International Journal of Innovative Computing, Information and Control
Volume **10**, Number **3**, June **2014**
ICIC International ©2014 ISSN 1349-4198
pp. 909-918

PATH FOLLOWING AND STABILIZATION OF UNDERACTUATED SURFACE VESSELS BASED ON ADAPTIVE HIERARCHICAL SLIDING MODE

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Received May 2013; revised September 2013

Abstract. *Path following of underactuated surface vessels is important practice in ship motion control field. Some of current studies have restrictions on forms of desired trajectory of which straight line or sinusoid curve was excluded. In this paper, adaptive robust controllers for path following of underactuated surface vessels are proposed based on hierarchical sliding mode method which suits various forms of curve. Adaptive technique is employed to deal with uncertain parameters. The proposed control strategy is robust to uncertain parameters and time-varying disturbances. Stabilization is presented as a special case. The numerical simulations are conducted to illustrate the effectiveness of the proposed methodology. Lyapunov stability theorem guarantees the stability of the closed-loop system.*

Keywords: Underactuated surface vessel (USV), Path following (PF), Stabilization, Adaptive hierarchical sliding mode

1. **Introduction.** With the development of marine industry, path following of underactuated surface vessels has become more and more important. There are various challenges on the problem of path following of underactuated surface vessels, such as the underactuation of ship system, strong nonlinearities, large inertia and time delay of ship motion. It attracts considerable attention from control community due to the difficulties and practical value of path following [1].

Two nonlinear control laws for ships were derived based on backstepping technique under the assumption that the reference surge velocity was always positive [2]. Furthermore, the orientation of ships was not controlled. In [3], the ship motion model was transformed into a triangular-like form which made it possible to use integrator backstepping to develop a tracking control law. Neither the extraneous disturbances nor uncertain parameters were considered. A local exponential tracking result for standard chain form systems was proposed based on recursive technique [4]. In [5], a practical time-varying feedback control law was introduced through combined averaging and backstepping techniques in which disturbances and uncertainties were not taken into account. Extending the work in [2], Toussaint et al. presented a systematic method for selecting outputs based on a geometrical analysis of the forces acting on the system [6]. However, the types of desired trajectory were limited. A global tracking result was obtained using cascade approach where the stability analysis relies on the linear time-varying theory [7]. Jiang obtained a result by exploiting the inherent cascade interconnected structure of the ship dynamics and generating explicit Lyapunov functions [8,9]. It is noted that in [7-9] the reference yaw rate cannot be zero, namely, the persistent excitation (PE) condition which is rather restrictive from a practical point of view, since the straight line cannot be tracked. Mazenc et al. modified the backstepping control approach in [3], and obtained a global uniform asymptotical stable result [10]. A controller was proposed for both stabilization and tracking based on Lyapunov's direct method and backstepping techniques [11]. An adaptive tracking controller was derived for an underactuated nonminimum phase model of a marine vehicle based on backstepping and dynamic surface control technique [12]. A controller was proposed for path following of underactuated surface vessels utilizing a guidance-based approach in [13]. A cascade method was proposed to obtain fast tracking controller under a weaker PE condition where stability analysis was given based on several newly developed stability criteria [14]. State- and output-feedback controllers were developed for path following of underactuated surface vessels based on backstepping and Lyapunov's direct method where a passive observer was employed to estimate velocities of sway and yaw, and the forward speed was constant [15]. Do et al. used a polar coordinate transformation to interpret the path following error dynamics in a triangular form, to which Lyapunov's direct method and backstepping technique can be applied [16]. Lipschitz continuous projection algorithm was used to update the estimation of the unknown parameters to avoid parameter drift instability due to time-varying environmental disturbances. Dong and Guo proposed three global smooth time-varying control laws with the aid of different techniques for underactuated surface vessels [17]. Ghommam et al. considered the problem of controlling the underactuated surface vessels using two independent side thrusters [18]. Two transformations were employed to transform the system into a pure cascade form, and a time-invariant discontinuous feedback law was derived. An adaptive supervisory control algorithm was designed which combines logic-based switching with iterative Lyapunov-based techniques for tracking of underactuated surface vessels [19]. An unscented Kalman filter (UKF) based tracking controller was proposed for underactuated surface vessel based on backstepping technique [20]. UKF was used to update the estimation of the uncertain parameters online to avoid the parameters' drift due to time-varying added mass matrices. McNinch et al. proposed a tracking controller for unmanned surface vessel based on sliding-mode control [21]. Combined model predictive control (MPC) and LOS projection, Oh and Sun presented a way-point tracking controller of underactuated surface vessels [22]. Chwa proposed a global tracking control method for underactuated ships with input and velocity constraints using the dynamic surface control where the control structure was formed in a modular way that cascaded kinematic structure and dynamic linearizations can be achieved similarly as in the backstepping technique [23].

