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INPUT FEEDBACK CONTROL OF MANIPULATED VARIABLES IN MODEL PREDICTIVE CONTROL

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Abstract. *The implementation of manipulated variables in model predictive control (MPC) is generally considered as an ideal control, which means the outputs of MPC are applied to real process completely. However, actually, due to actuator capacity of the bottom PID control or other constraints hidden in process plant, the control actions of manipulated variables actually applied may not be equal to the outputs of the MPC controller, so if MPC ignores the difference between actual implementation and theoretical calculation of manipulated variables, it will cause poor control performance. The problem cannot be solved by the traditional output or state feedback method. In this paper, an input feedback control algorithm is proposed for MPC. During the calculation of control law, the previous actual implementation values of manipulated variables are used to replace the previous theoretical calculation values through input feedback. The input feedback control in MPC strategy is illustrated on two cases: nonideal bottom PID control and false loose constraints. The simulation results demonstrate the effectiveness of the proposed input feedback control strategy.*

Keywords: Model predictive control, Input feedback, Nonideal control, False loose constraints

1. **Introduction.** Model predictive control (MPC) has been widely adopted in the petrochemical industry for controlling large, multi-variable processes. MPC solves an online optimization to determine inputs, which considers the current conditions of plant, any disturbances affecting operation, and imposed safety and physical constraints. The successful implementation [1-4] of MPC in industry is due to its distinct advantages.

In particular, MPC can achieve approximately optimal control performance even under actual constraints. This compelling feature has attracted significant attention for the analysis and synthesis of different forms of MPC. Recently, the application requirements on considering actual constraints and the implementation environment have triggered increasing attention of MPC towards new directions. Zhu et al. [5] presented stability results for discrete-time MPC system subject to an input amplitude constraint. Lee and Kouvaritakis [6] examined the stabilizable regions of receding horizon predictive control (RHPC) with input constraints. Li et al. [7] transformed a robust MPC strategy under chance constraints into a stochastic program under joint probabilistic constraints. Mayne et al. [8] discussed the stability and optimality of constrained MPC systematically. Wan and Kothare [9] developed an efficient robust constrained MPC algorithm with a time varying terminal constraint set for systems with model uncertainty and input constraints. Su et al. [10] proposed a constrained decoupling generalized predictive control algorithm for MIMO system considering constraints of inputs and their increments. Mhaskar et al. [11] proposed a Lyapunov-based predictive control design to solve the problem of stabilization of nonlinear systems subject to state and control constraints. Xia et al. [12] proposed parameter-dependent Lyapunov functions to design a state feedback controller for robust constrained MPC problem. Cannon et al. [13] provided a method of handling probabilistic constraints of Robust MPC and ensuring closed loop stability through the use of an extension of the concept of invariance. Adetola et al. [14] combined parameter adjustment mechanism with robust MPC algorithms for the constrained nonlinear system. Veselý et al. [15] designed a robust output/state MPC for linear polytopic systems with input constraints using Lyapunov function approach. Xia et al. [16] considered the problem of constrained infinite-horizon model predictive control for fuzzy-discrete systems. Pérez et al. [17] proposed an explicit solution to the MPC of linear systems subject to non-convex polyhedral constraints. Rahideh and Shaheed [18] presented a constrained output feedback MPC for nonlinear systems using unscented Kalman filter. Li and Shi [19] studied the robust output feedback MPC problem for a constrained linear system subject to periodical measurement losses and external disturbances. Zhang et al. [20] presented a distributed MPC algorithm for polytopic uncertain systems subject to actuator saturation. Fagiano and Teel [21] investigated a terminal state equality constraint for MPC, where the terminal state/input pair is not fixed a priori in the optimization. The above MPC strategies are mainly output or state feedback control subject to the input, output or state constraints.

In real industrial process, the outputs of MPC controller are always downloaded to the setpoints of bottom PID control, and the control action that actually applied to the process is implemented by bottom PID control. Usually the implementation on manipulated variables of model predictive control is generally considered as an ideal control. However, due to dynamic characteristics of local bottom PID control loop, the control actions on manipulated variables actually applied may not be equal to the outputs of the MPC controller. If the MPC system is running under the situation for long time, it will cause poor control performance.

The feedback method is always utilized to compensate the errors between prediction model and actual process. The generally utilized feedback methods are output feedback and state feedback. As said in the previous paragraph, there are errors between actual implementation u_p and theoretical calculation u of manipulated variables, and they can not be compensated by output feedback and state feedback methods. An input feedback method is utilized to solve the problem in this paper. During the calculation of control law $\Delta u(k)$, the previous actual implementation values $u_p(k-1)$ of manipulated variables are used to replace the previous theoretical calculation values $u(k-1)$ through input feedback, which guarantee well control performance when $u \neq u_p$. The input feedback control in MPC strategy will be illustrated on two cases: nonideal bottom PID control and false loose constraints.

The paper is organized as follows. Section 2 analyzes the implementation methods of manipulated variables in MPC through bottom PID control. Section 3 gives the input feedback control algorithm of MPC. Section 4 introduces the input feedback control algorithm applied in nonideal bottom PID control. Section 5 gives the input feedback control algorithm applied in false loose constraints problem. Finally, the major conclusions are drawn in Section 6.

2. **Implementation Methods Analysis of Manipulated Variables in MPC.** The control structure of MPC is seen in Figure 1. The output of the MPC controller $u(k)$ is given by adding the control law $\Delta u(k)$ with the previous control action $u(k-1)$. The control action actually applied is $u_p(k)$. Usually the implementation of model predictive

control is considered as ideal control which accounts for $u(k) = u_p(k)$. In some instances, there may be problems of nonideal control or false loose constraints, which will cause $u(k) \neq u_p(k)$ and means the outputs of MPC cannot be downloaded to the process completely.

