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QUANTIZATION OF ARTIFICIAL NEURON (QUANTUM CURRENT, MODEL OF POLARITON ON AXON)

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Abstract. *We proposed the positive hypotheses of neural interferences based on physiological knowledge of neurons, ephapse as engineering models. The neural interferences of axons and synaptic ones as ephapse are propagated by polaritons, which were a kind of quasi particles. The polaritons were essentially massive vector photon with spin 1. The polaritons, relativistic particles were strictly governed by Proca equation or quaternary Schrödinger equation. The polaritons were connecting between two ionic currents on phospholipid membrane of neuron. That membrane on their axons was propagating their excitations and action potentials using polaritons. The* Na^+ *currents, into insides of membranes of axons, caused the* K^+ *current's flow to outside of axons through charged or non-charged quantized polarization wave (polaritons). Various interferences, such as ephapse, synaptic and the other interferences, were intermediated by polaritons. The polaritons were able to go through myelin sheaths by quantum effect. The polariton carried amount of information,* 9.38×10^{12} *bits/polariton, at 300 Kelvin. We recognized to be required at least* $0.693k_BT$ *joules of energy to convey one bit of information. Those quantum interferences were utilized commonly to adjust our neural and brain's functions.*
Keywords: Polariton, Qusai particles, Polarization vectors, Sodium ionic currents, Potassium ionic currents, Wave function, Axon, Neural net, Quantum interferences, Ephapse, Dielectric materials, Proca equation

1. Introduction.

Many excellent experiments for neuro-function and neural conduction have been performed by usage of micro-needles for neurons, and we have been understanding notable phenomena for neuro-physiology and anatomy. Among all, one of the most famous researches is performed by Hodgikin and Huxley, whose research is based on physical cable theory, ionic currents (Na^+, K^+), local currents and conductions of action potentials [1]. Their model can be able to explain many phenomena of neuro-electrical physiology. In pathological area, Arvanitaki discovered the phenomena of ephapse, which is an interference between many neural axons. When he stimulated one neuron and made action potentials (impulses) arise on the stimulated neuron, that impulses affected on another axon despite of having no direct connections between two axons. As his discovery and experiments are thought he made up an artificial synapse.

However, the ephapse has been believed not to be in the case of healthy neuro-fibers. It is said that ephapse was found in pathological neural axons, i.e., for example, neuralgia and causalgia. Therefore, the axon's or synaptic interferences have been regarded as an evidence of wrong symptom.

We have been drawing negative images for the ephapse, whose sign is pathological neuron or symptom of demyelination.

We would like to propose a positive hypothesis for ephapse or interferences of neurons in this paper: our healthy brain or normal neurons actively utilize electromagnetic interactions, (for example, leakage current, polarization of membrane, noise current and ephapse), so as to adjust neuron's functions between each neuron and to accomplish integrated brain's functions. Note that we do not intend to discuss whether our neuron's model is correct or not from standpoints of biology. We would like to only discuss from engineering standpoints. In the other word, we have an interest in following question: if it were the interferences between each neuron and the brain utilized a fine adjustment of its functions, in spite of those weak electromagnetic interaction, we would like to show how neuron's images change and what a biophysical principle governs our neural networks and what a mathematical expression is for our neural networks. Therefore, we have been searching for way of suitable descriptions of those weak electromagnetic nano- or meso-phenomena.

First of all in this paper, we intend to mention basic idea and theoretical requests to introduce quantum method and concepts of quasi particles polaritons. Then we show quantum mechanism of neural-conduction based on dielectric of myelin sheath. We assert, information for neural interferences as ephapse is propagated by polaritons, and they are a kind of quasi particles, i.e., quantized polarization waves. We conclude that polaritons, massive vector photon with spin 1. Polaritons are closely related to many ionic currents, (Na^+, K^+, Cl^- current), when neurons and axons propagate action potentials (impulses). Moreover, polaritons run on neural membranes along to axon, and they go easily through myelin sheath by quantum tunnel effects of themselves. We propose the ideas that those quantum interferences are useful to adjust and harmonize our neural functions and brain's conditions [2-5]. One of my purposes is to study effects of quantum neural-interferences, and our computers, which are constructed by our quantum neurons, sometimes make mistakes as human being doing.

2. Polaritons' Model of Axon.

Axons of neurons have a series of polarization's processes: in short, the polarization, depolarization and re-polarization by Na^+- and K^+-currents penetrating axon's membranes.

If we observe the changes of magnitude of polarization vectors, we notice approximately to enable to describe the changes of action potentials on axons as the rotating polarization vectors (Figures 1A-1D). Figure 1-A shows the change of polarization vectors, whose vectors mean directions of ionic current and their magnitude. If we observe those polarization vectors, we know it is safe to express classically as rotation of those vectors.

We think to regard phenomena of neural conductions of action potentials (impulse) as propagation of the quantized polarizations vectors, which means the traveling quasi particles, polariton. Their motions (rotation of vectors and propagating polarization vectors) and the series of processes (polarization-depolarization-repolarization, etc.) are caused by mainly ionic currents (Na^+-current, K^+-current, etc.).

Those currents become sources of polaritons, whose rotating vectors propagate on the neural membrane, and triggers of those two ionic currents arise the polarization waves, and the quantized polarization waves correspond to quasi particles, polaritons.

