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REDUNDANT INPUT GUARANTEED COST SWITCHED TRACKING CONTROL FOR OMNIDIRECTIONAL REHABILITATIVE TRAINING WALKER

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Abstract. *In this study, the problem of guaranteed cost switched tracking control on the omnidirectional rehabilitative training walker is examined. The nonlinear redundant input method is proposed when one wheel actuator fault occurs. The aim of the study is to design an asymptotically stable controller that can guarantee the safety of the user and ensure tracking on a training path planned by a physical therapist. The guaranteed cost control and the asymptotically zero state detectable concept of the walker are presented, and the model of redundant degree is constructed. The cost performance index is defined to restrain initial oversize input forces. A switching approach is employed. A controller that can satisfy asymptotic stability is obtained using a common Lyapunov function for admissible uncertainties resulting from an actuator fault. Simulation results confirm the effectiveness of the proposed method and verify that the walker can provide safe sequential motion when one wheel actuator is at fault.*
Keywords: Omnidirectional walker, Rehabilitative training, Guaranteed cost switched control, Cost performance index, Redundant input degree

1. Introduction.

Many countries are facing the challenge of an aging society. Individually, elderly people face a range of challenges generally associated with the inability to perform normal daily household tasks. They often develop disabling movement disorders, which impair their ability to walk [1]. In addition, the elderly are prone to injuries, primarily as a result of falls, which also limit their mobility. Many elderly patients can expect a better recovery if rehabilitation training that includes highly repetitive walking starts as soon as possible after the injury. However, training may be delayed and patients may lose their best chance to recover because medical institutes or rehabilitation centers may not have sufficient physiotherapists and nurses to support the needs of all patients. To ensure that patients recover to the greatest extent possible, gait rehabilitation and walking support systems for the elderly and disabled are important [2]. The development of a walking training machine that can efficiently conduct various training programs is highly desirable. Considerable research into the development of robotic walkers has been undertaken [3-6].

Omnidirectional walker (ODW) can help patients undergoing rehabilitation training move in any direction on a flat surface, and can be programmed to follow specific training trajectories that make up recovery training programs for various illnesses and injuries.

FIGURE 1. Omnidirectional walker

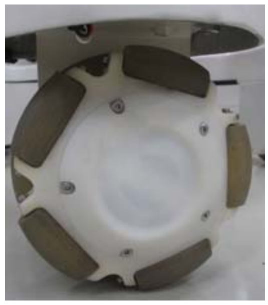

FIGURE 2. Omnidirectional wheel

Images of the ODW and an omnidirectional wheel are presented in Figures 1 and 2, respectively.

The effectiveness of ODW in rehabilitation has been verified through clinical tests [7,8], which has motivated research into various aspects of ODW performance. [9] studied the problem of robust control using a kinematic equation, however, the kinetic equation was not analyzed. Consequently, the analysis could not account for patients with different weights. [10,11] researched tracking control on mobile robots using a kinematic equation. It was found that tracking neglects the mass and inertia matrix when the loads are overweight, and consequently tracking precision is reduced. An adaptive control method for the ODW was discussed in [12-14]. The kinetic equation was improved by considering the center-of-gravity shifts and load fluctuations caused by users. The adaptive controller obtained favorable tracking. However, many parameters needed to be adjusted in the controller.

The ODW is similar to other wheeled mobile robots. To operate effectively in real-world applications, the control algorithm must guarantee that they can follow a prescribed path accurately. Note that all the above mentioned results were obtained under the assumption that the actuator motors are fault-free. However, in real-world applications, actuator motors often fail. Therefore, to guarantee performance safety, the controller must be reliable [15,16]. To ensure system reliability, many modern control applications, particularly in aircraft and robotic systems, are characterized by redundant actuators [17,18].

From previous studies, we know that the ODW has a redundant degree of freedom. Actuators are very important in transferring the controller output to the plant. Therefore, determining how to maintain both stability and a bound of a certain cost in the presence of actuator failures is a worthwhile endeavor. Previous research assumed that actuators never fail and can reliably stabilize wheeled mobile robots, including the ODW. This assumption ignores safety issues. We propose the use of the redundant input method to examine safety issues that may occur when an actuator fails.

In this study, we examine the ODW safety issues taking the following into consideration.

(i) Many modern control applications are characterized by the presence of redundant actuators. The redundant actuator input is very important when one actuator fails. In this study, a redundant input model is constructed by separating the corresponding columns of the control matrix. Furthermore, a redundant input switched model is established to facilitate switching from a failed actuator to a functioning actuator.

