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ADAPTIVE TRACKING CONTROL OF UNCERTAIN MIMO SWITCHED NONLINEAR SYSTEMS

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Abstract. *This paper is devoted to adaptive tracking control for a class of uncertain MIMO switched nonlinear systems. A novel systematic recursive algorithm is proposed to design adaptive update laws and an adaptive controller by multiple Lyapunov functions method combining with backstepping recursive technique. And then, by means of stability theory about the switched systems and the average dwell time conception, two kinds of switching signals are constructed to ensure that all the signals in the closed-loop systems are bounded and the outputs of the switched systems converge to a small neighborhood of the desired trajectories. The effectiveness is investigated by a simulation example.*
Keywords: Switched nonlinear systems, MIMO, Adaptive tracking control, Backstepping design, Average dwell time, Boundedness

1. Introduction.

In the last decades, switched systems have attracted more and more attention due to their significance in the modelling of many engineering applications, such as chemical processes, robot manipulators and power systems. So far, many remarkable achievements about the stability analysis and synthesis have been made in the field of switched systems commonly by two kinds of methods: one is to find a common Lyapunov function to ensure stability of the switched systems under arbitrary switching laws; the other is to use a multiple Lyapunov functional technique to stabilize the switched systems under some designed switching laws, see, e.g., [1-7] and the references therein.

In contrast to switched linear systems, the control problem of switched nonlinear systems presents a much more challenging task because of structural complexity. In recent years, much interest in switched nonlinear systems is reflected by many works for the systems in lower triangular forms via extending the backstepping recursive method for non-switched nonlinear systems in lower triangular forms to the switched ones, see, e.g., [8, 9] and the references therein. On the other hand, there may often be uncertain parameters in practical systems. Some plants are even characterized by the unstructured uncertainties in industrial control environment (e.g., [10]). Accordingly, robust and (or) adaptive control design approaches have been addressed for switched systems in lower triangular forms. [11] overcame the limitation in [9] and solved the case of robust stabilization under arbitrary switching laws for switched lower triangular systems with uncertainties. In [12], an adaptive backstepping technique was adopted for a class of switched nonlinear systems with constant unknown parameters in lower triangular forms. However, no systematic recursive approach was given to obtain the common virtual adaptive control law under arbitrary switching. By multiple Lyapunov function and average dwell time method, [13, 14] proposed an adaptive neural control scheme to deal with uncertainties for a class of switched nonlinear systems in lower triangular forms, and reached the Lyapunov stability.

Despite these efforts using backstepping recursive technique, it is worth mentioning that most of the existing results about the switched lower triangular systems are limited to single-input/single-output (SISO) ones. However, due to the interconnections between various states and inputs, the control design is difficult for multi-input/multi-output (MIMO) non-switched nonlinear systems (e.g., [15, 16]), much less for the switched ones. Hence, it is nontrivial to extend the results from SISO switched systems in lower triangular forms to the MIMO ones. In comparison with the results for SISO switched nonlinear systems, there are relatively fewer results available for MIMO switched nonlinear systems at present, only see [17, 18]. In addition, both of the papers stabilized the considered switched systems under arbitrary switching signals by designing common controller with high-gains, which is undesirable for practical systems.

Motivated by the aforementioned works in literature, this paper extends the adaptive backstepping recursive technique to a class of MIMO switched nonlinear systems in lower triangular forms with uncertainties. A novel systemic adaptive output feedback control approach is developed to guarantee all the signals in the closed-loop switched system to be globally uniformly ultimately bounded and the output of the controlled switched system converges to a small neighborhood of the origin. The main advantages of the proposed control scheme are as follows: 1) By virtue of multiple Lyapunov function theory, respective controller and proper adaptive law for each switched subsystem are designed under different coordinate transformation during the iterative process, which overcomes the problem of high-gain of controller resulting from finding common coordinate transformation; 2) Two kinds of different switching laws only depending on time are developed by virtue of average dwell time conception, such that the control problem is solved for certain class of MIMO switched nonlinear systems combining with the designed adaptive controllers.

The outline of this paper is as follows. Preliminary knowledge and the problem formulation are given in Section 2. Section 3 presents a direct adaptive tracking control design by backstepping method and summarizes the eventual results. A numerical example is treated to illustrate the effectiveness of the design approach in Section 4. Finally, a conclusion is given in Section 5.

Notations: $\mathbf{N}$ is the set of positive integers. $\mathbf{R}^m$ is m-dimension real vector space. We use N and M for the sets $\{1, \cdots, n\}$ and $\{1, \cdots, m\}$, respectively, $|\cdot|$ for the absolute value of scalars, $\hat{(\cdot)}$ for the estimate of unknown parameter vector $(\cdot)$, and $\tilde{(\cdot)} := \hat{(\cdot)} - (\cdot)$ for the estimate error, $\star_i = (\star_{i1}, \cdots, \star_{im})^T \in \mathbf{R}^m$. Define $\mathrm{Tanh}(z_i) = \mathrm{diag}\{\tanh(\frac{z_{i1}}{\varepsilon}), \cdots, \tanh(\frac{z_{im}}{\varepsilon})\}$, $\varepsilon > 0$, $e_i = (1, 1, \cdots, 1)^T \in \mathbf{R}^m$ for integer indices $i \in N$.