(A) Figure 1-A shows the feature of "the changes of magnitude of polarization vectors". According to the conduction of action potentials along to axons, the polarization vectors rapidly change their shapes, directions and magnitude (Figure 1-A).

(B) The process of conductions of action potentials hypothesizes to be shown as rotation of polarization vectors, if we thought the polarization vectors travel along to longitudinal direction of axons (Figure 1-B).
(C) This picture shows each phase of action potentials, which are mainly made up by those currents, sodium ion's currents, potassium ion's currents and sodium pomp (Figure 1-C).
(D) The inverted phase of polarization vectors (depolarization phase, center of Figure 1-D) is pictured, and the polarization vectors are propagating on the membrane of axon.

Those axon's membranes are constructed by phospholipid bilayer, which has characteristics of strong dielectric materials. Those dielectric materials can efficiently conduct the polarization's waves, or its quantized quasi-particles, polaritons. After all, the quantized

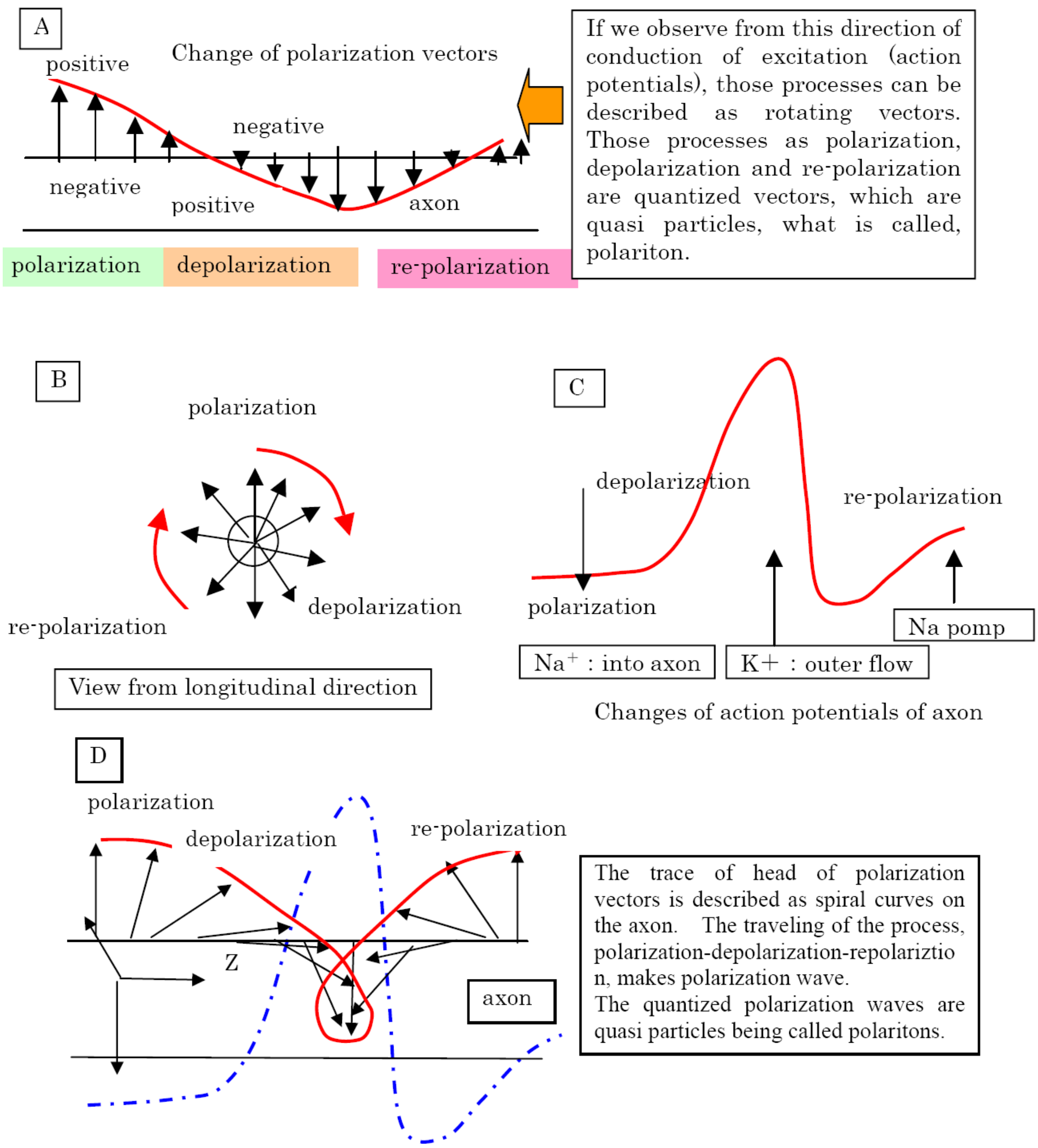
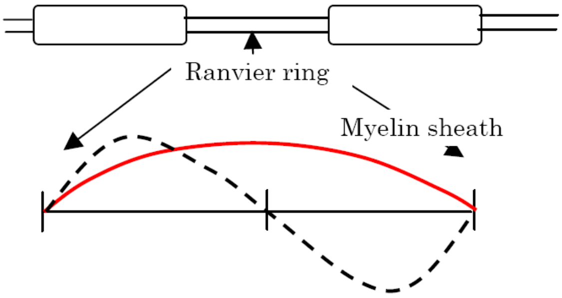


FIGURE 1. Theory of rotating polarization vectors



If polaritons exist in the Ranvier Ring, it is good to consider the polaritons, in initial state, being confined between myelin sheaths. So, we are simply able to apply box potential model for the confinement of polaritons. The continuous line is the ground state of polaritons, which are bosons, the dashed line is correspond to the first excited state.