2.1. **Nonideal bottom PID control problem.** In the industrial process, due to influences of the actuator capacity or the characteristics of bottom PID control loop, it will cause $u(k) \neq u_p(k)$ which means that $u_p(k) = \alpha u(k)$ ($\alpha \in (0, 1)$) in Figure 1.

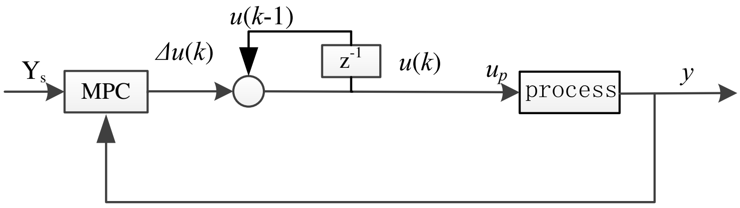
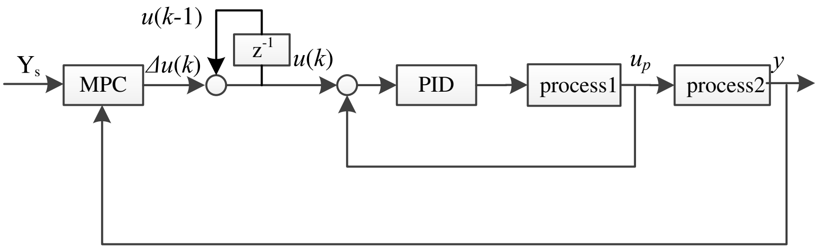
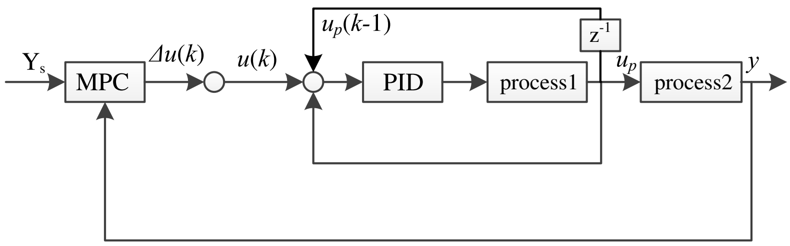


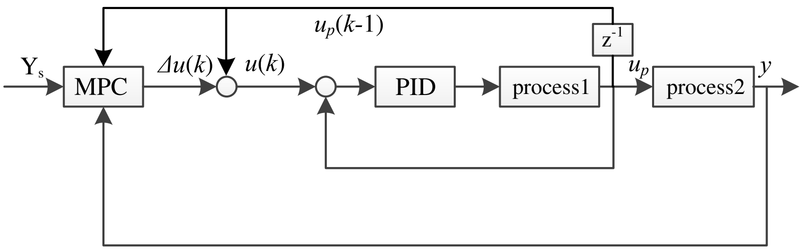
Figure 1. MPC control structure



(a)



(b)



(c)

Figure 2. Three kinds of MPC-PID cascade control structure. (a) The common MPC structure, (b) the common MPC structure, (c) the common MPC structure.

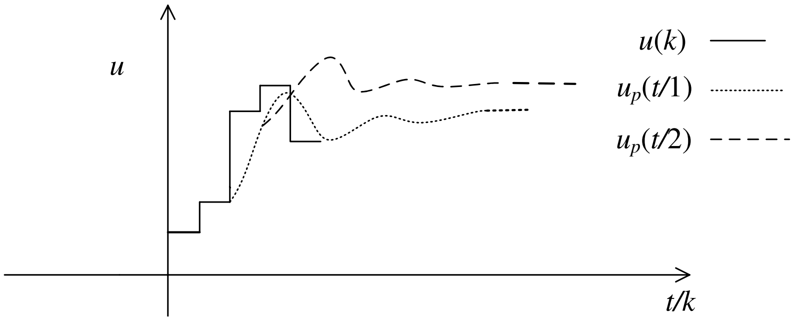


Figure 3. Control realization characteristics for manipulated variables of MPC-PID cascade control in two sample time

As seen in Figure 2(a), in the instance ‘transparent control’, the control action $u(k)$ is obtained by $\Delta u(k) + u(k-1)$, $\Delta u(k)$ is the control law, $u(k-1)$ is the control action of the previous step. For each step, $u(k)$ is the setpoint of the bottom PID control loop, the output of the bottom PID control loop is $u_p(k)$. In the theoretical study of MPC, it assumes that the bottom PID control is an ideal control which accounts for $u(k) = u_p(k)$. As said in Section 1, in industrial unit, due to the dynamic characteristic of the bottom PID control loop, it may cause $u(k) \neq u_p(k)$.

Let us consider a ‘temperature – flow rate’ control system with MPC-PID cascade structure. Generally, the sample time of the outer MPC loop is about 1 minute, and the complete response time of inner flow PID control loop is about 10 minutes. So in each sample time of the outer loop, the output of MPC controller, that is the value of flow rate, may not be realized completely by the inner PID control loop which accounts for $u \neq u_p$ in Figure 2(a). Figure 3 shows the output of MPC controller $u(k)$ and the actual output of the bottom PID control loop ($u_p(k)$) in the former two steps, obviously they are not equal. In the first step, the output of MPC controller $u(k)$ is the setpoint of the inner PID control loop. It does not reach the steady state in one sampling period of the MPC loop. In the second step, the PID control loop executs the output of MPC controller ignoring whether the first output of the MPC controller is realized or not.

2.2. **False loose constraints problem.** In the problem of false loose constraints, at the moment k, the previous step $u^*(k-1)$ that actually applied to the process can be measured: if $\Delta u_i^*(k-1) > 0$ or $\Delta u_i^*(k-1) < 0$, and the measured actual control action $\Delta u_{p,i}(k-1) < \Delta u_i^*(k-1)$ or $\Delta u_{p,i}(k-1) > \Delta u_i^*(k-1)$, it represents that $\Delta u_i^*(k-1)$ is not be downloaded entirely and the nominal constraints become false loose constraints, as seen in Figure 4.