(ii) Under a milder assumption, the separated term $M^{-1}\Delta B_\sigma(\theta)f_\sigma(t)$ is considered in relation to the uncertainty of the system. The controller is obtained for admissible control actuator failures, i.e., the resulting design could tolerate the failure and maintain the stability of the system by constructing a common Lyapunov function.

(iii) Reliability goals are intended to ensure system performance and are not exclusively related to safety. In this study, when an actuator fails, the corresponding column of the control matrix will be separated from the original control matrix. The structure of system will be changed. Therefore, maintaining system performance is important. The sufficient condition of quadratic guaranteed cost is derived for admissible uncertainties that an actuator fault causes.

(iv) As an application, the guaranteed cost switched tracking control on the ODW is considered. On the basis of the redundant input switched model, the efficiency of the proposed scheme is demonstrated. The proposed method can be extended to solve safety problems in other wheeled mobile robots.

The remainder of this paper is organized as follows. In Section 2, the model of the ODW with redundant degrees of freedom is formulated. The main results that provide a solution to the guaranteed cost tracking control problem are presented in Section 3. Simulation results are given in Section 4, and concluding remarks are provided in Section 5.

2. **Redundant Input Switched Model of the Omnidirectional Walker.** The coordinate settings and structure used to develop the safety switch control for the ODW are shown in Figure 3. We will deduce the redundant input switched model on the basis of the kinematic equation and the kinetic equation in [19].

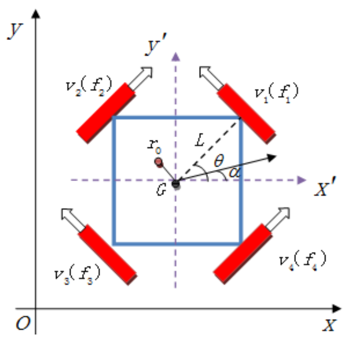

FIGURE 3. Structure of omnidirectional walker

In Figure 3,
$\Sigma(X, O, Y)$: Absolute coordinate system
$\Sigma(X', G, Y')$: Translation coordinate system
v : Speed of the ODW
v_i : Speed of an omnidirectional wheel
f_i : Force on each omnidirectional wheel
G : Center of gravity of the walker
α : Angle between the x' axis and the direction of v
L : Distance from the center of gravity of the walker to each omnidirectional wheel
θ : Angle between the x' axis and the position of the first omnidirectional wheel
r_0 : Distance between G and the center of gravity due to the load

The kinematic model is expressed as

$$\begin{cases} v_1 = -v_x \sin\theta + v_y \sin\left(\frac{\pi}{2} - \theta\right) + L\dot{\theta} \\ v_2 = v_x \cos\theta + v_y \cos\left(\frac{\pi}{2} - \theta\right) - L\dot{\theta} \\ v_3 = -v_x \sin\theta + v_y \sin\left(\frac{\pi}{2} - \theta\right) - L\dot{\theta} \\ v_4 = v_x \cos\theta + v_y \cos\left(\frac{\pi}{2} - \theta\right) + L\dot{\theta} \end{cases} \tag{1}$$

where $v_x = v\cos\alpha$; $v_y = v\sin\alpha$.

The kinetic model is expressed as

$$\begin{cases} (M+m)\ddot{x}_G = -f_1\cos\left(\frac{\pi}{2}-\theta\right) + f_2\cos\theta - f_3\cos\left(\frac{\pi}{2}-\theta\right) + f_4\cos\theta \\ (M+m)\ddot{y}_G = f_1\sin\left(\frac{\pi}{2}-\theta\right) + f_2\sin\theta + f_3\sin\left(\frac{\pi}{2}-\theta\right) + f_4\sin\theta \\ (I_0+mr_0^2)\ddot{\theta} = Lf_1 - Lf_2 - Lf_3 + Lf_4 \end{cases} \tag{2}$$

where M is the mass of the ODW, m is the user's equivalent mass, which varies according to the user's weight and walking disability and I_0 is the inertia of mass. The subscripts Lf_1, Lf_2, Lf_3, and Lf_4 represent the torque acting on the wheels.

From (1), we get the velocity restraint Equation (3)

$$v_1 + v_2 = v_3 + v_4 \tag{3}$$

As can be seen from the differential Equations (2), the system is nonlinear because the direction angle θ changes over time. Although four control forces are found, f_1, f_2, f_3, and f_4, only three are independent because of the linear relationship (3). This implies that the walker has a redundant degree of freedom.

