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STABILITY ANALYSIS OF LONGITUDINAL CONTROL OF A PLATOON OF VEHICLES BY CONSIDERING LAGS SUBJECT TO COMMUNICATION DELAY

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Abstract. *The problem of controlling a string of vehicles moving in one dimension is considered so that they all follow a lead vehicle with a constant spacing between successive vehicles. Due to practical design and implementation, stability and string stability of a platoon of vehicles in the simultaneous presence of lags, communication delay are investigated. Since the delays for transmitting signals will not be fixed, the communication time varying delay is considered. A sufficient condition for the stability criteria of the addressed problem is obtained. This paper provides a practical means to evaluate a platoon system by applying a hierarchical platoon controller and provides a reasonable proposal to design the controller. Finally, a simulation example of multiple vehicle platoon control with the effect of communication time varying delays is exploited to demonstrate the effectiveness of the proposed model and method.*
Keywords: Automated highway vehicle, Delay effect, Stability, String stability

1. **Introduction.** Highway congestion is imposing an intolerable burden on urban residents. Increasing in traffic density causes the traffic congestion on the roads, which will reduce travel time, traffic safety, air pollution, and energy consumption. There are various approaches to reduce congestion such as vehicle platoon control. One possible solution to this problem is to use the Adaptive Cruise Control (ACC) concept. ACC system has been proposed as an enhancement over existing cruise controllers on ground vehicles.

Large scale systems have been object of attention in system and control theory since the seventies [1]. The control theory community has put much effort into the development of methods for controlling vehicle formations. This interest is due to the variety of applications that have been made possible by modern technological advances, for example, in satellite formation flying [2], car platoons [3] or unmanned aerial vehicles [4,5].

This is a problem that is of primary interest to formation control applications, especially to platoons of vehicles. Considerable amount of research has been conducted on robustness to disturbance and stability issues of double integrator networks. Due to practical design and implementation, the actuator lag must be considered [6-8]. Xiao and Gao investigated the effect of self delayed information, from the command (throttle input and brake pedal) to the torque available at the tires, on string stability and did not consider the effect of communication delays and their effect on the stability [7].

For multi agent systems, many properties may become significant in addition to the usual stability and robustness analysis such as string stability must be considered. The term string stability was initially introduced in 1974 [9]. String stability explains how errors are propagated through the group of vehicles as a result of disturbances or the reference trajectory of the formation lead. String stability has been widely used in automated highway systems [10-12]. The relationship between string stability and spacing policies has been an important issue [13,14]. There are two major strategies for controlling of a platoon of vehicles, constant time headway control and spacing distance control. In spacing distance control, the distance between two adjacent vehicles in the platoon is controlled to be the same as a pre specified constant, independent of the velocity in the platoon travelling. Since constant spacing strategies are usually used due to the very high achievable traffic capacity, the desired trajectory of the platoon is considered to be of a constant-velocity type.

The control system for the vehicle platoon consists of the guidance model describing interaction between vehicles and the individual vehicle control. Among various cooperative control strategies, consensus algorithm has been broadly studied and extensively used in the multi-agent system, to guarantee that all agents reach an agreement on the convergence point. The research on the consensus algorithm can be referred to [15-18] and references therein. In the past, researchers have investigated the effects of communication structures, or information flow, on system stability using graph theory [19-21]. This paper is inspired by consensus algorithm that derived the states to a common value. Therefore, by adding an offset term to states, the desired inter vehicle spacing is achieved. Due to practical design, the actuator lag is considered while the multi-agent system is applied only in double integrator model. Hence, this makes a novel contribution.

Because of serious challenges induced by the communication networks, such as delays and packet dropouts [22-24], special attention is required when synthesizing the stabilizing control system. Due to limited bandwidth and possible data collisions, transmission delay in a networked control system is unavoidable. The stability analysis of such systems becomes quickly intractable as the number of agents increases and the delays enlarge. It has now been well known that the existence of time delays is commonly encountered in many dynamic systems, and time delay has become an important source of instability and performance degradation [25,26]. Time delays resulting from interconnection links have also been paid much attention to multi-agent systems because of the practical background [27,28].

Despite the huge amount of relevant literature to date, there is no apparent stability analysis methodology for platoon of vehicle with respect to the communication time varying delay. The purpose of this paper is to analyze stability and string stability by considering the transmission delay and the lags. Hence, this paper provides a practical means to evaluate a platoon system. The earlier analysis of homogenous string stability is conducted without consideration of these constraints. Also, the most studies considered the control law whereby the controller uses the relative distance and velocity information with respect to the preceding vehicle [5-10]. However, the control law is considered so that it only uses the relative distance for controlling a string of vehicles.

The remainder of the paper is structured as follows. Section 2 briefly introduces the longitudinal vehicle model that considers the lags. Section 3 deals with the stability analysis. Section 4 communication structures is described. Section 5 presents an analysis of string stability. In Section 6 simulations are presented to show the efficiency of the proposed method. Section 7 gives the conclusion.

2. **Vehicle Model.** Considering a group of vehicles in dense traffic will not be overtaken and assume each vehicle uses only the information of the preceding vehicle. The formation control of $N+1$ homogeneous string of vehicles is considered so that they all follow a lead vehicle, as shown in Figure 1.

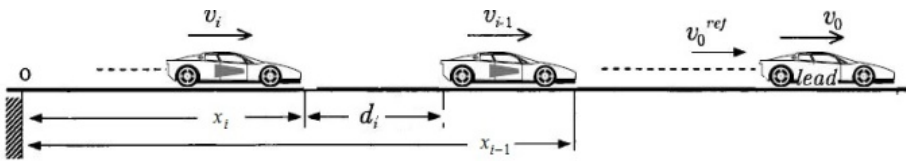

FIGURE 1. 1-D of vehicle string

The position, velocity and acceleration of lead vehicle are denoted by $x_0(t)$, $v_0(t)$, $a_0(t)$, respectively. $x_i(t)$, $v_i(t)$, $a_i(t)$ denote the position, velocity and acceleration of the i-th vehicle, respectively.

A model for the motion of a vehicle must take into account the power train, tire forces, aerodynamic drag forces, rolling resistance forces and gravitational forces. The power train consists of the internal combustion engine, the torque converter, the transmission and the wheels [29]. The dynamics of the i-th following vehicle can be modeled by the following non-linear differential equations [30-32].

$$\dot{a}_i = f_i(v_i, a_i) + g_i(v_i)c_i \tag{1}$$

where c_i is the engine input and $f_i(v_i, a_i)$ and $g_i(v_i)$ are given by

$$\begin{aligned} f_i(v_i, a_i) &= -\frac{1}{\tau_i}\left(a_i + \frac{\sigma A_i c_{di}}{2m_i}v_i^2 + \frac{d_{mi}}{m_i}\right) - \frac{\sigma A_i c_{di} v_i a_i}{m_i} \\ g_i(v_i) &= \frac{1}{\tau_i m_i} \end{aligned}$$

where σ is the specific mass of the air, A_i, c_{di}, d_{mi}, m_i and τ_i are the cross-sectional area, drag coefficient, mechanical drag, mass and engine time constant of the i-th vehicle, respectively, $\sigma A_i c_{di}/2m_i$ stands for the air resistance. Note that the vehicles considered here are not necessarily identical.

