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# A COMPARATIVE STUDY FOR DIMENSIONING OF FOOTINGS WITH RESPECT TO THE CONTACT SURFACE ON SOIL

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Abstract. *This paper presents a comparative study among three types of footings of different shape to obtain the more economical dimension of the contact surface, i.e., a comparison is realized among the rectangular footings, square and circular in terms of the contact area with soil, when load that must withstand said structural member is applied. The models presented in this paper satisfy the following conditions. 1) The minimum stress should be equal to or greater than zero, because the soil is not capable of withstanding tensile stresses. 2) The maximum stress must be equal to or less than the allowable capacity that can be capable of withstanding the soil. According to the results, tables show that the circular footings are more economic, subsequently the rectangular footings and last the square footings.*
**Keywords:** Circular footings, Rectangular footings, Square footings, Allowable capacity of soil, Contact surface, More economical dimension

## 1. Introduction.

The foundation is the part of the structure which transmits the loads to the floor. Each building demands the need to solve a problem of foundation. The foundations are classified into superficial and deep, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality and its constructive systems [1,2].

A superficial foundation is a structural member whose cross section is of large dimensions with respect to height and whose function is to transfer the loads of a building at depths relatively short, less than 4 m approximately with respect to the level of the natural ground surface [3,4].

Superficial foundations, whose constructive systems generally do not present major difficulties, may be of various types according to their function; isolated footing, combined footing: strip footing, or mat foundation [5].

In the design of superficial foundations, the specific cases of isolated footings are of three types in terms of the application of the loads. 1) Footings subject to concentric axial load. 2) Footings subject to axial load and moment in one direction (unidirectional bending). 3) Footings subject to axial load and moment in two directions (bidirectional bending) [1-4].

The hypothesis used in the classical model is developed by trial and error, i.e., a dimension is proposed, and last the expression of the bidirectional bending is used to obtain the stress acting on the contact surface of footings, which must satisfy the following conditions. 1) The minimum stress should be equal to or greater than zero, because the soil is not capable of withstanding tensile stresses. 2) The maximum stress must be equal to or less than the allowable capacity that can withstand the soil [1-5].

Lately, a mathematical model is developed to take into account the real pressure of soil acting on the contact surface of the rectangular footing when applying the load that must support said structural member [6]. Also a full mathematical model is presented for design of rectangular footings to obtain: 1) the moment around of an axis a'-a' that is parallel to axis "X-X" and moment around an axis b'-b' that is parallel to axis "Y-Y"; 2) the shear forces by bending (unidirectional shear force); 3) the shear forces by penetration (bidirectional shear force) for footings that are supporting to a rectangular column or a circular column for footings subject to axial load and moment in two directions (bidirectional bending) [7].

In recent years, mathematical models have been developed to obtain the more economical dimension of rectangular footings, square and circular subjected to an axial load and moments in two directions (bidirectional bending) that comply with the aforementioned conditions [8-10].

This paper presents a comparative study between three types of footings to obtain the more economical dimension of the contact surface, i.e., a comparison is realized among the rectangular footings, square and circular subjected to axial load and moment in two directions (bidirectional bending), in terms of the contact area on soil, when load that must withstand said structural member is applied, where there are two conditions that must satisfy: the first condition is that the minimum stress should be equal to or greater than zero, because the soil is not able to withstand tensile stresses, and the second condition is that the maximum stress should be equal to or less than the allowable capacity of the soil. Also, each model presents the point where maximum stress and minimum appear.

## 2. Mathematical Development of the Models.

### 2.1. General equation of the bidirectional bending.

Figure 1 shows a footing of general shape subject to axial load and moment in two directions (bidirectional bending) where pressures are different in contact surface; such pressures vary linearly [11-16].