FIGURE 2. Polariton on Ranvier ring

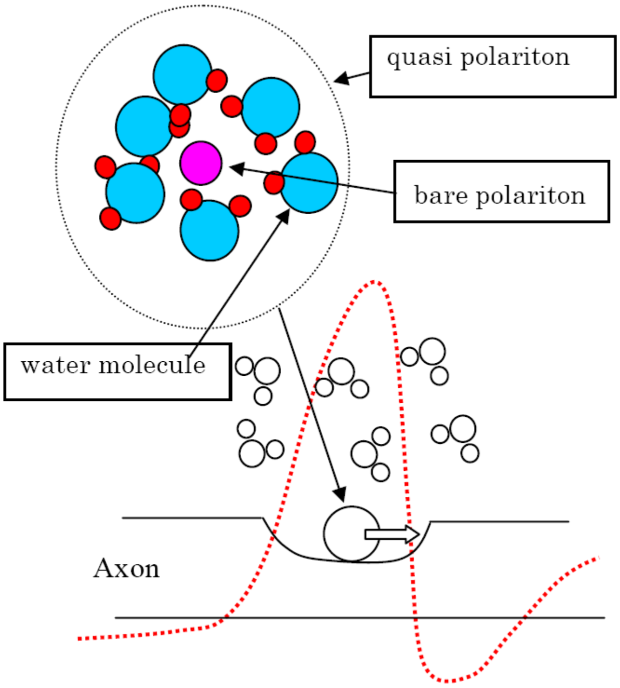


Figure 3 shows the bare polariton to attract many water molecules by the electrostatic forces. The bare polariton changes into dressed particle called quasi polariton. It is difficult to measure the mass of the bare polariton. Therefore, commonly we don't know the bare mass, but we can only detect the quasi (dressed) polariton's mass.
The quasi polariton flows along to an axon. Many polaritons are related to various phenomena, i.e., ephapse, neural conduction, tunnel effect and an interference between each neurons.
Thus, polaritons mean quantized polarization waves.

FIGURE 3. Quasi polariton traveling on axon

polarization vectors run along to longitudinal direction of axon with their rotating motions. The real polaritons are quasi particles covered with a lot of water molecules and ions, which are made by electro-static interactions between bare polariton and waters' (Figure 3).

3. **Characteristic of Polaritons and Quasi Particles.** We are able to estimate physical characteristics of quasi polaritons. Considering saltatory conduction of excitations and action potentials, we can estimate a range of the existence of polaritons to be almost equal to the width of Ranvier ring, whose length is said to be about 1μm (Figure 2). When the wave length of ground state of wave function is considered to be the width of Ranvier ring 1μm, the polaritons mass can be easily calculated by following relation: the equation says

$$p = \frac{\hbar}{\lambda} = mv, \tag{1}$$

where we adopt the conducting velocity of myelinated axon $v = 100$m/s, and the wave length of wave function of ground state of polaritons, is about equal to the width of Ranvier ring 1μm. This calculation for polariton's bare mass results in 6.7×10^{-30}kg.

We know, mass of the bare polaritons have at most about ten times as heavy as that of electron mass. The kinetic energy of a free bare polariton moving along to an axon is estimated as

$$E_K = \frac{1}{2}mv^2 = 2.07 \times 10^{-7} \text{ (eV per a polariton)}. \tag{2}$$

That polariton's kinetic energy is so smaller than any specific energies, i.e., thermal energy at 300K $= 3.0 \times 10^{-2}$eV, ATP hydrolysis $= 2.0 \times 10^{-1}$eV (Table 1). Its energy is indicated 10^{-6} times smaller than hydrogen bonds of water molecules, and that energy is ten times larger than kinetic energy of electron moving at 100m/s speeds. Polaritons work as intermediates of electromagnetic interaction being called as propagation of polarization waves, and so those bare quantized waves (those are called quantum particles), which are massive photons, have an average mass 6.7×10^{-30}kg with spin 1.

Table 1. Kinetic energy and thermal fluctuation

	Energy (eV)
Polariton's kinetic energy	2.0×10^{-7}eV
Electron's kinetic energy at 100m/s	3.2×10^{-8}eV
Hydrogen bond	1.0×10^{-1}eV
Thermal energy at 300K	3.0×10^{-2}eV
ATP hydrolysis	2.0×10^{-1}eV

[Nano-machine shows good efficiency at room templature, and an input energy almost equals thermal fluctuation.]