The following control law has been adopted

$$c_i = u_i m_i + 0.5\sigma A_i c_{di} v_i^2 + d_{mi} + \tau_i \sigma A_i c_{di} v_i a_i \tag{2}$$

where u_i is the additional input signal to be designed. Obviously, this control law can be achieved by feedback linearization, since after introducing (2), the third equation in (1) becomes

$$\tau_i \dot{a}_i + a_i = u_i \tag{3}$$

The feedback linearization controller in (2) plays the role of the first layer controller in our architecture. It helps to simplify the system model by excluding some characteristic parameters of the vehicle from its dynamics.

For simplicity, all the vehicles are supposed to be identical, although this assumption may be neither entirely necessary to the discussion nor satisfied in practice. Therefore,

$$\tau_i = \tau, \quad \forall i = 1, \ldots, N$$

3. **Stability Analysis.** Before analysis of any aspect of a platoon, it is important that the stability of a platoon subject to communication time varying delays is analyzed. In this paper we focus on the approach where it is assumed that every vehicle of the formation has identical dynamics and is controlled locally. In this situation, the formation can be schematized as a graph that mimics the information flow, and it is possible to prove a formation stability criterion which requires the evaluation of systems of the order of the agents, instead of the full order formation. Graph theory is mentioned as a way of interpreting the systems which are object of this paper. Actually graph theory can be

also of further use in this situation, as it can give guidelines in the choice of the pattern matrices. It is out of the scope of this article to give a complete account of graph theory.

Graph Theory

For a group of N vehicles, the interaction for all vehicles can be naturally modeled by a directed graph $g = (v, w)$, where $v = \{v_1, v_2, \ldots, v_N\}$ and $w \subseteq v^2$ represent, respectively, the vehicle set and the edge set. Each edge denoted as (v_i, v_j) means that vehicle j can access the state information of vehicle i, but not necessarily vice versa. Accordingly, vehicle i is a neighbor of vehicle j. An edge is undirected if $(v_i, v_j) \in w$ implies $(v_j, v_i) \in w$. We use N_i to denote the neighbor set of vehicle j. A directed path is a sequence of edges in a directed graph of the form $(v_1, v_2), (v_2, v_3), \ldots$, where $v_i \in v$. A directed graph has a directed spanning tree if there exists at least one vehicle that has a directed path to any other vehicle.

The interaction graph can be mathematically represented by two matrices: the adjacency matrix $A = [a_{ij}] \in R^{n\times n}$ with $a_{ij} > 0$ if $(v_j, v_i) \in w$ and $a_{ij} = 0$ otherwise, and the (non-symmetric) Laplacian matrix $L = [l_{ij}] \in R^{n\times n}$ with $l_{ii} = \sum\limits_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. Here we assume that $a_{ii} = 0$, $\forall i = 1, \ldots, N$. For undirected graphs, we assume that $a_{ij} = a_{ji}$. It is easy to verify that it has at least one zero eigenvalue with a corresponding eigenvector 1, where 1 is an all-one column vector with a compatible size. A path in a directed graph is a sequence $v_0, v_1, \ldots, v_f$ of distinct nodes such that (v_{j-1}, v_j) is an edge for $j = 1, 2, \ldots, f$, $f \in Z^+$. There exists a path from node v_i to node v_j, we say that v_j is reachable from v_i. If a node v_i is reachable from every other node in g, then we say it is globally reachable. The following definitions are borrowed from [33].

Definition 3.1. *For a group of N vehicles, a vehicle is called a leader if the vehicle has no neighbor. A vehicle is called a follower if the vehicle has a neighbor.*

To study a leader-following problem, we also concern another graph $\bar{g}$ associated with the system consisting of N agents and one leader (labeled 0). Similarly, we define a diagonal matrix $B \in R^{n\times n}$ to be a leader adjacency matrix associated with $\bar{g}$ with diagonal elements b_i, where $b_i = a_{i0}$ for some constant $a_{i0} > 0$ if node 0 (i.e., the leader) is a neighbor of node i and $b_i = 0$ otherwise.

3.1. **Control law.** With the help of graph theory, the robust consensus stability of the multi-agent system with communication delays is obtained. In multi-agent consensus applications, group of agents need to agree upon certain quantities of interest that depends on the state of all the agents [34], which is a kind of interesting collective phenomena in physics, nature and society [35].

String stability cannot be guaranteed when constant spacing policy is used. Therefore, for achieving string stability condition, we need information of the leading vehicle. Hence, a centralized control law is considered whereby this controller only uses the relative distance information with respect to the preceding vehicle as well as velocity information of the lead vehicle.

Due to the coupling delays, each agent cannot instantly get the information from others or the leader. Thus, for agent i $(i = 1, \ldots, N)$, a neighbor-based coupling rule can be expressed as follows:

$$\begin{aligned} u_i(t) = K &\left(\sum_{j\in N_i} a_{ij}(x_j(t-r) - x_i(t-r)) \right) \\ &+ K\left(b_i(x_0(t-r) - x_i(t-r))\right) + D(v_0 - v_i(t)) \end{aligned} \tag{4}$$

where the communication time-varying delay $r(t) > 0$ is a continuously bounded differentiable function with

$$0 < r(t) < \beta$$

The input can be written in a matrix form:

$$u = -K(L+B)x(t-r) - D(v - v_0 1) + KB1x_0(t-r) \tag{5}$$

We will demonstrate the convergence of the dynamics system; that is, $x_i \to x_0$ $v_i \to v_0$ as $t \to \infty$.

3.2. **Vehicle offsets.** The goal of the controller as defined above is to drive the states to a common value. In this problem definition, we do not concern about the final value so long as the vehicles share it. For some applications, such as orienting underwater vehicles, this is an understandable goal. For other applications, such as relative satellite positioning, it is necessary to add an offset term to states to achieve the desired inter vehicle spacing. An offset function d is defined which describes the inter vehicle spacing.

We assume that d is defined so that for all $i, j, k, d_{ij} + d_{jk} = d_{ik}$. One way to generate such a function is to define an offset d_{0i} for each vehicle relative to an arbitrary reference.

As mentioned before, the way is considered that all vehicles follow a lead vehicle with a constant spacing between successive vehicles. Consequently for achievement the propose, we consider d_{0i} as the following form

$$d_{0i} = \sum_{j=1}^{i} (D_{j-1,j} + L_{j-1}) \tag{6}$$

The desired geometry of the platoon is specified by the desired spaces $D_{i-1,i}$ for $i = 1, 2, \ldots, N$ where $D_{i-1,i}$ is the desired value of $x_{i-1}(t) - x_i(t) - L_{i-1}$, L_{i-1} is the length of $i-1$ vehicle. The control objective is to preserve a rigid formation, i.e., to make neighboring vehicles keep their pre-specified desired spaces and to make agent 1 follow its desired trajectory $x_0^*(t) - D_{0,1} - L_0$.

Therefore, a centralized control law is considered as the following form:

$$\begin{aligned} u_i(t) = &K\left(\sum_{j\in N_i} a_{ij}(x_j(t-r) - x_i(t-r) - d_{ji})\right) \\ &+ K\left(b_i(x_0(t-r) - x_i(t-r) - d_{0i})\right) + D(v_0 - v_i(t)) \end{aligned} \tag{7}$$

The goal of the controller as defined above is to drive the states to follow a lead vehicle with a desired spacing between successive vehicles.

3.3. **Convergence analysis.** In this section, we will focus on the convergence analysis of group vehicles.

