International Journal of Innovative
Computing, Information and Control
Volume 10, Number 4, August 2014

ICIC International ©2014 ISSN 1349-4198
pp. 1291–1303

MODE-INDEPENDENT GUARANTEED COST CONTROL OF SINGULAR MARKOVIAN DELAY JUMP SYSTEMS WITH SWITCHING PROBABILITY RATE DESIGN

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Received July 2013; revised November 2013

Abstract. *This paper considers the guaranteed cost control of singular Markovian jump systems (SMJSs) with time delay whose mode signal is inaccessible. The main contribution is to develop an approach to mode-independent guaranteed control, where the switching probability rate is also designed. New sufficient conditions of such controller are proposed in terms of linear matrix inequalities with some equation constraints, which also characterize the switching probability rate. Finally, numerical examples are used to demonstrate the effectiveness and advantage of the proposed method.*

Keywords: Singular Markovian jump systems, Mode-independent control, Guaranteed cost control, Linear matrix inequalities (LMIs)

1. **Introduction.** As we know, singular system was introduced by H. H. Rosenbrock [1] in 1974. Many practical systems such as electric power system, economic systems, mechanical engineering systems and robotics, can be described by singular systems. During the past years, a lot of attention has been paid to this system. Many important results have emerged, such as [2, 3], and the references cited therein. Due to delay existing in many industrial systems, it has been a hot topic. In the past decades, all kinds of delayed singular systems were studied, and many invaluable results were obtained, see, e.g., [4, 5, 6, 7]. It is seen that there are no jumping parameters. When the structure of singular systems changes abruptly, a kind of systems named to be SMJSs is very appreciated to describe such phenomenon, which is an important branch in stochastic system. Up to now, many important results were reported in [8, 9, 10, 11, 12, 13, 14].

On the other hand, many inevitable factors in the actual control systems will result in system having uncertain parameters. In this case, we usually desire to achieve a minimum value of performance index. However, if the uncertainty of a system is considered, the conclusion will be more conservative, and the robustness of system performance will be destroyed. Instead, a strategy named as guaranteed cost control [15, 16, 17] is an effective method to overcome this defect. However, by investing the existing references, it is seen that the guaranteed cost control is realized by mode-dependent controllers. Such controllers have an ideal assumption that their modes need to be available online. This assumption will have the application scope limited. Compared with mode-dependent controller, mode-independent control is very appropriate to deal with such general problem, in which system mode is not used. Moreover, it is seen that almost the existing results of MJSs see, e.g., [18, 19, 20], and the references cited there in, have an assumption that the switching probability rate is given beforehand. In some practical cases, one may have more freedom to choose an appropriate switching probability rate. For normal state-space

MJSs, [21] firstly studied the stabilization problem of MJSs, whose matrix of the adopted Lyapunov function should be positive-definite. Although the proposed result is necessary and sufficient, it is not applied to SMJSs. That is because the derivative matrix of SMJSs is singular, and it makes the matrix of Lyapunov function only nonsingular instead of being positive-definite. In this case, it is said that the method of [21] cannot be used to deal with SMJSs similarly. Based on the mentioned facts, it is meaningful to study the guaranteed cost control of SMJSs realized by mode-independent controller where the switching probability rate is also designed. To the best of our knowledge, there are still no results available in the literature.

In this paper, the mode-independent guaranteed cost control of singular Markovian delay jump systems with switching probability rate design is firstly considered. Under the switching probability rate and mode-independent controller given beforehand, a sufficient condition guaranteeing the closed-loop system stochastically admissible in addition to cost function bounded is proposed. Based on this, an existence condition to solve such problem is developed in terms of LMIs with some equation constraints. In order to make the condition within the desired framework ultimately, some novel techniques for dealing with switching probability rate are exploited. Because of the switching probability rate designed, it will be less conservative. Finally, numerical example demonstrates the utility and superiority of the presented methods.

Notation: $\mathbb{R}^{n\times m}$ denotes $n \times m$ dimension matrix. In symmetric block matrices, '$*$' stands for the corresponding position about matrix's transpose. $\mathrm{diag}\{\cdots\}$ is used to indicate block-diagonal matrix, considering $\deg(\cdot)$ as a symbol of the biggest polynomial order number and also $\det(D)$ is a sign of the determinant of phalanx D, and $(D)^\star = D + D^T$.

2. **Problem Formulation.**

Considering a class of SMJSs with time delay as follows:

$$\begin{cases} E\dot{x}(t) = A(\eta_t)x(t) + A_d(\eta_t)x(t-\tau) + B(\eta_t)u(t) \\ x(t) = \varphi(t), \quad t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is state variables, $u(t) \in \mathbb{R}^n$ is the control input, and time delay τ satisfies $\tau \geq 0$. Matrix E is singular with rank $E = r \leq n$, and $A(\eta_t), A_d(\eta_t), B(\eta_t) \in \mathbb{R}^{n\times n}$ are known matrix. The jump signal $\eta_t = i \in \mathbb{S} = \{1, 2, 3, \ldots, N\}$ is defined as

$$\Pr\{\eta_{t+h} = j|\eta_t = i\} = \begin{cases} \pi_{ij}h + o(h) & i \neq j \\ 1 + \pi_{ii}h + o(h) & i = j \end{cases} \tag{2}$$

where $h > 0$, $\lim_{h\to 0^+}(o(h)/h) = 0$, and $\pi_{ij} \geq 0$, if $i \neq j$, $\pi_{ii} = -\sum_{j=1,\ j\neq i}^{N} \pi_{ij}$, and $\Pi = (\pi_{ij})_{N\times N}$.