General expression for bidirectional flexure of footings is:

$$\sigma = \frac{P}{A} \pm \frac{M_x C_y}{I_x} \pm \frac{M_y C_x}{I_y} \tag{1}$$

where $A$ is the contact area of the footing, $P$ is the axial load applied at the center of gravity of the footing, $M_x$ is the moment around the axis "$X$", $M_y$ is the moment around the axis "$Y$", $C_x$ is the distance in the direction "$X$" which is measured from the axis "$Y$" to the fiber in study, $C_y$ is the distance in the direction "$Y$" which is measured from the axis "$X$" to the fiber in study, $I_y$ is the moment of inertia about axis "$Y$" and $I_x$ is the moment of inertia about axis "$X$".

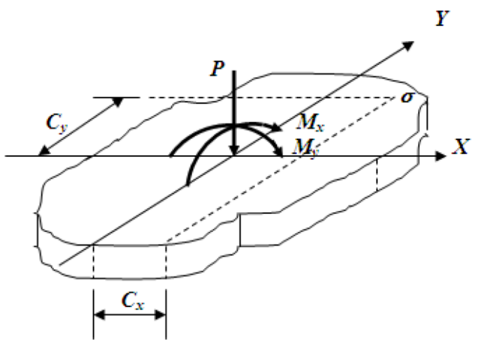


FIGURE 1. General shape footing due to real load

Any type of footings must satisfy the following conditions: the first condition is that the minimum stress should be equal to or greater than zero, because the soil is not able to withstand tensile stresses; the second condition is that the maximum stress should be equal to or less than the allowable capacity of the soil.

2.2. **Rectangular footing.** Figure 2 shows a rectangular footing due to a real load subjected to an axial load and moment in two directions (bidirectional bending) where pressures are different in the four corners of the contact surface [8,11-16].

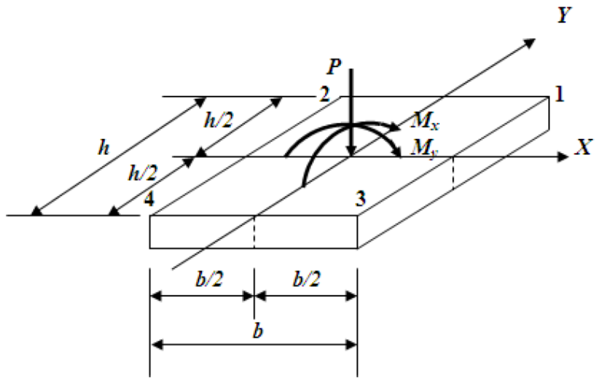


FIGURE 2. Rectangular footing due to real load

Geometric properties for rectangular footings are: $A = bh$, $C_y = h/2$, $C_x = b/2$, $I_x = bh^3/12$ and $I_y = hb^3/12$, which are substituted into Equation (1) to find the stresses at each corner of rectangular footings:

$$\sigma_1 = \frac{P}{bh} + \frac{6M_x}{bh^2} + \frac{6M_y}{b^2h} \tag{2}$$

$$\sigma_2 = \frac{P}{bh} + \frac{6M_x}{bh^2} - \frac{6M_y}{b^2h} \tag{3}$$

$$\sigma_3 = \frac{P}{bh} - \frac{6M_x}{bh^2} + \frac{6M_y}{b^2h} \tag{4}$$

$$\sigma_4 = \frac{P}{bh} - \frac{6M_x}{bh^2} - \frac{6M_y}{b^2h} \tag{5}$$

where $\sigma_1 = \sigma_{\max}$ is the maximum stress and $\sigma_4 = \sigma_{\min}$ is the minimum stress.

For a rectangular footing there are two conditions: the first is that the minimum stress should be zero, since the soil is not capable of withstanding tensile stresses and the second is that the maximum stress is the load capacity that can withstand the soil.

2.2.1. *General conditions.* Figure 3 presents a rectangular footing rectangular due to equivalent load. The normal solicitations of components are $P$, $M_x$, $M_y$, which are equivalent to an axial force $P$ acting in the point action with coordinates $(e_x, e_y)$.