We assume the leader has the desired constant velocity (i.e., v_0). The desired trajectory of the i-th vehicle is

$$x_i^*(t) = x_0^* - D_{0,i} - \sum_{j=l}^{i-1} L_j = x_0^* - \sum_{j=1}^{i} (D_{j-1,j} + L_{j-1}) \tag{8}$$

To facilitate analysis, the following tracking error has been defined:

$$\tilde{x}_i = x_i - x_i^* \quad \to \quad \dot{\tilde{x}}_i = \dot{x}_i - \dot{x}_i^* \quad \to \quad \ddot{\tilde{x}}_i = \ddot{x}_i \tag{9}$$

Substituting (9) into (7), and using $x_{i-1}^* - x_i^* = D_{i-1,i} - L_{i-1}$ we get

$$u_i(t) = K\left(\sum_{j\in N_i} a_{ij}\left(\tilde{x}_j(t-r) - \tilde{x}_i(t-r)\right)\right) + K\left(b_i\left(-\tilde{x}_i(t-r)\right)\right) + D\left(-\dot{\tilde{x}}_i(t)\right) \tag{10}$$

Combining the open loop dynamics (3) with the control law (10), we get

$$\tau_i \dot{\tilde{x}}_i(t) + \ddot{\tilde{x}}_i(t) = K\left(\sum_{j\in N_i} a_{ij}\left(\tilde{x}_j(t-r) - \tilde{x}_i(t-r)\right)\right) + K\left(b_i\left(-\tilde{x}_i(t-r)\right)\right) + D(-\dot{\tilde{x}}_i(t)) \tag{11}$$

Because 1 is eigenvector of Laplacian matrix, the dynamics system can be rewritten as the following form

$$\dot{\varepsilon} = C\varepsilon(t) + E\varepsilon(t-r) \tag{12}$$

where

$$\varepsilon(t) = \begin{pmatrix} \tilde{x} \\ \dot{\tilde{x}} \\ \ddot{\tilde{x}} \end{pmatrix}, \quad C = \begin{bmatrix} 0_{N\times N} & I_N & 0_{N\times N} \\ 0_{N\times N} & 0_{N\times N} & I_N \\ 0_{N\times N} & -DI_N/\tau & -I_N/\tau \end{bmatrix},$$

$$E = \begin{bmatrix} 0_{N\times N} & 0_{N\times N} & 0_{N\times N} \\ 0_{N\times N} & 0_{N\times N} & 0_{N\times N} \\ -KH/\tau & 0_{N\times N} & 0_{N\times N} \end{bmatrix}, \quad H = L + B$$

where I_N and $0_{N\times N}$ are the identity matrix and the zero matrix of dimension N, respectively.

Before the discussion, we introduce some basic concepts or results for time-delay systems [36].

Consider the following system:

$$\begin{cases} \dot{x} = f(x_t) \\ x(\theta) = \varphi(\theta), \ \theta \in [-\beta, 0] \end{cases} \tag{13}$$

where $x_t(\theta) = x(t+\theta)$, $\forall\theta \in [-\beta, 0]$ and $f(0) = 0$.

Lemma 3.1. ***(Lyapunov-Razumikhin Theorem).*** *Let φ_1, φ_2 and φ_3 be continuous, nonnegative, nondecreasing functions with $\varphi_1(s) > 0$, $\varphi_2(s) > 0$, $\varphi_3(s) > 0$ for $s > 0$ and $\varphi_1(0) = \varphi_2(0) = 0$. For system (12), suppose that the function $f : C([-\beta, 0], R^n) \to R$ takes bounded sets of $C([-\beta, 0], R^n)$ in bounded sets of R^n. If there is a continuous function $V(x,t)$ such that*

$$\varphi_1(\|x\|) \le V(t,x) \le \varphi_2(\|x\|), \quad t \in R, \quad x \in R^n \tag{14}$$

In addition, there exists a continuous nondecreasing function $\varphi(s)$ with $\varphi(s) > s$, $s > 0$ such that

$$\begin{aligned} &\dot{V}(t,x)|_{\textit{the dynamics system}} \le -\varphi_3\left(\|x\|\right), \\ &\textit{if} \ \ V(t+\theta, x(t+\theta)) < \varphi(V(t, x(t))), \quad \theta \in [-\beta, 0] \end{aligned} \tag{15}$$

Then the solution $x = 0$ is uniformly asymptotically stable.

$V(x,t)$ is called Lyapunov-Razumikhin function if it satisfies Equations (14) and (15) in Lemma 3.1.

Lemma 3.2. *The matrix $H = L + B$ is positive stable if and only if node 0 is globally reachable in $\bar{g}$.*

Therefore, if node 0 is globally reachable in $\bar{g}$, H is positive stable, and from Lyapunov theorem, there exists a positive definite matrix $\bar{P} \in R^{n\times n}$ such that

$$\bar{P}H + H^T\bar{P} = I_n \tag{16}$$

Let $Z = \begin{bmatrix} 2(D-1) & 1-\tau \\ 1-\tau & 2(1-\tau) \end{bmatrix}$, $\gamma = \min\{eigenvalue(Z)\}$, $\bar{\mu} = \max\{eigenvalue(\bar{P} HH^T\bar{P})\}$ and $\bar{\lambda} = \min\{eigenvalues(\bar{P})\}$ now we give the main result as follows:

Theorem 3.1. *For system (12), take*

$$K < \frac{2\bar{\mu}}{\gamma\bar{\lambda}} \quad and \quad D > 1 + \frac{1-\tau}{4} \tag{17}$$

Then, when β is sufficiently small

$$\lim_{t\to\infty} \varepsilon(t) = 0$$

If and only if node 0 *is globally reachable in $\bar{g}$.*

Proof: Node 0 is globally reachable, H is positive stable and $\bar{P}$ is positive definite matrix satisfying (16). Take a Lyapunov-Razumikhin function $V(\varepsilon) = \varepsilon^T P\varepsilon$, where

$$P = \begin{bmatrix} \frac{D\bar{P}}{\tau^2} & \frac{\bar{P}}{\tau^2} & \frac{\bar{P}}{\tau} \\ \frac{\bar{P}}{\tau^2} & \frac{D\bar{P}}{\tau^2} & \frac{\bar{P}}{\tau} \\ \frac{\bar{P}}{\tau} & \frac{\bar{P}}{\tau} & \frac{\bar{P}}{\tau} \end{bmatrix} \quad \text{where } D > \tau$$

is positive definite.

Then we continue $\dot{V}(\varepsilon)\big|_{(12)}$.

By Lyapunov-Razumikhin formula [37],

$$\varepsilon(t-r) = \varepsilon(t) - \int_{-r}^{0} \dot{\varepsilon}(t+s)ds = \varepsilon(t) - C\int_{-r}^{0} \varepsilon(t+s)ds - E\int_{-2r}^{-r} \varepsilon(t+s)ds \tag{18}$$

Thus, from $E^2 = 0$, the delayed differential Equation (12) can be rewritten as

$$\dot{\varepsilon} = F\varepsilon - EC\int_{-r}^{0} \varepsilon(t+s)ds \tag{19}$$

where $F = C + E$.