To facilitate description of the model, we define the following matrix. The kinetic model can be expressed as

$$\dot{x} = M^{-1}B(\theta)u(t) \tag{4}$$

where

$$\dot{x} = \ddot{X}_G, \quad X_G = \begin{bmatrix} x_G \\ y_G \\ \theta \end{bmatrix}, \quad M = \begin{bmatrix} M+m & 0 & 0 \\ 0 & M+m & 0 \\ 0 & 0 & I_0+mr_0^2 \end{bmatrix}$$

$$B(\theta) = \begin{bmatrix} -\sin\theta & \cos\theta & -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta & \cos\theta & \sin\theta \\ L & -L & -L & L \end{bmatrix}, \quad u(t) = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix}$$

Definition 2.1. *For a nonlinear system*

$$\dot{x}(t) = f(x,t) + Bu(t) \tag{5}$$

For control matrix $B \in R^{n\times m}$, if $rank(B) = \min\{n,m\}$, then B has redundant degree of d ($d = \max\{n,m\} - \min\{n,m\}$). Control actuators with less d are at fault, the other actuators can sustain regular motion of the system (5), we can state the system can realize safety using redundant degrees of freedom. Furthermore, if the system (5) can track the prescribed path sequentially when control actuators of less d are at fault, we can say the system (5) can guarantee tracking cost.

Remark 2.1. *In fact, the control matrix B has a nullspace of d dimension in which $u(t)$ can be perturbed without affecting the system dynamic. These redundant actuators are not functional when all redundant actuators are fault-free. However, if some actuators of less d are at fault, the redundant actuators are very important to maintain safe sequential motion. Many modern control applications have redundant actuators to promote system safety.*

The problem of safety control in the presence of actuator failures that may occur in the control channels is considered. The walker can realize safety switched tracking control through its redundant degree of freedom. The fault actuator input force will be separated from Equation (4) to obtain the following redundant input switched model.

$$\dot{x} = M^{-1}B_\sigma(\theta)u_\sigma(t) + M^{-1}\Delta B_\sigma(\theta)f_\sigma(t) \tag{6}$$

where $\sigma : R_+ \to M = \{1, 2, 3, 4\}$ is the switching signal to be designed. $u_\sigma(t)$ denotes the control input force of the ith subsystem. $f_\sigma(t)$ is a constant that represents the input force of actuator failure, and satisfies

$$-\left|f\right|_{\max} \le f_\sigma(t) \le \left|f\right|_{\max} \tag{7}$$

Remark 2.2. *Many modern control applications are characterized by multiple actuators. Modern aircrafts, especially military planes, often use redundant actuators to improve maneuverability and reliability [20]. The ODW has a redundant actuator that is not important when the ODW is in normal motion; however, it is very important to guarantee the safety of the ODW when an actuator is at fault. Therefore, construction of the redundant model (6) becomes very important.*

Remark 2.3. *As in [21], the generalized force τ can be decomposed as the dissipative force, the control force, and the random excitation force. In this study, the control input $u(t)$ in (4) can be decomposed as $u_\sigma(t)$ and $f_\sigma(t)$ in (6). $u_\sigma(t)$ is caused by the control input acting on the system, and $f_\sigma(t)$ is caused by the control input of an actuator failure. It is worth noting that $f_\sigma(t)$ will produce the uncertain term of $M^{-1}\Delta B_\sigma(\theta)f_\sigma(t)$. It is well known that mechanical systems are often subject to uncertainty, which can significantly affect system performance. The purpose of this study is to solve the ODW safety problem under the milder assumption of uncertainty $M^{-1}\Delta B_\sigma(\theta)f_\sigma(t)$.*

When an actuator fails during the course of system operation, $\Delta B_\sigma(\theta)$ is the corresponding column of the control matrix $B(\theta)$ and regards uncertainty with the milder assumption form as time-varying parameter

$$\Delta B_\sigma(\theta) = S_\sigma H_\sigma(\theta) Q_\sigma \tag{8}$$

where $H_\sigma(\theta)$ satisfies

$$H_\sigma^T(\theta) H_\sigma(\theta) \le I \tag{9}$$

Remark 2.4. *Obviously, the assumption of $\Delta B_\sigma(\theta)$ follows the form (8), which is weaker than the corresponding assumption of in [22]. In [22], the external uncertainty is treated as a constant. In fact, the form (8) has the same expression as Assumption 2.2 in [23], i.e., the uncertainty satisfies the norm-bounded condition.*

Corresponding to the switching signal σ, for any $j \in M$, we can derive the switching sequence

$$\sigma(t) = j, \quad t_k \le t < t_{k+1}, \quad k \in N \tag{10}$$

The jth subsystem is switched on at t_k and switched off at t_{k+1}.