Some of the works mentioned above cannot track straight line, while some of them cannot track sinusoid curve path. As a matter of fact, it is necessary to design path following controller for arbitrary path for practical implementation. The characteristics of ship motion are of large inertia, delay and strong nonlinearities, and the uncertainties and time-varying disturbance should be taken into account as well. In this paper, a simple robust controller for path following of underactuated surface vessels based on hierarchical sliding-mode technique [24] is proposed. Uncertain parameters and 2-order wave disturbance are taken into consideration. The proposed controllers can track both straight line and sinusoid curve. Stabilization of underactuated surface vessels is presented as a special case. Lyapunov stability theorem guarantees the stability of the closed-loop system.

2. **Problem Statement and Preliminaries.** We consider a model of the three degree-of-freedom (3-DoF) ship motion in the horizontal plane with two actuators, the propeller and the rudder. The model comprises the kinematics model and the dynamics model can be described as [25]

$$\begin{aligned}
\dot{x} &= u\cos\psi - v\sin\psi \\
\dot{y} &= u\sin\psi + v\cos\psi \\
\dot{\psi} &= r \\
\dot{u} &= \frac{m_{22}}{m_{11}}vr - \frac{d_{11}}{m_{11}}u + \frac{1}{m_{11}}\tau_1 \\
\dot{v} &= -\frac{m_{11}}{m_{22}}ur - \frac{d_{22}}{m_{22}}v \\
\dot{r} &= \frac{m_{11}-m_{22}}{m_{33}}uv - \frac{d_{33}}{m_{33}}r + \frac{1}{m_{33}}\tau_2
\end{aligned} \tag{1}$$

where (x, y) denote the coordinates of the surface vessels in earth-fixed frame. ψ is the yaw angle; u, v and r are the velocity of surge, sway and yaw, respectively. m_{ii} $(i = 1, 2, 3)$ and d_{jj} $(j = 1, 2, 3)$ are given by ship inertia and damping matrices. The surge force τ_1 and yaw moment τ_2 are the control inputs. Define

$$\theta_1 = \frac{m_{22}}{m_{11}}, \quad \theta_2 = \frac{d_{11}}{m_{11}}, \quad \theta_3 = -\frac{m_{11}}{m_{22}}, \quad \theta_4 = \frac{d_{22}}{m_{22}}, \quad \theta_5 = \frac{m_{11}-m_{22}}{m_{33}}, \quad \theta_6 = \frac{d_{33}}{m_{33}}.$$

From Equation (1) we have

$$\begin{aligned}
\dot{u} &= \theta_1 vr - \theta_2 u + \frac{1}{m_{11}}\tau_1 \\
\dot{v} &= \theta_3 ur - \theta_4 v \\
\dot{r} &= \theta_5 uv - \theta_6 r + \frac{1}{m_{33}}\tau_2
\end{aligned} \tag{2}$$

Employing the coordinate transformation [3], we have

$$\begin{aligned}
z_1 &= x\cos\psi + y\sin\psi \\
z_2 &= -x\sin\psi + y\cos\psi \\
z_3 &= \psi
\end{aligned} \tag{3}$$

Similarly

$$\begin{aligned}
z_{1d} &= x_d\cos\psi_d + y_d\sin\psi_d \\
z_{2d} &= -x_d\sin\psi_d + y_d\cos\psi_d \\
z_{3d} &= \psi_d
\end{aligned} \tag{4}$$

where (x_d, y_d) and ψ_d are the desired path and orientation.