In this paper, we will discuss the problem of guaranteed cost control realized by a mode-independent controller

$$u(t) = Kx(t) \tag{3}$$

where K is controller gain to be determined, and the switching probability rate with property (2) is also designed.

Definition 2.1. *For SMJS (1), define a cost function*

$$J = \mathscr{E}\left(\int_0^{+\infty} [x^T(t)S_i x(t) + u^T(t)R_i u(t)]dt\right) \tag{4}$$

where S_i, R_i are given positive-definite matrix. J^* is called an upper bound of the cost function, if there is a constant J^* to make the closed-loop system stable and $J \leq J^*$. Namely, $u(t)$ is called the guaranteed cost controller.

Definition 2.2. *Singular Markovian jump system (1) with $u(t) = 0$ is said to be regular and impulse free for any constant time delay τ, if pairs $(E, A(\eta_t))$ and $(E, A(\eta_t) + A_d(\eta_t))$ are regular and impulse free for every $\eta_t \in \mathbb{S}$.*

3. **Main Results.**

Proposition 3.1. *Consider system (1) with cost function (4), if there exist P_i, Z, Q_i Q and $\mu \geq |\pi_{ii}|_{\max}$, such that the following conditions hold for all $i \in \mathbb{S}$*

$$P_i^T E = E^T P_i \geq 0 \tag{5}$$

$$\begin{bmatrix} M_i^{11} & M_i^{12} & \tau \bar{A}_i^T Z & I & K^T \\ * & M_i^{22} & \tau A_{di}^T Z & 0 & 0 \\ * & * & -Z & 0 & 0 \\ * & * & * & -S_i^{-1} & 0 \\ * & * & * & * & -R_i^{-1} \end{bmatrix} < 0 \tag{6}$$

$$Q_i < Q \tag{7}$$

where

$$M_i^{11} = (P_i^T \bar{A}_i)^\star + \sum_{j=1}^{N} \pi_{ij} E^T P_j + Q_i + \mu\tau Q - E^T Z E$$

$$M_i^{12} = P_i^T A_{di} + E^T Z E, \ \ M_i^{22} = -Q_i - E^T Z E, \ \ \bar{A}_i = A_i + B_i K$$

System (1) is stochastically stable, and cost function (4) satisfies

$$\begin{aligned} J \leq & \varphi^T(0) E^T P_i \varphi(0) + \int_{-\tau}^{0} x^T(\alpha) Q_i x(\alpha) d\alpha \\ & + \int_{-\tau}^{0} \int_{\beta}^{0} \left[\dot{x}(\alpha) E^T \tau Z E \dot{x}(\alpha) + \mu x^T(\alpha) Q x(\alpha) \right] d\alpha d\beta \end{aligned} \tag{8}$$

Proof: Firstly, we prove system (1) is regular and impulse-free. From (6), one has

$$\left[P_i^T (\bar{A}_i + A_{di}) \right]^\star + \sum_{j=1}^{N} \pi_{ij} E^T P_j + \mu\tau Q < 0 \tag{9}$$

by premultiplying and postmultiplying (6) with $\begin{bmatrix} I & I & 0 & 0 & 0 \end{bmatrix}$ and its transform respectively. Moreover, there are two nonsingular matrices M and N satisfying

$$\begin{aligned} \bar{E} = MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad M(\bar{A}_i + A_{di})N = \begin{bmatrix} A_i^{11} & A_i^{12} \\ A_i^{21} & A_i^{22} \end{bmatrix}, \\ M^{-T} P_i N = \begin{bmatrix} P_i^{11} & P_i^{12} \\ P_i^{21} & P_i^{22} \end{bmatrix} \end{aligned} \tag{10}$$

Based on condition (5) by premultiplying and postmultiplying (5) by N^T and N, it is known that $P_i^{12} = 0$. Let

$$N^T Q N = \begin{bmatrix} Q^{11} & Q^{12} \\ Q^{21} & Q^{22} \end{bmatrix} > 0 \tag{11}$$

which illustrates that $Q^{22} > 0$. Based on (10) and (11), (9) becomes

$$\begin{bmatrix} \Upsilon_i^{11} & \Upsilon_i^{12} \\ \Upsilon_i^{21} & [(A_i^{22})^T P_i^{22}]^\star + \mu\tau Q^{22} \end{bmatrix} < 0 \tag{12}$$

where

$$\Upsilon_i^{11} = \left[(A_i^{11})^T P_i^{11} + (A_i^{21})^T P_i^{21}\right]^\star + \mu\tau Q^{11} + \sum_{j=1}^{N} \pi_{ij} P_j^{11}$$

$$\Upsilon_i^{12} = (A_i^{21})^T P_i^{22} + (P_i^{11})^T A_i^{22} + (P_i^{21})^T A_i^{22} + \sum_{j=1}^{N} \pi_{ij} P_j^{12} + \mu\tau Q^{12}$$

$$\Upsilon_i^{21} = (A_i^{12})^T P_i^{11} + (A_i^{22})^T P_i^{21} + (P_i^{22})^T A_i^{21} + \mu\tau Q^{21}$$

