International Journal of Innovative Computing, Information and Control
Volume **10**, Number **4**, August **2014**
ICIC International ©2014 ISSN 1349-4198
pp. 1305–1312

A NEW FUZZY ASSESSMENT BASED ON INTERVAL VALUE TO EVALUATE THE AGGREGATIVE RISK RATE IN SOFTWARE DEVELOPMENT

Huey-Ming Lee[1], Lily Lin[2] and Jin-Shieh Su[3]

[1]Department of Information Management
[3]Department of Applied Mathematics
Chinese Culture University
No. 55, Hwa-Kung Road, Yang-Ming-San, Taipei 11114, Taiwan
{ hmlee; sjs }@faculty.pccu.edu.tw

[2]Department of International Business
China University of Technology
No. 56, Sec. 3, Hsing-Lung Road, Taipei 11695, Taiwan
lily@cute.edu.tw

Received July 2013; revised January 2014

Abstract. *In this study, we propose the fuzzy sense of interval value* $[m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}]$ *instead of the single value* m_{ij} *applied on assessment of aggregated risk rate. The proposed fuzzy assessment method on the risk rate analysis in software development can really reflect the interviewee's incomplete and uncertain thought. The results show that the proposed method provides a more accurate measurement of uncertainty than the existing ones to reduce the degree of subjectivity of the evaluators.*
Keywords: Risk assessment, Interval value, Triangular fuzzy number

1. **Introduction.** Risk assessment is a common first step and also the most important step in a risk management process. Risk assessment is the determination of quantitative or qualitative value of risk related to a concrete situation and a recognized threat. In a quantitative sense, it is the probability at such a given point in a software development process that predicted goals could not be achieved with the available resources. Due to the complexity of risk factors and the compounding uncertainty associated with future sources of risk, risk is normally not treated with mathematical rigor during the early software development [1]. Risks result in project problems such as schedule and cost overrun, so risk minimization is a very important project management activity [15,16]. Up to now, there are many papers investigating risk identification, risk analysis, risk priority, and risk management planning [1-5,7,15,16].

Based on [2-5,7,15,16], Lee [9] classified the risk factors into six attributes, divided each attribute into some risk items, built up the hierarchical structured model of aggregative risk and the evaluating procedure of structured model, ranged the grade of risk for each risk item into eleven ranks, and presented the procedure to evaluate the rate of aggregative risk using two stages fuzzy assessment method. Chen [6] ranged the grade of risk for each risk item into thirteen ranks, and defuzzified the trapezoid or triangular fuzzy numbers by the median. Lee [10] presented two algorithms for group decision making to tackle the rate of aggregative risk in fuzzy circumstances. Lee *et al.* [11] presented the other algorithm to evaluate the rate of aggregative risk. In [12], Lee and Lin presented the other method to tackle the risk rate in software development process. Lee and Lin [13] proposed the computational rule inferences to tackle the presumptive rate of aggregative risk in software development in fuzzy circumstances. The presented method of presumptive rate of aggregative risk directly uses the fuzzy numbers rather than the linguistic values to presume, and is defuzzified by the centroid method. In [14], Lin and Lee presented the fuzzy sense on sampling survey to do aggregated assessment analysis.

In this paper, we present the fuzzy sense of the closed interval value $[m_{ij1}-\Delta_{ij1}, m_{ij2}+\Delta_{ij2}]$ instead of the single value m_{ij} on assessment for the sub-item X_{ij} to do the rate of aggregated risk. The presented fuzzy assessment method on the risk rate analysis in software development process can really reflect the evaluator's incomplete and uncertain thought. The computing risk rate is close to the human thinking. Section 2 shown preliminaries. The presented assessment method is in Section 3. Section 4 is the example implementation. Finally, Section 5 makes conclusions of our work.

2. **Preliminaries.**

For the proposed algorithm, all pertinent definitions of fuzzy sets are given below [17-20].

Definition 2.1. *Let $\tilde{a}$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a fuzzy point if its membership function is*

$$\mu_{\tilde{a}}(x) = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{if } x \neq a \end{cases} \tag{1}$$

Definition 2.2. *Let $[a, b; \alpha]$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a level α fuzzy interval, $0 \le \alpha \le 1$, $a < b$, if its membership function is*

$$\mu_{[a,b;\alpha]}(x) = \begin{cases} \alpha, & \text{if } a \le x \le b \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

If $b = a$, we call $[a, a; \alpha]$ a level α fuzzy point at a.