2.3. **Realization characteristics analysis for manipulated variables.** In the above two situations $u \neq u_p$, as seen in Figure 3 and Figure 4. In the calculation of the control law, the MPC controller assumes that $u = u_p$, so there are errors between the ideal and the actual situation, and the control performance may be affected by these errors.

As seen in Figure 2(b), a new control structure is utilized to solve this problem. The control action $u_p(k-1)$ that actually applied to the process is utilized to add with $\Delta u(k)$, and the sum value is set as the setpoint of the inner PID control loop. However, the method is of the following problem: in the calculation of the control law $\Delta u(k)$, the previous historical control action is $u(k-1)$, it is inconsistency to use $u_p(k-1)$ adding with $\Delta u(k)$ instead of $u(k-1)$. As seen in Figure 5, the setpoint of the inner PID control loop exceeds the desired MPC controller output. This method will be unstable in the actual process.

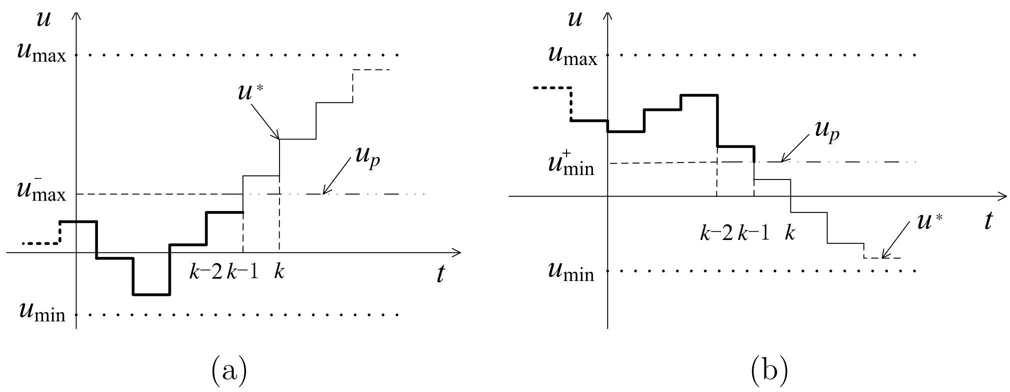


FIGURE 4. False and loose constraint. (a) $\Delta u_{p,i}(k-1) < \Delta u_i^*(k-1)$, (b) $\Delta u_{p,i}(k-1) > \Delta u_i^*(k-1)$.

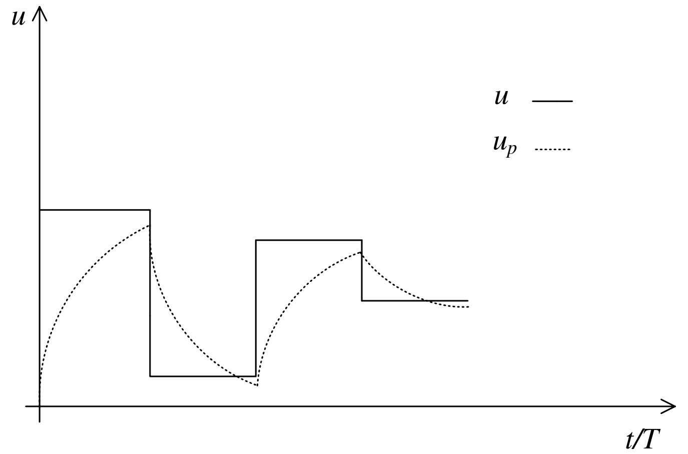


FIGURE 5. Realization characteristics for operating variables

To solve this problem, an input feedback control is proposed, and the control structure is seen in Figure 2(c). In general, the control action $u(k-1)$ of the previous step is used in the calculation of the control law $\Delta u(k)$. In input feedback control structure, the control action $u(k-1)$ is replaced by the control action $u_p(k-1)$ which is actually applied to the process. The next section will give this modified method.

3. **Model Predictive Control Algorithm with Input Feedback Structure.** Let us consider a process with r outputs and m inputs described by the following state space model.

$$\begin{cases} x(k+1|k) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{1}$$

where, $x(k) \in R^n$ is the state vector at the moment k, $x(k+i\,|k) \in R^n$ are the predictive state vector at the moment $(k+i)$. The inputs $u(k) \in R^m$ are the manipulated variables, the outputs $y(k) \in R^r$ are the controlled variables; $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{r\times n}$ are the corresponding matrices of the system.

According to the state feedback MPC algorithm, P is the prediction horizons and M is the control horizon, and $M \leq P$, $\Delta u(k) = u(k) - u(k-1)$ is the increment of input variable, if $i \geq M$, $\Delta u(k+i) = 0$, model (1) can be represented by the increment form.

$$\begin{cases} x(k+1|k) = Ax(k) + B[u(k-1) + \Delta u(k)] \\ y(k) = Cx(k) \end{cases} \tag{2}$$

The matrix form expression for the prediction of the state is

$$\begin{bmatrix} x(k+1|k) \\ x(k+2|k) \\ \vdots \\ x(k+P|k) \end{bmatrix} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^P \end{bmatrix} x(k) + \begin{bmatrix} B \\ AB+B \\ \vdots \\ \sum_{i=0}^{P-1} A^i B \end{bmatrix} u(k-1)$$

$$+ \begin{bmatrix} B & & & \\ AB+B & B & & \\ \vdots & \vdots & \ddots & \\ \sum_{i=0}^{M-1} A^i B & \sum_{i=0}^{M-2} A^i B & \cdots & B \\ \sum_{i=0}^{M} A^i B & \sum_{i=0}^{M-1} A^i B & \cdots & AB+B \\ \vdots & \vdots & & \vdots \\ \sum_{i=0}^{P-1} A^i B & \sum_{i=0}^{P-2} A^i B & \cdots & \sum_{i=0}^{P-M} A^i B \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+M-1) \end{bmatrix} \tag{3}$$