Obviously, we get

$$(A_i^{22})^T P_i^{22} + (P_i^{22})^T A_i^{22} < 0 \tag{13}$$

Then, system (1) is regular and impulse-free. Now, choose a Lyapunov function

$$V(t, \eta_t) = V_1(t, \eta_t) + V_2(t, \eta_t) + V_3(t, \eta_t) + V_4(t, \eta_t) \tag{14}$$

where

$$V_1(t, \eta_t) = x^T(t) E^T P_i x(t), \quad V_2(t, \eta_t) = \int_{t-\tau}^{t} x^T(\alpha) Q_i x(\alpha) d\alpha$$

$$V_3(t, \eta_t) = \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha) E^T \tau Z E \dot{x}(\alpha) d\alpha d\beta, \quad V_4(t, \eta_t) = \mu \int_{-\tau}^{0}\int_{t+\beta}^{t} x^T(\alpha) Q x(\alpha) d\alpha d\beta$$

Letting the weak infinitesimal generator $\mathcal{L}$, one has

$$\begin{aligned}
\mathcal{L}V(t, \eta_t) &= \lim_{h\to 0} \frac{1}{h}\mathscr{E}(V(t+h, \eta_{t+h}) - V(t, \eta_t)) \\
&= \dot{x}^T(t) E^T P_i x(t) + x^T(t) P_i^T E \dot{x}(t) + x^T(t) \sum_{j=1}^{N} \pi_{ij} E^T P_j x(t) + x^T(t) Q_i x(t) \\
&\quad - x^T(t-\tau) Q_i x(t-\tau) + \int_{t-\tau}^{t} x^T(\alpha) \sum_{j=1}^{N} \pi_{ij} Q_j x(\alpha) d\alpha + \tau^2 \dot{x}^T(t) E^T Z E \dot{x}(t) \\
&\quad \int_{t-\tau}^{t} \dot{x}(\alpha) E^T Z E \dot{x}(\alpha) d\alpha + x^T(t) \mu\tau Q x(t) - \mu \int_{t-\tau}^{t} x^T(\alpha) Q x(\alpha) d\alpha \\
&\le \xi^T(t) \Psi_i \xi(t) < 0
\end{aligned} \tag{15}$$

where

$$\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-d) \end{bmatrix}^T, \quad \Psi_i = \begin{bmatrix} \Psi_i^{11} & \Psi_i^{12} \\ * & \Psi_i^{22} \end{bmatrix}$$

$$\Psi_i^{11} = (P_i^T \bar{A}_i)^\star + \bar{A}_i^T \tau^2 Z \bar{A}_i + Q_i + \mu\tau Q - E^T Z E + \sum_{j=1}^{N} \pi_{ij} E^T P_j$$

$$\Psi_i^{12} = P_i^T A_{di} + \bar{A}_i^T \tau^2 Z A_{di} + E^T Z E, \quad \Psi_i^{22} = -Q_i + A_{di}^T \tau^2 Z A_{di} - E^T Z E$$

By [23], it is concluded that system is stochastically stable and stochastically admissible. Moreover, by computing, it is obtained that

$$\begin{aligned}
J_T &= \mathscr{E}\int_0^T \left[x^T(t) S_i x(t) + u^T(t) R_i u(t) + \mathcal{L}V(t, \eta_t)\right] dt - \mathscr{E}\int_0^T \mathcal{L}V(t, \eta_t) dt \\
&\le \mathcal{E}\int_0^T \xi^T(t) \bar{\Psi}_i \xi(t) dt + V(0) \le V(0)
\end{aligned} \tag{16}$$

where $\bar{\Psi}_i = \Psi_i + \mathrm{diag}\{S_i + K^T R_i K, 0\}$. Letting $T \to \infty$, one has

$$J = \mathscr{E} \int_0^{+\infty} \left[\xi^T(t) \bar{\Psi}_i \xi(t) + V(0) \right] dt \le V(0) \tag{17}$$

which implies the upper bound of the cost function is $J^* = V(0)$. By condition (6), it is concluded that $\bar{\Psi}_i < 0$. This completes the proof.

Remark 3.1. *Proposition 3.1 gives an existence condition with guaranteed cost controller (3) given beforehand, where an upper bound of the system performance index is found. However, it should be pointed out that the unknown switching probability rate of system is related to some variable, which makes $\pi_{ij} E^T P_j$ nonlinear. Moreover, mode-independent control gain K and mode-dependent Lyapunov matrix P_i have strong coupling which should be decoupled first. Such nonlinearities cannot be solved directly, which should be considered carefully.*

Theorem 3.1. *Consider system (1) with cost function (4), if there exist matrices W_i, H_i, $\hat{Q}$, $\check{Q}$, $\bar{Q}$, $\bar{Q}_i$, $\tilde{P}_i$, $\bar{Z}$, $\tilde{Q}_i$, scalars $\bar{\mu}$ and $\bar{\pi}_{ij} \ge 0$ with $i \ne j$, such that the following conditions hold for all $i \in \mathbb{W}$*