2. **Problem Statement and Preliminaries.** Consider a class of MIMO switched nonlinear systems in the following form:

$$\begin{cases} \dot{x}_i = G_{i,\sigma(t)}(\bar{x}_i)x_{i+1} + f_{i,\sigma(t)}(\bar{x}_i), \quad i = 1, 2, \cdots, n-1 \\ \dot{x}_n = G_{n,\sigma(t)}(\bar{x}_n)u + f_{n,\sigma(t)}(\bar{x}_n) \\ y = x_1 \end{cases} \tag{1}$$

where $x_i, u, y \in \mathbf{R}^m$ ($m \in \mathbf{N}, i \in N$) are the state vector, control input vector and the system output vector, respectively. $\bar{x}_i = [x_1^T, x_2^T, \cdots, x_i^T]^T \in \mathbf{R}^{mi}$. The right-continuous function $\sigma(t) : [0, +\infty) \to \mathbf{S} = \{1, 2, \cdots, s\}$ is a piecewise constant switching signal. For any switching time sequence $0 = t_1 < \cdots < t_k < \cdots$, when $t \in [t_k, t_{k+1})$, $\sigma(t) = \sigma(t_k) = i_k \in \mathbf{S}$, which means the i_kth subsystem is activated and the remaining subsystems are inactivated. For all $i \in N$ and $j \in \mathbf{S}$, bounded matrices $G_{i,j}(\cdot) \in \mathbf{R}^{m\times m}$ are known invertible smooth control gains. $f_{i,j}(\cdot) \in \mathbf{R}^m$ are unknown smooth nonlinear function vectors and satisfy the following assumption.

Assumption 1. For $\forall i \in N$, $j \in \mathbf{S}$, the unknown function vectors $f_{i,j}(\cdot)$ can be expressed as

$$f_{i,j}(\bar{x}_i) = F_{i,j}(\bar{x}_i)\theta_i + \Delta F_{i,j}(\bar{x}_i)$$

where $F_{i,j}(\cdot) \in \mathbf{R}^{m\times q_i}$ ($q_i \in \mathbf{N}$) are known smooth nonlinear function, $\theta_i \in \mathbf{R}^{q_i}$ are unknown constant parameter vectors, $\Delta F_{i,j}(\cdot) = [f_{i1,j}(\cdot), f_{i2,j}(\cdot), \cdots, f_{im,j}(\cdot)]^T$ are unknown smooth functions satisfying

$$|f_{il,j}(\bar{x}_i)| \le \phi_{il}$$

where ϕ_{il} are unknown nonnegative constant parameters, $j \in \mathbf{S}$, $l \in M$, $i \in N$. For convenience, we let $\phi_i = (\phi_{i1}, \cdots, \phi_{im})^T \in \mathbf{R}^m$ for any $i \in N$.

Assumption 2. The state vector $\bar{x}_n$ is available for feedback. The desired signal y_d is continuously differentiable, and y_d and its derivative $\dot{y}_d$ are bounded.

Definition 2.1. [1] *A switching signal $\sigma(t)$ has average dwell-time τ_α if there exist numbers $N_0 \ge 0$, $\tau_\alpha > 0$ such that*

$$\forall T \ge t \ge t_1 : N_\sigma(T,t) \le N_0 + \frac{T-t}{\tau_\alpha}$$

where N_0 is the chatter bound, $N_\sigma(T,t)$ is the number of switches occurring in the interval $[t,T)$. As commonly used in the literature, we choose $N_0 = 0$.

Lemma 2.1. [19] *The following inequality holds for any $\varepsilon > 0$ and any $\eta \in \mathbf{R}$*

$$0 \le |\eta| - \eta \tanh(\eta/\varepsilon) \le k_p\varepsilon$$

where k_p is a constant that satisfies $k_p = e^{-(k_p+1)}$, i.e., $k_p = 0.2785$.

3. Adaptive Controller Design and Stability Analysis.

The purpose of this section is to present a systematic design procedure for adaptive control of switched system (1) by combining multiple Lyapunov functions and average dwell time. The bounded stability of all signals in closed-loop system is achieved. As usual, the recursive procedure contains n steps as follows.

Step 1: For the x_1-equations of switched system (1), define error variable $z_1 = x_1 - y_d$. Then for the positive function

$$V_{1,\sigma(t)} = \frac{1}{2} z_1^T z_1 \tag{2}$$

the time derivative is

$$\dot{V}_{1,j} = z_1^T G_{1,j}(x_1)x_2 + z_1^T[F_{1,j}(x_1)\theta_1 + \Delta F_{1,j}(x_1) - \dot{y}_d] \tag{3}$$

where $\sigma(t) = j \in \mathbf{S}$. Let $z_2 = x_2 - \alpha_1$, with the assumption and lemma, the virtual control law α_1 chosen as

$$\alpha_1 = G_{1,j}^{-1}\left[-K_{1,j}z_1 - F_{1,j}\hat{\theta}_1 + \dot{y}_d - \mathrm{Tanh}(z_1)\hat{\phi}_1\right] \tag{4}$$

makes Formula (3) satisfy

$$\dot{V}_{1,j} \le -z_1^T K_{1,j} z_1 + z_1^T G_{1,j} z_2 + z_1^T F_{1,j}\tilde{\theta}_1 + z_1^T \mathrm{Tanh}(z_1)\tilde{\phi}_1 + k_p\varepsilon e_1^T \phi_1 \tag{5}$$

where $K_{1,j} = K_{1,j}^T > 0$ are known proper constant matrices.

Step i $(2 \le i \le n-1)$: Define vectors $\Theta_i = (\theta_1^T, \theta_2^T, \cdots, \theta_i^T)^T$ and $\Phi_i = (\phi_1^T, \phi_2^T, \cdots, \phi_i^T)^T$. Noting that the error variable $z_i = x_i - \alpha_{i-1}$ and α_{i-1} is a function of $\bar{x}_{i-1}$, $\hat{\Theta}_{i-1}$, $\hat{\Phi}_{i-1}$, $y_d^{(i-1)}$, then from x_i-equations of system (1), the dynamic equation for z_i is

$$\begin{aligned} \dot{z}_i &= G_{i,j}(\bar{x}_i)x_{i+1} + F_{i,j}(\bar{x}_i)\theta_i - \dot{\alpha}_{i-1} + \Delta F_{i,j}(\bar{x}_i) \\ &= G_{i,j}(\bar{x}_i)x_{i+1} + \Sigma_{i,j}^T\Theta_i - \sum_{\varsigma=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_\varsigma^T}G_s x_{\varsigma+1} - \omega_{i-1} \\ &\quad -\frac{\partial\alpha_{i-1}}{\partial(y_d^{(i-1)})^T}y_d^{(i)} + \Delta F_{i,j}(\bar{x}_i) - \sum_{\varsigma=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_\varsigma^T}\Delta F_{\varsigma,j}(\bar{x}_\varsigma) \end{aligned} \tag{6}$$