The permitted maximum eccentricity that the tensile stresses are not presented in the soil is 1/6 the side of the footing. Then the eccentricity is defined as follows [8,11-15]:

$$e_x = \frac{M_y}{P} = \frac{b}{6} \to P = \frac{6M_y}{b} \tag{6}$$

$$e_y = \frac{M_x}{P} = \frac{b}{6} \to P = \frac{6M_x}{h} \tag{7}$$

where $e_x$ is the eccentricity in the direction "$X$", and $e_y$ is the eccentricity in the direction "$Y$".

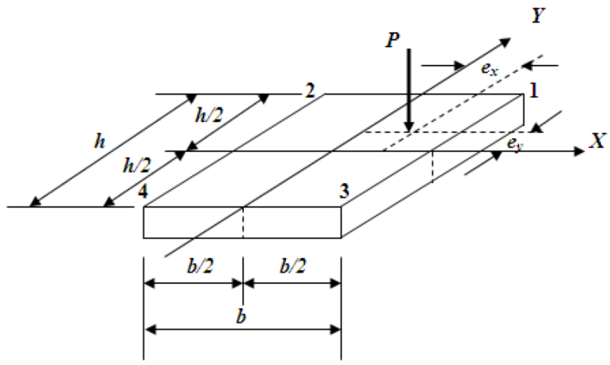


FIGURE 3. Rectangular footing due to equivalent load

The values of $P$ are equal in Equations (6) and (7):

$$\frac{6M_y}{b} = \frac{6M_x}{h} \tag{8}$$

The value $b$ is found from Equation (8):

$$b = \frac{M_y h}{M_x} \tag{9}$$

2.2.2. *First condition.* The minimum stress is zero:

$$\sigma_{\min} = \sigma_4 = 0 \tag{10}$$

Equation (10) is substituted into Equation (5):

$$0 = \frac{P}{bh} - \frac{6M_x}{bh^2} - \frac{6M_y}{b^2 h} \tag{11}$$

Equation (11) is presented as follows:

$$0 = Pbh - 6M_x b - 6M_y h \tag{12}$$

Equation (9) is substituted into Equation (12), and we have the following:

$$0 = P\left(\frac{M_y h}{M_x}\right) h - 6M_x\left(\frac{M_y h}{M_x}\right) - 6M_y h \tag{13}$$

Equation (13) is simplified:

$$0 = Ph - 6M_x - 6M_x \tag{14}$$

Then of Equation (14) is found "$h$" as follows:

$$h = \frac{12M_x}{P} \tag{15}$$

When Equation (15) is substituted into Equation (9), "$b$" is obtained:

$$b = \frac{12M_y}{P} \tag{16}$$

Therefore, of Equations (15) and (16) are found the dimensions of a rectangular footing, when the pressure of soil on the footing is zero.

2.2.3. *Second condition.* The maximum stress is the loading capacity of the soil:

$$\sigma_1 = \sigma_{\max} \tag{17}$$

Equation (17) is substituted into Equation (2):

$$\sigma_{\max} = \frac{P}{bh} + \frac{6M_x}{bh^2} + \frac{6M_y}{b^2h} \tag{18}$$

Equation (18) is presented as follows:

$$\sigma_{\max} b^2 h^2 = Pbh + 6M_x b + 6M_y h \tag{19}$$

Equation (9) is substituted into Equation (19) as the following:

$$\sigma_{\max} \left(\frac{M_y h}{M_x}\right)^2 h^2 = P\left(\frac{M_y h}{M_x}\right) h + 6M_x \left(\frac{M_y h}{M_x}\right) + 6M_y h \tag{20}$$

Equation (20) is simplified:

$$\sigma_{\max} M_y h^3 - PM_x h - 12M_x^2 = 0 \tag{21}$$

Then of Equation (21) is solved to obtain value "$h$", and this is substituted into Equation (9) to find the value "$b$". These are dimensions of a rectangular footing when the pressure is the loading capacity of the soil.