Note that $2a^Tb \le a^T\psi a + b^T\psi^{-1}b$ holds for any appropriate positive definite matrix ψ. Then, with $a = C^TE^TP\varepsilon$, $b = \varepsilon(t+s)$ and $\psi = P^{-1}$ then

$$\begin{aligned} \dot{V}\big|_{(12)} &= \varepsilon^T(F^TP + PF)\varepsilon - 2\varepsilon^TPEC\int_{-r}^{0} \varepsilon(t+s)ds \\ &\le \varepsilon^T(F^TP + PF)\varepsilon + r\varepsilon^TPECP^{-1}C^TE^TP\varepsilon + \int_{-r}^{0} \varepsilon^T(t+s)P\varepsilon(t+s)ds \end{aligned}$$

Take $\varphi(s) = qs$ for some constant $q > 1$. In the case of

$$V(\varepsilon(t+\theta)) < qV(\varepsilon(t)), \quad -\tau \le \theta \le 0$$

we have

$$\dot{V} \le -\varepsilon^TQ\varepsilon + r\varepsilon^T(PECP^{-1}C^TE^TP + qP)\varepsilon$$

where

$$Q = -(F^T P + PF) = \begin{bmatrix} \frac{K}{\tau^2} I_n & \frac{K}{\tau^2} H^T \bar{P} & \frac{K}{\tau^2} H^T \bar{P} \\ \frac{K}{\tau^2} \bar{P} H & \frac{2(D-1)}{\tau^2} \bar{P} & \frac{(1-\tau)}{\tau^2} \bar{P} \\ \frac{K}{\tau^2} \bar{P} H & \frac{(1-\tau)}{\tau^2} \bar{P} & \frac{2(1-\tau)}{\tau^2} \bar{P} \end{bmatrix}$$

When delay is sufficiently small, if the matrix Q will be positive definite, then the system (12) is asymptotically stable. To find the condition on control parameters so that the matrix Q become positive definite, we use two following lemmas:

Lemma 3.3. *For any symmetric matrix, M, in the form of*

$$M = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$$

If A is invertible then the following properties hold
$M \succ 0 \, iff \, A \succ 0$ and $C - B^T A^{-1} B \succ 0$ [38].

By using Lemma 3.3, the matrix Q is positive definite if and only if the following condition holds:

$$\frac{1}{\tau^2}\left(\begin{bmatrix} 2(D-1) & 1-\tau \\ 1-\tau & 2(1-\tau) \end{bmatrix} \otimes \bar{P}\right) - \frac{K}{\tau^2}\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \otimes \bar{P} H H^T \bar{P}\right) > 0 \qquad (20)$$

Lemma 3.4. *Let $A \in \Re^{n\times n}$ have eigenvalues λ_i, $i = 1, \ldots, n$ and let $B \in \Re^{m\times m}$ have the eigenvalues μ_j, $j = 1, \ldots, m$. Then the mn eigenvalues of $A \otimes B$ are*
$\lambda_1\mu_1, \ldots, \lambda_1\mu_m, \lambda_2\mu_1, \ldots, \lambda_2\mu_m, \ldots, \lambda_n\mu_m$ [39].

According to Lemma 3.4, the matrix Q is positive definite if and only if the following condition satisfies:

$$\gamma\bar{\lambda} > 2\bar{\mu}K$$

This condition is reasonable if the matrix is positive definite. Thus, Z is positive definite if and only if the following conditions hold:

$$\tau < 1 \text{ and } D > 1 + \frac{1-\tau}{4}$$

Because the lag in the vehicles is always less than one, this condition always holds. Hence, the matrix Z is positive definite if and only if $D > 1 + 0.25\,(1-\tau)$.

$\lambda_{\min}$ denotes the minimum eigenvalue of matrix Q. If we take

$$\tau < \frac{\lambda_{\min}}{\|PECP^{-1}C^T E^T P\| + q\,\|P\|}$$

Then $\dot{V}(\varepsilon) \le -\eta\varepsilon^T\varepsilon$ for some $\eta > 0$, therefore, the conclusion follows by Lemma 3.1.

Here, we complete the proof of the theorem.

4. **Communication Structures.**

A data communication network is assumed to exit among the vehicle. Figure 2 shows the communication structure that is investigated in this paper. In the figure, vehicle 0 is the platoon lead. These communication structures can be extended to N vehicle formation. This structure can be described as directional communication between follower vehicles.

As the communication structure adjacency matrix, Laplacian matrix and leader adjacency matrix are the following form:

$$A = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 \\ -1 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & -1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & 0 \end{bmatrix}$$

FIGURE 2. Communication network in a platoon of vehicles

Communication and control are related namely in networked control systems. Networked control systems are control systems composed of the plant, actuators, sensors, and controllers which are connected to the plant via some forms of communication. To transmit a continuous-time signal over a network, the signal must be sampled, encoded in a digital format, transmitted over the network, and finally the data must be decoded at the receiver side. This process is considerably different from the usual uniform sampling in digital control. The overall delay between sampling and eventual decoding at the receiver can be highly time-varying because both the network access delays. It includes the time taken for a shared network to accept data as well as the transmission delays, i.e., the time during which data are in transit inside the network. It depends on highly variable data transmission conditions such as congestion or channel quality. Generally, two transmission channels appear sensor-to-controller delay and a controller-to-actuator delay. The sensor acts in a time-driven variable manner, while the controller and actuator with the zero order- hold operate in an event-driven manner. It means that the controller and the actuator update their outputs as soon as they receive a new sample. The two transmission channels may be considered as an equivalent to a single transmission channel with the sum of both delays. Therefore, it is supposed that only one transmission channel is available. It means that such a delay is considered as a sensor-to-controller delay.

5. **String Stability Analysis.** In addition to analysis of stability, i.e., convergence analysis of a platoon of vehicles for multi agents system, how errors are propagated through the string due to disturbances has to be considered. The question of string stability/error amplification is usually answered by looking at the transfer functions that relate the spacing errors between two successive vehicle pairs. The string-stability ensures that range errors decrease as they propagate along the vehicle stream. The string stability performance is mainly determined by the structure of the controller and the used headway policy. In other words, the string stability performance is mostly dependent on the information received and used by the longitudinal controllers.

By considering communication structure the control law is the following form

$$u_i(t) = K(\tilde{x}_j(t-r) - \tilde{x}_i(t-r)) + D(-\dot{\tilde{x}}_i) \quad \forall i = 1, \ldots, N \tag{21}$$

The overall schematic diagram of the control system is shown in Figure 3.

The velocity and spacing error dynamics models can be derived based on the vehicle dynamics model (3)

$$\tau_i \tilde{x}_i(t) + \ddot{\tilde{x}}_i(t) = K(\tilde{x}_{i-1}(t-r) - \tilde{x}_i(t-r)) - D\dot{\tilde{x}}_i(t) \tag{22}$$

In the analysis of string stability, the communication time delay is assumed constant and equal to the bounded value of it (i.e., β).