The actual motion trajectory is X_G, and the desired motion trajectory is X_d; therefore, tracking error $e(t)$ is

$$e(t) = X_G - X_d \tag{11}$$

$$\ddot{e}(t) = \ddot{X}_G - \ddot{X}_d \tag{12}$$

where

$$e(t) = \begin{pmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{pmatrix} = \begin{pmatrix} x_G - x_d \\ y_G - y_d \\ \theta_G - \theta_d \end{pmatrix}, \quad E(t) = \dot{e}(t) \tag{13}$$

The error state equation is

$$\dot{E}(t) = M^{-1}B_\sigma(\theta)u_\sigma(t) - \ddot{X}_d + M^{-1}\Delta B_\sigma(\theta)f_\sigma(t) \tag{14}$$

The cost function associated with the system (14) is

$$J = \int_0^\infty e^T(t) P_\sigma e(t) dt \tag{15}$$

For each $j \in M$, $P_\sigma > 0$ is a symmetric constant matrix.

Remark 2.5. *This study examines the problem of tracking control. Therefore, tracking error is an important performance index. When an actuator is at fault, the corresponding column of the control matrix will be separated from the original control matrix $B(\theta)$. Therefore, the guaranteed tracking cost is given as (15) such that the tracking error $e(t)$ is restricted an upper bounded when the structure of the system (4) is changed as a result of actuator failure. This approach (15) has the advantage of providing the upper bound on a given performance index, and thus the system performance degradation incurred by the model parameter uncertainties is guaranteed to be less than this bound.*

3. The Design of the Safety Switched Controller.

Our purpose is to design a safety controller that can track the paths defined in walking training programs to ensure compliance with the rehabilitation regime when one wheel actuator fault occurs. The guaranteed cost control for the switched error state Equation (14) is stated as follows.

Definition 3.1. *If there exist control laws u_σ^*, a switching law $\sigma = \sigma(t)$, and a positive scalar J^* such that for all admissible actuator failures, the switched error state Equation (14) is asymptotically stable and the corresponding value of the cost function (15) satisfies $J \leq J^*$, then J^* is said to be a guaranteed cost value and u_σ^* is said to be a guaranteed cost control law.*

Remark 3.1. *The safety control can be realized by switching law σ when the actuator fault has been detected. Therefore, three non-fault functioning actuators can maintain the programmed sequential motion of a walker that normally operates with four actuators. In the authors' previous studies, the corresponding fault column was separated from the control matrix $B(\theta)$ and regarded as extrinsic bounded interference. However, the controller is short of expression of nonlinear term $M^{-1}\Delta B_\sigma(\theta) f_\sigma(t)$, and adjusting the control parameters to track the training path is difficult. At the same time, the initial input forces must increase to compensate for the tracking error. This condition is dangerous for both the walker and trainer. The functioning actuators will be damaged by the disproportionate input forces, and the walker will lose stability. Here, safety switched control and the cost function have been proposed to address this issue.*

Definition 3.2. *The walker error tracking system*

$$\begin{aligned} &\dot{E}(t) = M^{-1}B(\theta)u(t) - \ddot{X}_d \\ &y(t) = e(t) \end{aligned}$$

is called asymptotically zero state detectable if for any $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$, such that when $|y(t+s)| < \delta$ holds for some $t \geq 0$, $h > 0$, and $0 \leq s < h$, we have $\|E(t)\| < \varepsilon$.

Remark 3.2. *Definition 3.2 is the asymptotic zero state detectability concept. Detectability is a useful property when addressing asymptotic stability. Here, asymptotic zero state detectability is a limited-time norm observability.*

Next, we design control laws for subsystems and a switching law by employing a common Lyapunov function technique to solve the problem of guaranteed cost control for admissible actuator failures.

Theorem 3.1. *Considering the redundant input error state Equation (14), suppose that there exist scalar symmetric matrices $T_\sigma > 0$, $P_\sigma > 0$, $R_\sigma > 0$, a positive constant $\varepsilon_\sigma > 0$, and that all the subsystems of (13) are asymptotically zero state detectable. Then, the* *control input (16)*

$$u_\sigma(t) = B_\sigma^{-1}(\theta)MT_\sigma^{-1}\Big[T_\sigma\ddot{X}_d - \frac{\varepsilon_\sigma}{4}f_\sigma(t)T_\sigma M^{-1}S_\sigma S_\sigma^T M^{-1}T_\sigma E(t) - \frac{1}{\varepsilon_\sigma}f_\sigma(t)\widehat{E}^T(t)Q_\sigma^T Q_\sigma - \left(\widehat{E}^T(t)e^T(t)+I\right)P_\sigma e(t) - R_\sigma E(t)\Big] \tag{16}$$

and switching law $\sigma(t) = j$ solve the problem of guaranteed cost switched tracking control for actuator failures.