where

$$\Sigma_{i,j}^T = \left[-\frac{\partial\alpha_{i-1}}{\partial x_1^T}F_{1,j}, \cdots, -\frac{\partial\alpha_{i-1}}{\partial x_{i-1}^T}F_{i-1,j}, F_{i,j}\right] \tag{7}$$

$$\omega_{i-1} = \frac{\partial\alpha_{i-1}}{\partial\hat{\Theta}_{i-1}^T}\dot{\hat{\Theta}}_{i-1} + \frac{\partial\alpha_{i-1}}{\partial\hat{\Phi}_{i-1}^T}\dot{\hat{\Phi}}_{i-1} \tag{8}$$

Let $z_{i+1} = x_{i+1} - \alpha_i$. Then the time derivative of the positive function

$$V_{i,j} = \frac{1}{2}z_i^T z_i \tag{9}$$

along dynamic Equation (6) gives that

$$\begin{aligned} \dot{V}_{i,j} &\le z_i^T G_{i,j} z_{i+1} + z_i^T\left[G_{i,j}\alpha_i + \Sigma_{i,j}^T\Theta_i - \sum_{\varsigma=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_\varsigma^T}G_\varsigma x_{\varsigma+1} - \omega_{i-1}\right] \\ &\quad - z_i^T\frac{\partial\alpha_{i-1}}{\partial(y_d^{(i-1)})^T}y_d^{(i)} + |z_i^T\Delta F_{i,j}(\bar{x}_i)| + \left|z_i^T\sum_{\varsigma=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_\varsigma^T}\Delta F_{\varsigma,j}(\bar{x}_\varsigma)\right| \end{aligned} \tag{10}$$

In addition, with assumption and lemma, we know that

$$|z_i^T\Delta F_{i,j}(\bar{x}_i)| + \left|z_i^T\sum_{\varsigma=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_\varsigma^T}\Delta F_{\varsigma,j}(\bar{x}_\varsigma)\right| \le z_i^T\Pi_{i,j}^T\Phi_i + \frac{5}{4}k_p\varepsilon E_i^T\Phi_i \tag{11}$$

where

$$E_i^T = \left[me_1^T, \cdots, me_{i-1}^T, \frac{4}{5}e_i^T\right] \tag{12}$$

$$\Pi_{i,j}^T = [T_{i-1,1}, \cdots, T_{i-1,i-1}, \mathrm{Tanh}(z_i)]_{m\times mi} \tag{13}$$

and for any $\varsigma = 1, \cdots, i-1$,

$$T_{i-1,\varsigma} = (a_{r,h})_{m\times m}, \quad r, h \in M$$

$$a_{r,h} = \frac{1}{4}\left(\frac{\partial\alpha_{i-1,r}}{\partial x_{\varsigma,h}}\right)^2\tanh\left(\frac{z_{i,r}}{\varepsilon}\left(\frac{\partial\alpha_{i-1,r}}{\partial x_{\varsigma,h}}\right)^2\right) + \tanh\left(\frac{z_{i,r}}{\varepsilon}\right)$$

Combining with (10) and (11), the virtual control law chosen as

$$\begin{aligned} \alpha_i = G_{i,j}^{-1}\bigg[&-K_{i,j}z_i - G_{i-1,j}^T z_{i-1} - \Sigma_{i,j}^T\hat{\Theta}_i + \sum_{\varsigma=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_\varsigma^T}G_\varsigma x_{\varsigma+1} \\ &+\frac{\partial\alpha_{i-1}}{\partial(y_d^{(i-1)})^T}y_d^{(i)} - \Pi_{i,j}^T\hat{\Phi}_i + t_{i-1}\bigg] \end{aligned} \tag{14}$$

leads to

$$\begin{aligned}\dot{V}_{i,j} \le z_i^T G_{i,j} z_{i+1} - z_i^T K_{i,j} z_i - z_i^T G_{i-1,j} z_{i-1} + z_i^T \Sigma_{i,j}^T \tilde{\Theta}_i \\ + z_i^T \Pi_{i,j}^T \tilde{\Phi}_i + z_i^T (t_{i-1} - \omega_{i-1}) + \frac{5}{4} k_p \varepsilon E_i^T \Phi_i\end{aligned} \tag{15}$$

where $K_{i,j} = K_{i,j}^T > 0$, and t_{i-1} is the adjustment function to be determined later.

Step n: Noting that the error variables $z_n = x_n - \alpha_{n-1}$ and α_{n-1} is a function of $\bar{x}_{n-1}$, $\hat{\Theta}_{n-1}$, $\hat{\Phi}_{n-1}$, $y_d^{(n-1)}$, then from x_n-equations of system (1), the dynamic equation for z_n is

$$\begin{aligned}\dot{z}_n = G_{n,j}(\bar{x}_n) u + \Sigma_{n,j}^T \Theta_n - \sum_{\varsigma=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_\varsigma^T} G_\varsigma x_{\varsigma+1} - \omega_{n-1} \\ - \frac{\partial \alpha_{n-1}}{\partial (y_d^{(n-1)})^T} y_d^{(n)} + \Delta F_{n,j}(\bar{x}_n) - \sum_{\varsigma=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_\varsigma^T} \Delta F_{\varsigma,j}(\bar{x}_\varsigma)\end{aligned} \tag{16}$$

where $\Sigma_{n,j}$, ω_{n-1} are defined in (7), (8) with $i = n$, respectively. The actual control input chosen as