The Laplace transforms can be used for analyzing with the conventional notation and the regular assumption of zero initial conditions for the derivation of transfer functions. Therefore, by taking the Laplace transformation on both sides of (22), we obtain

$$(\tau s^3 + s^2 + Ds + Ke^{-\beta s})\tilde{X}_i(s) = Ke^{-\beta s}\tilde{X}_{i-1}(s)$$

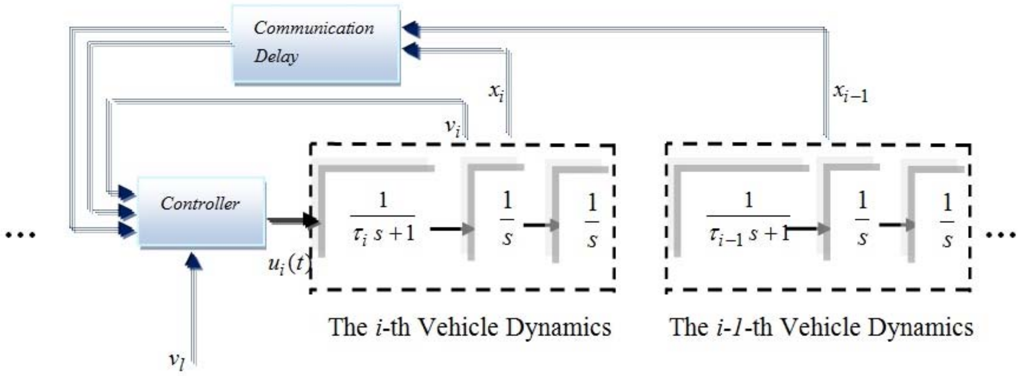

FIGURE 3. Overall control system structure

Therefore,

$$G(s) = \frac{\tilde{X}_i(s)}{\tilde{X}_{i-1}(s)} = \frac{Ke^{-\beta s}}{\tau s^3 + s^2 + Ds + Ke^{-\beta s}} \tag{23}$$

This describes the spacing error dynamics model of two successive vehicles in the string of vehicles.

It is clear that the range error output must be smaller than or equal to the range error input to avoid range errors propagate indefinitely along the string. For this uniform vehicle string, a string-stability definition is widely used [40,41] and is described as follows:

$$|G(j\omega)| \leq 1 \quad \forall \omega > 0$$

where $G(j\omega)$ is derived from the spacing error transform function (23) by substituting $s = j\omega$.

Theorem 5.1. *The condition $|G(j\omega)| \leq 1$ is satisfied for $\forall \omega > 0$ if the following condition holds*

$$K\beta + \sqrt{K^2\beta^2 + 2K} < D < \frac{1}{2\tau} - 0.5K\tau \tag{24}$$

Proof: $|G(j\omega)|$ can be expressed as $|G(j\omega)| = \sqrt{p/q}$ that

$$\begin{aligned} p &= K^2 \\ q &= \tau^2\omega^6 + (1-2D\tau)\omega^4 + D^2\omega^2 + K^2 + 2K\tau\omega^3\sin(\beta\omega) - 2KD\omega\sin(\beta\omega) \\ &\quad - 2K\omega^2\cos(\beta\omega) \end{aligned}$$

The magnitude of $|G(j\omega)|$ is less than 1 if the following condition is satisfied

$$q-p = \tau^2\omega^6 + (1-2D\tau)\omega^4 + D^2\omega^2 + 2K\tau\omega^3\sin(\beta\omega) - 2KD\omega\sin(\beta\omega) - 2K\omega^2\cos(\beta\omega) \geq 0$$

Taking into account the fact that $\sin(r\omega) \leq r\omega$, we obtain

$$q-p \geq \tau^2\omega^6 + (1-2D\tau)\omega^4 + (D^2 - 2KD\beta)\omega^2 + 2K\tau\omega^3\sin(\beta\omega) - 2K\omega^2\cos(\beta\omega)$$

By taking into account this fact $-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}$. Therefore, if the following condition is satisfied, then $q-p$ will be positive. In other word, we considered the worse case.

$$\tau^2\omega^6 + (1-2D\tau)\omega^4 + (D^2 - 2KD\beta)\omega^2 - 2K\omega^2\sqrt{1+\tau^2\omega^2} > 0 \tag{25}$$

The spacing errors have most of their energy at the region of low frequencies, which is also called the key region of the string stability [42]. Hence, at the low frequencies, we can assume $\sqrt{1+\tau^2\omega^2} \cong 1 + 0.5\tau^2\omega^2$.

$$\tau^2\omega^6 + (1-2D\tau-K\tau^2)\omega^4 + (D^2 - 2KD\beta - 2K)\omega^2 > 0 \tag{26}$$

While the coefficients are more than zero, the inequality (26) is satisfied. Therefore, if the coefficients $1-2D\tau-K\tau^2$ and $D^2-2KD\beta-2K$ are positive, then the inequality (26) is hold. By using elementary calculation, we can show these constraints will be satisfied if the following conditions are hold.

$$D < 0.5/\tau - 0.5K\tau \text{ and } D > K\beta + \sqrt{K^2\beta^2 + 2K}$$

Proof of the theorem is completed.

As mentioned earlier, the spacing errors have most of their energy at the region of low frequencies. At the law frequencies, the value of ω is far smaller than the values of the parameters. Hence, the coefficient $D^2 - 2KD\beta - 2K$ is the most important value of the polynomial at low frequency. The negative effect of the communication delay on string stability is shown by Theorem 5.1.

6. Simulation.

The automotive longitudinal control is generally composed by two loops: an inner force (acceleration) control loop which compensates the nonlinear vehicle dynamics (acceleration and brake systems), and an outer inter-distance control loop which is responsible for guaranteeing a good tracking of the desired inter distance reference. The inner control loop, i.e., the throttle/brake control loop, is a nontrivial control problem. The difficulty is the complexity and lack of symmetry of the throttle and brake subsystems. In addition, the vehicle dynamics is highly nonlinear and behaves differently than our idealized. Thus, based on published experimental result [43], we can accept $\tau_i\dot{a}_i + a_i = u_i(t)$. In this paper, we assume an inner controller performance. The values are only chosen for the relevant parameters in the simulation. As mentioned before, we focused on the upper level controller and did not consider the issues related to lower level.

In order to validate the performance of the proposed control algorithm, computer simulations have been carried out for the platoon system contained with ten followers, i.e., $N = 10$. As stated before, the ability of maintaining the distance between vehicles is important for the safety of the vehicle platoon system. The most important disturbances in a platoon control system include the lead vehicle acceleration/deceleration. In other words, a disturbance means any source which ensures that the vehicle string cannot maintain a constant velocity.

In the simulations, the desired vehicle space was set as $D_{i-1,i} = 8\text{m}$ and the length of vehicle as $L_i = 4\text{m}$, other parameters used in the simulations are the same with [6,44], namely, the specific mass of the air $\sigma_i = 1\text{N/m}^3$, the cross sectional area $A_i = 2.2\text{m}^2$, the drag coefficient $c_{di} = 0.35$, vehicle mass $m_i = 1500\text{kg}$, and the mechanical drag $d_{mi} = 150\text{N}$. We assume that, without loss of the generality, the leading vehicle initially travels at a steady-state velocity of $v_{initial} = 20\text{m/s}, a_{initial} = 0\text{m/s}^2$ then; the leading vehicle begins to accelerate to another steady-state velocity $v_{final} = 40\text{m/s}, a_{final} = 0\text{m/s}^2$ at $t = 20\,\text{sec}$ the time varying communication delay $r(t) = 0.03\,|\cos(t)|$.

It is not hard to obtain $\bar{\lambda} = 0.2554$, $\bar{\mu} = 1.5964$ (By solving the Lyapunov Equation (16)). By taking $D = 4.5$, the value of γ is equal to 1.6486. Regarding to (17), if $K < 0.14$ the system is stable.

