ is a disturbance difference, $\bar{\eta}$ is an integer number and

$$\eta > 0$$

Generally, in $\tilde{\omega} = \omega - \hat{\omega}$ there is no prior information about the derivative of the disturbance ω. When the disturbance varies slowly relative to the observer dynamics, it is reasonable to suppose that $\dot{\omega} = 0$ and then we have $\dot{\tilde{\omega}} + \dot{\hat{\omega}} = 0 \Rightarrow \dot{\tilde{\omega}} = -\dot{\hat{\omega}}$. Choose the form of Lyapunov function as follows:

$$V = \frac{1}{2}x_1^T x_1 + \frac{1}{2}x_2^T M x_2 + \frac{1}{2}\tilde{\omega}^2 \tag{32}$$

Its time derivative along the state trajectory is given by

$$\dot{V} = x_1^T\dot{x}_1 + x_2^T M\dot{x}_2 + \tilde{\omega}\dot{\tilde{\omega}} = x_1^T\dot{x}_1 + x_2^T M\dot{x}_2 - \tilde{\omega}\dot{\hat{\omega}} \tag{33}$$

Substituting Equation (23) in (33) yields

$$\dot{V} = -x_1^T x_1 + x_2^T\left((B_0 - k_0 - M)x_1 - (B_0 - M)x_2 + x_1 + H_1 P - H_0 + \omega\right) - \tilde{\omega}\dot{\hat{\omega}} \tag{34}$$

In Equation (34) we chose the input pressure P as

$$P = \left(-(B_0 - k_0 - M)x_1 + (B_0 - M)x_2 - x_1 + H_0 - \hat{\omega} - \eta\right)(H_1)^{-1} \tag{35}$$

Equation (34) can be written, after substituting Equation (35) and making use of Equation (30) and assumption in (31) as

$$\begin{aligned}
&\dot{V} = -x_1^T x_1 + x_2^T(\tilde{\omega} - \eta) - \tfrac{c_2}{M}\tilde{\omega}^2 = -x_1^T x_1 - x_2^T(\eta - \tilde{\omega}) - \tfrac{c_2}{M}\tilde{\omega}^2 \\
&\dot{V} \le \min\left(-x_2^T(\eta + \bar{\eta}), -x_2^T(\eta - \bar{\eta})\right)
\end{aligned} \tag{36}$$

According to the minimum law, $\min(a,b) = \frac{a+b}{2} - \frac{|a-b|}{2}$ we can write (36) as

$$\dot{V} \leq \min\left(-x_2^T(\eta+\bar{\eta}), -x_2^T(\eta-\bar{\eta})\right) = -x_2^T(\eta+\bar{\eta}) = -x_2^T\beta \tag{37}$$

where $\beta = \eta + \bar{\eta}$. This shows that the closed-loop system is passive from the input, viz.β, to the output, viz.y. According to relations of passivity and asymptotic stability, let $\beta = x_2$ and the closed-loop system is stable. Substitute H_0 in (35) and consider $x_1 = e$ and $x_2 = \dot{e} + e$ where $e = x - x_d$ the controller P is given by

$$P = \frac{M\ddot{x}_d + B_0\dot{x} - M\dot{e} + K_0 x - e + (Mg - F_0) - \hat{\omega} - \eta}{F_1 - B_1\dot{x} - K_1 x} \tag{38}$$

Substituting Equation (35) in (21), we get

$$u = (B_0 - M)\dot{e} + (K_0 - 1)e - \eta + \hat{\omega} \tag{39}$$

Then we can write for Equation (23) following

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2 \\ \dot{x}_2 &= -M^{-1}(x_1 + \eta + \tilde{\omega}) \\ y &= x_2 \end{aligned} \tag{40}$$

If Formula (40) is passive and can be written in general form

$$\begin{aligned} \dot{x} &= f(x) + g(x)\eta \\ y &= h(x) \end{aligned} \tag{41}$$

then we have according to a Kalman-Yacubovitch-Popov (KYP) Lemma, the following statement

$$\frac{\partial V}{\partial X} f(x) \leq 0, \quad \frac{\partial V}{\partial X} g(x) = h^T(x) \tag{42}$$