$$\begin{bmatrix} \Phi_i^{11} & \Phi_i^{12} & \Phi_i^{13} & \Phi_i^{14} & \Phi_i^{15} & \Phi_i^{16} \\ * & -(G)^\star & 0 & \Phi_i^{24} & \Phi_i^{25} & 0 \\ * & * & \Phi_i^{33} & \tau X_i^T A_{di}^T & 0 & 0 \\ * & * & * & -\bar{Z} & 0 & 0 \\ * & * & * & * & \Phi_i^{55} & 0 \\ * & * & * & * & * & \Phi_i^{66} \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} -E\tilde{P}_i^T E^T - W_i & E\tilde{P}_i^T E_R + U^T \tilde{Q}_i^T V^T E_R \\ * & -E_R^T \tilde{P}_j E_R \end{bmatrix} < 0 \tag{19}$$

$$W_i H_i = I, \ \ \hat{Q}\check{Q} = I \tag{20}$$

$$\begin{bmatrix} -\hat{Q} & \bar{\mu} I \\ * & -\bar{Q} \end{bmatrix} < 0 \tag{21}$$

$$\bar{\mu} \ge \sum_{j=1, j\ne i}^{N} \bar{\pi}_{ij} \tag{22}$$

$$\bar{Q} < \bar{Q}_i \tag{23}$$

where

$$\Phi_i^{11} = (A_i G)^\star + (B_i Y)^\star - E\tilde{P}_i E^T - E\tilde{P}_i^T E^T + \bar{Z}, \ \ \Phi_i^{12} = A_i G + B_i Y + X_i^T - G^T$$

$$\Phi_i^{13} = A_{di} X_i + E\tilde{P}_i E^T + E\tilde{P}_i^T E^T - \bar{Z}, \ \ \Phi_i^{14} = \tau (A_i G)^T + \tau (B_i Y)^T$$

$$\Phi_i^{15} = \begin{bmatrix} X_i^T & Y^T & X_i^T & X_i^T \end{bmatrix}, \ \ \Phi_i^{16} = \begin{bmatrix} \bar{\pi}_{i1} I & \dots & \bar{\pi}_{i(i-1)} I & \bar{\pi}_{i(i+1)} I & \dots & \bar{\pi}_{iN} I \end{bmatrix}$$

$$\Phi_i^{24} = \tau G^T A_i^T + \tau Y^T B_i^T, \ \ \Phi_i^{25} = \begin{bmatrix} 0 & Y^T & 0 & 0 \end{bmatrix}$$

$$\Phi_i^{33} = -X_i^T - X_i + \bar{Q}_i - E\tilde{P}_i E^T - E\tilde{P}_i^T E^T + \bar{Z}, \ \ \Phi_i^{55} = -diag\left\{ \bar{S}_i, \bar{R}_i, \bar{Q}_i, \frac{1}{\tau}\check{Q} \right\}$$

$$\Phi_i^{66} = -diag\{H_i, \dots, H_i, \dots, H_i\}, \ \ X_i = \tilde{P}_i E^T + V\tilde{Q}_i U$$

$E = E_L E_R^T$ with $E_R \in \mathbb{R}^{n\times r}$ and $E_L \in \mathbb{R}^{n\times r}$. $V \in \mathbb{R}^{n\times(n-r)}$ and $U \in \mathbb{R}^{(n-r)\times n}$ are any given and satisfy $EV = UE = 0$, system (1) with controller (3) is stochastically stable, and cost function (4) satisfies (8). In this case, the control gain and switching probability rate are given as

$$K = YG^{-1} \tag{24}$$

$$\pi_{ij} = \bar{\pi}_{ij}^2, \quad \pi_{ii} = -\sum_{i=1, i \neq j}^{N} \pi_{ij} \tag{25}$$

Proof: Letting $X_i = P_i^{-1}$, pre- and post-multiplying (5) with X_i^T and (6) with $\text{diag}\{X_i^T, X_i^T, I, I, I\}$ and the corresponding transposes respectively, one has

$$EX_i = X_i^T E^T \geq 0 \tag{26}$$

$$\begin{bmatrix} \Omega_i^{11} & \Omega_i^{12} & \tau X_i^T \bar{A}_i^T & X_i^T & X_i^T K^T \\ * & \Omega_i^{22} & \tau X_i^T A_{di}^T & 0 & 0 \\ * & * & -Z^{-1} & 0 & 0 \\ * & * & * & -S_i^{-1} & 0 \\ * & * & * & * & -R_i^{-1} \end{bmatrix} < 0 \tag{27}$$

where

$$\begin{aligned} \Omega_i^{11} =& (\bar{A}_i X_i)^\star + X_i^T \left(\sum_{j=1}^{N} \pi_{ij} E^T P_j \right) X_i + X_i^T Q_i X_i + X_i^T \tau \mu Q X_i - X_i^T E^T Z E X_i \\ \Omega_i^{12} =& A_{di} X_i + X_i^T E^T Z E X_i, \quad \Omega_i^{22} = -X_i^T Q_i X_i - X_i^T E^T Z E X_i \end{aligned}$$