The error system can be defined as

$$\begin{aligned}
z_{1e} &= z_1 - z_{1d} \\
z_{2e} &= z_2 - z_{2d} \\
z_{3e} &= z_3 - z_{3d}
\end{aligned} \tag{5}$$

In this paper, the control objective is to design simple robust control law τ_1 and τ_2 to make the (x, y) follow (x_d, y_d) as closely as possible.

3. **Controller Design.** The control design contains two steps: we firstly design the yaw moment controller based on adaptive hierarchical sliding mode; then, we design the surge force controller based on a common sliding mode. The hierarchical sliding mode is comprised of two first-level sliding mode surface. In order to state the control algorithm clearly, we incorporate the stability analysis into controller design procedures.

Step 1: Firstly, design the yaw moment τ_2 with a hierarchical sliding-mode controller, the first first-level sliding mode surface can be defined as follows:

$$\sigma_1 = c_1 z_{2e} + \dot{z}_{2e} \tag{6}$$

where c_1 is the positive constant.

By differentiating Equation (6), we have

$$\begin{aligned}\dot{\sigma}_1 &= c_1\dot{z}_{2e} + \ddot{z}_{2e} \\ &= c_1\dot{z}_{2e} + \theta_3 ur - \theta_4 v - (u + z_2 r)r - z_1\left(\theta_5 uv - \theta_6 r + \frac{1}{m_{33}}\tau_2\right) - \ddot{z}_{2d}\end{aligned} \tag{7}$$

We can obtain

$$\tau_{eq21} = -\frac{m_{33}}{z_1}\left(-c_1\dot{z}_{2e} - \hat{\theta}_3 ur + \hat{\theta}_4 v + (u + z_2 r)r + z_1\hat{\theta}_5 uv - z_1\hat{\theta}_6 r + \ddot{z}_{2d}\right) \tag{8}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ $(i = 1, 2, \ldots, 6)$ are the errors of adaptive parameters.

Secondly, define the second first-level sliding mode surface as follows:

$$\sigma_2 = c_2 z_{3e} + \dot{z}_{3e} \tag{9}$$

where c_2 is the positive constant.

Differentiating Equation (9), we have

$$\begin{aligned}\dot{\sigma}_2 &= c_2\dot{z}_{3e} + \ddot{z}_{3e} \\ &= c_2\dot{z}_{3e} + \theta_5 uv - \theta_6 r + \frac{1}{m_{33}}\tau_2 - \ddot{z}_{3d}\end{aligned} \tag{10}$$

Then we define

$$\tau_{eq22} = m_{33}\left(-c_2\dot{z}_{3e} - \hat{\theta}_5 uv + \hat{\theta}_6 r + \ddot{z}_{3d}\right) \tag{11}$$

Lastly, define second-level sliding mode surface as follows:

$$S_1 = a\sigma_1 + b\sigma_2 \tag{12}$$

The switching control law of yaw moment is

$$\tau_{sw2} = \frac{-\eta_1 \mathrm{sgn}(S_1) - k_1 S_1 + a\frac{z_1}{m_{33}}\tau_{eq22} - \frac{b}{m_{33}}\tau_{eq21}}{\frac{b}{m_{33}} - a\frac{z_1}{m_{33}}} \tag{13}$$

where a, b, η_1 and k_1 are the positive controller parameters.

The total control law for yaw moment can be described as

$$\tau_2 = \tau_{eq21} + \tau_{eq22} + \tau_{sw2} \tag{14}$$

Step 2: We design control law for surge force based on common sliding mode technique, and define the sliding mode surface as

$$S_2 = c_3 z_{1e} + \dot{z}_{1e} \tag{15}$$

where c_3 is the positive constant.