4.2. **Nonlinear disturbance observer (NDO).** Estimation of nonlinear uncertainty can be pursued in two ways: firstly, through developing a bounding function on the size of the uncertainty, linearly parameterize the bounding function (or the uncertainty directly) in terms of a number of unknown parameters, and then estimate the parameters using adaptation laws; secondly, through generating the uncertainties as the output of some exogenous system, allow estimation and compensation under certain conditions. In either of these ways, certain structure property about the uncertainty or its bounding function is needed. It is a matter of fact, that a model of the exosystems must be included into the controller to reach the design goals. Owing to that end, a block diagram of a proposed PBCNDO is shown in Figure 2. As seen in the figure, the composite controller consists of two parts: a controller without or having poor disturbance attenuation ability and a disturbance observer. As the dynamics of PM are nonlinear and hard to model precisely, the design of the model-based control algorithm is more cumbersome. Besides the modeling uncertainties, external disturbances are inevitable in real environments which degrade the control performance. Therefore, the controller should have a robust capability to achieve the desired objective. Since the PM dynamics show highly nonlinear behavior, it is difficult to estimate the proper norm bound and, thus, the usual robust control method for PM control often results in a conservative design. Therefore, the use of a disturbance observer resolves these difficulties. Considering the external disturbances and the model error in Equation (23), the PM model is given by:

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2 \\ \dot{x}_2 &= \tfrac{1}{M}\left(-M\ddot{x}_d + M\dot{e} - B_0\dot{x} - K_0 x - (Mg - F_0)\right) + \tfrac{H_1}{M}P + \tfrac{1}{M}\omega \end{aligned} \tag{43}$$

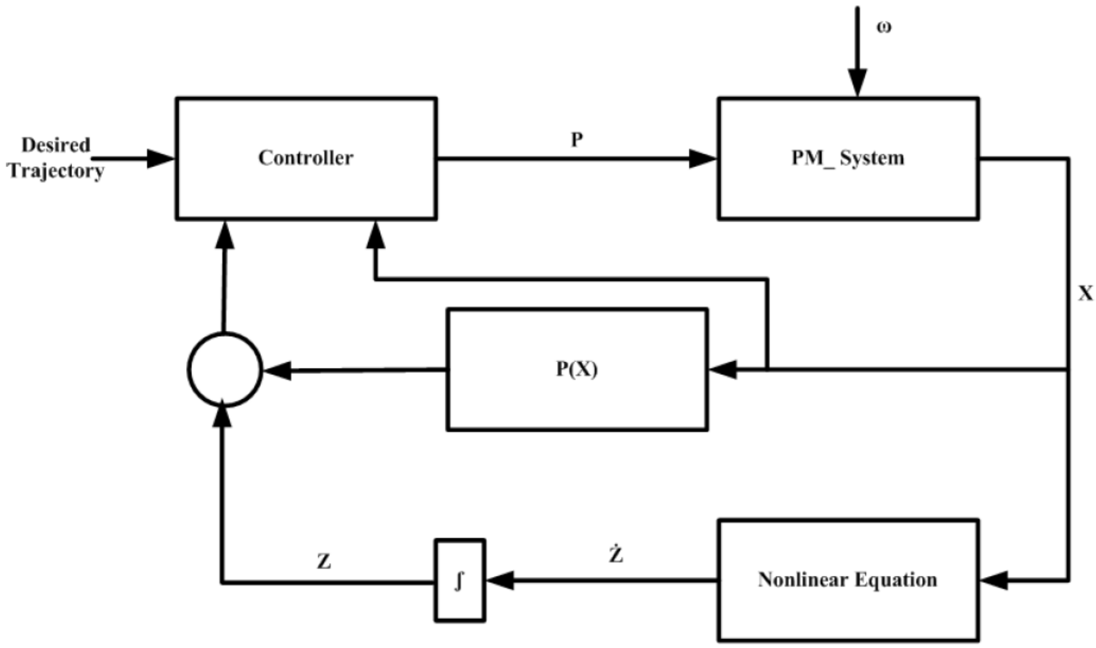

FIGURE 2. The proposed PBCNDO

This can be written generally as

$$\dot{X} = f(X) + g_1(X)P + g_2(X)\omega \tag{44}$$

In this paper, a nonlinear disturbance observer to estimate the disturbance ω of system (43) is proposed as

$$\dot{\hat{\omega}} = \alpha\left(\dot{X} - f(X) - g_1(X)P - g_2(X)\hat{\omega}\right) \tag{45}$$