Definition 2.3. *α-level set of the triangular fuzzy number $\tilde{A} = (p, q, r)$ is*

$$\begin{aligned} A(\alpha) &= \{x \mid \mu_{\tilde{A}}(x) \ge \alpha\} \\ &\equiv [A_L(\alpha), A_R(\alpha)] \end{aligned} \tag{3}$$

where

$$\begin{aligned} A_L(\alpha) &= p + (q-p)\alpha, \\ A_R(\alpha) &= r - (r-q)\alpha, \quad \alpha \in [0,1] \end{aligned} \tag{4}$$

Let F be the family of all these fuzzy sets $\tilde{A}$ on $R = (-\infty, \infty)$. Let $\tilde{A} \in F$, and then from the decomposition theory [20], we can represent $\tilde{A}$ as

$$\begin{aligned} \tilde{A} &= \bigcup_{0 \le \alpha \le 1} \alpha A(\alpha) \\ &= \bigcup_{0 \le \alpha \le 1} [A_L(\alpha), A_R(\alpha); \alpha] \end{aligned} \tag{5}$$

As in Yao and Wu [17], we may define the signed distance from $[A_L(\alpha), A_R(\alpha); \alpha]$ to $\tilde{0}$ as

$$d([A_L(\alpha), A_R(\alpha); \alpha], \tilde{0}) = \frac{1}{2}[A_L(\alpha) + A_R(\alpha)] \tag{6}$$

Since $\tilde{A} \in F$, $A_L(\alpha)$ and $A_R(\alpha)$ exist and are integrable for $\alpha \in [0, 1]$, from (5), we have the following definition.

Definition 2.4. *Let $\tilde{A} \in F$, and we define the signed distance of $\tilde{A}$ measured from $\tilde{0}$ as*

$$d(\tilde{A}, \tilde{0}) = \frac{1}{2} \int_0^1 [A_L(\alpha) + A_R(\alpha)] d\alpha \tag{7}$$

Let $\tilde{A} = (p, q, r)$. We denote

$$C(\tilde{A}) = \frac{\int_p^r x\mu_{\tilde{A}}(x)dx}{\int_p^r \mu_{\tilde{A}}(x)dx} \tag{8}$$

as the defuzzification of $\tilde{A}$ by the centroid method.

Remark 2.1. *If $\tilde{A} = (p, q, r)$, then the left endpoint and the right endpoint of the α-cut of $\tilde{A}$ are $A_L(\alpha) = p + (q - p)\alpha$ and $A_R(\alpha) = r - (r - q)\alpha$, respectively. The centroid of $\tilde{A}$ is $C(\tilde{A}) = \frac{1}{3}(p + q + r)$, and the signed distance of $\tilde{A}$ is $d(\tilde{A}, \tilde{0}) = \frac{1}{4}(2q + p + r)$.*

Proposition 2.1. *Let $\tilde{A}_j = (a_j, b_j, c_j)$, $\tilde{B}_j = (p_j, q_j, r_j)$, $j = 1, 2$. If $0 \le A_{jL}(\alpha) < A_{jR}(\alpha)$, $0 \le B_{jL}(\alpha) < B_{jR}(\alpha)$, for $\alpha \in [0, 1]$, $j = 1, 2$, then we have the following three properties.*

(1^0) $\tilde{A}_1 \otimes \tilde{B}_1$
$= \bigcup_{0\le\alpha\le1} [A_{1L}(\alpha)B_{1L}(\alpha), A_{1R}(\alpha)B_{1R}(\alpha); \alpha]$

(2^0) $(\tilde{A}_1 \otimes \tilde{B}_1) \oplus (\tilde{A}_2 \otimes \tilde{B}_2)$
$= \bigcup_{0\le\alpha\le1} [A_{1L}(\alpha)B_{1L}(\alpha) + A_{2L}(\alpha)B_{2L}(\alpha), A_{1R}(\alpha)B_{1R}(\alpha) + A_{2R}(\alpha)B_{2R}(\alpha); \alpha]$

(3^0) $k(\tilde{A}_1 \otimes \tilde{B}_1)$
$= \bigcup_{0\le\alpha\le1} [kA_{1L}(\alpha)B_{1L}(\alpha), kA_{1R}(\alpha)B_{1R}(\alpha); \alpha], \text{for } k > 0$

Proposition 2.2. *Let $\tilde{A}_j = (a_j, b_j, c_j)$, $\tilde{B}_j = (p_j, q_j, r_j)$, $j = 1, 2$. Then, we have*

$$\begin{aligned} &d([(\tilde{A}_1 \otimes \tilde{B}_1) \oplus (\tilde{A}_2 \otimes \tilde{B}_2)], \tilde{0}) \\ =\,&d(\tilde{A}_1 \otimes \tilde{B}_1, \tilde{0}) + d(\tilde{A}_2 \otimes \tilde{B}_2, \tilde{0}) \end{aligned}$$

We can easily show Proposition 2.2 by Definition 2.4 and Proposition 2.1.