Therefore, the proposal minimum dimension of a rectangular footing is the following: the dimension greater obtained the first condition by Equations (15) and (16) or the second condition by Equations (9) and (21).

2.3. **Square footing.** Figure 4 shows a square footing due to a real load subjected to an axial load and moment in two directions (bidirectional bending) where pressures are different in the four corners of the contact surface.

Geometric properties for square footings are: $A = L^2$, $C_y = L/2$, $C_x = L/2$, $I_x = L^4/12$ and $I_y = L^4/12$, which are substituted into Equation (1) to find the stresses at each corner of square footings [9,11-16]:

$$\sigma_1 = \frac{P}{L^2} + \frac{6M_x}{L^3} + \frac{6M_y}{L^3} \tag{22}$$

$$\sigma_2 = \frac{P}{L^2} + \frac{6M_x}{L^3} - \frac{6M_y}{L^3} \tag{23}$$

$$\sigma_3 = \frac{P}{L^2} - \frac{6M_x}{L^3} + \frac{6M_y}{L^3} \tag{24}$$

$$\sigma_4 = \frac{P}{L^2} - \frac{6M_x}{L^3} - \frac{6M_y}{L^3} \tag{25}$$

where $\sigma_1 = \sigma_{\max}$ is the maximum stress and $\sigma_4 = \sigma_{\min}$ is the minimum stress.

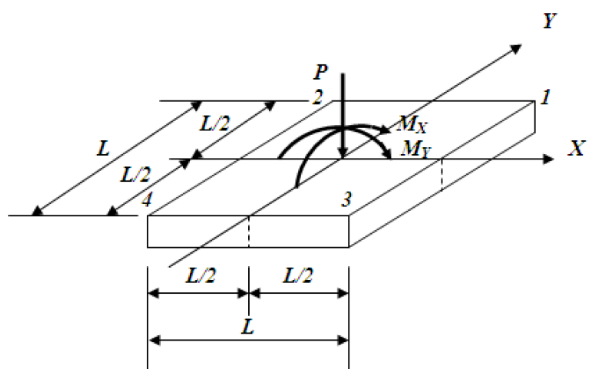


FIGURE 4. Square footing due to real load

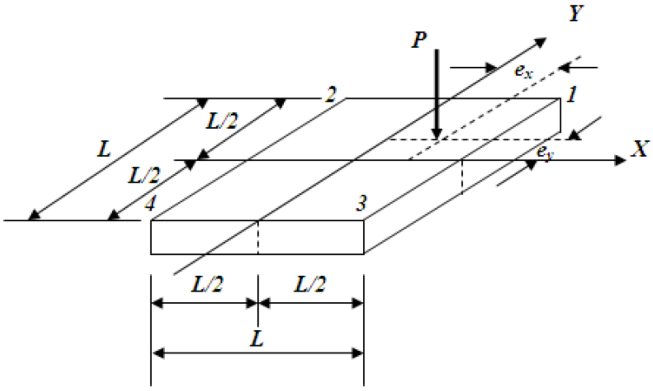


FIGURE 5. Square footing due to equivalent load

For a square footing there are two conditions: the first is that the minimum stress should be zero, since the soil is not capable of withstanding tensile stresses and the second is that the maximum stress is the load capacity that can withstand the soil.

2.3.1. *General conditions.* Figure 5 presents a square footing due to equivalent load. The normal solicitations of components are $P$, $M_x$, $M_y$, which are equivalent to an axial force $P$ acting in the point action with coordinates $(e_x, e_y)$.