Define an auxiliary vector $z = \hat{\omega} - \rho(X)$ where $z \in R^2$, $\rho(X)$ is a nonlinear function to be designed and the non-linear observer gain α is defined as $\alpha = \frac{\partial\rho(X)}{\partial X}$. Taking the time derivative of z gives us

$$\dot{z} = -\alpha\left(f(X) + g_1(X)P + g_2(X)(z + \rho(X))\right) \tag{46}$$

Let $\tilde{\omega} = \omega - \hat{\omega}$, generally, and there is no a priori information about the derivative of the disturbance ω. When the disturbance varies slowly relative to the observer dynamics, it is reasonable to suppose that $\dot{\omega} = 0$, and then we have $\dot{\tilde{\omega}} + \dot{\hat{\omega}} = 0$ [21]. Now according to Equation (45) we write

$$\dot{\tilde{\omega}} + \dot{\hat{\omega}} = \dot{\tilde{\omega}} + \alpha\left(g_2(X)\omega - g_2(X)\hat{\omega}\right) = \dot{\tilde{\omega}} + \alpha g_2(X)\tilde{\omega} = 0 \tag{47}$$

If we choose the gain α as a constant matrix

$$\alpha = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \tag{48}$$

where $c_1 > 0$ and $c_2 > 0$, in this case $\rho(X)$ is given by

$$\rho(X) = c_1x_1 + c_2x_2 \tag{49}$$

Substituting (48) and (49) in (46), the disturbance observer can be designed as

$$\begin{aligned}\hat{\omega} &= z + \rho(X) \\ \dot{z} &= -\left[\begin{array}{l} c_1(-x_1 + x_2) + \frac{c_2}{M}(-M\ddot{x}_d + M\dot{e} - B_0\dot{x} - K_0 x - (Mg - F_0) + H_1 P) \\ -\frac{c_2}{M}(z + c_1 x_1 + c_2 x_2) \end{array}\right]\end{aligned} \tag{50}$$

Substituting Equation (38) in (50), we can write Equation (50) as the following

$$\begin{aligned}\hat{\omega} &= z + \rho(X) \\ \dot{z} &= -\left[\left(-c_1 - \tfrac{c_2}{M}\right)x_1 + c_1 x_2 - \tfrac{c_2}{M}\eta\right]\end{aligned} \tag{51}$$

Now if we take the error definition $e = x - x_d$, $x_1 = e$ and $x_2 = \dot{e} + e$ in consideration, then Equation (51) can be written as the following

$$\begin{aligned}\hat{\omega} &= z + \rho(X) \\ \dot{z} &= -\left[-\tfrac{c_2}{M}e + c_1\dot{e} - \tfrac{c_2}{M}\eta\right] = -\left[-\alpha_1 e + \alpha_2\dot{e} - \alpha_0\eta\right]\end{aligned} \tag{52}$$

where $\alpha_0 = \alpha_1 = \frac{c_2}{M}$ and $\alpha_2 = c_1$. Considering (38) and (43), the passive controller law combined with disturbance observer can be deduced as

$$P = \frac{M\ddot{x}_d + B_0\dot{x} - M\dot{e} + (K_0 - 1)x + x_d + \eta - (F_0) - \frac{\hat{\omega}}{M} + Mg}{F_1 - B_1(\dot{e} + \dot{x}_d) - K_1(e + x_d)} \tag{53}$$

5. **Simulation Results and Discussion.** The objective of this section is to demonstrate the robustness of the proposed controller. However, the feedback linearization control law designed above may have quite poor disturbance attenuation ability. If we consider that unknown variable load is applied to this system, it is obvious that with the increase of the load, the position error significantly increases. For evaluation, the performance of the proposed controller, a sinusoidal trajectory-tracking problem, was set as in Equation (54) and a nonlinear disturbance observer was presented to estimate the unknown exogenous disturbances. The exogenous disturbance in our simulation system was considered as Coulomb and Viscous friction. When we use our proposed controller (PBC) combined with a nonlinear disturbance observer (NDO) or (PBCNDO), the external non-linear disturbance can be approximately estimated by this nonlinear disturbance observer (NDO). The desired sinusoidal trajectory is given by

$$x = 0.015\sin(0.5\pi(t - 1)) + 0.023 \tag{54}$$

The PM coefficients used for the simulation are shown in Table 1. The parameters $F(P)$, $K(P)$ and $B(P)$ of column C1 and C2 are chosen assuming $\pm 10\%$ error in the evaluation values (C). The simulation interval time was selected as 10 ms.