Regarding to Theorem 3.1, by taking two different control parameters, stability and instability of platoon of vehicles are shown in Figure 4 and Figure 5, respectively.

$\delta_i = x_{i-1} - x_i - \mathit{length\,of\,vehcle} - d_{ref}$ denotes the spacing error of the i-th vehicle which is the deviation between the spacing and desired spacing.

Figure 4 demonstrates the effectiveness of the proposed controller and indicates that we can only use the relative distance for controlling a string of vehicles.

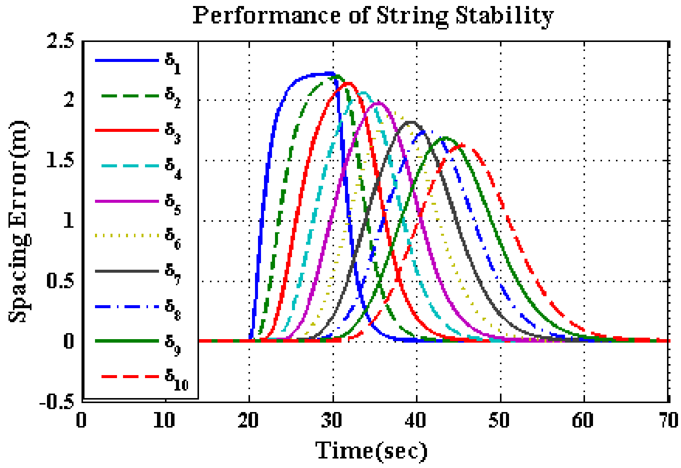

FIGURE 4. Stable behavior for ten followers subject to time varying communication delay by taking $D = 1.9$

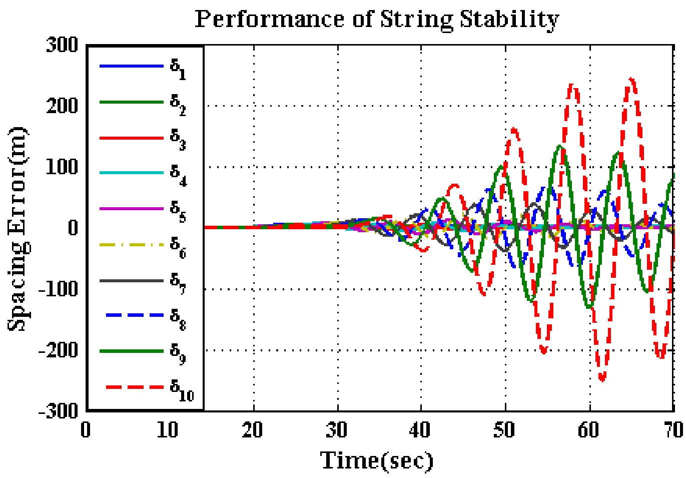

FIGURE 5. Unstable behavior for ten followers subject to time varying communication delay by taking $D = 0.6$

Note the conditions (17) are a sufficient condition on the control parameters. If we choose another P matrix, different conditions will be got. Hence, instability necessarily does not occur if the control parameters do not satisfy the conditions (17).

Next to confirm the results of Theorem 5.1 that express the practical string stable conditions with consideration of the lags and communication delay, several numerical simulations have been conducted. Table 1 presents the relationship between control parameters, the lag and communication delay by three cases.

TABLE 1. Parameters for the platoon

Parameters	Case I	Case II	Case III
τ	0.1	0.1	0.1
β	0.03	0.03	0.03
K	2	2	2
D	2.5	2.06	1.5
Condition	$D > \rho$	$D = \rho$	$D < \rho$

Here $\rho = K\beta + \sqrt{K^2\beta^2 + 2\,K}$ and at the all of three cases $D < 0.5/\tau - 0.5K\tau$.

Three cases, i.e., string stability, critical string stability, and string instability, have been simulated by considering three groups of specific parameters. Figure 6 illustrates Case I in Table 1 which represents the string stable condition $D > \rho$ and it also shows the spacing errors of the vehicles in the string smoothly reduce upstream. If the initial inter vehicle spacing is the same for all vehicles, this ensures that if the first following vehicle does not collide with the leading vehicle, and then there will not be a collision upstream between successive vehicles.

Figure 7 illustrates Case II in Table 1, which represents the critical string stable condition. Figure 8 illustrates Case III in Table 1 which represents the string unstable condition of $D < \rho$.

Figure 7 clearly demonstrates when the relative velocity information was not used by the controller; string stability was achieved more difficult (compared with [6,7]). In other words, compared with a controller whereby used information of the relative velocity, the control parameters must be chosen in a shorter domain to achieve same response. On the other hand, information of the relative velocity needs to use extra sensor to obtain information of the relative velocity respect to the preceding vehicle. Hence, by using this controller law, we do not need this sensor in practice.

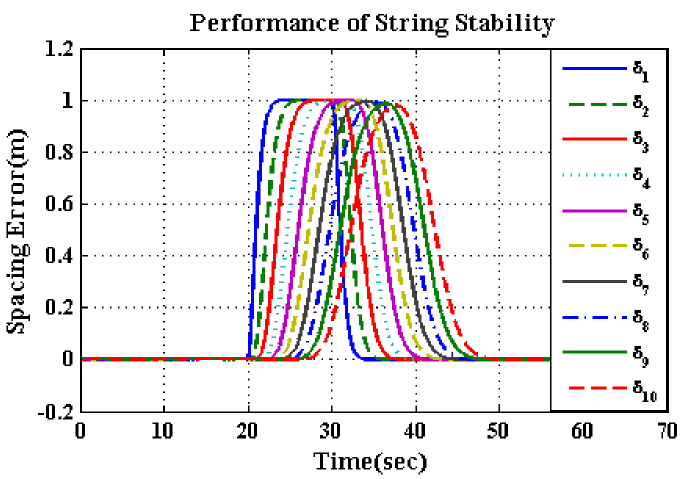

FIGURE 6. Performance of string stability of ten followers under condition $D > \rho$

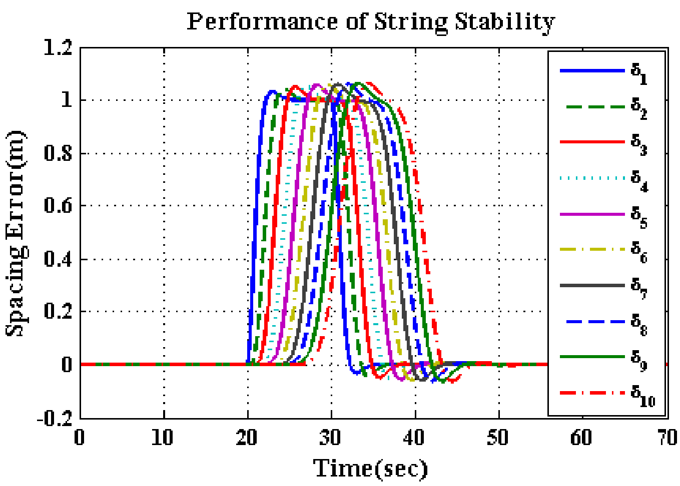

FIGURE 7. Performance of critical string stability of ten followers under condition $D = \rho$