Moreover,

$$J^* = V(t_0) \tag{17}$$

where $\widehat{E}(t) = (E^T(t)E(t))^{-1}E^T(t)$.

Proof: Define the Lyapunov function

$$V(t) = \frac{1}{2}E^T(t)T_\sigma E(t) + \frac{1}{2}e^T(t)P_\sigma e(t)$$

The time derivative of $V(t)$ along the trajectory of system (14) is given by

$$\begin{aligned}\dot{V}(t) &= E^T(t)T_\sigma\dot{E}(t) + \dot{e}^T(t)P_\sigma e(t)\\ &= E^T(t)T_\sigma[M^{-1}B_\sigma(\theta)u_\sigma(t) - \ddot{X}_d + M^{-1}\Delta B_\sigma(\theta)f_\sigma(t)] + E^T(t)P_\sigma e(t)\\ &\le E^T(t)T_\sigma M^{-1}B_\sigma(\theta)u_\sigma(t) - E^T(t)T_\sigma\ddot{X}_d + \frac{\varepsilon_\sigma}{4}f_\sigma(t)E^T(t)T_\sigma M^{-1}S_\sigma S_\sigma^T M^{-1}T_\sigma E(t)\\ &\quad + \frac{1}{\varepsilon_\sigma}f_\sigma(t)Q_\sigma^T Q_\sigma + E^T(t)P_\sigma e(t)\\ &= E^T(t)\Big[T_\sigma M^{-1}B_\sigma(\theta)u_\sigma(t) - T_\sigma\ddot{X}_d + \frac{\varepsilon_\sigma}{4}f_\sigma(t)T_\sigma M^{-1}S_\sigma S_\sigma^T M^{-1}T_\sigma E(t)\\ &\quad + \frac{1}{\varepsilon_\sigma}f_\sigma\widehat{E}^T(t)Q_\sigma^T Q_\sigma + \left(\widehat{E}^T(t)e^T(t)+I\right)P_\sigma e(t)\Big] - e^T(t)P_\sigma e(t)\end{aligned}$$

From (16)

$$\dot{V}(t) = -E^T(t)R_\sigma E(t) - e^T(t)P_\sigma e(t) \le 0$$

Thus, according to the asymptotical zero state detectability, the error state Equation (14) is asymptotically stable.

In the following section, we show that the system satisfies the cost performance upper bound.

According to the switching sequence (10), we have

$$\begin{aligned}J &= \int_0^\infty e^T(t)P_\sigma e(t)dt\\ &= \int_0^\infty \left[e^T(t)P_\sigma e(t) + \dot{V}(t) - \dot{V}(t)\right]dt\\ &= \int_0^\infty \left[-E^T(t)R_\sigma E(t)\right]dt - \int_0^\infty \dot{V}(t)dt\\ &\le -\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}}\dot{V}(t_k)dt\\ &= V(t_0)\end{aligned}$$

Therefore,

$$J^* = V(t_0)$$

is a cost performance upper bound of system (14). This completes the proof of Theorem 3.1.

Remark 3.3. *The safety controller is easily solved by a common Lyapunov function, and the performance upper bound, required to prevent the initial oversize input forces caused by the initial tracking error, can be obtained from $V(t)$. We propose a method that is much easier to satisfy than the partial differential inequalities in [24,25].*

4. **The Simulation Results.** In this section, the proposed redundant input guaranteed cost switched control algorithm is verified by the ODW circular path tracking simulation.

Consider actuators that may fail during the course of system operation. Suppose that for any t, only one actuator fails. Without loss of generality, we consider that $\sigma(t) = 1$ and $\sigma(t) = 4$. We will show how the redundant input guaranteed cost control problem is solved by switching between subsystems.