Table 1. PM coefficient sets used for the simulation

Coefficient	**C (Evaluation)**	**C1 (0.9×C)**	**C2 (1.1×C)**
$F0(\times 10^2)$	-1.0336	-0.9302	-1.137
$F1(\times 10^{-6})$	719.75	647.78	791.73
$Km0(\times 10^4)$	1.5010	1.351	1.651
$Km1(\times 10^{-2})$	-5.703	-5.133	-6.2733
$Kn0(\times 10^2)$	2.598	2.338	2.8578
$Kn1(\times 10^{-2})$	0.858	0.772	0.9438
$B0i$	52.08	46.87	57.29
$B1i(\times 10^{-6})$	-124.5	-112.1	-137.0
$B0d$	-3.19	-2.87	-2.51
$B1d(\times 10^{-6})$	90.22	81.20	99.24

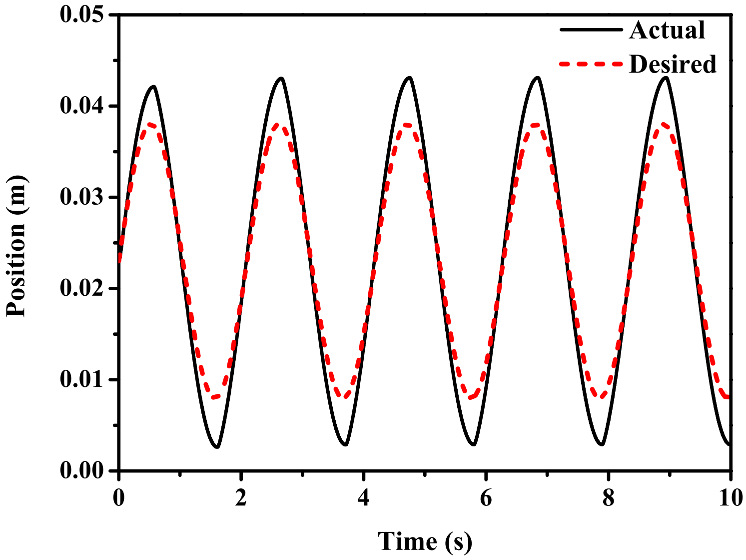

FIGURE 3. PBC-Sine trajectory tracking result without NDO ($\eta = 50 \times (\dot{e} + e)$)

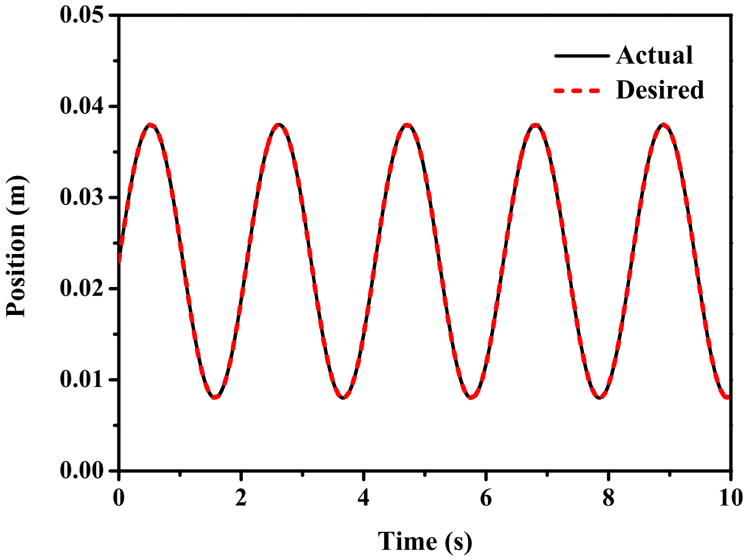

FIGURE 4. PBC-Sine trajectory tracking result without NDO ($\eta = 500 \times (\dot{e} + e)$)