Differentiating Equation (15), we have

$$\begin{aligned}\dot{S}_2 &= c_3\dot{z}_{1e} + \ddot{z}_{1e} \\ &= c_3\dot{z}_{1e} + \theta_1 vr - \theta_2 u + \frac{1}{m_{11}}\tau_1 + (v - z_1 r)r + z_2\left(\theta_5 uv - \theta_6 r + \frac{1}{m_{33}}\tau_2\right) - \ddot{z}_{1d}\end{aligned} \tag{16}$$

Then we have

$$\tau_{eq1} = m_{11}\left(-c_3\dot{z}_{1e} - \hat{\theta}_1 vr + \hat{\theta}_2 u - (v - z_1 r)r - z_2\left(\hat{\theta}_5 uv - \hat{\theta}_6 r + \frac{1}{m_{33}}\tau_2\right) + \ddot{z}_{1d}\right) \quad (17)$$

The switching control yaw for surge force is

$$\tau_{sw1} = m_{11}(-\eta_2 \operatorname{sgn}(S_2) - k_2 S_2) \quad (18)$$

where η_2 and k_2 are the positive controller parameters.

The total control law of surge force can be described as

$$\tau_1 = \tau_{eq1} + \tau_{sw1} \quad (19)$$

The adaptive control laws are designed as

$$\begin{aligned}
\dot{\hat{\theta}}_1 &= S_2 vr \\
\dot{\hat{\theta}}_2 &= -S_2 u \\
\dot{\hat{\theta}}_3 &= aS_1 ur \\
\dot{\hat{\theta}}_4 &= -aS_1 v \\
\dot{\hat{\theta}}_5 &= (-aS_1 z_1 + bS_1 + z_2 S_2)uv \\
\dot{\hat{\theta}}_6 &= (az_1 S_1 - bS_1 - z_2 S_2)r
\end{aligned} \quad (20)$$

Stability Analysis: Choose the following Lyapunov function

$$V = \frac{1}{2}\left(S_1^2 + S_2^2 + \sum_{i=1}^{6}\tilde{\theta}_i\right) \geq 0 \quad (21)$$

Differentiating Equation (21), we have

$$\begin{aligned}
\dot{V} &= S_1\dot{S}_1 + S_2\dot{S}_2 - \sum_{i=1}^{6}\dot{\hat{\theta}}_i\tilde{\theta}_i \\
&= -k_1 S_1^2 - \eta_1|S_1| - k_2 S_2^2 - \eta_2|S_2| \leq 0
\end{aligned} \quad (22)$$

According to Equations (21) and (22), the closed-loop system is stable.

4. **Simulation Results.** We conduct three simulations to testify the proposed controllers. The first case is straight line following without disturbances. The second case is sinusoid curve following with time-varying disturbances. The third case is the stabilization of underactuated surface vessels. The model parameters are $m_{11} = 200$, $m_{22} = 250$, $m_{33} = 80$, $d_{11} = 70$, $d_{22} = 100$, $d_{33} = 50$ [26]. The controller parameters are $c_1 = 1$, $c_2 = 1.2$, $c_3 = 12$, $a = 0.001$, $b = 100$, $k_1 = 100$, $k_2 = 100$, $\eta_1 = 0.001$, $\eta_2 = 0.001$. The initial values are $(x_0, y_0, \psi_0, u_0, v_0, r_0) = (2, 0, 0, 0, 0, 0)$.

a) Straight-Line Following

The desired trajectory is $x_d = t$, $y_d = t$. Figure 1 depicts the tracking performance of the proposed controllers. From Figure 1 we can see, the straight-line tracking ability of proposed control strategy is quite well. Figure 2 depicts the velocity of surge, sway and yaw. Figure 3 shows the surge force and yaw moment.

b) Sinusoid-Curve Tracking

The desired trajectory is $x_d = t$, $y_d = \sin(0.01t)$. 2-order wave drift forces are considered which are modeled as slowly varying bias terms (Wiener processes) [27]: $\dot{\tau}_{wu} = w_1$, $\dot{\tau}_{wv} = w_2$, $\dot{\tau}_{wr} = w_3$. Here w_i $(i = 1, 2, 3)$ are Gaussian white noise processes. Figure 4 depicts the tracking performance of the proposed controllers. From Figure 4 we can see that the sinusoid curve tracking performance of the proposed controllers are quite well under