Proposition 2.3. *Let $\tilde{A} = (p, q, r)$ be a triangular fuzzy number. We have*

(1^0) *If the membership function of $\tilde{A}$ is not an isosceles triangle, then, based on the maximum membership grade principle, to defuzzify $\tilde{A}$ by the signed distance is better than that by the centroid method.*

(2^0) *If the membership function of the triangular fuzzy number $\tilde{A}$ is an isosceles triangle, then to defuzzify $\tilde{A}$ by the signed distance is equal to that by the centroid method based on the maximum membership grade principle.*

Let $[m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}]$ be a closed interval, where $0 \le \Delta_{ij1} < m_{ij}$, $0 \le \Delta_{ij2}$. If the decision maker takes a value which coincides with m_{ij}, then the error is zero. If the value deviates from m_{ij} farther from both sides of m_{ij}, the error is larger. If the value lies at one of the two endpoints $m_{ij} - \Delta_{ij1}$ and $m_{ij} + \Delta_{ij2}$ then the error will attain the maximum. From the fuzzy point of view, we can express the error by the confidence level. If the error is 0 then confidence level is 1. The farther the value is from both sides of m_{ij}, the less the confidence level is. The confidence level is zero. At the two endpoints $m_{ij} - \Delta_{ij1}$ and $m_{ij} + \Delta_{ij2}$. Corresponding to the closed interval $[m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}]$, we characterize the normal triangular fuzzy number as follows:

$$\tilde{m}_{ij} = (m_{ij} - \Delta_{ij1}, m_{ij}, m_{ij} + \Delta_{ij2}) \tag{9}$$

The membership grade of m_{ij} in $\tilde{m}_{ij}$ is 1. The farther the point in the interval $[m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}]$ from both sides of m_{ij}, the lower the membership grade is. The membership grade and the confidence level have the same properties. Therefore, if we make a correspondence between the membership grade and the confidence level, it is reasonable to set up a triangular fuzzy number in Equation (9) corresponding to $[m_{ij}-\Delta_{ij1}, m_{ij}+\Delta_{ij2}]$, for $i = 1, 2, \ldots, 6$; $j = 1, 2, \ldots, n_i$. We have the following proposition:

Proposition 2.4. *Let* $0 < \Delta_{ij1} < m_{ij}$, $0 < \Delta_{ij2}$, *then there is a triangular fuzzy number* $\tilde{m}_{ij} = (m_{ij} - \Delta_{ij1}, m_{ij}, m_{ij} + \Delta_{ij2})$ *corresponding to the closed interval* $[m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}]$.

Definition 2.5. *Let* $X, Y \subseteq R$ *be universal sets, then*

$$\tilde{R} = \{((x, y), \mu_{\tilde{R}}(x, y)) | (x, y) \subseteq X \times Y\} \tag{10}$$

is called a fuzzy relation on $X \times Y$.

3. The Proposed Fuzzy Risk Assessment Method.

We present the fuzzy assessment method as follows:

3.1. Assessment form for the risk items.

The criteria ratings of risk are linguistic variables with linguistic values $V_1, V_2, \ldots, V_7$, where V_1 = extra low, V_2 = very low, V_3 = low, V_4 = middle, V_5 = high, V_6 = very high, V_7 = extra high. These linguistic values are treated as the following normal triangular fuzzy numbers,

$$\begin{aligned} &\tilde{V}_1 = (0, 0, 1/6) \\ &\tilde{V}_k = \left(\frac{k-2}{6}, \frac{k-1}{6}, \frac{k}{6}\right), \text{ for } k = 2, 3, \ldots, 6 \\ &\tilde{V}_7 = (5/6, 1, 1) \end{aligned} \tag{11}$$

Now, we defuzzify $\tilde{V}_1, \tilde{V}_2, \ldots, \tilde{V}_7$ by the signed distance method, and we have $d(\tilde{V}_1, \tilde{0}) = 0.0417$, $d(\tilde{V}_2, \tilde{0}) = 0.1667$, $d(\tilde{V}_3, \tilde{0}) = 0.3333$, $d(\tilde{V}_4, \tilde{0}) = 0.5$, $d(\tilde{V}_5, \tilde{0}) = 0.6667$, $d(\tilde{V}_6, \tilde{0}) = 0.8333$, $d(\tilde{V}_7, \tilde{0}) = 0.9583$.