$$\begin{aligned}u = G_{n,j}^{-1} \Bigg[-K_{n,j} z_n - G_{n-1,j}^T z_{n-1} - \Sigma_{n,j}^T \hat{\Theta}_n + \sum_{\varsigma=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_\varsigma^T} G_\varsigma x_{\varsigma+1} \\ + \frac{\partial \alpha_{n-1}}{\partial (y_d^{(n-1)})^T} y_d^{(n)} - \Pi_{n,j}^T \hat{\Phi}_n + t_{n-1} \Bigg]\end{aligned} \tag{17}$$

makes the time derivative of the positive function

$$V_{n,j} = \frac{1}{2} z_n^T z_n \tag{18}$$

along dynamic Equation (16) satisfy

$$\begin{aligned}\dot{V}_{n,j} \le -z_n^T K_{n,j} z_n - z_n^T G_{n-1,j}^T z_{n-1} + z_n^T \Sigma_{n,j}^T \tilde{\Theta}_n \\ + z_n^T \Pi_{n,j}^T \tilde{\Phi}_n + z_n^T (t_{n-1} - \omega_{n-1}) + \frac{5}{4} k_p \varepsilon E_n^T \Phi_n\end{aligned} \tag{19}$$

where $K_{n,j} = K_{n,j}^T > 0$ are known proper constant matrices. E_n^T, $\Pi_{n,j}^T$ are defined in (12) and (13) with $i = n$, respectively. t_{n-1} is the adjustment function to be determined later.

The control performance and the trajectories of all signals in the closed-loop switched system will be summarized in the following theorem.

Theorem 3.1. *Consider the switched MIMO nonlinear system (1) with Assumptions 1 and 2. If there exist continuously differentiable positive definite functions $V_j(t) : [0, \infty) \to [0, \infty)$ and a constant $\mu_j > 0$, $j \in \mathbf{S}$ such that at any switching time t_k ($k \in \mathbf{N}$),*

$$V_{i_k}(t_k) \le \mu_k V_{i_{k-1}}(t_k^-) \tag{20}$$

and the switching signals possess the average dwell time

$$\tau_\alpha \le \frac{\ln q}{\lambda} \tag{21}$$

where $q = \max_{k \in \mathbf{N}} \{\mu_k, 1\}$, and λ is a known positive constant. Then under the control input (17) and the appropriate adaptive laws given later, all the closed-loop signals z_i, $\hat{\theta}_i$, $\hat{\phi}_i$, $i \in N$ are uniformly ultimately bounded for any initial conditions.

Proof: Supposing that $t \in [t_k, t_{k+1})$ ($k \in \mathbf{N}$), then the i_kth subsystem is activated. Choose the augmented Lyapunov function candidate for switched system (1)

$$V_{i_k} = V_{1,i_k} + \cdots + V_{n,i_k} + \frac{1}{2} \tilde{\Theta}_n^T \Lambda_{n,i_k}^{-1} \tilde{\Theta}_n + \frac{1}{2} \tilde{\Phi}_n^T \Gamma_{n,i_k}^{-1} \tilde{\Phi}_n \tag{22}$$

where V_{i,i_k} $(i \in N, i_k \in \mathbf{S})$ are defined in (2), (9) and (18). $\Lambda_{n,i_k} = \mathrm{diag}\{\lambda_{1,i_k}, \cdots, \lambda_{n,i_k}\}$, $\Gamma_{n,i_k} = \mathrm{diag}\{\gamma_{1,i_k}, \cdots, \gamma_{n,i_k}\}$ and $\lambda_{i,i_k} = \lambda_{i,i_k}^T, \gamma_{i,i_k} = \gamma_{i,i_k}^T (i \in N)$ are known positive matrices. Invoking (5), (15) and (19), the time derivative of V_{i_k} along switched system (1) satisfies

$$\begin{aligned} \dot{V}_{i_k} \leq & -\sum_{i=1}^{n} z_i^T K_{i,i_k} z_i + \bar{z}_n^T \Xi_{n,i_k} \tilde{\Theta}_n + \bar{z}_n^T \Upsilon_{n,i_k} \tilde{\Phi}_n + \sum_{\varsigma=1}^{n-1} z_{\varsigma+1}^T (t_\varsigma - \omega_\varsigma) \\ & + k_p \varepsilon E^T \Phi_n - \tilde{\Theta}_n^T \Lambda_{n,i_k}^{-1} \dot{\hat{\Theta}}_n - \tilde{\Phi}_n^T \Gamma_{n,i_k}^{-1} \dot{\hat{\Phi}}_n \end{aligned} \tag{23}$$

where

$$\Xi_{n,i_k} = \begin{bmatrix} F_{1,i_k} & 0 & \cdots & 0 \\ -\frac{\partial \alpha_1}{\partial x_1^T} F_{1,i_k} & F_{2,i_k} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ -\frac{\partial \alpha_{n-1}}{\partial x_1^T} F_{1,i_k} & -\frac{\partial \alpha_{n-1}}{\partial x_2^T} F_{2,i_k} & \cdots & F_{n,i_k} \end{bmatrix}$$

$$\Upsilon_{n,i_k} = \begin{bmatrix} \mathrm{Tanh}(z_1) & 0 & \cdots & 0 & 0 \\ T_{1,1} & \mathrm{Tanh}(z_2) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ T_{n-2,1} & T_{n-2,2} & \cdots & \mathrm{Tanh}(z_{n-1}) & 0 \\ T_{n-1,1} & T_{n-1,2} & \cdots & T_{n-1,n-1} & \mathrm{Tanh}(z_n) \end{bmatrix}$$

$$E^T = \left[\left(1 + \frac{5}{4}(n-1)m\right) e_1^T, \left(1 + \frac{5}{4}(n-2)m\right) e_2^T, \cdots, \left(1 + \frac{5}{4}m\right) e_{n-1}^T, e_n^T \right]$$