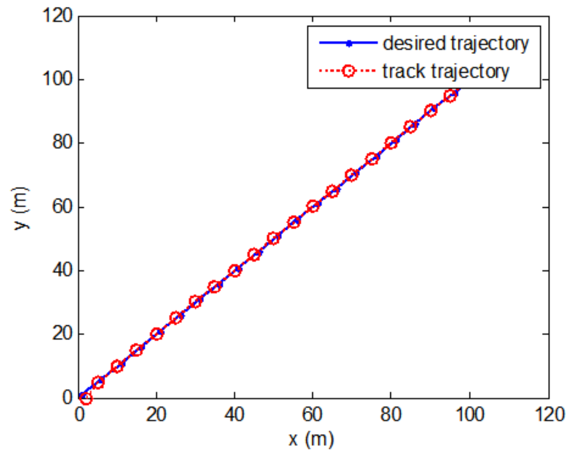

FIGURE 1. Tracking performance in straight-line case

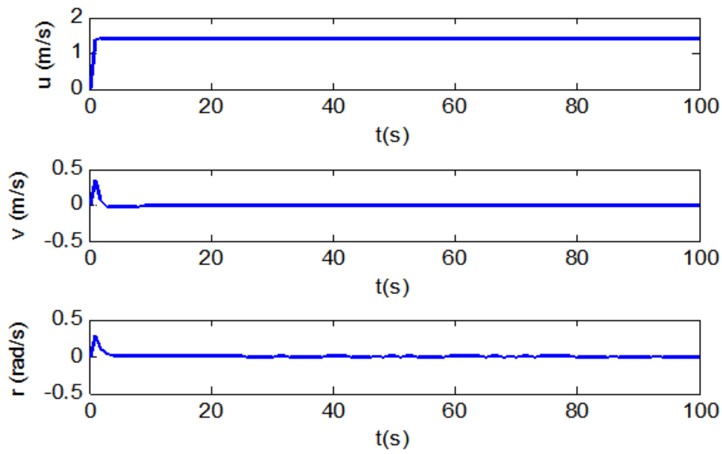

FIGURE 2. Velocity of surge, sway and yaw in straight-line case

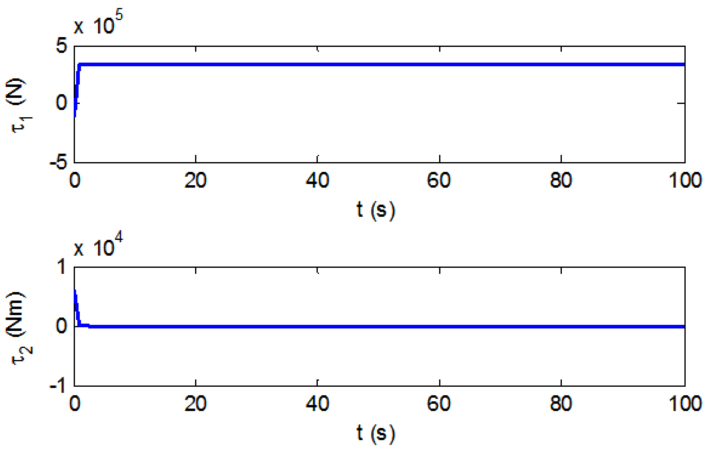

FIGURE 3. Surge force and yaw moment in straight-line case

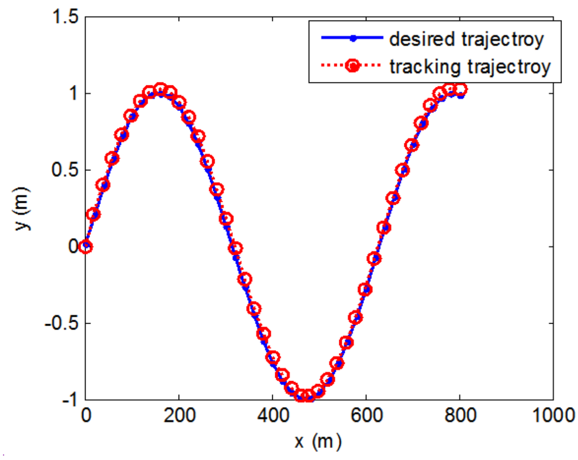

FIGURE 4. Tracking performance in sinusoid-curve case

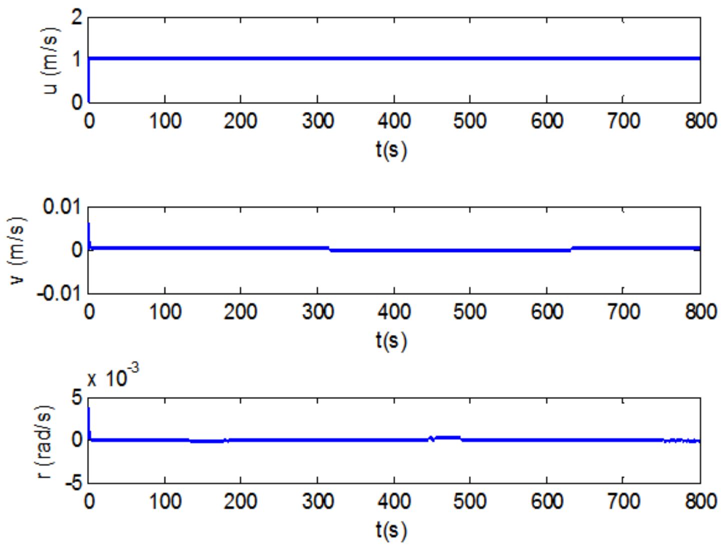

FIGURE 5. Velocity of surge, sway and yaw in sinusoid-curve case

time-varying disturbances. Figure 5 depicts the velocity of surge, sway and yaw. Figure 6 shows the surge force and yaw moment.

c) Stabilization case

Figure 7 depicts the stabilization performance of proposed controllers. Figure 8 shows the velocity of surge, sway and yaw, respectively. Figure 9 shows the control inputs.

The three cases share the same controller parameters, which demonstrate the adaptive properties of the proposed controllers. The parameters c_i $(i = 1, 2, 3)$ are related to the response time. The larger the value is, the faster to achieve the sliding mode surface. The parameters η_i $(i = 1, 2)$ are needed to be chosen carefully to balance the robustness and chattering problem.

5. **Conclusions.** In this paper, we propose adaptive robust controllers for path following of underactuated surface vessels with uncertain parameters and external disturbances. Coordinate transformation is employed to avoid the singular problem. Two sliding mode