The simulation results of the study indicate that the parameter η affected the stability and the tracking performance of the PM strongly. The simulation results for different η values for our proposed controller PBC without integrating the nonlinear disturbance observer (NDO) are shown in Figure 3 and Figure 4. As shown in Figure 3 it is obvious that the tracking performance of the system using a PBC is not satisfactory when there are modeling uncertainties and perturbations. On the other hand, as shown in Figure 4, by increasing the parameter η, the deviation between the actual and the desired trajectory is significantly reduced.

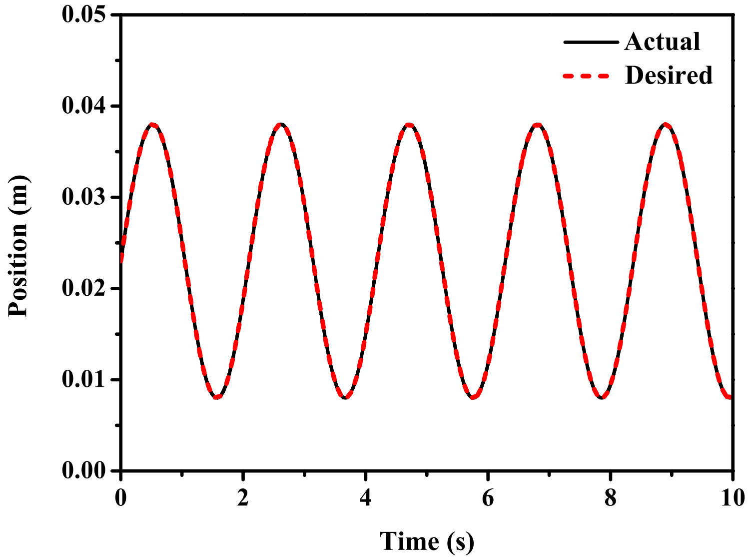

FIGURE 5. PBCNDO-Sine trajectory tracking result ($\eta = 50 \times (\dot{e} + e)$)

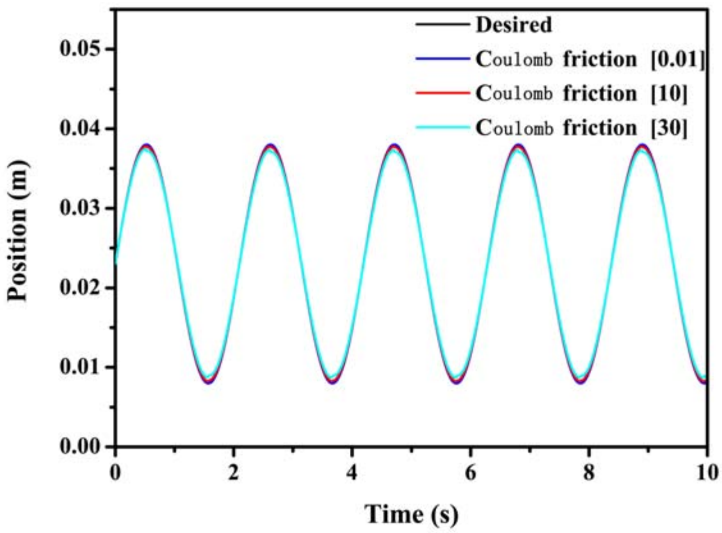

FIGURE 6. Tracking result for different friction's parameters

When using our proposed PBCNDO method, the influence of disturbances is estimated by a disturbance observer and then compensated for based on the estimate. At that point, the effects of disturbances and uncertainties are reduced significantly, as depicted in Figure 5. As we see in this figure, a small η can achieve a satisfactory tracking performance.