Those massive photons have serious problems. Generally speaking, bio-nanomachines show good efficiency at room temperature, and their input energies almost equal thermal fluctuation. According to the above common nano-machine's examples, we think that the polariton's kinetic energy should be nearly equal to thermal noise energy. If polaritons are always exposed under water rich circumstance, whose temperature indicates about room temperature $T = 300$K, the energy of thermal noise reaches the value.

$$3/2k_BT = 6.3 \times 10^{-21}\text{J} = 3.9 \times 10^{-2}\text{eV}. \tag{3}$$

Judging from standpoint both Equation (2) and Equation (3), we guess the bare polariton's kinetic energy is almost 10^{-5} times smaller than thermal noise. Those conditions cause serious problems, because of preventing polaritons from normal neural conductions and from traveling action potentials. Thus, the polaritons' kinetic energy is so small that polartions cannot work efficiently under water rich environmental like as human body, since polaritons' motions are interfered with thermal fluctuation and noise. At least, the polaritons, which are against thermal noise, are needed to become 10^5 times heavier than their average mass. Though that mass 6.7×10^{-30}kg is bare polariton's mass, we are able to estimate the quasi polariton mass (dressed mass), which mean the bare polariton to be covered with some ions and water molecules. Thus, the bare polariton is requested to become average 10^5 times heavier than its bare mass (Figure 3). Then the bare polariton needs to wear the water molecules, and an average quasi polariton's mass is guessed as

$$m_T \approx \frac{3k_BT}{v^2} = 1.3 \times 10^{-24}\text{(kg)}. \tag{4}$$

Water molecule's mass is 3.1×10^{-26}kg, and the depressed polariton can sufficiently resist the thermal noise under room temperature at 300Kelvin, if each bare polariton can attract the 41 water molecules at least. Thus, the quasi particle, polariton means

(dessed polariton; qusai polariton) = (bare polariton) + (dressed mass, water molecules).

$$m_T \approx 6.7 \times 10^{-30}\text{kg} + 1.3 \times 10^{-24}(\text{kg}). \tag{5}$$

Note that we can detect that dressed polariton's mass, which are covered with many water molecules, but we cannot measure the bare polariton's mass. The polariton with that quasi particle's mechanism can gain an energy of polariton as strong as that of thermal noise.

According to statistical mechanics, it is said that an order of fluctuation of particles is almost $N^{0.5}$. If we assume the length of human's axon to reach about 1m, and size of water molecule to have 2.0×10^{-10}m (2Å) at its length, the 5.0×10^9 water molecules, at least, exist at the length 1m per an axon. In this case, the particles' average fluctuation is about $N^{0.5}$, i.e., 7.0×10^4 numbers' water molecules. The fluctuation of 7.0×10^4 numbers' particle corresponds to about 10^{-5}m at length, whose value is larger than the width of Ranvier ring, 1μm. Since the quasi polariton's size is smaller than the width of Ranvier ring and particles' fluctuation and the width of Ranvier ring is less than the value of fluctuation, many of quasi polariotns can occupy their positions on both Ranvier ring. Moreover, that result gives us a suggestion that wave functions of polaritons make an invasion to an interior portion of myelin sheath. Thus, polariton's momentum fluctuation is given as

(region of Polariton's existence of ground state) < (length of fluctuation of statistics)

$$\Delta p \cong \frac{\hbar}{\Delta x} \to 1.0^{-29}(\text{kg m/s}). \tag{6}$$

That mass fluctuation is 1.0^{-31}kg, whose value is hundredth part of bare polariton's mass.

4. **Information of Polariton.** Generally speaking, the thermal noise is against neural conductions of polariton being a kind of electrical signals. On the other hand, heat generates some sort of undesirable electrical signals. J. B. Johnson, discovered the electrical fluctuations caused by heat, in terms of a fluctuation voltage produced across a resistor. That fluctuation voltage (noise voltage) is called thermal noise and a hot resistor is a potential source of noise power. In this case, the most noise power N is described as

$$N = k_B T W, \tag{7}$$

where k_B is Boltzmann constant, T means temperature of resistor in degree Kelvin, and W is the band width of noise in cycles per second. Obviously the bandwidth W depends only on the properties of our measuring device. Notice that the noise power is given by Equation (7), where T is temperature of the object. The thermal noise constitutes a minimum noise which we must accept, and additional noise sources only make the situation of apparatus and measurement worse. The noise determines the power required to send messages (conduct on axon). In order to transmit C bits/s, we must have a signal power P related to noise power N by a relation. Referencing Equation (7), we have

$$C = W \log\left(\frac{1+P}{N}\right) = W \log\left(\frac{1+P}{k_B T W}\right). \tag{8}$$

The P is a given signal power. If the P/k_BTW becomes very small compared with unity, Equation (8) gives the following relations: Equation (8) becomes

$$C = \frac{1.44P}{k_B T} \tag{9}$$

or

$$P = 0.693 k_B T C. \tag{10}$$

Equation (10) says that, even when we use a very wide band width, we need at least a power $0.693k_BT$ joule per second to send one bit per second, so that on the average we must use an energy $0.693k_BT$ joule for each bit of information we transmit ($C = 1$). At 300 Kelvin, we obtain the signal power 1.7×10^{-2}eV (per/s)/(bit/s) from Equation (10). The thermal noise of Equation (3) is larger than the value of $0.693k_BT$, 1.7×10^{-2}eV (per/s)/(bit/s), so polariton needs to have the same level of energy as or larger than thermal noise in order to convey the neural information according to classical mechanics. However, the polariton is a quantum particle and massive photon with spin 1, and we should apply quantum effects to Equation (7). H. Nyquist proposed to give an expression for thermal noise applied to all frequencies of light. His expression for thermal noise in a bandwidth W_i was

$$N_i = \frac{\hbar\omega_i W_i}{\exp\left(\hbar\omega_i/k_BT\right) - 1}. \tag{11}$$