Thus, when $\sigma(t) = 1$, the error state system (14) can be rewritten as

$$\dot{E}(t) = M^{-1}B_1(\theta)u_1(t) - \ddot{X}_d + M^{-1}\Delta B_1(\theta)f_1(t)$$

where

$$B_1(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & \cos\theta \\ \sin\theta & \cos\theta & \sin\theta \\ -L & -L & L \end{bmatrix}, \quad u_1(t) = \begin{bmatrix} f_2 \\ f_3 \\ f_4 \end{bmatrix}, \quad \Delta B_1(\theta) = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ L \end{bmatrix}$$

$\Delta B_1(\theta)$ represents time-varying parameter uncertainty with the form

$$\Delta B_1(\theta) = S_1 H_1(\theta) Q_1$$

and

$$S_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad H_1(\theta) = \begin{bmatrix} -\frac{1}{2}\sin\theta & 0 & 0 \\ \frac{1}{2}\cos\theta & 0 & 0 \\ \frac{1}{2}L & 0 & 0 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

Similarly, when $\sigma(t) = 4$, we have

$$\dot{E}(t) = M^{-1}B_4(\theta)u_4(t) - \ddot{X}_d + M^{-1}\Delta B_4(\theta)f_4(t)$$

where

$$B_4(\theta) = \begin{bmatrix} -\sin\theta & \cos\theta & -\sin\theta \\ \cos\theta & \sin\theta & \cos\theta \\ L & -L & -L \end{bmatrix}, \quad u_4(t) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, \quad \Delta B_4(\theta) = \begin{bmatrix} \cos\theta \\ \sin\theta \\ L \end{bmatrix}$$

and $\Delta B_4(\theta) = S_4 H_4(\theta) Q_4$.

$$S_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad H_4(\theta) = \begin{bmatrix} \frac{1}{2}\cos\theta & 0 & 0 \\ \frac{1}{2}\sin\theta & 0 & 0 \\ \frac{1}{2}L & 0 & 0 \end{bmatrix}, \quad Q_4 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

S_σ and Q_σ can be adjusted with the tracking performance that represent the effects on the ODW of the separating term $M^{-1}\Delta B_\sigma(\theta)f_\sigma(t)$. Therefore, according to (16) and (17), the controller and cost performance upper bound can be obtained.

In a typical rehabilitation scenario, the ODW must follow a predefined path that consists of a series of circular and linear paths. Here, to verify the tracking performance of the proposed method rigorously, we assume that the walker follows a circular path. The path X_d is described by

$$\begin{aligned} x_d(t) &= x_0 + r\cos\left[\tfrac{1}{r}\left(\tfrac{a}{3}t^3 + \tfrac{b}{2}t^2\right)\right] \\ y_d(t) &= y_0 + r\sin\left[\tfrac{1}{r}\left(\tfrac{a}{3}t^3 + \tfrac{b}{2}t^2\right)\right] \\ \theta_d(t) &= \theta_0 + \tfrac{\pi}{30}t \end{aligned}$$

where x_0, y_0, and θ_0 specify the initial values of the circle. The physical parameters of the ODW used in the simulation are $M = 58$kg, $L = 0.4$m, $r = 3$m, $r_0 = 0.2$m, $I_0 = 27.7$kg.m^2, and load $m = 55$kg. The parameters $a = -11.3 \times 10^{-2}$ and $b = 11.3 \times 10^{-1}$ can be changed relative to the requirements of the patient.

As an example, we consider the failure of the fourth actuator. The controller parameters are $T_4 = diag\{\ 50 \quad 20 \quad 0.95\ \}$, $P_4 = diag\{\ 200 \quad 100 \quad 200\ \}$, and $R_4 = diag\{\ 2600 \quad 2050 \quad 95\ \}$. The simulation results are given in the following figures.