The permitted maximum eccentricity that the tensile stresses are not presented in the soil is 1/6 the side of the footing. Then the eccentricity is defined as follows:

$$e_x = \frac{M_y}{P} = \frac{L}{6} \tag{26}$$

$$e_y = \frac{M_x}{P} = \frac{L}{6} \tag{27}$$

2.3.2. *First condition.* The minimum stress is zero:

$$\sigma_{\min} = \sigma_4 = 0 \tag{28}$$

Equation (28) is substituted into Equation (25):

$$0 = \frac{P}{L^2} - \frac{6M_x}{L^3} - \frac{6M_y}{L^3} \tag{29}$$

Equation (29) is simplified:

$$0 = PL - 6M_x - 6M_y \tag{30}$$

Then from Equation (30) is found "$L$":

$$L = \frac{6M_y + 6M_x}{P} \tag{31}$$

Therefore, of Equation (31) is found the dimension of a square footing, when the pressure of soil on the footing is zero.

2.3.3. *Second condition.* The maximum stress is the loading capacity of the soil:

$$\sigma_1 = \sigma_{\max} \tag{32}$$

Equation (32) is substituted into Equation (22):

$$\sigma_{\max} = \frac{P}{L^2} + \frac{6M_x}{L^3} + \frac{6M_y}{L^3} \tag{33}$$

Equation (33) is presented as follows:

$$\sigma_{\max} L^3 - PL - 6(M_x + M_y) = 0 \tag{34}$$

Then of Equation (34) is solved to obtain value "$L$", and this is the dimension of a square footing when the pressure is the loading capacity of the soil.

Therefore, the proposal minimum dimension of a square footing is the following: the dimension greater obtained the first condition by Equation (31) or the second condition by Equation (34).

2.4. **Circular footing.** Figure 6 shows a circular footing due to a real load subjected to an axial load and moment in two directions (bidirectional bending) where pressures are different in the entire contact surface.

2.4.1. *General conditions.* Figure 7 presents a typical circular footing to obtain the stresses in any point on the contact surface of said structural member due to pressure exerted by the soil.

Geometric properties for circular footings are: $A = \pi d^2/4$, $C_y = y$, $C_x = x$, $I_x = d^4/64$ and $I_y = d^4/64$, which are substituted into Equation (1) to find the stresses on the contact surface of circular footings [9,11-16]:

$$\sigma(x, y) = \frac{4P}{\pi d^2} + \frac{64M_x y}{\pi d^4} + \frac{64M_y x}{\pi d^4} \tag{35}$$

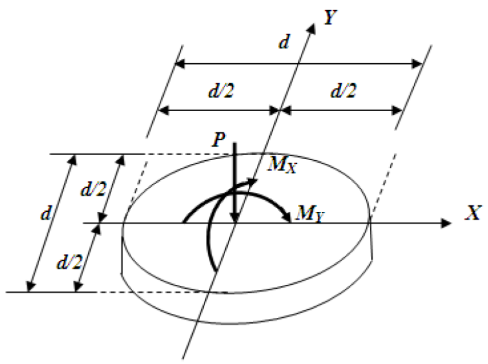


FIGURE 6. Circular footing due to real load

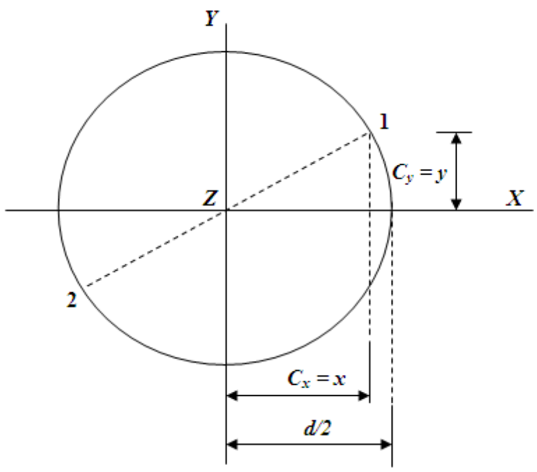


FIGURE 7. Circular footing in plan

Geometric properties of the circle are used [17,18]:

$$x = \sqrt{4d^2 - y^2} \tag{36}$$

Equation (36) is substituted into Equation (35), we obtain the following:

$$\sigma(y) = \frac{4P}{\pi d^2} + \frac{64M_x y}{\pi d^4} + \frac{64M_y\sqrt{4d^2 - y^2}}{\pi d^4} \tag{37}$$