Quantum effects become important when one polariton energy is comparable to or larger than k_BT. If a polariton energy $\gg k_BT$, then most noise power N_i is given as

$$N_i \approx x(1+x+x^2\cdots)\hbar\omega_i W_i = \left[\frac{\hbar\omega_i}{k_BT}\exp\left(-\frac{\hbar\omega_i}{k_BT}\right)\right]k_BTW_i, \quad x \equiv \exp\left(-\frac{\hbar\omega_i}{k_BT}\right). \tag{12}$$

We take sum for the suffix i and an average of Equation (12),

$$\langle N\rangle = \langle E_iW_i\rangle\, k_BT \tag{13}$$

Taking the relations in Equation (14), we will obtain the similar expression to classical result

$$\langle E_i\rangle \equiv \left[\frac{\hbar\omega_i}{k_BT}\exp\left(-\frac{\hbar\omega_i}{k_BT}\right)\right], \quad \langle W_i\rangle = const, \tag{14}$$

from Equation (13), and if $\langle E_i\rangle = 1$, then Equation (13) is

$$\langle N\rangle = \langle E_i\rangle\, k_BTW \Rightarrow \langle N\rangle = k_BTW. \tag{15}$$

Note that Equation (15) means approximately a quantum expression of the most noise power which is different from Equation (7). The frequency above, being the exact expression for thermal noise of Equation (11), departs fundamentally from the expression valid at low frequency of Equation (7). It is said that there are the quantum limitations other than the imposed thermal noise as Equation (11) or Equation (13). It turns out that ideally $0.693k_BT$ joule per second to send one bit per second is still the limit, and

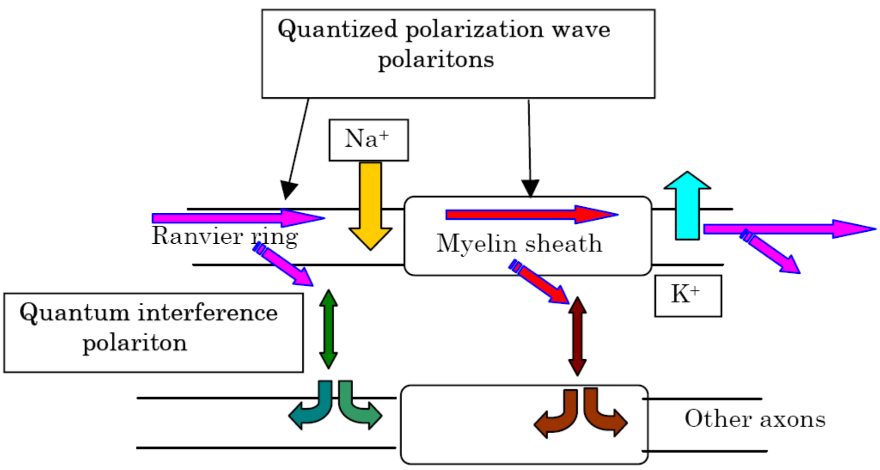


FIGURE 4. Na^+, K^+ ionic currents and roles of polaritons

it is impossible to change the above limiting value. The energy per polariton is $h\nu$, and ideally the energy per bit is $0.693k_BT$. Thus, ideally polariton can carry information, and we can know the bits per polariton at 300 Kelvin,

$$\frac{\hbar\omega}{0.693k_BT} = 2.31 \times 10^{-3}\nu \text{ (bits/polariton).} \tag{16}$$

If we can use frequency of thermal noise, then the polariton carries amount of information, 9.38×10^{12} bits/polariton, at 300 Kelvin from Equation (15). We recognize to be required at least $0.693k_BT$ joules of energy to convey one bit of information.

5. **Description of Polariton.** Polaritons, have an electromagnetic interaction, to be massive photon with spin 1. If the polaritons are traveling along to z-axis, those polaritons having right-handed polarized light are expressed as summation and superposition between state of x-polarized light and that of y-polarized light. This right-handed polarized photon is given as

$$\begin{aligned} |\boldsymbol{E}(z,t)\rangle &= E_0\boldsymbol{\varepsilon_x}\exp i(kz-\omega t) + E_0\boldsymbol{\varepsilon_y}i(kz-\omega t+\pi/2) \\ &= E_0\boldsymbol{\varepsilon_x}\exp i(kz-\omega t) + iE_0\boldsymbol{\varepsilon_y}\exp i(kz-\omega t) \\ &= |\pi_x\rangle\exp i(kz-\omega t) + i\,|\pi_y\rangle\exp i(kz-\omega t) \\ &\quad |\pi_x\rangle = E_0\boldsymbol{\varepsilon_x}, \quad |\pi_y\rangle = E_0\boldsymbol{\varepsilon_y}, \end{aligned} \tag{17}$$

with the ε_i vectors of polarized light. We attempt to practice normalization right-handed polarized light:

$$|\boldsymbol{E}(z,t)\rangle = \frac{1}{\sqrt{2}}(|\pi_x\rangle + i\,|\pi_y\rangle)\exp i(kz-\omega t). \tag{18}$$