Therefore, the predictive output can be represented by the following equation.

$$Y_c(k) = S_x x(k) + S_u u(k-1) + S_{\Delta U} \Delta U(k) + E(k) = Y_0(k) + S_{\Delta U} \Delta U(k) \tag{4}$$

where,

$$Y_c(k) = \begin{bmatrix} y_c(k+1) \\ y_c(k+2) \\ \vdots \\ y_c(k+p) \end{bmatrix}, \quad \Delta U(k) = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+M-1) \end{bmatrix},$$

$$S_x = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^P \end{bmatrix}, \quad S_u = \begin{bmatrix} CB \\ C(AB+B) \\ \vdots \\ C\sum_{i=0}^{P-1} A^i B \end{bmatrix},$$

$$S_{\Delta U} = \begin{bmatrix} CB & & & \\ C(AB+B) & CB & & \\ \vdots & \vdots & \ddots & \\ C\sum_{i=0}^{M-1} A^i B & C\sum_{i=0}^{M-2} A^i B & \cdots & CB \\ C\sum_{i=0}^{M} A^i B & C\sum_{i=0}^{M-1} A^i B & \cdots & C(AB+B) \\ \vdots & \vdots & & \vdots \\ C\sum_{i=0}^{P-1} A^i B & C\sum_{i=0}^{P-2} A^i B & \cdots & C\sum_{i=0}^{P-M} A^i B \end{bmatrix},$$

$$Y_0(k) = \begin{bmatrix} y_0(k+1) \\ y_0(k+2) \\ \vdots \\ y_0(k+P) \end{bmatrix} = S_x x(k) + S_u u(k-1).$$

The general expression for such an objective function will be

$$V(k) = \|Y_c(k) - Y_s(k)\|_Q^2 + \|\Delta U(k)\|_R^2 \tag{5}$$

where $Y_s(k) = \begin{bmatrix} y_s(k+1) \\ y_s(k+1) \\ \vdots \\ y_s(k+P) \end{bmatrix}$ is the reference trajectory.

The objective (5) can be represented as

$$V(k) = \|Y_c(k) - Y_s(k)\|_Q^2 + \|\Delta U(k)\|_R^2 = \text{Const} + \Delta U(k)^{\mathrm{T}} H \Delta U(k) + 2F^{\mathrm{T}} \Delta U(k) \tag{6}$$

where $H = S_{\Delta U}^{\mathrm{T}} Q S_{\Delta U} + R$, $F = S_{\Delta U}^{\mathrm{T}} Q Y_z(k)$, $Y_z(k) = Y_0(k) - Y_s(k)$. The term "Const" is a constant value, it has no influence on objective function, so the objective function can be described as

$$J(k) = \Delta U(k)^{\mathrm{T}} H \Delta U(k) + 2F^{\mathrm{T}} \Delta U(k) \tag{7}$$

The constraints of inputs and outputs variables in the system can be represented as:

$$u_{\min} \le u \le u_{\max}, \quad y_{\min} \le y \le y_{\max} \tag{8}$$

The matrix form of the constraints is

$$D_1 \Delta U(k) \le d_1, \quad D_2 \Delta U(k) \le d_2 \tag{9}$$

where

$$D_1 = \begin{bmatrix} \Psi \\ -\Psi \end{bmatrix}, \quad D_2 = \begin{bmatrix} S_{\Delta U} \\ -S_{\Delta U} \end{bmatrix},$$

$$d_1 = \begin{bmatrix} U_{\max} - U(k-1) \\ U(k-1) - U_{\min} \end{bmatrix}, \quad d_2 = \begin{bmatrix} Y_{\max} - Y_0(k) \\ Y_0(k) - Y_{\min} \end{bmatrix},$$

$$U(k-1) = \begin{bmatrix} u(k-1) \\ \vdots \\ u(k-1) \end{bmatrix}, \quad U_{\max} = \begin{bmatrix} u_{\max} \\ \vdots \\ u_{\max} \end{bmatrix}, \quad U_{\min} = \begin{bmatrix} u_{\min} \\ \vdots \\ u_{\min} \end{bmatrix},$$

$$Y_{\max} = \begin{bmatrix} y_{\max} \\ \vdots \\ y_{\max} \end{bmatrix}, \quad Y_{\min} = \begin{bmatrix} y_{\min} \\ \vdots \\ y_{\min} \end{bmatrix},$$

$$Y_0(k) = \begin{bmatrix} y_0(k+1) \\ \vdots \\ y_0(k+P) \end{bmatrix}, \quad \Psi = \begin{bmatrix} I & & & \\ I & I & & \\ \vdots & \vdots & \ddots & \\ I & I & \cdots & I \end{bmatrix}.$$

The control law can be obtained by minimizing the quadratic functional J

$$\begin{aligned} &\min_{\Delta U(k)} J(k) = \tfrac{1}{2} \Delta U(k)^{\mathrm{T}} 2H \Delta U(k) + 2F^{\mathrm{T}} \Delta U(k) \\ &\text{s.t.} \quad \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \Delta U(k) \le \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \end{aligned} \tag{10}$$

The solution minimizing (10) gives an optimal suggested control increment sequence $\Delta U^*(k)$. The first output of $\Delta U^*(k)$ is $\Delta u^*(k)$. In the next step, the control law is calculated using the receding horizon concept. The outer loop manipulated variable is

$u(k) = u(k-1) + \Delta u^*(k)$. As seen in Figure 2(a), the output of MPC controller $u(k)$ is the setpoint of the inner PID control loop.

In the theory study of model predictive control, it is assumed that the implementation of manipulated variables is an ideal control which accounts for the MPC output u that equals the inner PID loop output u_p. In real industrial process, due to the dynamic characteristic of the inner PID control loop, the MPC output u may not equal the inner PID control loop output u_p. In this situation, there is a deviation for manipulated variables between prediction model (2) and the actual system, the real system response may deviate from the expected output.