Maximum and minimum stresses are found, from the first derivative of Equation (37), that is [17,18]:

$$\frac{d\sigma(y)}{dy} = \frac{64M_x}{\pi d^4} - \frac{64M_y y}{\pi d^4\sqrt{4d^2 - y^2}} \tag{38}$$

Subsequently, Equation (38) is equal to zero to find the value of "$y$", where the stress is a maximum or minimum:

$$y = \frac{M_x d}{\pm 2\sqrt{M_x^2 + M_y^2}} \tag{39}$$

After, Equation (39) is replaced into Equation (36) to obtain the values of "$x$":

$$x = \frac{M_y d}{\pm 2\sqrt{M_x^2 + M_y^2}} \tag{40}$$

Therefore, the coordinates value, when the stress is maximum:

$$x = \frac{M_y d}{2\sqrt{M_x^2 + M_y^2}}; \quad y = \frac{M_x d}{2\sqrt{M_x^2 + M_y^2}} \tag{41}$$

And, the coordinates value, when the stress is minimal:

$$x = -\frac{M_y d}{2\sqrt{M_x^2 + M_y^2}}; \quad y = -\frac{M_x d}{2\sqrt{M_x^2 + M_y^2}} \tag{42}$$

Thus, Equation (43) is used to find the maximum and minimum stresses in circular footings as follows:

$$\sigma_1 = \frac{4P}{\pi d^2} + \frac{32\sqrt{M_x^2 + M_y^2}}{\pi d^3} \tag{43}$$

$$\sigma_2 = \frac{4P}{\pi d^2} - \frac{32\sqrt{M_x^2 + M_y^2}}{\pi d^3} \tag{44}$$

where $\sigma_1 = \sigma_{\max}$ is the maximum stress and $\sigma_2 = \sigma_{\min}$ is the minimum stress.

2.4.2. *First condition.* The minimum stress is zero:

$$\sigma_{\min} = \sigma_2 = 0 \tag{45}$$

Equation (45) is substituted into Equation (44) as follows:

$$0 = \frac{4P}{\pi d^2} - \frac{32\sqrt{M_x^2 + M_y^2}}{\pi d^3} \tag{46}$$

Equation (46) is simplified, and we obtain:

$$0 = 4Pd - 32\sqrt{M_x^2 + M_y^2} \tag{47}$$

Then from Equation (47) is found "$d$":

$$d = \frac{32\sqrt{M_x^2 + M_y^2}}{4P} \tag{48}$$

Therefore, of Equation (48) is found the diameter of a circular footing, when the pressure of soil on the footing is zero.

2.4.3. *Second condition.* The maximum stress is the loading capacity of the soil:

$$\sigma_1 = \sigma_{\max} \tag{49}$$

Equation (49) is substituted into Equation (43) as follows:

$$\sigma_{\max} = \frac{4P}{\pi d^2} + \frac{32\sqrt{M_x^2 + M_y^2}}{\pi d^3} \tag{50}$$

Equation (50) is simplified, and we obtain:

$$\sigma_{\max}\pi d^3 - 4Pd - 32\sqrt{M_x^2 + M_y^2} = 0 \tag{51}$$

Then of Equation (51) is solved to obtain value "$d$", and this is the dimension of a circular footing when the pressure is the loading capacity of the soil.

Therefore, the proposal minimum diameter of a circular footing is the following: the dimension greater obtained the first condition by Equation (48) or the second condition by Equation (51).

3. **Application.** An example is presented for rectangular footings, square and circular subjected to an axial load and moment in two directions (bidirectional bending) to show the differences.