Consider the adaptive laws for $\hat{\Theta}_n$, $\hat{\Phi}_n$ as

$$\dot{\hat{\Theta}}_n = \Lambda_{n,i_k} (\Xi_{n,i_k}^T \bar{z}_n - \beta \hat{\Theta}_n) \tag{24}$$

$$\dot{\hat{\Phi}}_n = \Gamma_{n,i_k} (\Upsilon_{n,i_k}^T \bar{z}_n - \rho \hat{\Phi}_n) \tag{25}$$

and the adjustment function t_ς $(\varsigma = 1, \cdots, n-1)$ as

$$\begin{aligned} t_\varsigma = & \frac{\partial \alpha_\varsigma}{\partial \hat{\Theta}_\varsigma^T} \Lambda_{\varsigma,i_k} (\Xi_\varsigma^T \bar{z}_\varsigma - \beta \hat{\Theta}_\varsigma - \Delta_\varsigma z_{\varsigma+1}) - \frac{\partial \alpha_\varsigma}{\partial \bar{x}_{\varsigma-1}^T} \bar{F}_{\varsigma-1} \Lambda_{\varsigma-1,i_k} P_{\varsigma-1} \xi_\varsigma \\ & + \frac{\partial \alpha_\varsigma}{\partial \hat{\Phi}_\varsigma^T} \Gamma_{\varsigma,i_k} (\Upsilon_\varsigma^T \bar{z}_\varsigma - \rho \hat{\Phi}_\varsigma - \Psi_\varsigma z_{\varsigma+1}) - \frac{\partial \alpha_\varsigma}{\partial \bar{x}_{\varsigma-1}^T} \bar{T}_{\varsigma-1} \Gamma_{\varsigma-1,i_k} Q_{\varsigma-1} \xi_\varsigma \end{aligned} \tag{26}$$

where

$$\Delta_\varsigma = \left(\frac{\partial \alpha_\varsigma}{\partial x_1^T} F_{1,j}, \frac{\partial \alpha_\varsigma}{\partial x_2^T} F_{2,j}, \cdots, \frac{\partial \alpha_\varsigma}{\partial x_\varsigma^T} F_{\varsigma,j} \right)^T$$

$$\bar{F}_{\varsigma-1} = \left(F_{1,j}^T, F_{2,j}^T, \cdots, F_{\varsigma,j}^T \right)^T$$

$$P_{\varsigma-1} = \begin{bmatrix} \frac{\partial \alpha_1}{\partial \hat{\theta}_1^T} & \frac{\partial \alpha_2}{\partial \hat{\theta}_1^T} & \cdots & \frac{\partial \alpha_{\varsigma-1}}{\partial \hat{\theta}_1^T} \\ 0 & \frac{\partial \alpha_2}{\partial \hat{\theta}_2^T} & \cdots & \frac{\partial \alpha_{\varsigma-1}}{\partial \hat{\theta}_2^T} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial \alpha_{\varsigma-1}}{\partial \hat{\theta}_{\varsigma-1}^T} \end{bmatrix}, \quad Q_{\varsigma-1} = \begin{bmatrix} \frac{\partial \alpha_1}{\partial \hat{\phi}_1^T} & \frac{\partial \alpha_2}{\partial \hat{\phi}_1^T} & \cdots & \frac{\partial \alpha_{\varsigma-1}}{\partial \hat{\phi}_1^T} \\ 0 & \frac{\partial \alpha_2}{\partial \hat{\phi}_2^T} & \cdots & \frac{\partial \alpha_{\varsigma-1}}{\partial \hat{\phi}_2^T} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial \alpha_{\varsigma-1}}{\partial \hat{\phi}_{\varsigma-1}^T} \end{bmatrix}$$

$$\Psi_\varsigma = (T_{\varsigma,1}, \cdots, T_{\varsigma,\varsigma})^T$$

$$\bar{T}_{\varsigma-1} = (\mathrm{Tanh}(z_1), \mathrm{Tanh}(z_2), \cdots, \mathrm{Tanh}(z_{\varsigma-1}))^T$$

$$\xi_\varsigma = (z_2^T, z_3^T, \cdots, z_\varsigma^T)^T$$

and $\beta > 0$, $\rho > 0$ are known constants. Substituting (24)-(26) into (23), we have

$$\begin{aligned} \dot{V}_{i_k} &\leq -\sum_{i=1}^{n} z_i^T K_{i,i_k} z_i + \beta \tilde{\Theta}_n^T \hat{\Theta}_n + \rho \tilde{\Phi}_n^T \hat{\Phi}_n + k_p \varepsilon E^T \Phi_n \\ &\leq -\lambda V_{i_k} + c \end{aligned} \tag{27}$$

where $\lambda = \min\limits_{i \in N, i_k \in \mathbf{S}} \left\{ 2\lambda_{\min}(K_{i,i_k}), \frac{\beta}{\lambda_{\max}(\Lambda_{n,i_k}^{-1})}, \frac{\rho}{\lambda_{\max}(\Gamma_{n,i_k}^{-1})} \right\}$, $c = \frac{\beta}{2}\Theta_n^T\Theta_n + \frac{\rho}{2}\Phi_n^T\Phi_n + k_p\varepsilon E^T\Phi_n$. Integrating both sides of (27) leads to

$$V_{i_k}(t) \leq V_{i_k}(t_k) e^{-\lambda(t-t_k)} - \frac{c}{\lambda} e^{-\lambda(t-t_k)} + \frac{c}{\lambda}$$