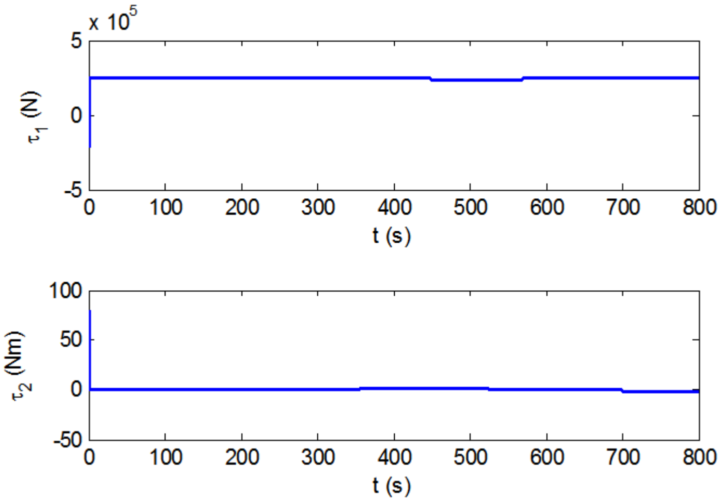

FIGURE 6. Surge force and yaw moment in sinusoid-curve case

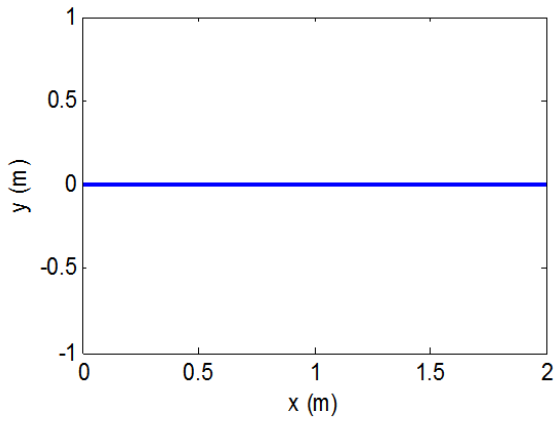

FIGURE 7. Stabilization performance

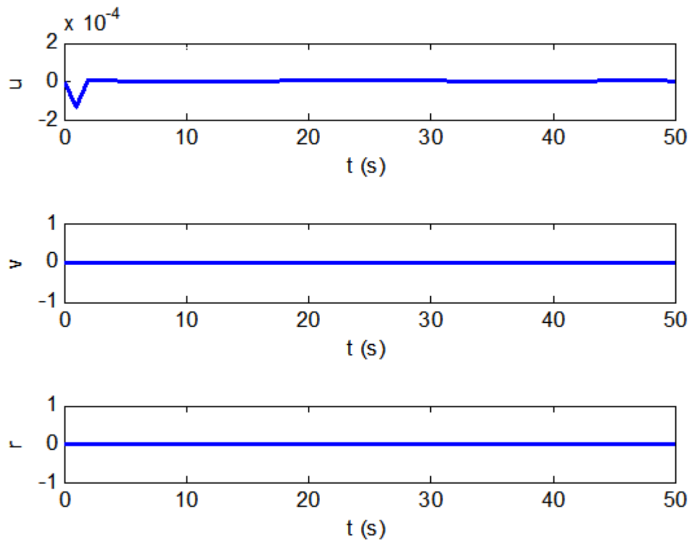

FIGURE 8. Velocity of surge, sway and yaw in stabilization case

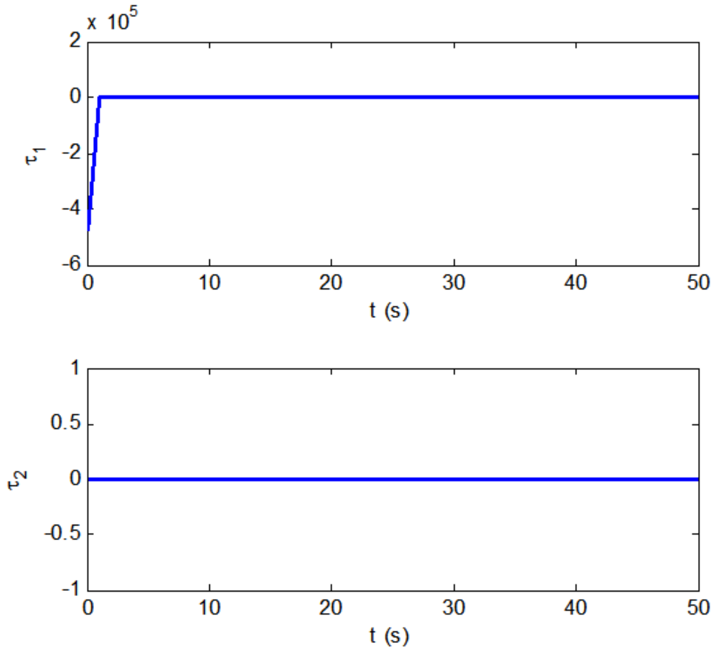

FIGURE 9. Surge force and yaw moment in stabilization case

controllers are designed: a common sliding mode controller for the surge force control and a hierarchical controller for the yaw moment control. 2-order wave drift forces are considered to testify the robustness of the proposed controllers. The closed-loop stability of the system is proved by Lyapunov stability theorem. The effectiveness of proposed controllers is validated by simulation results.

Acknowledgment. This work is supported by the Special Research Fund of Ministry of Education of China for the Doctoral Program (Grant No.: 20110073110009) and the National Natural Science Foundation of China (Grant Nos.: 51179019, 51279106).

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