We obtain an expression for right-handed polarization state. Using this expression (18), we practice to differentiate with variable z,

$$\frac{\partial^2\,|\boldsymbol{E}(z,t)\rangle}{\partial z^2} = -k^2\,|\boldsymbol{E}(z,t)\rangle\,, \tag{19}$$

and then we multiply both sides by $-\hbar^2/2m$ and add $-V|E(Z,t)\rangle$. We notice following relation:

$$E = \hbar\omega = (\hbar k)^2 + V. \tag{20}$$

We multiply the state vector to both side on Equation (20). Finally, we obtain Schrödinger equation, which describes motion of three components of polariton, with time dependent factors as shown in Equation (21).

$$i\hbar\frac{\partial\,|\boldsymbol{E}(z,t)\rangle}{\partial t} = \left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \hat{V}(z,t)\right]|\boldsymbol{E}(z,t)\rangle\,. \tag{21}$$

Performing derivations as well as the previous procedure, we obtain the relativistic expression of polariton. We use a relation

$$\begin{aligned} &\hat{E}^2\,|\boldsymbol{E}(z,t)\rangle = m^2c^4\,|\boldsymbol{E}(z,t)\rangle + \hat{p}^2c^2\,|\boldsymbol{E}(z,t)\rangle + V\,|\boldsymbol{E}(z,t)\rangle \\ &\because \hat{E} = i\hbar\partial/\partial t, \quad \hat{p} = i\hbar\partial/\partial z, \end{aligned} \tag{22}$$

which is named Klein-Gordon equation. Its quantum expression is given as

$$(\hbar\omega)^2 = m^2c^4 + c^2(\hbar k)^2 + V. \tag{23}$$

Equation (23) means a relativistic spin 1 (vector) particle moving under potential V. Note that common Klein-Gordon equation has one component, scalar particle, but, the Klein-Gordon equation of Equation (22) possesses three components vectors. An electromagnetic theory says, in quantum mechanics, that vector potential $\mathbf{A}$ and scalar potential ϕ are more essential elements than electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$. Thus, according to Maxwell equations, the electromagnetic fields $\mathbf{E}$ & $\mathbf{B}$ are described by the vector and scalar potentials $\mathbf{A}$ & ϕ:

$$\begin{aligned}
&\mathbf{B}(x,t) = rot\mathbf{A}(x,t) \\
&\mathbf{E}(x,t) = -grad\phi - \tfrac{1}{c}\tfrac{\partial \mathbf{A}(x,t)}{\partial t} \\
&A^{\mu} = (\phi(x,t), \mathbf{A}(x,t)).
\end{aligned} \tag{24}$$

Equation (24) teaches that those vectors and scalar potential ($\mathbf{A}$ & ϕ) obey the Klein-Gordon equation, because $\mathbf{B}$ & $\mathbf{E}$ is satisfied with the Klein-Gordon equation. We introduce strength of an electromagnetic field $F^{\mu\nu}$, whose expression connects quaternary A^{μ} with both electromagnetic fields $\mathbf{B}$ & $\mathbf{E}$. The $F^{\mu\nu}$ is defined as

$$\begin{aligned}
&F^{\mu\nu} \equiv \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} = \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{pmatrix} \\
&\because \mathbf{B}(x,t) = (B^1, B^2, B^3), \quad \mathbf{E}(x,t) = (E^1, E^2, E^3), \quad A^{\mu} = (\phi, \mathbf{A}).
\end{aligned} \tag{25}$$

The polariton of massive photon, quantized particle with spin 1, whose equation of four components is similar to the Klein-Gordon equation of massless photon. The polariton's Lagrangian density is given as

$$\ell = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m^2 A_{\mu} A^{\mu} - j_{\mu} A^{\mu}, \tag{26}$$

whose expression gives rise to Proca equation (relativistic massive vector's equation) by applying variational principle for Equation (26). The Proca equation with an interaction between polariton and current j^{μ}

$$\begin{aligned}
&\partial_{\mu} F^{\mu\nu} + m^2 A^{\nu} = j^{\nu} \\
&\because j^{\nu}(x) = (\rho(\boldsymbol{x}, t), \boldsymbol{i}(\boldsymbol{x}, t))
\end{aligned}, \tag{27}$$

is automatically satisfied with Lorentz condition, if that source term $j^{\mu} = 0$ or current conservation law holds correct. (in Equation (27), we use natural unit system). So under Lorentz condition, Equation (27) becomes simple form:

$$\left(\partial_{\mu} \partial^{\mu} + m^2\right) A^{\nu} = j^{\nu}. \tag{28}$$

Comparing Equation (28) with Equation (22), we notice the corresponding relation between term of $V\mathbf{E}(x,t)$ and the j^{μ} current. If we consider the current j^{μ} is generated by major two ionic currents, sodium current J_{Na} and potassium current J_K, the total current j^{μ} through axon's membrane becomes as

$$\left(\partial_{\mu} \partial^{\mu} + m^2\right) A^{\mu} = j^{\nu}_{Na} + j^{\nu}_{K}. \tag{29}$$

We notice those currents to be a source of generating many polaritons.