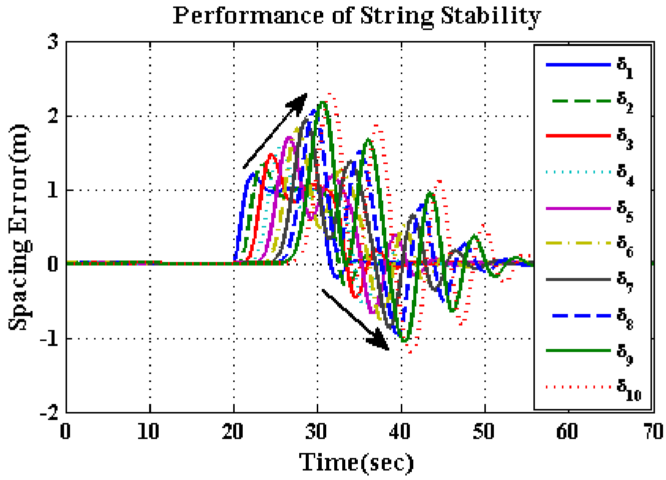

FIGURE 8. Performance of string instability of ten followers under condition $D < \rho$

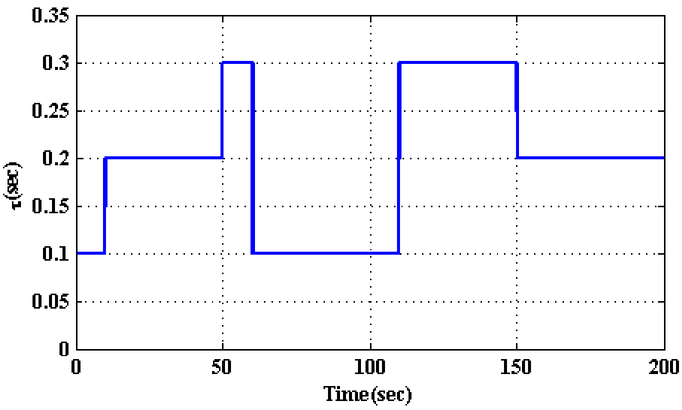

FIGURE 9. The time constant variation

A first-order lag model approximation of vehicle dynamics is considered. The time constant of the first-order models may be a function of many factors, such as vehicle speed and mass. Therefore, the controller must be designed so that it can accommodate, or at least be robust to, variations in vehicle dynamics. A numerical simulation result is presented to show the robustness of the control approach. Using the same scenario used in the sudden speed change simulation and fixing all relevant gains and parameters for the controllers, the time constants of vehicle models are varied in order to represent the various kinds of vehicle dynamics. The time constant ranges from 0.1 to 0.3 which is shown in Figure 9. In Figure 10, the spacing errors are presented by taking the control parameters as same as stability examination (Figure 4).

The ability of maintaining the distance between vehicles regardless the measurement signals noises is important for the safety of the vehicle platoon. In this paper, the robustness beside the measurement signals noises are considered. Even though the information is measurable via sensors, the extraneous noises were included unavoidably in the measurement process, and the overall performance of the platoon control system can be worse as a result of the noises. In this simulation, in order to take into account robustness against the noise in the measurement signals, the measurement noise of the proposed algorithm, the random numbers within $[-0.5, 0.5]$ (m) are added into the measurement signals of the relative distance. The performances of the vehicle platoon control system

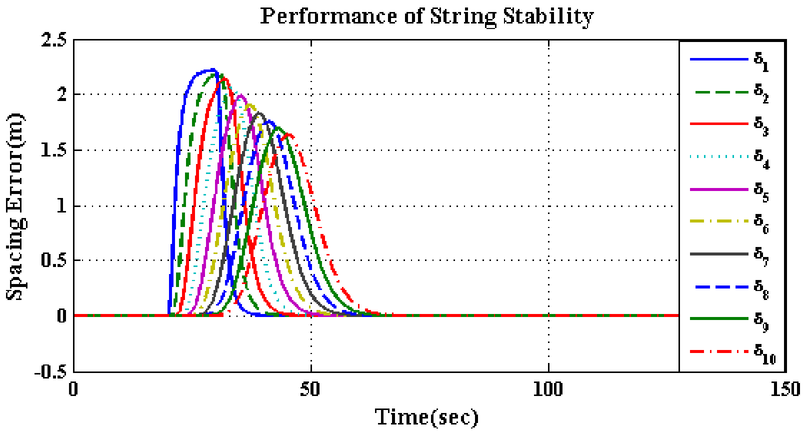

FIGURE 10. Effects of uncertainty in vehicle dynamics

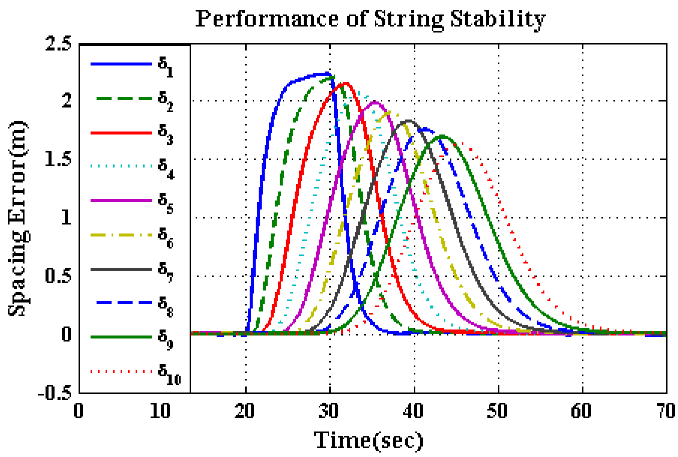

FIGURE 11. Performance of string stability of ten followers with the measurement noise

under the noise in the measurement signals is shown in Figure 11 which represents the string stable state. The parameters are chosen as Figure 4.

7. **Conclusion.** In this paper, the practical analysis of a platoon of ACC vehicles, which apply the constant time headway policy, is investigated by considering the lags of the actuators and sensors when building the vehicle longitudinal dynamics model and communication time varying delay. A hierarchical platoon controller design framework is established comprising a feedback linearization controller at the first layer and a centralized bidirectional controller at the second layer. This paper provides a practical means to evaluate the ACC systems by applying the modified consensus algorithm and provides a reasonable proposal to design the ACC controller. For the purpose of disturbance attenuation, string stability analysis is examined. The results have shown the decrees of error in the platoon if the $\rho < D < 0.5/\tau - 0.5K\tau$ is satisfied and the delays have a negative effect on string stability. At the end, the robustness of the proposed algorithm is validated through computer simulations under noise in the sensor measurement signal and engine time constant variation.

REFERENCES

[1] N. Sandell, P. Varaiya, M. Athans and M. Safonov, Survey of decentralized control methods for large scale systems, *IEEE Trans. on Automatic Control*, vol.23, no.2, pp.108-128, 1978.

[2] J. R. Carpenter, A preliminary investigation of decentralized control for satellite formations, *Proc. of the IEEE Conf. on Aerospace*, vol.7, pp.63-74, 2000.

[3] M. J. Woo and J. W. Choi, A relative navigation system for vehicle platooning, *Proc. of the 40th SICE Conf.*, pp.28-31, 2001.

[4] A. Betser, P. A. Vela, G. Pryor and A. Tannenbaum, Flying in formation using a pursuit guidance algorithm, *Proc. of the American Control Conf.*, vol.7, pp.5085-5090, 2005.

[5] J. M. Fowler and R. D'Andrea, A formation flight experiment, *IEEE Control Systems Magazine*, vol.23, no.5, pp.35-43, 2003.