In this study, we propose the fuzzy sense of interval value $[m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}] \subseteq [0, 1]$ instead of single value m_{ij} on assessment for the risk item X_{ij} to do the rate of aggregated risk, where $0 < \Delta_{ij1} < m_{ij}$, $0 < \Delta_{ij2}$. Since $m_{ij} \in [m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}]$ and $[m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}]$ is an interval, the evaluator could select a suitable value in $[m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}]$ for the risk item X_{ij}. Based on [12], we proposed the assessment form as shown in Table 1.

In Table 1,

$$\sum_{i=1}^{6} W_2(i) = 1, \;\; 0 \le W_2(i) \le 1, \tag{12}$$

for each $i = 1, 2, \ldots, 6$.

$$\sum_{i=1}^{n_k} W_1(k, i) = 1, \;\; 0 \le W_1(k, i) \le 1 \tag{13}$$

for $k = 1, 2, \ldots, 6$; $n_1 = 1$, $n_2 = 4$, $n_3 = 2$, $n_4 = 4$, $n_5 = 2$, $n_6 = 1$; $i = 1, 2, \ldots, n_k$.

$$\tilde{m}_{ki}^{(l)} = \left(m_{ki}^{(l)} - \Delta_{ki1}^{(l)}, m_{ki}^{(l)}, m_{ki}^{(l)} + \Delta_{ki2}^{(l)}\right) \tag{14}$$

$$\sum_{l=1}^{7} m_{ki}^{(l)} = 1, \;\; 0 \le m_{ki}^{(l)} \le 1 \tag{15}$$

$$0 \le \Delta_{ki1}^{(l)} < m_{ki}^{(l)}, \;\; 0 \le \Delta_{ki2}^{(l)}, \tag{16}$$

for $l = 1, 2, \ldots, 7$; $k = 1, 2, \ldots, 6$; $i = 1, 2, \ldots, n_k$. $\Delta_{ki1}^{(l)}$, $\Delta_{ki2}^{(l)}$ are determined by the evaluator.