In this paper, the input feedback method is used to solve this problem. The control action $u_p(k-1)$ that applied to the process is taken into the calculation of the next step control law $\Delta u(k)$ though input feedback. At the moment k, prediction model (2) is modified by

$$\begin{cases} \hat{x}(k+1|k) = A\hat{x}(k) + B[u_p(k-1) + \Delta u(k)] \\ \hat{y}(k) = C\hat{x}(k) \end{cases} \tag{11}$$

According to the modified prediction model (11), $u(k-1)$ in the prediction of the state vector (3) is replaced by $u_p(k-1)$.

$$\begin{aligned} \begin{bmatrix} \hat{x}(k+1|k) \\ \hat{x}(k+2|k) \\ \vdots \\ \hat{x}(k+P|k) \end{bmatrix} &= \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^P \end{bmatrix} \hat{x}(k) + \begin{bmatrix} B \\ AB+B \\ \vdots \\ \sum_{i=0}^{P-1} A^i B \end{bmatrix} u_p(k-1) \\ &+ \begin{bmatrix} B & & & \\ AB+B & B & & \\ \vdots & \vdots & \ddots & \\ \sum_{i=0}^{M-1} A^i B & \sum_{i=0}^{M-2} A^i B & \cdots & B \\ \sum_{i=0}^{M} A^i B & \sum_{i=0}^{M-1} A^i B & \cdots & AB+B \\ \vdots & \vdots & & \vdots \\ \sum_{i=0}^{P-1} A^i B & \sum_{i=0}^{P-2} A^i B & \cdots & \sum_{i=0}^{P-M} A^i B \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+M-1) \end{bmatrix} \end{aligned} \tag{12}$$

The prediction of the output Y_c in Equation (4) is replaced by $\hat{Y}_c$.

$$\hat{Y}_c(k) = \hat{Y}_0(k) + S_{\Delta U}\Delta U(k) \tag{13}$$

where $\hat{Y}_0(k) = S_x\hat{x}(k) + S_u u_p(k-1)$.

The objective function (7) of control performance J is replaced by $\hat{J}$.

$$\hat{J}(k) = \Delta U(k)^{\mathrm{T}} H \Delta U(k) + 2\hat{F}^{\mathrm{T}}\Delta U(k) \tag{14}$$

where $\hat{F} = S_{\Delta U}^{\mathrm{T}} Q \hat{Y}_z(k)$, $\hat{Y}_z(k) = \hat{Y}_0(k) - Y_s(k)$.

The constraints of inputs and outputs in Equation (9) are replaced by

$$D_1\Delta U(k) \le \hat{d}_1, \quad D_2\Delta U(k) \le \hat{d}_2 \tag{15}$$

where

$$\hat{d}_1 = \begin{bmatrix} U_{\max} - U_p(k-1) \\ U_p(k-1) - U_{\min} \end{bmatrix}, \quad \hat{d}_2 = \begin{bmatrix} Y_{\max} - \hat{Y}_0(k) \\ \hat{Y}_0(k) - Y_{\min} \end{bmatrix}, \quad U_p(k-1) = \begin{bmatrix} u_p(k-1) \\ \vdots \\ u_p(k-1) \end{bmatrix}.$$

The control law can be obtained by solving the quadratic programming problem

$$\begin{aligned} &\min_{\Delta U(k)} \hat{J}(k) = \tfrac{1}{2}\Delta U(k)^{\mathrm{T}} 2H \Delta U(k) + 2\hat{F}^{\mathrm{T}} \Delta U(k) \\ &\text{s.t.} \quad \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \Delta U(k) \le \begin{bmatrix} \hat{d}_1 \\ \hat{d}_2 \end{bmatrix} \end{aligned} \tag{16}$$

Equations (3), (4), (7), (9), (10) are replaced by Equations (12)-(16). The control action $u(k-1)$ in model predictive control algorithm is replaced by $u_p(k-1)$.

The control law is obtained by minimizing the quadratic programming problem (16). As seen in Figure 2(c), the control action actually applied to process $u_p(k-1)$ is taken into the control law calculation of the next step through input feedback.

4. Application in the Nonideal PID Control Problem.

4.1. The introduction of the nonideal PID control problem. The output of MPC controller u is applied to the process via the inner PID control loop. In the theory study of MPC, it assumes that the inner PID control loop is an ideal control which accounts for $u = u_p$. However, as seen in Figure 3, the complete response time of inner PID control loop is always longer than the sample time of the outer MPC loop, the output of MPC controller cannot be applied to the process completely, and it will cause nonideal control. In the actual processes, the output of MPC controller u is not be implemented by the inner PID control loop while the output u_p of the inner PID control loop is the actually implementation control action applied to the process, as seen in Figure 6. MPC does not measure the information of inner PID control loop in actual process. The control law is calculated in each step according to the ideal PID control. In the nonideal PID control situation, there is a deviation of manipulated variables between prediction model (2) and the actual system, the response of actual system may deviate from the expected output. The problem can be solved by using the predictive control method with input feedback structure proposed in the previous section.