[6] G. Guo and W. Yue, Hierarchical platoon control with heterogeneous information feedback, *IET Control Theory Appl.*, vol.5, no.15, pp.1766-1781, 2011.

[7] L. Xiao and F. Gao, Practical string stability of platoon of adaptive cruise control vehicles, *IEEE Trans. on Intelligent Transportation Systems*, vol.12, no.4, 2011.

[8] A. Ghasemi, R. Kazemi and S. Azadi, Stable decentralized control of platoon of vehicles with heterogeneous information feedback, *IEEE Trans. on Vehicular Technology*, vol.62, no.7, 2013.

[9] K. C. Chu, Optimal decentralized regulation for a string of coupled systems, *IEEE Trans. on Automatic Control*, vol.19, no.3, pp.243-246, 1974.

[10] D. Swaroop and J. K. Hedrick, String stability of interconnected systems, *IEEE Trans. on Automatic Control*, vol.41, no.3, pp.349-357, 1996.

[11] D. Swaroop and K. R. Rajagopal, A review of constant time headway policy for automatic vehicle following, *Proc. of the IEEE Conf. on Intelligent Transportation Systems*, Oakland, CA, USA, 2001.

[12] S. K. Yadlapalli, S. Darbha and K. R. Rajagopal, Information flow and its relation to stability of the motion of vehicles in a rigid formation, *IEEE Trans. on Automatic Control*, vol.51, no.8, pp.1315-1319, 2006.

[13] K. Santhanakrishnan and R. Rajamani, On spacing policies for highway vehicle automation, *IEEE Trans. on Intelligent Transportation Systems*, vol.4, no.4, pp.198-204, 2003.

[14] J. Zhou and H. Peng, Range policy of adaptive cruise control vehicle for improved flow stability and string stability, *IEEE Trans. on Intelligent Transportation Systems*, vol.6, no.2, pp.229-237, 2005.

[15] R. Olfati-Saber and R. M. Murray, Consensus problems in networks of agents with switching topology and time-delay, *IEEE Trans. on Automatic Control*, vol.53, no.9, pp.1520-1533, 2004.

[16] W. J. Dong and J. A. Farrell, Cooperative control of multiple nonholonomic mobile agents, *IEEE Trans. on Automatic Control*, vol.53, no.6, pp.1434-1448, 2008.

[17] W. Ren and R. W. Beard, *Distributed Consensus in Multi-Vehicle Cooperative Control*, Springer-Verlag, London, 2008.

[18] Z. H. Qu, *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles*, Springer-Verlag, London, 2009.

[19] S. K. Yadlapalli, S. Darbha and K. R. Rajagopal, Information flow and its relation to stability of the motion of vehicles in a rigid formation, *IEEE Trans. on Automatic Control*, vol.51, no.8, pp.1315-1319, 2006.

[20] J. A. Fax and R. M. Murray, Information flow and cooperative control of vehicle formations, *IEEE Trans. on Automatic Control*, vol.49, no.9, pp.1465-1476, 2004.

[21] A. Gattami and R. M. Murray, A frequency domain condition for stability of interconnected MIMO systems, *Proc. of the American Control Conf.*, pp.3723-3728, 2004.

[22] M. Cui, Z. Wu, X. Xie and P. Shi, Modeling and adaptive tracking for a class of stochastic Lagrangian control systems, *Automatica*, vol.49, no.3, pp.770-779, 2013.

[23] J. Lian, C. Mu and P. Shi, Asynchronous H_∞ filtering for switched stochastic systems with time-varying delay, *Information Sciences*, vol.224, pp.200-212, 2013.

[24] J. Lian, P. Shi and Z. Feng, Passivity and passification for a class of uncertain switched stochastic time-delay systems, *IEEE Trans. on Cybernetics*, vol.43, no.1, pp.3-13, 2013.

[25] H. Gao, J. Lam, C. Wang and Y. Wang, Delay-dependent output-feedback stabilization of discrete-time systems with time-varying state delay, *Proc. of the IEEE Conf. on Control Theory and Applications*, vol.151, no.6, pp.691-698, 2004.

[26] J. P. Richard, Time-delay systems: An overview of some recent advances and open problems, *Automatica*, vol.39, no.10, pp.1667-1694, 2003.

[27] M. G. Earl and S. H. Strogatz, Synchronization in oscillator networks with delayed coupling: A stability criterion, *Phys. Rev. E*, vol.67, 2003.

[28] S. Low, F. Paganini and J. Doyle, Internet congestion control, *IEEE Control Systems Magazine*, vol.32, pp.28-43, 2002.

[29] R. Rajamani, *Vehicles Dynamics and Control*, Springer, New York, USA, 2006.

[30] J. Moskwa and J. Hedrick, Modeling and validation of automotive engines for control algorithm development, *Journal of Dynamic Systems, Measurement, and Control*, vol.114, pp.278-285, 1992.

[31] X. Liu, S. Mahal, A. Goldsmith and J. Hedrick, Effects of communication delays on string stability in vehicle platoons, *Proc. of the IEEE Conf. on ITS*, Oakland, CA, USA, 2001.

[32] S. Sheikholeslam and C. A. Desoer, Longitudinal control of a platoon of vehicles, *IEEE American Control Conference*, 1990.

[33] Y. Cao and W. Ren, Containment control with multiple stationary or dynamic leaders under a directed interaction graph, *Proc. of IEEE Conf. Decision Control*, Shanghai, China, pp.3014-3019, 2009.

[34] R. Olfati-Saber, J. A. Fax and R. M. Murray, Consensus and cooperation in networked multi-agent systems, *Proc. of IEEE*, vol.95, pp.215-233, 2007.

[35] I. M. Havel, Sixty years of cybernetics: Cybernetics still alive, *Kybernetika*, vol.44, pp.314-327, 2008.

[36] J. K. Hale and S. M. V. Lunel, Introduction to the theory of functional differential equations 99, in *Applied Mathematical Sciences*, New York, Springer, 1991.

[37] J. Hu, On robust consensus of multi-agent systems with communication delays, *Kybernetika*, vol.45, no.5, pp.768-784, 2009.

[38] J. Gallier, *The Schur Complement and Symmetric Positive Semidefinite (and Definite) Matrices*, 2010.

[39] A. J. Laub, Matrix analysis for scientists and engineers, *Society for Industrial and Applied Mathematics*, 2004.

[40] D. YanaKiev and I. Kanellakopoulos, A simplified framework for string stability analysis in AHS, *Proc. of the 13th IFAC World Congress*, pp.177-182, 1996.

[41] P. Cook, Conditions for string stability, *System and Control Letters*, vol.54, pp.991-998, 2005.

[42] P. Seiler, A. Pant and K. Hedrick, Disturbance propagation in vehicle strings, *IEEE Trans. on Automatic Control*, vol.49, no.10, pp.1835-1841, 2004.

[43] G. Naus, R. Vugts, J. Ploeg, R. Molengraft and M. Steinbuch, String-stable CACC design and experimental validation: A frequency-domain approach, *IEEE Trans. on Vehicular Technology*, vol.59, pp.4268-4279, 2010.

[44] A. Ghasemi, R. Kazemi and S. Azadi, Directional control of a platoon of vehicles for comfort specification by considering parasitic time delays and lags, *PROMET – Traffic & Transportation*, vol.25, no.5, pp.413-420, 2013.