TABLE 1. Contents of the hierarchical structure model

Risk attribute	Risk item	Weight-2	Weight-1	Linguistic variables						
				V_1	V_2	V_3	V_4	V_5	V_6	V_7
X_1: Personal		$W_2(1)$								
	X_{11}: Personal shortfalls, key person(s) quit		$W_1(1,1)$	$\tilde{m}_{11}^{(1)}$	$\tilde{m}_{11}^{(2)}$	$\tilde{m}_{11}^{(3)}$	$\tilde{m}_{11}^{(4)}$	$\tilde{m}_{11}^{(5)}$	$\tilde{m}_{11}^{(6)}$	$\tilde{m}_{11}^{(7)}$
X_2: System requirement		$W_2(2)$								
	X_{21}: Requirement ambiguity		$W_1(2,1)$	$m_{21}^{(1)}$	$\tilde{m}_{21}^{(2)}$	$\tilde{m}_{21}^{(3)}$	$m_{21}^{(4)}$	$\tilde{m}_{21}^{(5)}$	$m_{21}^{(6)}$	$\tilde{m}_{21}^{(7)}$
	X_{22}: Developing the wrong software function		$W_1(2,2)$	$\tilde{m}_{22}^{(1)}$	$\tilde{m}_{22}^{(2)}$	$\tilde{m}_{22}^{(3)}$	$\tilde{m}_{22}^{(4)}$	$\tilde{m}_{22}^{(5)}$	$\tilde{m}_{22}^{(6)}$	$\tilde{m}_{22}^{(7)}$
	X_{23}: Developing the wrong user interface		$W_1(2,3)$	$\tilde{m}_{23}^{(1)}$	$\tilde{m}_{23}^{(2)}$	$\tilde{m}_{23}^{(3)}$	$\tilde{m}_{23}^{(4)}$	$\tilde{m}_{23}^{(5)}$	$\tilde{m}_{23}^{(6)}$	$\tilde{m}_{23}^{(7)}$
	X_{24}: Continuing stream requirement changes		$W_1(2,4)$	$\tilde{m}_{24}^{(1)}$	$\tilde{m}_{24}^{(2)}$	$\tilde{m}_{24}^{(3)}$	$\tilde{m}_{24}^{(4)}$	$\tilde{m}_{24}^{(5)}$	$\tilde{m}_{24}^{(6)}$	$\tilde{m}_{24}^{(7)}$
X_3: Schedules and budgets		$W_2(3)$								
	X_{31}: Schedule not accurate		$W_1(3,1)$	$\tilde{m}_{31}^{(1)}$	$\tilde{m}_{31}^{(2)}$	$\tilde{m}_{31}^{(3)}$	$\tilde{m}_{31}^{(4)}$	$\tilde{m}_{31}^{(5)}$	$\tilde{m}_{31}^{(6)}$	$\tilde{m}_{31}^{(7)}$
	X_{32}: Budget not sufficient		$W_1(3,2)$	$\tilde{m}_{32}^{(1)}$	$\tilde{m}_{32}^{(2)}$	$\tilde{m}_{32}^{(3)}$	$\tilde{m}_{32}^{(4)}$	$\tilde{m}_{32}^{(5)}$	$\tilde{m}_{32}^{(6)}$	$\tilde{m}_{32}^{(7)}$
X_4: Developing technology		$W_2(4)$								
	X_{41}: Gold-plating		$W_1(4,1)$	$\tilde{m}_{41}^{(1)}$	$\tilde{m}_{41}^{(2)}$	$\tilde{m}_{41}^{(3)}$	$\tilde{m}_{41}^{(4)}$	$\tilde{m}_{41}^{(5)}$	$\tilde{m}_{41}^{(6)}$	$\tilde{m}_{41}^{(7)}$
	X_{42}: Skill levels inadequate		$W_1(4,2)$	$\tilde{m}_{42}^{(1)}$	$\tilde{m}_{42}^{(2)}$	$\tilde{m}_{42}^{(3)}$	$\tilde{m}_{42}^{(4)}$	$\tilde{m}_{42}^{(5)}$	$\tilde{m}_{42}^{(6)}$	$\tilde{m}_{42}^{(7)}$
	X_{43}: Straining hardware		$W_1(4,3)$	$\tilde{m}_{43}^{(1)}$	$\tilde{m}_{43}^{(2)}$	$\tilde{m}_{43}^{(3)}$	$\tilde{m}_{43}^{(4)}$	$\tilde{m}_{43}^{(5)}$	$\tilde{m}_{43}^{(6)}$	$\tilde{m}_{43}^{(7)}$
	X_{44}: Straining software		$W_1(4,4)$	$\tilde{m}_{44}^{(1)}$	$\tilde{m}_{44}^{(2)}$	$\tilde{m}_{44}^{(3)}$	$\tilde{m}_{44}^{(4)}$	$\tilde{m}_{44}^{(5)}$	$\tilde{m}_{44}^{(6)}$	$\tilde{m}_{44}^{(7)}$
X_5: External resource		$W_2(5)$								
	X_{51}: Shortfalls in externally furnished components		$W_1(5,1)$	$\tilde{m}_{51}^{(1)}$	$\tilde{m}_{51}^{(2)}$	$\tilde{m}_{51}^{(3)}$	$\tilde{m}_{51}^{(4)}$	$\tilde{m}_{51}^{(5)}$	$\tilde{m}_{51}^{(6)}$	$\tilde{m}_{51}^{(7)}$
	X_{52}: Shortfalls in externally performed tasks		$W_1(5,2)$	$\tilde{m}_{52}^{(1)}$	$\tilde{m}_{52}^{(2)}$	$\tilde{m}_{52}^{(3)}$	$\tilde{m}_{52}^{(4)}$	$\tilde{m}_{52}^{(5)}$	$\tilde{m}_{52}^{(6)}$	$\tilde{m}_{52}^{(7)}$
X_6: Performance		$W_2(6)$								
	X_{61}: Real-time performance shortfalls		$W_1(6,1)$	$\tilde{m}_{61}^{(1)}$	$\tilde{m}_{61}^{(2)}$	$\tilde{m}_{61}^{(3)}$	$\tilde{m}_{61}^{(4)}$	$\tilde{m}_{61}^{(5)}$	$\tilde{m}_{61}^{(6)}$	$\tilde{m}_{61}^{(7)}$

3.2. **Evaluating the rate of risk by two stages.**

Step 1: By the first stage:

Let $V = \{V_1, V_2, \dots, V_7\}$ be the set of the criteria rating of risk for each item. By fuzzy relation [20] on $X_i \times V$, we can form a fuzzy assessment matrix with fuzzy numbers M_i for $X_i \times V$ as follows, for $i = 1, 2, \dots, 6$. Thus, we have

$$M_i = \begin{bmatrix} \tilde{m}_{i1}^{(1)} & \tilde{m}_{i1}^{(2)} & \dots & \tilde{m}_{i1}^{(7)} \\ \tilde{m}_{i2}^{(1)} & \tilde{m}_{i2}^{(2)} & \dots & \tilde{m}_{i2}^{(7)} \\ \vdots & \vdots & & \vdots \\ \tilde{m}_{in_i}^{(1)} & \tilde{m}_{in_i}^{(2)} & \dots & \tilde{m}_{in_i}^{(7)} \end{bmatrix} \tag{17}$$

Let

$$(\tilde{t}_{i1}, \tilde{t}_{i2}, \dots, \tilde{t}_{i7}) = (W_1(i,1), W_1(i,2), \dots, W_1(i,n_i)) \circ M_i \tag{18}$$

where

$$\tilde{t}_{il} = \left(W_1(i,1) \otimes \tilde{m}_{i1}^{(l)}\right) \oplus \left(W_1(i,2) \otimes \tilde{m}_{i2}^{(l)}\right) \oplus \dots \oplus \left(W_1(i,n_i) \otimes \tilde{m}_{in_i}^{(l)}\right) \tag{19}$$

Then, we can have

$$\tilde{t}_{il} = \left(\sum_{j=1}^{n_i} W_1(i,j)\left(m_{ij}^{(l)} - \Delta_{ij1}^{(l)}\right), \sum_{j=1}^{n_i} W_1(i,j)\left(m_{ij}^{(l)}\right), \sum_{j=1}^{n_i} W_1(i,j)\left(m_{ij}^{(l)} + \Delta_{ij2}^{(l)}\right)\right) \tag{20}$$

Defuzzified $\tilde{t}_{il}$ by signed distance, we have

$$d(\tilde{t}_{il},\tilde{0})=\sum_{j=1}^{n_i}W_1(i,j)\left(m_{ij}^{(l)}\right)+\frac{1}{4}\sum_{j=1}^{n_i}W_1(i,j)\left(\Delta_{ij2}^{(l)}-\Delta_{ij1}^{(l)}\right) \tag{21}$$

Let

$$P_{il}=\frac{d(\tilde{t}_{il},\tilde{0})}{\sum_{l=1}^{7}d(\tilde{t}_{il},\tilde{0})} \tag{22}$$

Step 2: By the second stage:
Let

$$(\tilde{u}_1,\tilde{u}_2,\ldots,\tilde{u}_7)=(W_2(1),W_2(2),\ldots,W_2(6))\circ\begin{bmatrix}\tilde{t}_{11}&\tilde{t}_{12}&\cdots&\tilde{t}_{17}\\ \tilde{t}_{21}&\tilde{t}_{22}&\cdots&\tilde{t}_{27}\\ \vdots&\vdots&&\vdots\\ \tilde{t}_{61}&\tilde{t}_{62}&\cdots&\tilde{t}_{67}\end{bmatrix} \tag{23}$$

where

$$\tilde{u}_k=(W_2(1)\otimes\tilde{t}_{1k})\oplus(W_2(2)\otimes\tilde{t}_{2k})\oplus\cdots\oplus(W_2(6)\otimes\tilde{t}_{6k}) \tag{24}$$

Then, we can express $\tilde{u}_k$ as $\tilde{u}_k=(v_{k1},v_{k2},v_{k3})$, where

$$v_{k1}=\sum_{i=1}^{6}\left(W_2(i)\left(\sum_{j=1}^{n_i}W_1(i,j)\left(m_{ij}^{(k)}-\Delta_{ij1}^{(k)}\right)\right)\right)$$

$$v_{k2}=\sum_{i=1}^{6}\left(W_2(i)\left(\sum_{j=1}^{n_i}W_1(i,j)m_{ij}^{(k)}\right)\right)$$

$$v_{k3}=\sum_{i=1}^{6}\left(W_2(i)\left(\sum_{j=1}^{n_i}W_1(i,j)\left(m_{ij}^{(k)}+\Delta_{ij2}^{(k)}\right)\right)\right)$$

Defuzzified $\tilde{u}_k$ by signed distance, we have

$$\begin{aligned}d(\tilde{u}_k,\tilde{0})&=\sum_{i=1}^{6}\left(W_2(i)\left(\sum_{j=1}^{n_i}W_1(i,j)m_{ij}^{(k)}\right)\right)\\&\quad+\frac{1}{4}\sum_{i=1}^{6}\left(W_2(i)\left(\sum_{j=1}^{n_i}W_1(i,j)\left(\Delta_{ij2}^{(k)}-\Delta_{ij1}^{(k)}\right)\right)\right)\end{aligned} \tag{25}$$

Let

$$U_k=\frac{d\left(\tilde{u}_k,\tilde{0}\right)}{\sum_{k=1}^{7}d\left(\tilde{u}_k,\tilde{0}\right)} \tag{26}$$

Then, we have the following proposition.