Considering simulations for different friction's parameters, disturbance and uncertainties have been taken to demonstrate and illustrate the significance and novelty of our work as practical systems (see Figure 6). In this figure, different values (0.01, 10 and 30) from the Coulomb frictions were applied to show the robustness of the system. It can be observed that the proposed controller can track the desired trajectory in spite of

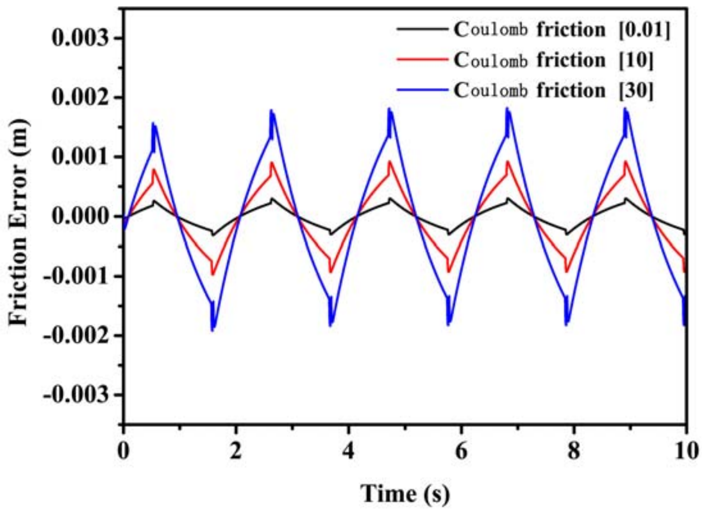

Figure 7. Effect of the variant viscous friction

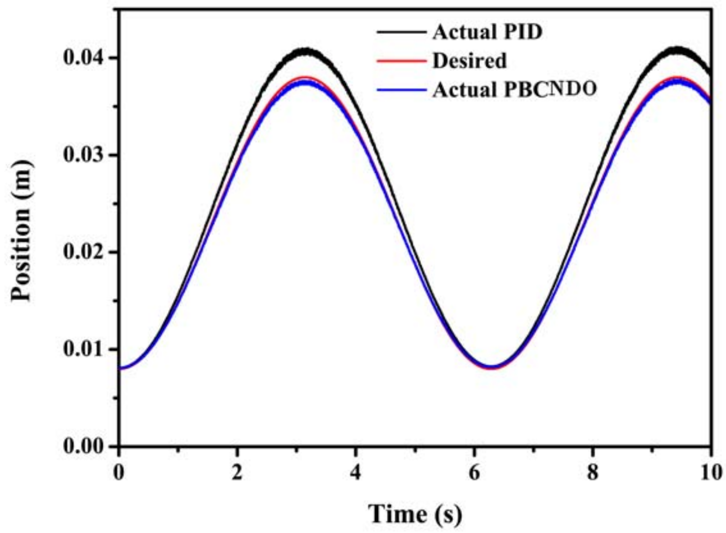

Figure 8. Comparison of PBCNDO controller with a conventional PID

these variant viscous frictions values with very small deviation. Figure 7 shows the effect of the variant viscous frictions values; it can be noticed that the friction error behaves proportional to the value of the viscous friction.

PID controllers are one of the most used types of controllers in practice; they can cope reasonably well with systems having different types of dynamics and their analogue implementation is easy to realize with operational amplifiers. However, in spite of their simplicity and the small number of parameters that have to be adjusted, PID controller is frequently poor for stability analysis and tuned. To show superior performance and to verify the effectiveness of the proposed PBCNDO controller over the conventional PID controller, a simulation comparison is performed Figure 8.