Moreover, taking into account (24) and letting $Z_i = G$, $Y = KG$, it is concluded that

$$\begin{bmatrix} \bar{\Omega}_i^{11} & \bar{\Omega}_i^{12} & \Omega_i^{12} & \tau G^T \bar{A}_i^T & \bar{\Omega}_i^{15} \\ * & -(Z_i)^\star & 0 & \tau Z_i^T \bar{A}_i^T & \bar{\Omega}_i^{25} \\ * & * & \Omega_i^{22} & \tau X_i^T A_{di}^T & 0 \\ * & * & * & -Z^{-1} & 0 \\ * & * & * & * & \bar{\Omega}_i^{55} \end{bmatrix} < 0 \tag{28}$$

where

$$\begin{aligned} \bar{\Omega}_i^{11} =& (\bar{A}_i G)^\star - X_i^T E^T Z E X_i + X_i^T \left(\sum_{j=1}^{N} \pi_{ij} E^T P_j \right) X_i + X_i^T Q_i X_i + X_i^T \tau \mu Q X_i \\ \bar{\Omega}_i^{12} =& \bar{A}_i Z_i + X_i^T - G^T, \quad \bar{\Omega}_i^{15} = \begin{bmatrix} X_i^T & G^T K^T \end{bmatrix}, \\ \bar{\Omega}_i^{25} =& \begin{bmatrix} 0 & Z_i^T K^T \end{bmatrix}, \quad \bar{\Omega}_i^{55} = -\text{diag}\{S_i^{-1}, R_i^{-1}\} \end{aligned}$$

implies (27) by pre- and post-multiplying (28) with the following matrix

$$\begin{bmatrix} I & \bar{A}_i & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & \tau \bar{A}_i & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & K & 0 & 0 & 0 & I \end{bmatrix} \tag{29}$$

and its transform respectively. Secondly, due to the nature of the switching probability rate, the following equality is always satisfied. That is

$$\sum_{j=1}^{N} \pi_{ij} W_i = 0 \tag{30}$$

Based on (30), it is obtained that (28) can be guaranteed by

$$\begin{bmatrix} \bar{\Phi}_i^{11} & \Phi_i^{12} & \bar{\Phi}_i^{13} & \Phi_i^{14} & \bar{\Phi}_i^{15} \\ * & -(G)^\star & 0 & \Phi_i^{24} & \bar{\Phi}_i^{25} \\ * & * & \bar{\Phi}_i^{33} & \tau X_i^T A_{di}^T & 0 \\ * & * & * & -Z^{-1} & 0 \\ * & * & * & * & \bar{\Omega}_i^{55} \end{bmatrix} < 0 \tag{31}$$

$$X_i^T(E^T P_j)X_i - X_i^T E^T - W_i < 0 \tag{32}$$

where

$$\begin{aligned} \bar{\Phi}_i^{11} &= (\bar{A}_i G)^\star + X_i^T Q_i X_i + \tau X_i^T \mu Q X_i - X_i^T E^T Z E X_i + \sum_{j=1, j\neq i}^{N} \pi_{ij} W_i \\ \bar{\Phi}_i^{13} &= A_{di} X_i + X_i^T E^T Z E X_i, \ \ \bar{\Phi}_i^{15} = \begin{bmatrix} X_i^T & Y^T \end{bmatrix}, \ \ \bar{\Phi}_i^{25} = \begin{bmatrix} 0 & Y^T \end{bmatrix} \\ \bar{\Phi}_i^{33} &= -X_i^T Q_i X_i - X_i^T E^T Z E X_i \end{aligned}$$

Unfortunately, there are still some problems in (27), such as nonlinear terms, for example, $X_i^T \tau \mu Q X_i$ and $X_i^T \left(\sum_{j=1}^N \pi_{ij} E^T P_j\right) X_i$. Especially, $\mu \geq |\pi_{ii}|_{\max}$ must be satisfied, which is a precondition. As for $X_i^T \tau \mu Q X_i$, by letting $\mu Q < \hat{Q}$, one has (21), where $\bar{Q} = Q^{-1}$ and $\bar{\mu} = \sqrt{\mu}$. On the other hand, for any given $a_m \in R^+$, where $R^+ = \{x|x > 0\}$ is a given set, and m is some positive integer, it is known that

$$\sqrt{a_1 + a_2 + \cdots + a_m} \leq \sqrt{a_1} + \sqrt{a_2} + \cdots + \sqrt{a_m} \tag{33}$$

Based on the established inequality (33), we obtain that condition (22) implies $\mu \geq |\pi_{ii}|_{\max}$ holds. Let

$$P_i = \hat{P}_i E + U^T \hat{Q}_i V^T \tag{34}$$

with $\hat{P}_i > 0$, $\hat{Q}_i$ being nonsingular matrix, and according to [24], it is concluded that $E_L^T \hat{P}_i E_L > 0$, where $UE = 0$, $EV = 0$, $E = E_L E_R^T$. Moreover,

$$X_i = P_i^{-1} = \left(\hat{P}_i E + U^T \hat{Q}_i V^T\right)^{-1} = \tilde{P}_i E^T + V \tilde{Q}_i U \tag{35}$$

$$X_i^T \left(E^T P_j\right) X_i = X_i^T \left[E_R \left(E_L^T \hat{P}_i E_L\right) E_R^T\right] X_i = X_i^T \left[E_R \left(E_R^T \tilde{P}_i E_R\right)^{-1} E_R^T\right] X_i \tag{36}$$

$$-X_i^T Q_i X_i \leq -X_i - X_i^T + Q_i^{-1} \tag{37}$$

Based on (35)-(37) and letting $Z^{-1} = \bar{Z}$, $Q^{-1} = \bar{Q}$, $R_i^{-1} = \bar{R}_i$, $S_i^{-1} = \bar{S}_i$, we have that (19) implies (32). This completes the proof.