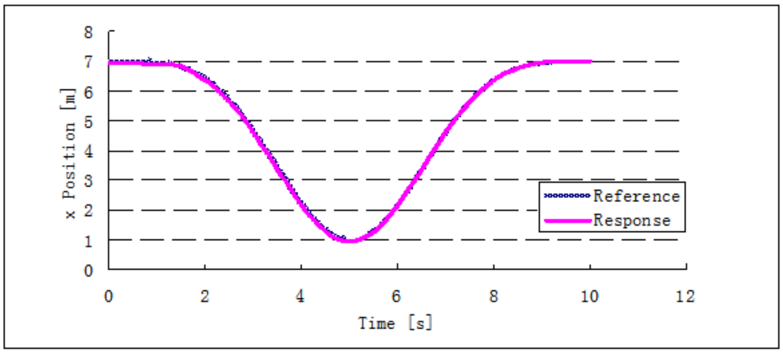

FIGURE 4. Trajectory tracking of x position

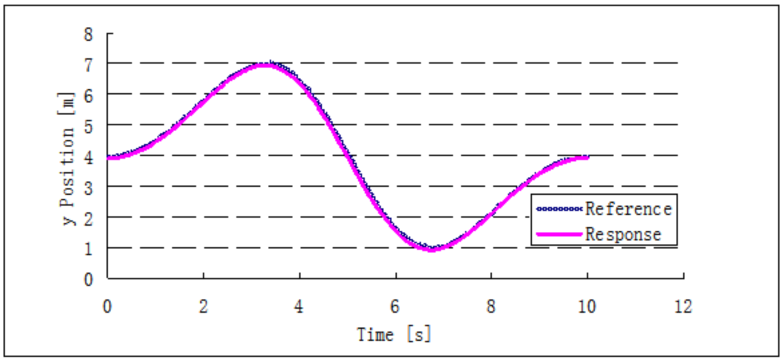

FIGURE 5. Trajectory tracking of y position

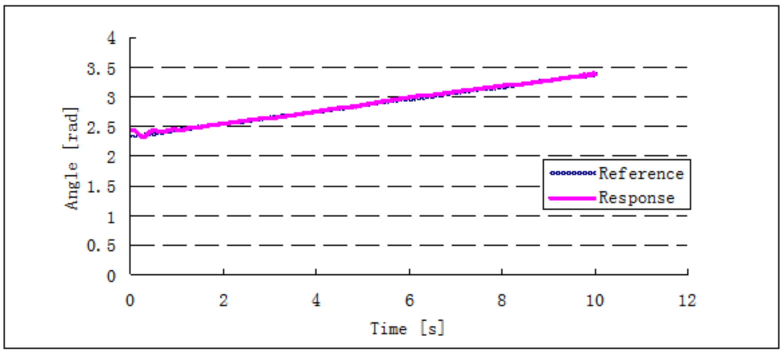

FIGURE 6. Trajectory tracking of angle

Figures 4, 5, and 6 plot the trajectories of the x and y positions and the orientation angle, respectively. The error state Equation (14) can realize asymptotic stability in limited time. The path tracking performance is shown in Figure 7. The switched control method can guarantee the walker's continuous safe motion when one actuator fails. Application of switching law $\sigma(t)$ resolves the safety problem with a performance upper bound

$$J^* = V(t_0) = 4.7672$$

To verify the effectiveness of redundant input switched control to deal with actuator failures, we conducted comparative simulations with the four input forces controller. If the redundant input method is not used to decompose the kinetic model (4), the error state equation is

$$\dot{E}(t) = M^{-1}B(\theta)u(t) - \ddot{X}_d \tag{18}$$

and the four control input forces f_i can be expressed as follows

$$u(t) = \widehat{B}(\theta)MT^{-1}(T\ddot{X}_d - Pe(t) - RE(t)) \tag{19}$$

$$\widehat{B}(\theta) = B^T(\theta)(B(\theta)B^T(\theta))^{-1} \tag{20}$$

Using the controller (19), the walker can follow the specified training trajectory X_d. However, one wheel actuator f_4 fails abruptly when the ODW is in motion, the walker must rely on the remaining three functioning actuators to maintain the training sequence. The results of the comparative simulation are presented in the following figures.

Figures 8, 9, 10, and 11 plot the tracking performance of the walker for the x position, y position, orientation angle, and circular path, respectively. It is evident that the walker cannot realize asymptotic stability and trajectory tracking. These simulation results demonstrate that the controller (19) is only effective when all the actuators are fault-free. In particular, as shown in Figure 11, the user may be in danger because the actual path is far from the desired path. Given such dramatic deviation from the desired path, the