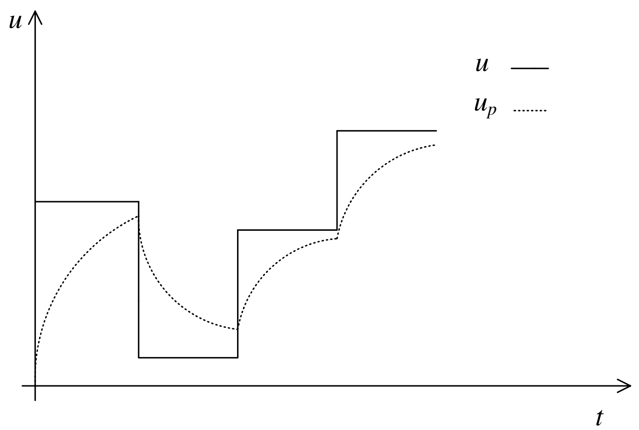


Figure 6. The output difference between MPC and actuator in nonideal control

4.2. **The simulation for the nonideal PID control problem.** In order to demonstrate the effectiveness of the proposed input feedback MPC in the nonideal PID control problem, the top product quality control of atmospheric tower is used as a simulation case. The linear discrete state space model of the controlled process is (see [22])

$$\begin{cases} X(k+1) = \begin{bmatrix} 0.8547 & 0.012 & 0 & 0.0752 \\ 0.242 & 0.6962 & -0.0284 & 0.0673 \\ 0.1076 & -0.0361 & 0.8863 & 0 \\ 0.217 & 0.0081 & -0.042 & 0.7703 \end{bmatrix} X(k) + \begin{bmatrix} -0.0949 \\ -0.0671 \\ -0.0401 \\ 0 \end{bmatrix} u(k) \\ y(k) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} X(k) \end{cases} \tag{17}$$

where, $X(k) = \begin{bmatrix} x_1(k) & x_2(k) & x_3(k) & x_4(k) \end{bmatrix}^{\mathrm{T}}$, $x_1(k)$, $x_2(k)$ and $x_3(k)$ represent the 11st, 13th, 15th tower plates temperature. $y(k) = x_4(k)$ represents the product density. $u(k)$ represents the cold reflux. Linear model is obtained at the steady-state point (the 11, 13, 15 tower plates temperature are 131°C, 154°C and 190°C), and the product density is $0.776 \times 1000\mathrm{kg} \cdot \mathrm{m}^{-3}$, the cold reflux is $127.5\mathrm{m}^3 \cdot \mathrm{h}^{-1}$, the sample time is $T_s = 60s$.

The initial state $x(0) = [0.01, 0.03, 0.05, 0.05]^{\mathrm{T}}$, $u(0) = 0$. The aim of the control system is to control the product density y at the setpoint 0.3 by the manipulated variable u. The constraint of the manipulated variable is $u \in [-3, 3]$, and the constraint of the output variable is $y \in [-0.05, 0.35]$. The manipulated variable of outer MPC loop is the setpoint of cold reflux. The inner PID control loop is flow control system. The matlab platform is used for simulation, the MPC controller is written with S-function, the sample time of the outer loop is 60s, and the inner loop is 0.5s. The simulation results are seen in Figure 7.

From Figure 7, we can see that, under the condition of nonideal PID control, the MPC without input feedback is of the slow dynamic response and the steady-state error, but the MPC with input feedback has the better control effect and no steady-state error.

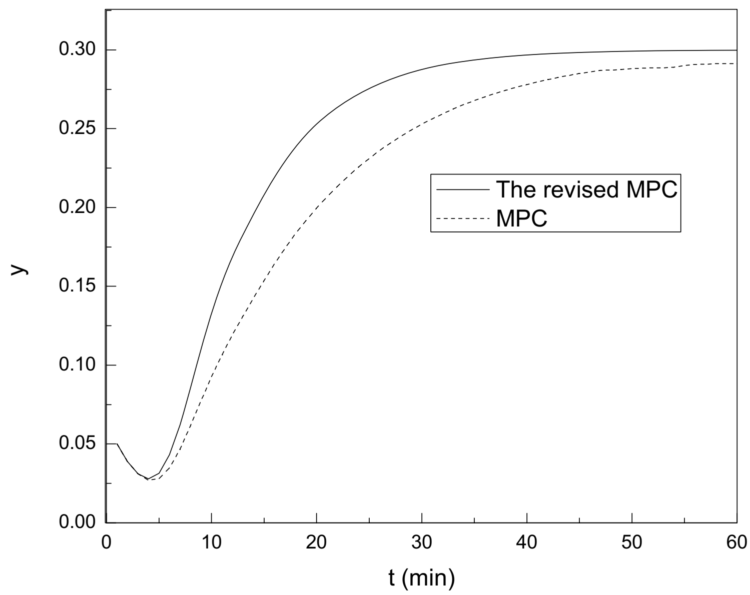


Figure 7. The control effect curve

5. Application in the False Loose Constraints Problem.

5.1. The introduction of the false loose constraints problem. Assuming the prediction model is accurate, nominal constraints of input and output variables are $u_{\min} \leq u \leq u_{\max}$ and $y_{\min} \leq y \leq y_{\max}$. At the moment $k-2$ $(k \geq 2)$, the control law is $\Delta u^*(k-2)$, and the control action $u^*(k-2) = u(k-3) + \Delta u^*(k-2)$ is downloaded to the inner PID control loop as setpoint. At the moment $(k-1)$, if the previous MPC output control action $u^*(k-2)$ equals the control action actually applied to the unit $u_p^*(k-2)$, the nominal constraints are correct. In the next moment k, the information of the implementation of the previous control law $\Delta u^*(k-1)$ is measured. If $\Delta u^*(k-1) > 0$, the measured actual control action increment $\Delta u_p(k-1) < \Delta u^*(k-1)$, or if $\Delta u^*(k-1) < 0$, the measured actual control action increment $\Delta u_p(k-1) > \Delta u^*(k-1)$, the nominal constraints are false loose constraints. Input feedback method is used to solve this problem. When the false loose constraints problem happen, $u_p(k-1)$ instead of $u(k-1)$ in MPC algorithm is used as introduced in Section 3, and constraints are adjusted as follows:

Constraints of input variables $u_{\min} \leq u \leq u_{\max}$

$$u_{\min} \leq u \leq u_{p,\max} \text{ or } u_{p,\min} \leq u \leq u_{\max} \tag{18}$$