Remark 3.2. *It is worth mentioning that compared with some existing results, Theorem 3.1 has the following advantages: 1) Different from* [21]*, nonlinear terms such as $\pi_{ij} E^T P_j$ are handled by technique (30), where the switching probability rate can be designed. Especially, novelty technique (34) is used to further linearize such related terms. In contrast to* [25, 26]*, it is said that the switching probability rate plays an important role in systems synthesis, whose effect is illustrated by numerical examples; 2) The coupling between mode-independent control gain K and mode-dependent matrix P_i is decoupled successfully; 3) The condition is expressed in terms of LMIs with equation constraints, which could be solved by some existing computing methods such as cone complementary linearization.*

Based on the proposed method, we have the following corollary similarly.

Corollary 3.1. *The unforce system (1) is stochastically stable, if there exist matrices W_i, H_i, Q_i, Q, $\bar{Q}$, Z, P_i, scalars $\bar{\mu}$ and $\bar{\pi}_{ij} \geq 0$ with $i \neq j$, satisfying (21) and (22), and the following LMIs hold for all $i \in \mathbb{S}$:*

$$P_i^T E = E^T P_i \geq 0 \tag{38}$$

$$\begin{bmatrix} \bar{\Upsilon}_i & P_i^T A_{di} + E^T ZE & \tau A_i^T Z & \Phi_i^{16} \\ * & -Q_i - E^T ZE & \tau A_{di}^T Z & 0 \\ * & * & -Z & 0 \\ * & * & * & \Phi_i^{66} \end{bmatrix} < 0 \tag{39}$$

$$E^T P_j - E^T P_i - W_i < 0 \tag{40}$$

$$Q_i < Q \tag{41}$$

$$W_i H_i = I, \ \ Q\bar{Q} = I \tag{42}$$

where

$$\bar{\Upsilon}_i = A_i^T P_i + P_i^T A_i + Q_i + \tau \hat{Q} - E^T ZE$$

Then, the switching probability rate is computed by (25).

4. **Numerical Example.** In this section, several numerical examples are used to prove superiority and correctness of the provided method.

Example 4.1. *Consider an SMJS of form (1) is obtained by*

$$A_1 = \begin{bmatrix} -1.2 & 0.3 \\ 2 & \theta \end{bmatrix}, \ A_{d1} = \begin{bmatrix} 0.3 & 0.2 \\ 0 & -0.1 \end{bmatrix}, \ A_2 = \begin{bmatrix} -0.5 & 1.1 \\ 1 & -1.5 \end{bmatrix}, \ A_{d2} = \begin{bmatrix} -0.5 & 0 \\ 0.22 & 0.1 \end{bmatrix}$$

where θ takes different values. Singular matrix E is

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

When θ takes different values, the comparisons between Corollary 3.1 and [25, 26] *are listed in Table 1. For this example, it is seen that our results are less conservative, if one can choose an appropriate switching probability rate.*

Table 1. The upper bounds of τ with different θ

θ	*1.8*	*–1.8*	*2.5*	*–2.5*
[25]	*1.16*	*1.98*	*1.28*	*1.94*
[26]	*1.95*	*2.04*	*2.08*	*2.36*
Corollary 3.1	*3.59*	*3.59*	*3.59*	*3.59*

Example 4.2. *Consider another SMJS, whose parameters are given as:*
Mode 1:

$$A_1 = \begin{bmatrix} -1.1 & 1.2 \\ 2.3 & -2.1 \end{bmatrix}, \ A_{d1} = \begin{bmatrix} 0.5 & -0.1 \\ 0.2 & 0.1 \end{bmatrix}, \ B_1 = \begin{bmatrix} -4.3 \\ -2 \end{bmatrix}, \ R_1 = 2.3, \ S_1 = 0.1$$

Mode 2:

$$A_2 = \begin{bmatrix} -1.8 & 1 \\ 2.5 & -1.5 \end{bmatrix}, \ A_{d2} = \begin{bmatrix} 0.8 & 0.2 \\ 0 & 0.1 \end{bmatrix}, \ B_2 = \begin{bmatrix} -3.1 \\ -3 \end{bmatrix}, \ R_2 = 1.2, \ S_2 = 0.1$$

The time delay is $\tau = 0.1$, and matrix E is

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Letting initial condition $x(0) = \begin{bmatrix} 1 & 0.6 \end{bmatrix}^T$, *the state response of the open-loop system is shown in Figure 1. It is seen that the open-loop system is unstable. By Theorem 3.1, one can design a mode-independent controller in addition to switching probability rate matrix* Π. *That is*

$$K = \begin{bmatrix} 0.4856 & 0.0516 \end{bmatrix}$$
$$\Pi = \begin{bmatrix} -0.1263 & 0.1263 \\ 0.1632 & -0.1632 \end{bmatrix}$$