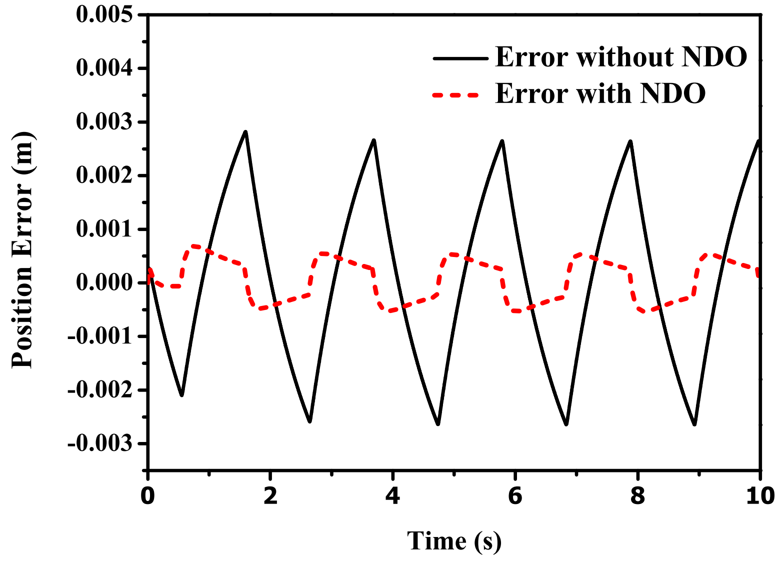

FIGURE 9. Position error comparison with and without disturbance observer

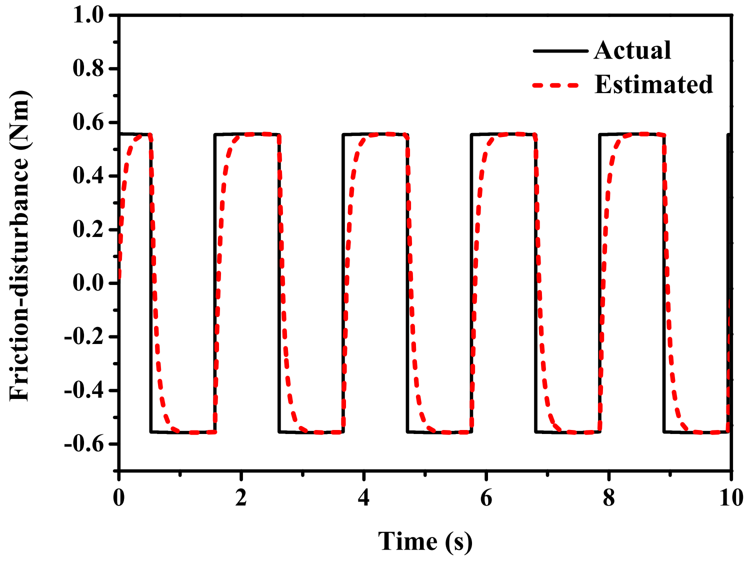

FIGURE 10. The time history of estimated and actual friction-disturbance

The major significance of the proposed PBCNDO controller lies in its high robustness against disturbance and superior performance over the conventional PID controller, and is easy of engineering implementation, which explores a convenient engineering method to improve the performance of the PM control system. The tracking error in the simulations associated with our composed controller of PBC and NDO is shown in Figure 9. It can be seen that, by using a properly designed nonlinear disturbance observer and a proper feedback, the tracking error or regulation error caused by disturbances can be suppressed significantly.

In Figure 10, we see the time history of the estimated disturbance-observer compared with the actual friction-disturbance. In this figure as we can see, the disturbance-observer tracks the actual friction-disturbance with slight time delay. The maximum mismatch between the actual friction and estimated frication is found to be near the peaks. This is an expected result due to the high rate of change at the peak, and the fact that the observer was designed to track piecewise constant trajectory.

6. **Conclusions.** Based on PBC design technology, a nonlinear controller for PM-System to achieve stability and other performance specifications under the assumption of a measurable disturbance is first proposed in this paper. To achieve robustness and stability a nonlinear disturbance observer for disturbance estimation combined with the proposed controller has been additionally designed. We demonstrate by decoupling estimation errors from disturbances and the controller passivity property that the overall closed-loop system is asymptotically stable. Furthermore, to integrate the disturbance observer with the controller, the disturbance in the control law is replaced by an estimation yielded by the disturbance observer. The use of a disturbance-observer can help improve stability and tracking performance of PM under closed-loop control and data show that the proposed control-approach is superior. Finally, simulation results show that the performance of the proposed controller based on PBC design methodology combined with NDO is significantly improved, and that the NDO achieves a superior ability for disturbance attenuation.

Acknowledgment. This work was supported by the Natural Science Foundation of China under Grant 61075095, the International Science and Technology Cooperation Program of China "Precision Manufacturing Technology and Equipment for Metal Parts" under Grant No. $2012DFG70640$ and the International Science and Technology Cooperation Program of Hubei Province "Joint Research on Green Smart Walking Assistance Rehabilitant Robot" under Grant No. $2012IHA00601$.

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