With the conditions (20) and (21), it further follows

$$\begin{aligned} V_{i_k}(t) \leq & \mu V_{i_{k-1}}(t_k^-) e^{-\lambda(t-t_k)} - \frac{c}{\lambda} e^{-\lambda(t-t_k)} + \frac{c}{\lambda} \\ \leq & \mu V_{i_{k-1}}(t_{k-1}) e^{-\lambda(t-t_{k-1})} - \mu \frac{c}{\lambda} \left[e^{-\lambda(t-t_{k-1})} - e^{-\lambda(t-t_k)} \right] - \frac{c}{\lambda} \left[e^{-\lambda(t-t_k)} - 1 \right] \\ \leq & \cdots \leq \mu^k V_{i_0}(t_0) e^{-\lambda(t-t_0)} - \frac{c}{\lambda} \sum_{\iota=0}^{k-1} \mu^{k-\iota} \left[e^{-\lambda(t-t_\iota)} - e^{-\lambda(t-t_{\iota+1})} \right] \\ & - \frac{c}{\lambda} \left[e^{-\lambda(t-t_k)} - 1 \right] \\ \leq & e^{k \ln q} e^{-k\lambda\tau_\alpha} V_{i_0}(t_0) + \frac{c}{\lambda} \sum_{\iota=0}^{k} q^{k-\iota} \leq V_{i_0}(t_0) + C \end{aligned} \tag{28}$$

where $V_{i_0}(t_0) = \frac{1}{2}\sum_{i=1}^{n} \left[z_i^T(0) z_i(0) + \tilde{\Theta}_i^T(0) \Lambda_{i,i_0}^{-1} \tilde{\Theta}_i(0) + \tilde{\Phi}_i^T(0) \Gamma_{i,i_0}^{-1} \tilde{\Phi}_i(0) \right]$, and $C = \frac{c}{\lambda}\sum_{\iota=0}^{k} q^{k-\iota}$. It can be seen from (22) and (28) that all signals of the closed-loop switched system (1) are uniformly ultimately bounded. This completes the proof.

Remark 3.1. *According to the stability theory of switched systems, a switched system might become unstable even if all the individual subsystems are stable. Thus, it is an important issue to restrict the admissible switching signals to achieve stability for the switched systems. In Theorem 3.1, condition (20) holding for* $\mu_k > 1$ ($k \in \mathbf{N}$) *means that at each switching time* t_k*, the energy function* V_{i_k} *of the activated subsystem exists unstable switching. Thus, the operation interval of the activated subsystem affects the stability of the whole switched system. In addition, if* $0 < \mu_k \leq 1$ *in condition (20), the energy function* V_{i_k} *of the activated subsystem processes stable switching. In this case, the activated interval may be arbitrary under the assumption that the activated subsystem is stabilizable. So according to the essence of average dwell time, it is possible to switch fastly and then compensate for it by switching slowly according to the value of* μ_k*. What is more, if the switching law is determined only by the stable switching time of the energy function* V_{i_k}*, i.e., condition (20) holds with* $\max_{k\in\mathbf{N}}\{\mu_k\} \leq 1$ *for each* t_k ($k \in \mathbf{N}$)*, there is no limitation on the dwell time of the activated subsystem, which may be described by the following proposition.*

Corollary 3.1. *Consider the switched MIMO nonlinear system (1) with Assumptions 1 and 2. If there exist continuously differentiable positive definite functions* $V_j(t) : [0, \infty) \to [0, \infty)$*,* $j \in \mathbf{S}$ *such that at any switching time* t_k ($k \in \mathbf{N}$)*,*

$$V_{i_k}(t_k) \leq V_{i_{k-1}}(t_k^-) \tag{29}$$

i_k, $i_{k-1} \in \mathbf{S}$ and $i_k \neq i_{k-1}$. Then under the control input (17), the adaptive laws (24), (25) and the adjustment function (26), all the closed-loop signals z_i, $\hat{\theta}_i$, $\hat{\phi}_i$, $i \in N$ are uniformly ultimately bounded for any initial conditions.

4. **Numerical Example.** To demonstrate the effectiveness of the proposed adaptive control algorithms, a numerical example with two MIMO switched subsystems is considered in this section:

$$\begin{cases} \dot{x}_1 = G_{1,\sigma(t)}(x_1)x_2 + F_{1,\sigma(t)}(x_1)\theta_1 + \Delta F_{1,\sigma(t)}(x_1), \\ \dot{x}_2 = G_{2,\sigma(t)}(\bar{x}_2)u + F_{2,\sigma(t)}(\bar{x}_2)\theta_2 + \Delta F_{2,\sigma(t)}(\bar{x}_2), \\ y = x_1 \end{cases} \tag{30}$$

where $x_1 = (x_{11}, x_{12})^T \in \mathbf{R}^2$, $x_2 = (x_{21}, x_{22})^T \in \mathbf{R}^2$, $\sigma(t) = 1, 2$ and

$$F_{1,1}(x_1) = \begin{bmatrix} x_{11}^2 & x_{11}x_{12} \\ x_{12} & x_{12}^3 \end{bmatrix}, \quad F_{1,2}(x_1) = \begin{bmatrix} x_{11}^3 & x_{12} \\ x_{11} & x_{11}x_{12} \end{bmatrix},$$

$$F_{2,1}(\bar{x}_2) = \begin{bmatrix} x_{11}x_{21} & x_{12}x_{22} \\ x_{12}\sin(x_{22}) & e^{x_{22}} \end{bmatrix}, \quad F_{2,2}(\bar{x}_2) = \begin{bmatrix} 2x_{11}x_{21} + x_{22} & x_{11} + x_{22} \\ x_{12}\cos(x_{11}) & x_{11}x_{21}^2 \end{bmatrix},$$

$$G_{1,1}(x_1) = \begin{bmatrix} 1.5 + \sin(x_{11}) & 0 \\ 0 & 2 + \cos(x_{12}) \end{bmatrix}, \quad G_{1,2}(x_1) = \begin{bmatrix} 2 - \cos(x_{11}) & 0 \\ 0 & 3 + \sin(x_{12}) \end{bmatrix},$$

$$G_{2,1}(\bar{x}_2) = \begin{bmatrix} 2.2 + \sin(x_{22}) & 0 \\ 0 & 1.7 - \sin(x_{21})\cos(x_{12}) \end{bmatrix},$$

$$G_{2,2}(\bar{x}_2) = \begin{bmatrix} 2.4 - \cos(x_{11})\cos(x_{22}) & 0 \\ 0 & 2 + \sin(x_{12}x_{21}) \end{bmatrix}$$