To derive non-relativistic polariton's equation from relativistic Equation (29), we need return from the wave function A^{μ} of natural unite to that of MKS unite:

$$A^{\mu}(\boldsymbol{x}, t) = \varphi^{\mu}(\boldsymbol{x}, t) \cdot \exp\left(-\frac{i}{\hbar} mc^2 t\right). \tag{30}$$

Then, we split the time dependent of A^{μ} into two terms, and then the one's term is containing the rest polariton's mass, and another is common wave term $\varphi(x,t)$. In the non-relativistic limit, the kinetic energy E_k is so smaller than energy of rest mass that we can reduce it to non-relativistic form as

$$E_K = E - mc^2, \quad E' \ll mc^2 \tag{31}$$

Non-relativistic kinetic energy E_k means

$$\left|i\frac{\partial\varphi^\mu}{\partial t}\right| \approx E_K\varphi^\mu \ll mc^2\varphi^\mu. \tag{32}$$

Hence, we have

$$\begin{aligned}\frac{\partial A^\mu}{\partial t} &\approx -i\frac{mc^2}{\hbar}\varphi^\mu \cdot \exp\left(-\frac{i}{\hbar}mc^2t\right)\\ \frac{\partial^2 A^\mu}{\partial t^2} &\approx \left[-i\frac{2mc^2}{\hbar}\frac{\partial\varphi^\mu}{\partial t} - i\frac{m^2c^4\varphi^\mu}{\hbar^2}\right]\cdot \exp\left(-\frac{i}{\hbar}mc^2t\right).\end{aligned} \tag{33}$$

Inserting all above approximations into following relativistic relation:

$$p^\mu p_\mu A^\nu + m^2c^2A^\nu = j^\nu/c, \tag{34}$$

we finally obtain the non-relativistic expression like Schrödinger equation. That result is non-relativistic polariton's relationship with quaternary components,

$$\begin{aligned}i\hbar\frac{\partial A^\mu}{\partial t} &= \left[-\frac{\hbar^2}{2m}\nabla^2 + \hat{V}\right]A^\mu\\ A^\mu = (\phi, \boldsymbol{A}), &\quad j^\nu\hbar^2/(2mc) \Leftrightarrow \hat{V}A^\nu.\end{aligned} \tag{35}$$

Then, A_0 is scalar potential ϕ, and we remove the rest mass term in the non-relativistic limit, and the final polariton's equations become a set of the quaternary Schrödinger equation:

$$\begin{aligned}i\hbar\frac{\partial\varphi^0}{\partial t} &= \left[-\frac{\hbar^2}{2m}\nabla^2 + \hat{V}\right]\varphi^0\\ i\hbar\frac{\partial\varphi^a}{\partial t} &= \left[-\frac{\hbar^2}{2m}\nabla^2 + \hat{V}\right]\varphi^a,\\ \because \varphi^\mu(x) &= (\varphi^0(\boldsymbol{x}, t), \varphi^a(\boldsymbol{x}, t)), \quad a = 1, 2, 3.\end{aligned} \tag{36}$$

Notice that that equation describes a motion of non charged polariton. As a charged polariton it is expected to obey the complex Klein-Gordon equation for electromagnetic interaction. We multiply Equation (28) by complex conjugate of A_ν, and take the complex conjugate of Equation (28) and multiply it by A_ν

$$\partial_\mu(A^*_\nu\partial^\mu A_\mu + A_\nu\partial^\mu A^{\nu*}) = j^{\nu*}A_\nu - j_\nu A^{\nu*}. \tag{37}$$

We can define a quaternary current vector J_μ using MKS unit system,

$$J^\mu \equiv \frac{ie\hbar}{2m}\left(A^*_\nu\partial^\mu A_\mu + A_\nu\partial^\mu A^{\nu*}\right), \tag{38}$$

and we are able to define the polariton's charge

$$Q \equiv \frac{ie\hbar}{2mc}\left(A^*_\nu\partial^0 A_\mu + A_\nu\partial^0 A^{\nu*}\right), \tag{39}$$

where the Q is time component of A_ν. And the polariton's field A_μ are divided into real part and imaginary part like Equation (40)

$$\begin{aligned}A^\mu &= \frac{1}{\sqrt{2}}\left(A^\mu_1 + iA^\mu_2\right)\\ j^\mu &= j^\mu_1 + ij^\mu_2\end{aligned} \tag{40}$$

If the two fields A_1^μ and A_2^μ separately satisfy a Klein-Gordon equation with the same rest mass m, then the equations can be replaced by one equation for a complex field,

$$\begin{aligned}\left(\partial_\nu\partial^\nu + \frac{m^2c^2}{\hbar^2}\right) A^\mu &= j^\mu \\ \left(\partial_\nu\partial^\nu + \frac{m^2c^2}{\hbar^2}\right) A^{\mu *} &= j^{\mu *}.\end{aligned} \tag{41}$$

According to pi-mesons example, expressions for positive charge's polariton, negative charge's polariton and for neutral particle should be paid attention to, and each of equations has following fields:

$$\begin{aligned}A_+^\mu &= A^{\mu *} = \frac{1}{\sqrt{2}}\left(A_1^\mu - iA_2^\mu\right) \\ A_-^\mu &= A^\mu = \frac{1}{\sqrt{2}}\left(A_1^\mu + iA_2^\mu\right) \\ A_0^\mu &= A^\mu = A^{\mu *}\end{aligned} \tag{42}$$

We adopt the same procedure from Equation (40) to Equation (42), and finally, we will reach non-relativistic similar form to Equation (36). We would like to emphasize that the neutral polariton is characterized by a real eave function, and the charged polaritons have to be represented by complex wave functions.