Constraints of upper limit and lower limit $u_L \leq u \leq u_H$

$$u_{p,L} \leq u \leq u_{p,H} \tag{19}$$

On the basis of constraints (15) $\hat{d}_1$, coefficient matrix of the input variables d_1 is modified as

$$\hat{d}_{p1} = \begin{bmatrix} U_{p,\max} - U_p(k-1) \\ U_p(k-1) - U_{p,\min} \end{bmatrix} \tag{20}$$

When $\Delta u^*(k-1) > 0$, if $\Delta u_p(k-1) < \Delta u^*(k-1)$, upper limit of nominal constraints $u_{\max}$ is too upper, and it is modified to $u_{p,\max}$. When $\Delta u^*(k-1) < 0$, if $\Delta u_p(k-1) > \Delta u^*(k-1)$, lower limit of nominal constraints $u_{\min}$ is too lower, and it is modified to $u_{p,\min}$. So the calculation of control law is according to the modified constraints. According to the algorithm for feasibility analysis and constraints adjustment of constrained optimal control [23], when the false loose constraints problem happens, the feasibility analysis for the model predictive control is according to the modified constraints of upper limit and lower limit $u_{p,L} \leq u \leq u_{p,H}$, and the weight of u is adjusted to maximum.

In summary, when the false loose constraints problem happens, in order to guarantee the performance of the control system, three aspects as follows need to be adjusted.

1) The information of the previous control action $u(k-1)$ is modified to the control action $u_p(k-1)$ that actually applied to the process.

2) Constraints of input variables $u_{\min} \leq u \leq u_{\max}$ is modified to the actual hard constraints of $u_{\min} \leq u \leq u_{p,\max}$ or $u_{p,\min} \leq u \leq u_{\max}$.

3) The upper limit and lower limit u_H, u_L are modified to $u_{p,H}$, $u_{p,L}$, and the weight of u is adjusted to maximum.

5.2. The simulation for the false loose constraints problem. In order to justify the proposed MPC algorithm for the false loose constraints problem, we consider the isothermal continuous stirred-tank reactor (CSTR) undergoing reaction A→B. The reactor has constant volume and its dynamics are described by (see [24])

$$\begin{aligned} \dot{x}_1 &= -k_1 x_1 - k_3 x_1^2 - x_1 u \\ \dot{x}_2 &= k_1 x_1 - k_2 x_2 - x_2 u \end{aligned} \tag{21}$$

which models the Van de Vusse series of reactions. There are one manipulated variable, two state variables and two output variables in the process model: the output variables

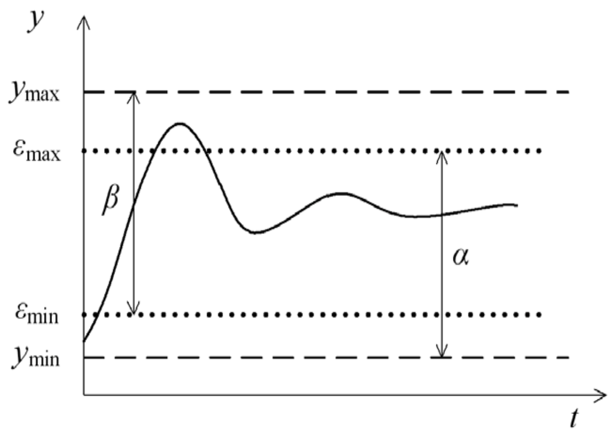


FIGURE 8. MPC based on zone control

are concentration of A, $y_1(x_1)$, concentration of B, $y_2(x_2)$, the manipulated variable are the dilution (feed) rate u. We linearize (21) around this desired steady-state point and discretize the result with a sampling time of 0.002min.

$$\begin{cases} x(k+1) = \begin{bmatrix} 0.95123 & 0 \\ 0.08833 & 0.81873 \end{bmatrix} x(k) + \begin{bmatrix} -0.0048771 \\ -0.0020429 \end{bmatrix} u(k) \\ y(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(k) \end{cases} \tag{22}$$

A simulation is performed that is started from $x(0) = [0.5, 0.1]^{\mathrm{T}}$. The zone-control algorithm is used here, and optimization variables α, β is given here. As seen in Figure 8, in the zone-control algorithm, output variables y is not a setpoint but in the zone $[y_{\min}, y_{\max}]$. And the more desirable zone is $[\varepsilon_{\min}, \varepsilon_{\max}]$.

The cost function (5) is replaced by (see [25,26])