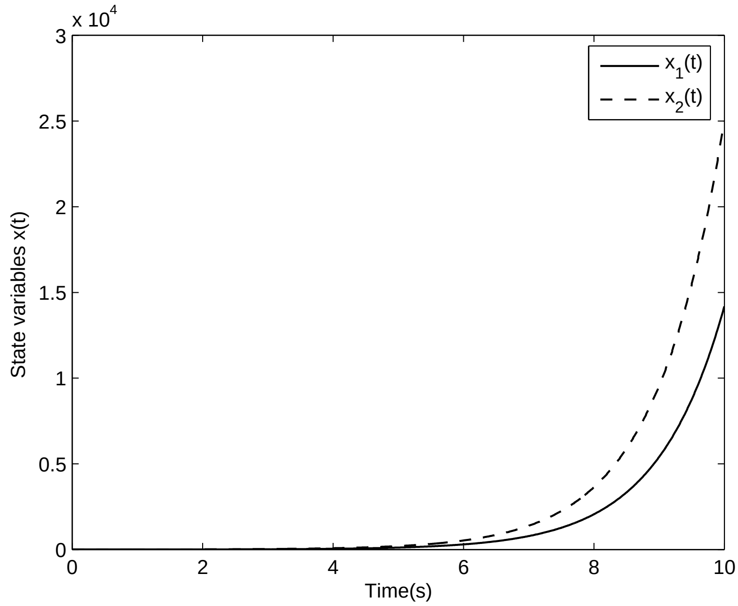

Figure 1. The simulation of open-loop system

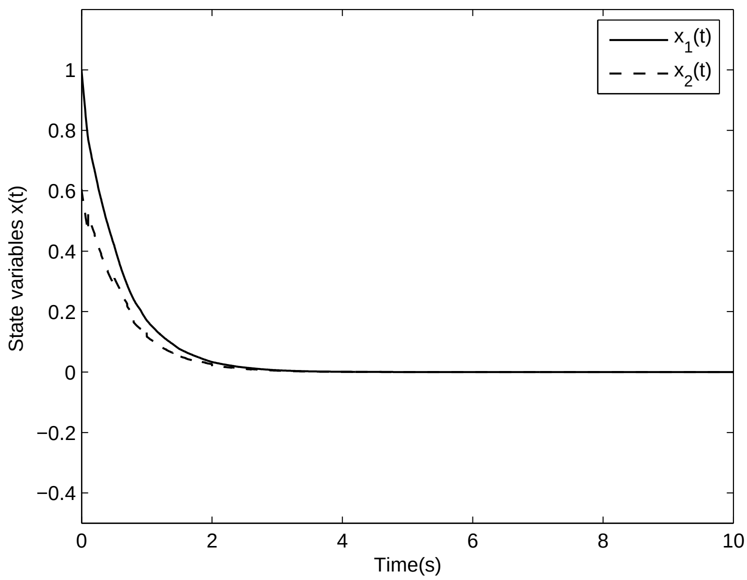

Figure 2. State response of closed-loop system

Applying these to the original system, the simulation of the closed-loop system is given in Figure 2, which is stable.

Example 4.3. *Consider the DC motor driving a load that changes randomly and abruptly, see Figure 3. The switching is driven by a continuous-time Markov process* $\{\eta_t, t > 0\}$ *taking values in a finite set* $N = \{1, 2\}$*. If we neglect the DC motor inductance* L_m *and let* $i(t), \varpi(t)$ *and* $u(t)$ *denote electric current, the speed of the shaft at time* t *and the voltage, respectively, based on the basic electrical and mechanic laws:*

$$\begin{cases} \dot{\varpi}(t) = -\dfrac{b_i}{J_i}\varpi(t) + \dfrac{K_t}{J_i}i(t) \\ u(t) = K_{\varpi}\varpi(t) + Ri(t) \end{cases} \tag{43}$$

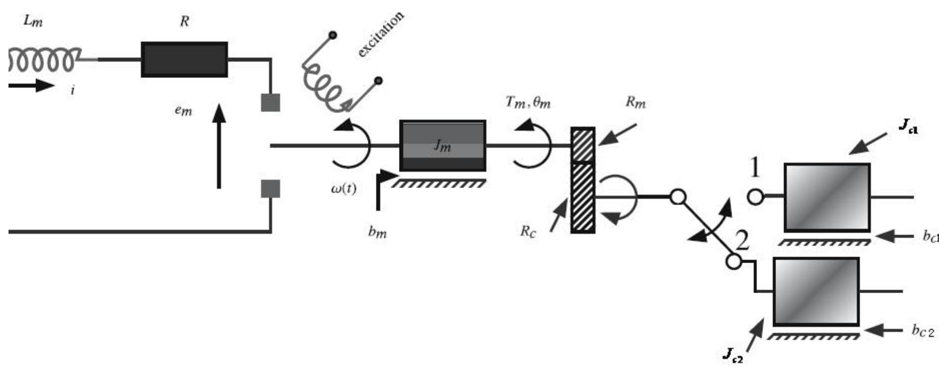

FIGURE 3. The block diagram of a DC motor

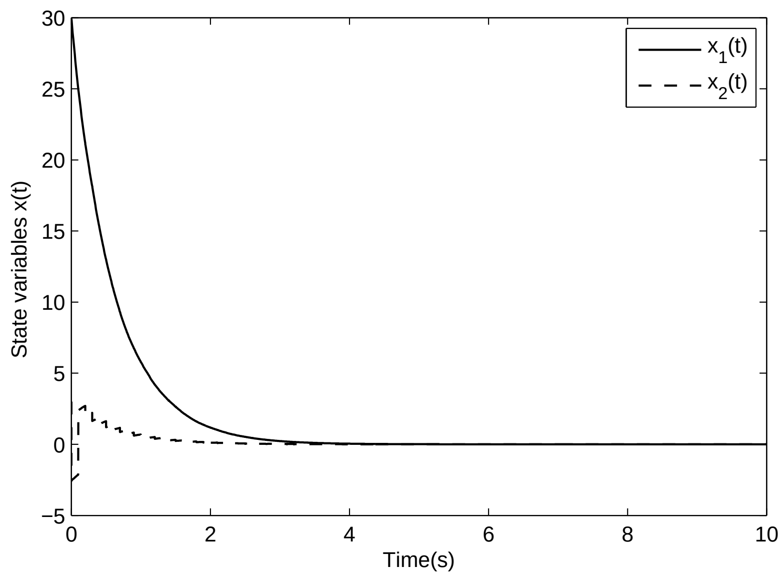

FIGURE 4. State response of the closed-loop system

where K_ϖ*,* K_t *represent the electromotive force constant and the torque constant, respectively.* R *is the electric resistor, and* J_i *and* b_i *are defined by:*