6. **Current and Polariton.** If the polariton with electric charge q, interacting with both sodium current J_{Na} and potassium current J_K, moves under electromagnetic fields, then a minimal interaction is written as

$$\begin{aligned}\nabla &\to \nabla - \frac{q}{c}\boldsymbol{G} \\ H &\to H - qG^0\end{aligned} \tag{43}$$

Then the above relation (43) is inserted into vector's type of Schrödinger Equation (35) or (36). Performing after simple calculations, we finally have the complex equation,

$$\begin{aligned}i\hbar\frac{\partial\varphi^\mu}{\partial t} &= \left(-\frac{\hbar^2}{2m}\nabla^2 + \hat{V}\right)\varphi^\mu \\ &\quad + \frac{q\hbar}{mc}i\left(\boldsymbol{G}\cdot\nabla + \frac{1}{2}\nabla\cdot\boldsymbol{G}\right)\varphi^\mu + \left(\frac{q^2}{2mc^2}\boldsymbol{G}^2 + qG^0\right)\varphi^\mu.\end{aligned} \tag{44}$$

The complexity of Equation (44) comes from possession of polariton's electric charge and an electromagnetic interaction, and that equation cannot be reduced to simple form like Equation (36) because of containing self-energy of polariton. The neutral polariton can convey only both momentum and energy, and does not carry electromagnetic charge, but, the charged polariton carries its momentum, energy and charged current. We need address as many body problems or quantum field theory since the charged polaritons have many interactions among others.

7. **Results and Conclusions.** We proposed a hypothesis of polariton for quantum neural conduction's theory on artificial axons. The polariton, which means a quasi particle, is considered to be real object, which carries momentum, energy, impulse, charge current, and those various quantum interferences. The model of polariton is described as the quantized polarization wave, being generated by an action potential of neural membrane on axons. The polariton flew from neural body to synapse along to the axon. The phenomenon is commonly known to be the neural conduction based on classical physiology. However, we regarded that classical process, (polarization-depolarization-re-polarization),

as the rotation of the quantized polarization vector. The classical conduction is translated as propagation of rotational quantized vector, whose phenomenon is equivalent to the propagation of polariton. We think, the quantized polarization wave gives rise to phenomena of the neuro-interferences, (for example, ephapse, causalgia, neuralgia), neural conductions and neural activities. The propagation of the quantized vector is described as that of propagation of polariton. The polariton is an essential carrier of neural information, conduction and interference of each neuron. The polariton is a kind of quasi particle. The polariton has various physical quantities, for example, mass about 10^{-25}kg, spin 1, massive photon, positive, neutral and negative charge.

To resist the thermal fluctuation and noise, each bare polariton need attract about 41 water molecules, and that phenomenon is known as hydration. Commonly we are only able to measure and to observe the physical characteristics of the hydrated polariton, which means quasi polariton. We think, that quasi mechanism is an important idea that, it is said nano machine to attain an excellent efficiency using same magnitude of energy as the thermal noise at room temperature. When the polariton is in the ground state, its state means the wavelength of polariton which lies in almost 1μm, and its range of existence is between 0.6μm and 10μm. The polariton satisfies the quaternary Schrödinger equation and complex Klein-Gordon equation. Strictly speaking, the polaritons motion is given in Proca field, with massive vector photon. Both inflow and outflow, which are both sodium ionic current and potassium ionic current through neural membrane, cause the neural conduction along to axon, and the arised polarization wave as polariton, travels along to axon and conveys action potential and an excitation's impulse. Generally speaking, an inflow of sodium ionic current causes an outflow of potassium ionic current from soma. Then the polariton electrically connects both ionic currents. Polariton is a real particle like an electron, anion and cation, and it is a dressed and medium particle being caused by rotation of polarization's phase. The rapid communication of information lies in a quantum tunnel effect, and polariton changes its mass and it gives rise to the tunnel current in myelin sheath. Both sodium and potassium ionic current are true sources of the polariton's currents, and those currents make the many polaritons arise on the dielectric phospholipid membrane of neuron. Those polaritons act on Ranvier ring, and they affect neighbor neurons, whose phenomena are defined as quantum neural interference. The ephapse is physiological phenomenon based on quantum interference caused by many polaritons. We think, they regularly work as a physiological functional adjustor, and that ephapse contributes to maintenance of homeostasis of neural networks and brain.

Macroscopic phenomena show us that each neuron receives an influence of the fluctuant electromagnetic field as shown in magneto-encephalogram. For example, each neuron is subject to electromagnetic phenomena like an induced electromotive force, and leak current. Those holistic electromagnetic effects of brain give rise to the many polaritons at the microscopic level. We believe that those effects make various activities of neural networks, like cooperation, adisaffection, divergence, and convergence. Dr. Shams reported in 2000, the sound induced by flash, which was a kind of illusions.

When the one short pulse of light was illuminated to our eyes and our ears were simultaneously stimulated by a short duration-sound twice for a short intervals, then our brains felt that the electric lamp put twice on a light. Though a visual area is away from an auditory area and both areas have anatomically independent routes of neural conductions, the stimulations of visual area affected on auditory area. We think, those phenomena to be examples of the illusion and of macroscopic neural interference, and polaritons of visual area affected on the neurons of auditory area and polaritons caused those mistakes and illusions of both neural areas.

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