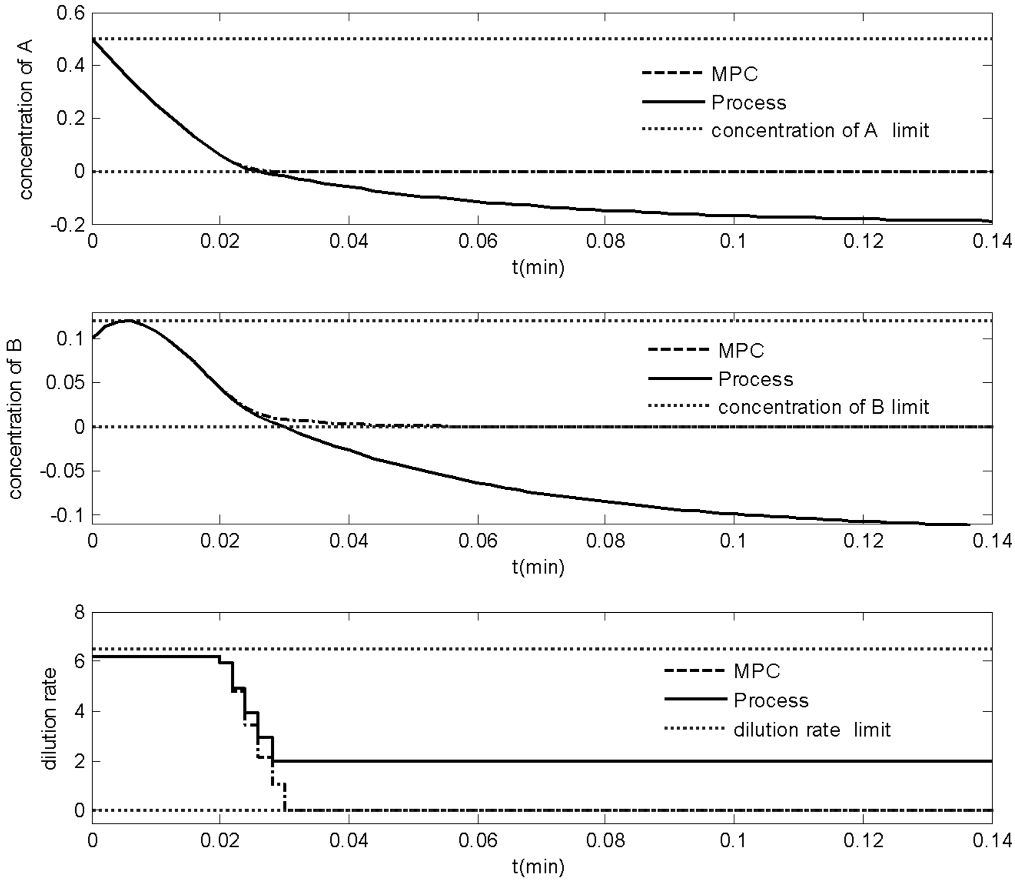
$$V(k) = \|Y_c(k) - \alpha(k)\|^2_{Q_1} + \|Y_c(k) - \beta(k)\|^2_{Q_2} + \|\Delta U(k)\|^2_R \tag{23}$$

The aim of the control system is the output variables that are controlled in the zone $y_1 \in [0, 0.5]$, $y_2 \in [0, 0.12]$ by the manipulated variable u. The constraint of the manipulated variable is $[0, 6.5]$, and the constraints of the output variables are $[-1, 1]$. Assume that the hard constraints of the manipulated variable in the inner loop is not 0 but 2. For the false loose constraints problem, we used Matlab software for the study, the simulation curves are seen in Figure 9.

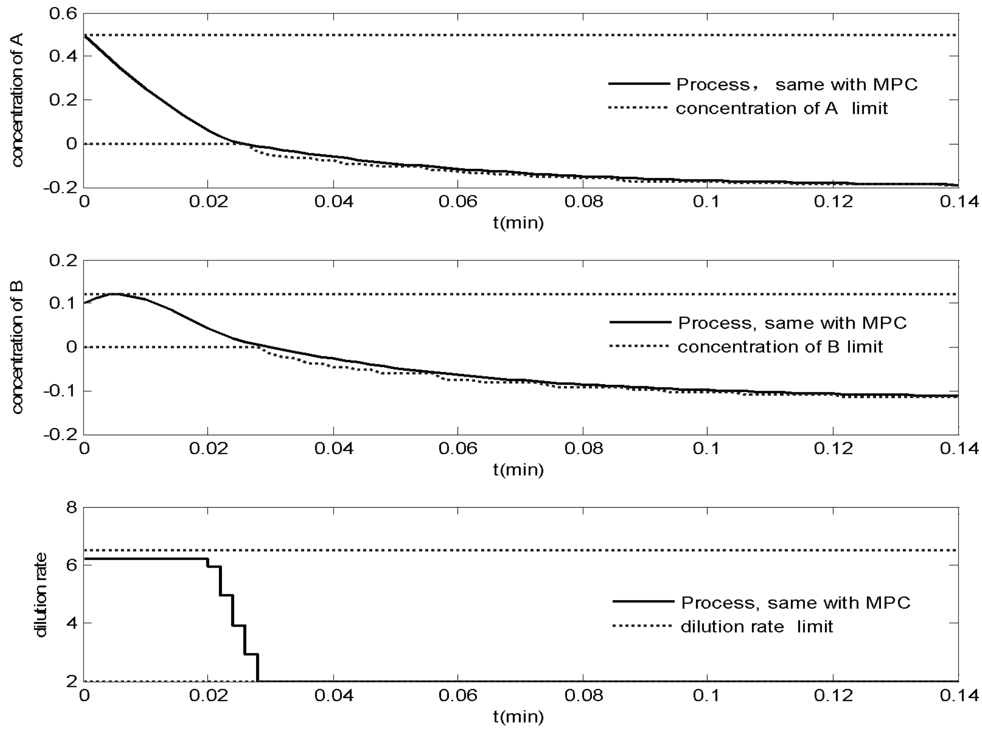
The MPC without input feedback is shown in Figure 9(a). If the MPC output is less than hard constraint 2, the control action applied to the process does not equal the MPC output, the control action applied is actually the hard constraint 2. It will lead to the difference between the actual outputs and the predicted outputs and then the actual outputs may exceed the constraints.

The MPC with input feedback is shown in Figure 9(b). When the false loose constraints happen, u is replaced by u_p in the modified prediction model. The hard constraint of the manipulated variable u is modified to 2, and the weight of u is adjusted to maximum. Based on the revised information, a feasibility analysis is carried out by MPC, and the unfeasible constraints of outputs will be relaxed, then the optimal control law is solved based on the modified constraints. Thus the constraints of inputs and outputs can be guaranteed.

6. **Conclusions.** This paper gives the implementation analysis of manipulated variables in model predictive control. Then to solve two problems in model predictive control:



(a)



(b)

FIGURE 9. Simulation results. (a) The common MPC algorithm, (b) the modified MPC algorithm.

nonideal bottom PID control and false loose constraints. A modified MPC algorithm with input feedback structure has been proposed. During the calculation of control law, the previous actual implementation values of manipulated variables are used to replace the previous theoretical calculation values through input feedback. The simulation results demonstrate the effectiveness of the proposed input feedback control strategy.

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