Proposition 3.1. *For assessment form as shown in Table 1, and from Equation (22) and Equation (26), we have*

(1) The risk rate for the attribute X_i respective to the criteria rating V_l is P_{il} as shown in Equation (22).

(2) The aggregative risk rate in software development respective to the criteria rating V_l is U_l as shown in Equation (26).

(3) The aggregative risk rate in software development is $S=\sum_{l=1}^{7}U_l\cdot d(\tilde{V}_l,\tilde{0})$.

4. **Example Implementation.** Suppose that we have the assessed weights of the risk attributes and items, and the rating for each risk item as shown in Table 2.

TABLE 2. Contents of the example

Attribute	Risk item	Weight-2	Weight-1	Linguistic variables						
				V_1	V_2	V_3	V_4	V_5	V_6	V_7
X_1		0.3								
	X_{11}		1	$\tilde{0}$	(0, 0.17, 0.34)	(0.7, 0.83, 0.96)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
X_2		0.3								
	X_{21}		0.4	$\tilde{0}$	(0.5, 0.53, 0.56)	(0.4, 0.47, 0.54)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
	X_{22}		0.4	$\tilde{0}$	(0.8, 0.89, 0.98)	(0.1, 0.11, 0.12)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
	X_{23}		0.1	(0.1, 0.25, 0.4)	(0.6, 0.75, 0.9)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
	X_{24}		0.1	(0.5, 0.61, 0.72)	(0.2, 0.39, 0.58)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
X_3		0.1								
	X_{31}		0.5	$\tilde{0}$	(0.1, 0.17, 0.24)	(0.7, 0.83, 0.96)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
	X_{32}		0.5	$\tilde{0}$	(0.5, 0.53, 0.56)	(0.4, 0.47, 0.54)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
X_4		0.1								
	X_{41}		0.3	$\tilde{0}$	(0.8, 0.89, 0.98)	(0.1, 0.11, 0.12)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
	X_{42}		0.1	$\tilde{0}$	(0.1, 0.17, 0.24)	(0.7, 0.83, 0.96)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
	X_{43}		0.3	$\tilde{0}$	(0.1, 0.17, 0.24)	(0.7, 0.83, 0.96)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
	X_{44}		0.3	$\tilde{0}$	(0.5, 0.53, 0.56)	(0.4, 0.47, 0.54)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
X_5		0.1								
	X_{51}		0.5	$\tilde{0}$	$\tilde{0}$	(0.8, 0.81, 0.82)	(0, 0.19, 0.38)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
	X_{52}		0.5	$\tilde{0}$	$\tilde{0}$	(0.7, 0.81, 0.92)	(0, 0.19, 0.38)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
X_6		0.1								
	X_{61}		1	0	(0.1, 0.17, 0.24)	(0.7, 0.83, 0.96)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$

In Table 2, $\tilde{0} = (0, 0, 0)$.

By the evaluating process in Section 3.2, we have

(1) The risk rate of the attribute X_i with respective to the criteria V_q are as shown in Table 3, for $i = 1, 2, \ldots, 6$.

TABLE 3. The value of P_{il}, $i = 1, 2, \ldots, 6$; $l = 1, 2, \ldots, 7$

Attribute	V_1	V_2	V_3	V_4	V_5	V_6	V_7
X_1	0	0.17	0.83	0	0	0	0
X_2	0.086	0.682	0.232	0	0	0	0
X_3	0	0.35	0.65	0	0	0	0
X_4	0	0.494	0.506	0	0	0	0
X_5	0	0	0.81	0.19	0	0	0
X_6	0	0.17	0.83	0	0	0	0

(2) The aggregative risk rate in software development respective to the criteria rating V_l is as the following, for $l = 1, 2, \ldots, 7$.

$$U_1 = 0.0258, \quad U_2 = 0.357, \quad U_3 = 0.5982$$

$$U_4 = 0.019, \quad U_5 = 0, \quad U_6 = 0, \quad U_7 = 0$$

(3) The aggregative risk rate in software development is $S = 0.269476$.