In addition, θ_i are unknown parameter vectors, and $\Delta F_{i,\sigma(t)}(\cdot)$ are unknown smooth functions satisfying Assumption 1, $i = 1, 2$. For simulation purpose, we assume $\theta_1 = (0.1, -0.2)^T$, $\theta_2 = (-0.3, 0.1)^T$, $\phi_1 = (0.2, 0.21)^T$, $\phi_2 = (0.15, 0.2)^T$. Following the proposed design procedure, the control objective is to design an adaptive controller u given by (17) and the parameter update laws for $\hat{\theta}_i$, $\hat{\phi}_i$ $(i = 1, 2)$ given by (24) and (25), such that the system output $y = x_1$ tracks the desired signal $y_d = [y_{d1}, y_{d2}]^T$, where $y_{d1} = 0.1\sin(t) + 0.3\sin(0.3t)$ and $y_{d2} = 0.2\cos(t) + 0.2\cos(0.1t)$. The corresponding parameters are adopted in the simulation: $K_{1,1} = K_{1,2} = \mathrm{diag}\{17, 22\}$, $K_{2,1} = K_{2,2} = \mathrm{diag}\{19, 15\}$, $\lambda_{1,1} = \lambda_{1,2} = \mathrm{diag}\{0.5, 0.5\}$, $\lambda_{2,1} = \lambda_{2,2} = \mathrm{diag}\{0.3, 0.1\}$, $\gamma_{1,1} = \gamma_{1,2} = \mathrm{diag}\{0.4, 0.6\}$, $\gamma_{2,1} = \gamma_{2,2} = \mathrm{diag}\{0.3, 0.2\}$, $\beta_1 = 0.5$, $\beta_2 = 0.3$, $\rho_1 = 0.5$, $\rho_2 = 0.6$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.2$. The initial values are chosen as $x(0) = (0.1, 0.25, -0.45, 0.2)^T$, $\hat{\theta}_1 = (0.3, -0.4)^T$, $\hat{\theta}_2 = (0.5, -0.1)^T$, $\hat{\phi}_1 = (0.2, 0.2)^T$, $\hat{\phi}_2 = (0.1, 0.2)^T$.

The simulation results are given with two different switching laws. Case 1: according to Theorem 3.1, the average dwell time τ_α satisfies $\tau_\alpha \leq 0.667$. A uniform switching with dwell time $\tau_\alpha = 0.6$ is considered in the simulation, see Figures 1-4. Case 2: according to Corollary 3.1, the switching law satisfies (29), i.e., a slow switching is chosen here, see Figures 5-8. From these simulation results, it can be seen that even though there are some unavailable factors on the nonlinear functions in the considered system, the proposed adaptive output feedback controller guarantees the desired performance for both states and tracking errors of the closed-loop switched system (30).