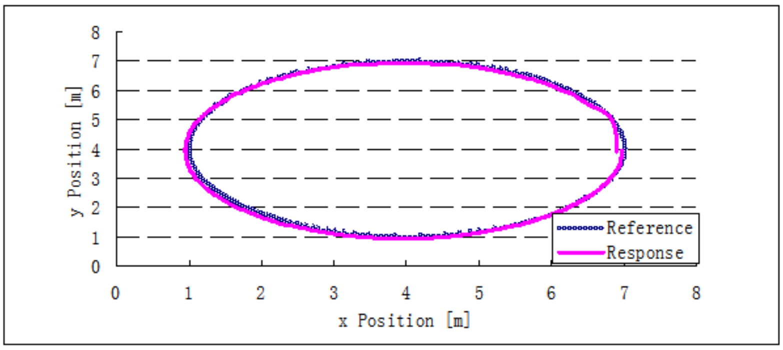

FIGURE 7. Path tracking of circle

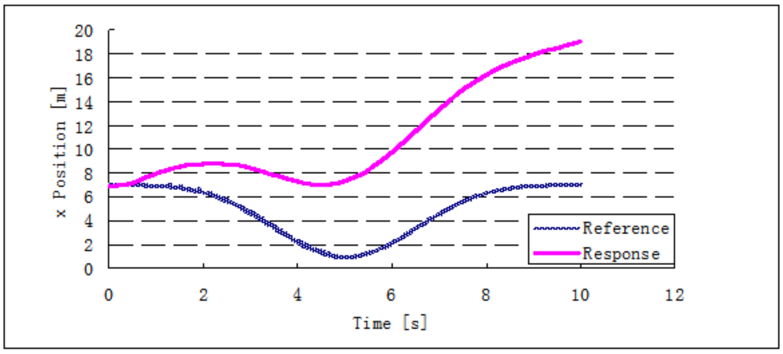

FIGURE 8. Trajectory tracking of x position

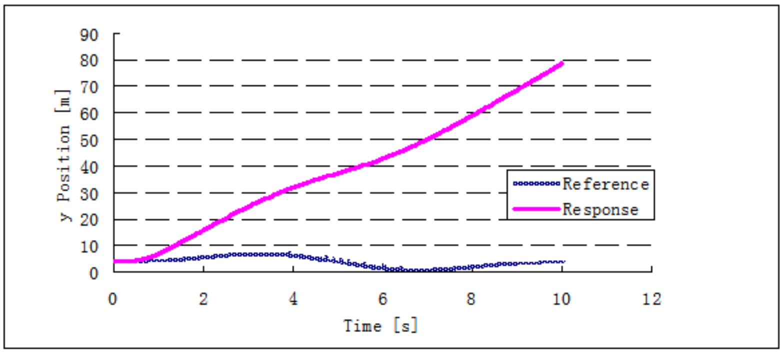

FIGURE 9. Trajectory tracking of y position

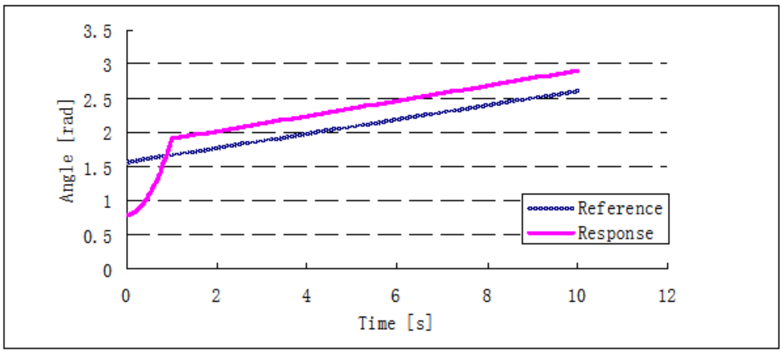

FIGURE 10. Trajectory tracking of angle

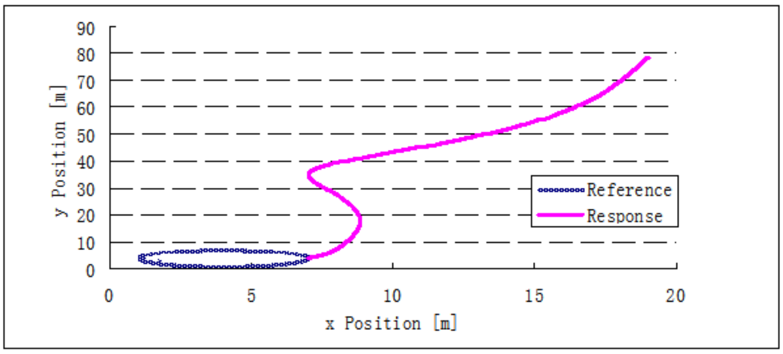

FIGURE 11. Path tracking of circle

ODW could bump into an obstacle and potentially even be a danger to others in the rehabilitation facility.

We developed the safety mechanism to ensure that patients would have the best opportunity to regain normal walking ability through a variety of ODW training programs. The redundant input switched control method exhibits good tracking performance for a training path planned by a physical therapist when one actuator fault occurs, and the user is safe even though an ODW actuator fails. Therefore, according to Definition 2.1, the proposed control method, which was verified by simulations, can ensure patient safety and satisfy the cost function.

5. **Conclusions.** Guaranteeing the safety of patients using the ODW when an actuator fault occurs is very important. The nonlinear redundant input guaranteed cost switched control method is proposed. By using the common Lyapunov function, the obtained

safety controller can stabilize the walker. The tracking results are consistent with a pre-programmed training path designed by a medical professional. The proposed method focuses on actuator failures, which have not been addressed in previous studies. Simulation results for a new synthesis design to resolve safety issues associated with actuator failures have demonstrated the effectiveness of the proposed method. It is probable that, besides the ODW, the proposed method can also be applied to other wheeled mobile robots.

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