(4) Comparison with Lee and Lin [12].

In [12], the rate of aggregative risk is 0.269475. By the proposed method in this study, the computed result is 0.269476. The relative error is $(0.269476 - 0.269475)/0.269475 = 0.0000037$. It is very small. However, we can apply the proposed method in this paper to evaluating the risk rate with respective to the criteria V_q, for $q = 1, 2, \ldots, 7$.

5. **Conclusions.** In this paper, we propose the fuzzy sense of interval value $[m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}]$ instead of the single value m_{ij} on assessment to do the rate of aggregated risk. The proposed fuzzy assessment method using interval valued is presented aiming at evaluating software risk rate. Not only the evaluators' incomplete and uncertainty thought is reflected, but also the assessment of evaluation with fuzzy numbers can reduce the degree of subjectivity of the evaluators.

Acknowledgment. This work was partially supported by the National Science Council of Taiwan, under grant numbers NSC-99-2410-H-034-019- and NSC-100-2410-H-034-007-MY2-. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] AFSC, Software risk abatement, *U. S. Air Force Systems Command, AFSC/AFLC Pamphlet 800-45*, Andrews AFB, MD, USA, pp.1-28, 1988.
[2] B. W. Boehm, A spiral model of software development and enhancement, *Computer*, pp.61-72, 1988.
[3] B. W. Boehm, *Software Risk Management*, CS Press, Los Alamitos, 1989.
[4] B. W. Boehm, Software risk management: Principles and practices, *IEEE Software*, vol.8, pp.32-41, 1991.
[5] R. N. Charette, *Software Engineering Risk Analysis and Management*, Mc Graw-Hill, New York, 1989.
[6] S. M. Chen, Evaluating the rate of aggregative risk in software development using fuzzy set theory, *Cybernetics and Systems: International Journal*, vol.30, pp.57-75, 1999.
[7] S. A. Conger, *The New Software Engineering*, Wadsworth Publishing Co., Belmont, CA, USA, 1994.
[8] A. Kaufmann and M. M. Gupta, *Introduction to Fuzzy Arithmetic Theory and Applications*, Van Nostrand Reinhold, New York, USA, 1991.
[9] H.-M. Lee, Applying fuzzy set theory to evaluate the rate of aggregative risk in software development, *Fuzzy Sets and Systems*, vol.79, pp.323-336, 1996.
[10] H.-M. Lee, Group decision making using fuzzy sets theory for evaluating the rate of aggregative risk in software development, *Fuzzy Sets and Systems*, no.80, pp.261-271, 1996.
[11] H.-M. Lee, S.-Y. Lee, T.-S. Lee and J.-J. Chen, A new algorithm for applying fuzzy set theory to evaluate the rate of aggregative risk in software development, *Information Sciences*, vol.153, pp.177-197, 2003.
[12] H.-M. Lee and L. Lin, A fuzzy assessment for software development risk rate, *ICIC Express Letters*, vol.4, no.2, pp.319-323, 2010.
[13] H.-M. Lee and L. Lin, Fuzzy risk presumptive evaluation in software development, *International Journal of Innovative Computing, Information and Control*, vol.7, no.7(A), pp.3881-3889, 2011.
[14] L. Lin and H.-M. Lee, Fuzzy assessment method on sampling survey analysis, *Expert Systems with Applications*, vol.36, no.3, pp.5955-5961, 2009.
[15] S. L. Pfleeger, *Software Engineering Theory and Practice*, Prentice-Hall, Inc., Upper Saddle River, New Jersey, USA, 1998.
[16] I. Sommerville, *Software Engineering*, 6th Edition, Pearson Education Limited, England, 2001.
[17] J.-S. Yao and K. Wu, Ranking fuzzy numbers based on decomposition principle and signed distance, *Fuzzy Sets and Systems*, vol.116, pp.275-288, 2000.
[18] L. A. Zadeh, Fuzzy sets, *Information and Control*, vol.8, pp.338-353, 1965.
[19] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, *Information Sciences*, vol.8, pp.199-249 (Part I), pp.301-357 (Part II), 1975; vol.9, pp.43-58 (Part III), 1976.
[20] H.-J. Zimmermann, *Fuzzy Set Theory and Its Applications*, 2nd Revised Edition, Kluwer Academic Publishers, Boston/Dordrecht/London, 1991.