$$\begin{cases} J_i = J_m + \dfrac{J_{ci}}{n^2} \\ b_i = b_m + \dfrac{b_{ci}}{n^2} \end{cases} \tag{44}$$

where J_m *and* J_{ci} *are the moments of the motor and the load.* b_m *and* b_{ci} *are the damping ratios with gear ratio* n*. Now we let* $x_1(t) = \varpi(t)$*,* $x_2(t) = i(t)$*, and one has*

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \dot{x}(t) = \begin{bmatrix} -\frac{b_i}{J_i} & \frac{K_t}{J_i} \\ k_\varpi & R \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{45}$$

When there is time delay $\tau = 0.1s$*, system (45) becomes (1). Without loss of generality, it is assumed to be*

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \dot{x}(t) = \begin{bmatrix} -\frac{b_i}{J_i} & \frac{K_t}{J_i} \\ k_\varpi & R \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix} x(t-\tau) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{46}$$

Letting $J_m = 0.5\mathrm{kg}\cdot\mathrm{m}$*,* $J_{c1} = 50\mathrm{kg}\cdot\mathrm{m}$*,* $J_{c2} = 150\mathrm{kg}\cdot\mathrm{m}$*,* $b_{c1} = 100$*,* $b_{c2} = 240$*,* $R = 1\Omega$*,* $b_m = 1$*,* $R_1 = 0.3$*,* $S_1 = 0.1$*,* $R_2 = 0.4$*,* $S_2 = 0.1$*,* $K_t = 3\mathrm{Nm/A}$*,* $K_\varpi = 1\mathrm{Vs/rad}$ *and* $n = 10$*, by Theorem 3.1, one can design a mode-independent controller in addition to switching probability rate matrix* Π*. That is*

$$K = \begin{bmatrix} -1.2837 & -4.3221 \end{bmatrix}$$
$$\Pi = \begin{bmatrix} -0.0193 & 0.0193 \\ 0.0307 & -0.0307 \end{bmatrix}$$

Letting initial condition $x(0) = \begin{bmatrix} 30 & 3 \end{bmatrix}^T$*, we have the state response of the closed-loop system, which is illustrated in Figure 4. In addition, the curve of system mode is demonstrated in Figure 5. Based on such simulations, it shows the desired controller in addition to switching probability rate is effective.*

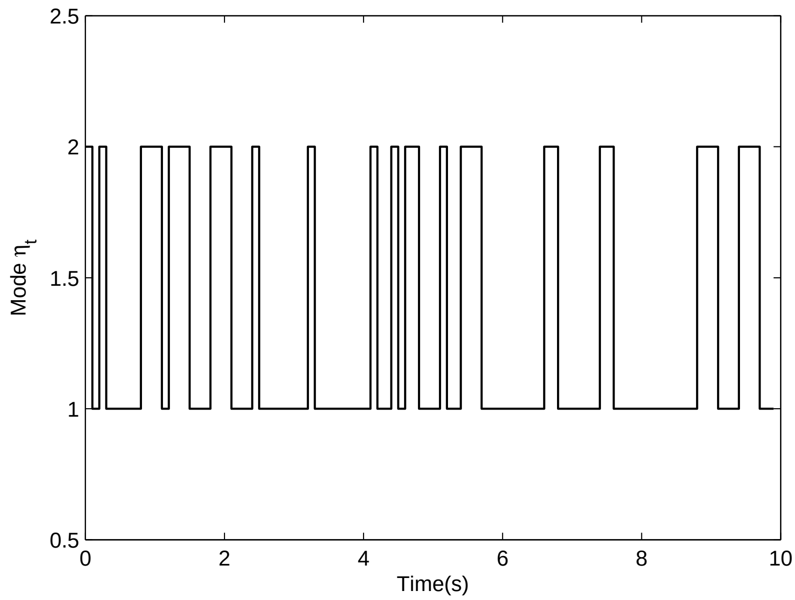

FIGURE 5. The curve of system mode

5. **Conclusion.** This paper has studied the guaranteed cost control problem for singular Markovian jump systems with time-delay via designing mode-independent state feedback controller and transition probability rates simultaneously. Based on an LMI approach with some equation constraints, such problems are solved using a mode-dependent Lyapunov function, where some novel techniques are developed to achieve the desired goals ultimately. Finally, numerical examples are used to show the utility and advantages of the developed theories.

Acknowledgment. This work was supported by the National Natural Science Foundation of China under Grants 61104066, 61203021 and 61374043, the China Postdoctoral Science Foundation funded project under Grant 2012M521086, the Program for Liaoning Excellent Talents in University under Grant LJQ2013040.

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