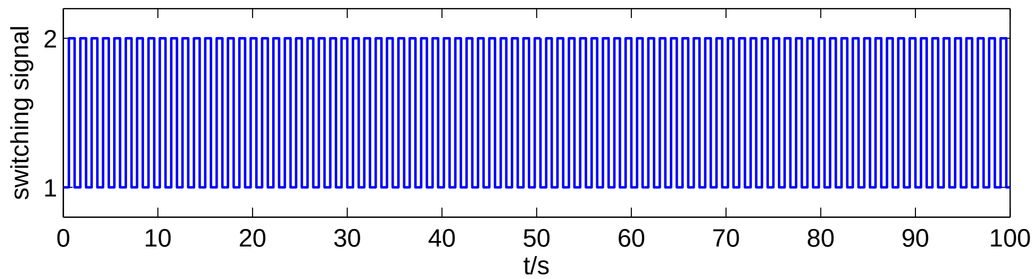

FIGURE 1. Switching signal with average dwell time $\tau_\alpha = 0.6$

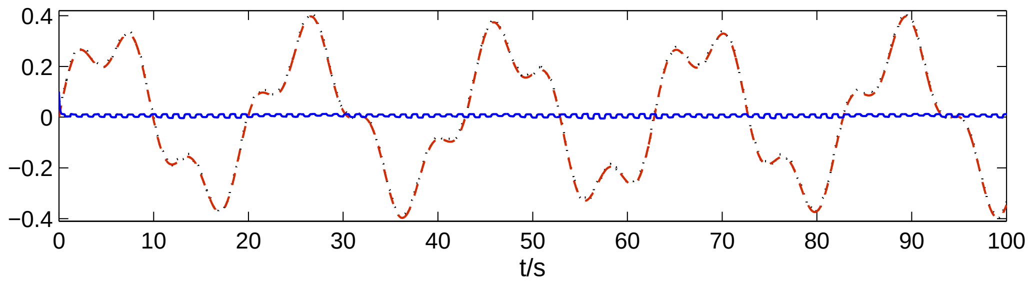

FIGURE 2. Tracking performance of output $y_1 = x_{11}$

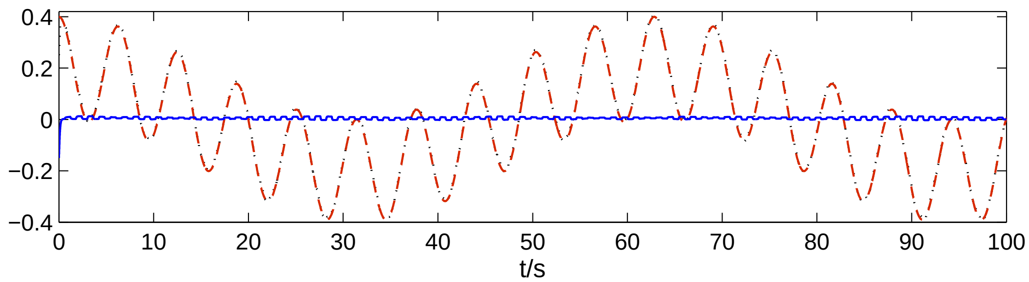

FIGURE 3. Tracking performance of output $y_2 = x_{12}$

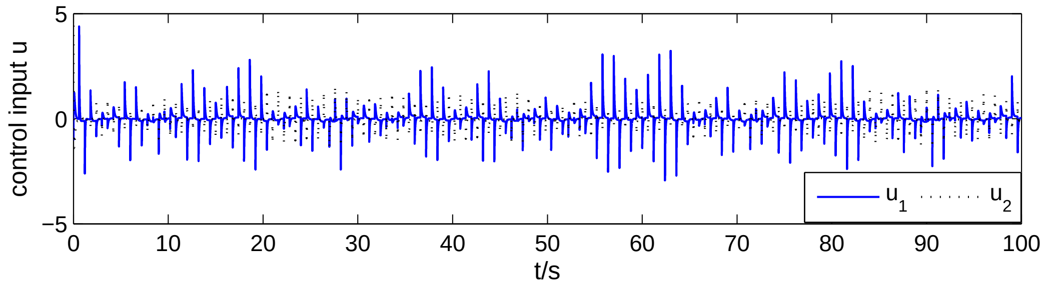

FIGURE 4. Control signal $u = (u_1, u_2)^T$ for case 1

5. **Conclusion.** An adaptive backstepping tracking control problem has been investigated for a class of uncertain MIMO switched nonlinear systems in lower triangular forms. Combing the adaptive control theory, multiple Lyapunov synthesis and average dwell time

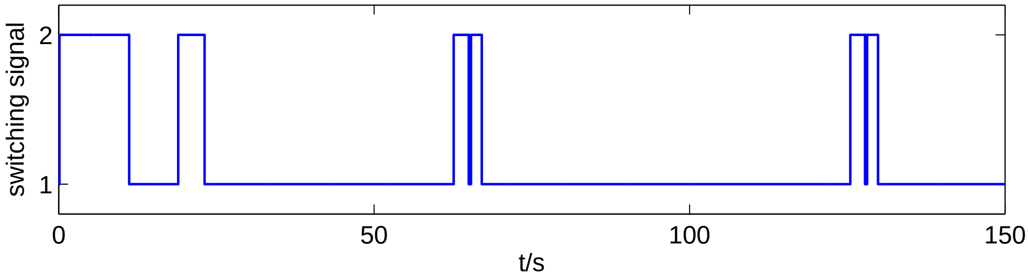

FIGURE 5. Switching signal for case 2

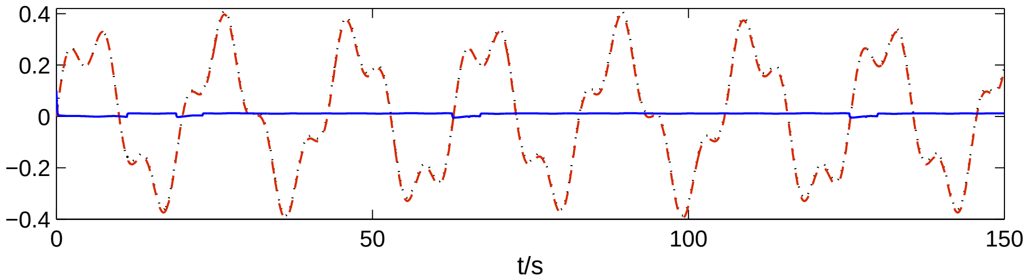

FIGURE 6. Tracking performance of output $y_1 = x_{11}$

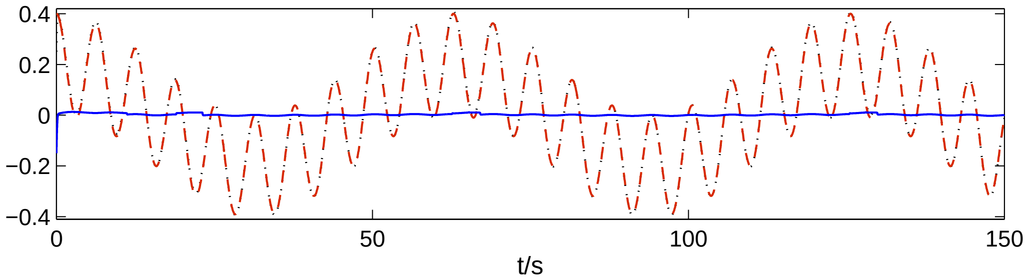

FIGURE 7. Tracking performance of output $y_2 = x_{12}$

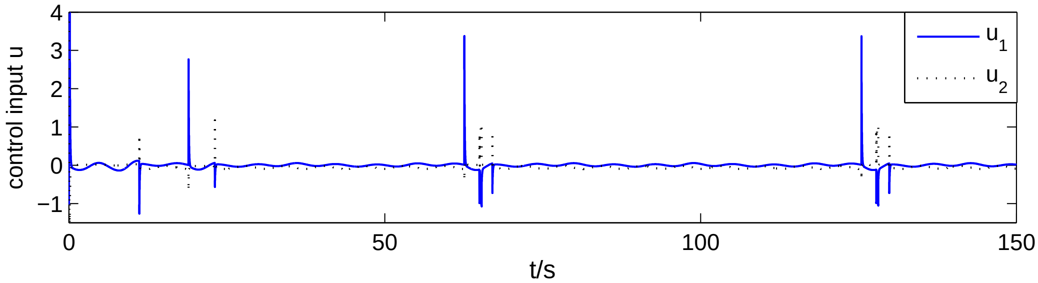

FIGURE 8. Control signal $u = (u_1, u_2)^T$ for case 2

conception, an adaptive output feedback controller is constructed recursively under different coordinate transformation to guarantee all the signals of the resulting closed-loop switched system are bounded and the tracking error converges to a small neighborhood of the origin. Finally, simulation studies have been presented to illustrate the effectiveness of the proposed control method.

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