3.1. **Rectangular footing.** Below, the three cases of rectangular footings are presented. The dimensions are obtained by means of Equations (15) and (16), when the minimum pressure is zero. The dimensions are found by means of Equations (9) and (21), where the maximum pressure is the soil loading capacity. The proposed dimensions are found taking into account the larger of the two above conditions. Once the dimensions of the footing are defined, the stresses generated by loads applied to the foundation are obtained to verify that these stresses are within the established parameters, i.e., the maximum stress is equal to or less than the load capacity of the soil, and the minimum stress is equal to or greater than zero, since the soil is not capable of withstanding the tensile stresses. The results are presented in Table 1.

3.2. **Square footing.** Below, the three cases of square footings are presented. The dimension is found through Equation (31), when the minimum pressure is zero. The dimension is obtained through Equation (34), where the maximum pressure is the soil loading capacity. The proposed dimension is found taking into account the larger of the two above conditions. Once the dimension of the footing is defined, the stresses generated by loads applied to the foundation are obtained to verify that these stresses are within the established parameters, i.e., the maximum stress is equal to or less than the load capacity of the soil, and the minimum stress is equal to or greater than zero, since the soil is not capable of withstanding the tensile stresses. The results are presented in Table 2.

3.3. **Circular footing.** Below, the three cases of circular footings are presented. The diameter is obtained through Equation (48), when the minimum pressure is zero. The diameter is found through Equation (51), where the maximum pressure is the soil loading capacity. The proposed diameter is found taking into account the larger of the two above conditions. Once the diameter of the footing is defined, the stresses generated by loads applied to the foundation are found to verify that these stresses are within the established parameters, i.e., the maximum stress is equal to or less than the load capacity of the soil, and the minimum stress is equal to or greater than zero, since the soil is not capable of withstanding the tensile stresses. The results are presented in Table 3.

Table 1. Dimensioning of rectangular footings

| Load capacity of the soil (kN/m$^2$) | Axial load in the footing $P$ (kN) | Moments (kN-m) | | Dimensions for zero minimum pressure (m) | | Dimensions for the maximum pressure (m) | | Dimensions proposed (m) | | Stresses generated by the loads (kN/m$^2$) | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $M_y$ | $M_x$ | $b$ | $h$ | $b$ | $h$ | $b$ | $h$ | $\sigma_{\max}$ | $\sigma_{\min}$ |
| Case 1 | | | | | | | | | | | |
| 245.25 | 686.70 | 68.67 | 98.10 | 1.20 | 1.71 | 1.81 | 2.58 | 1.85 | 2.60 | 236.13 | 49.44 |
| 196.20 | 686.70 | 68.67 | 98.10 | 1.20 | 1.71 | 1.98 | 2.83 | 2.00 | 2.85 | 192.86 | 48.07 |
| 147.15 | 686.70 | 68.67 | 98.10 | 1.20 | 1.71 | 2.24 | 3.20 | 2.25 | 3.20 | 146.37 | 44.44 |
| 98.10 | 686.70 | 68.67 | 98.10 | 1.20 | 1.71 | 2.67 | 3.81 | 2.70 | 3.85 | 95.45 | 36.69 |
| Case 2 | | | | | | | | | | | |
| 245.25 | 490.50 | 68.67 | 98.10 | 1.68 | 2.40 | 1.67 | 2.39 | 1.70 | 2.40 | 239.80 | 0.69 |
| 196.20 | 490.50 | 68.67 | 98.10 | 1.68 | 2.40 | 1.83 | 2.62 | 1.85 | 2.65 | 190.80 | 9.32 |
| 147.15 | 490.50 | 68.67 | 98.10 | 1.68 | 2.40 | 2.06 | 2.94 | 2.10 | 2.95 | 143.03 | 15.30 |
| 98.10 | 490.50 | 68.67 | 98.10 | 1.68 | 2.40 | 2.44 | 3.48 | 2.45 | 3.50 | 96.43 | 17.95 |
| Case 3 | | | | | | | | | | | |
| 245.25 | 490.50 | 98.10 | 147.15 | 2.40 | 3.60 | 1.77 | 2.66 | 2.40 | 3.60 | 113.50 | 0.00 |
| 196.20 | 490.50 | 98.10 | 147.15 | 2.40 | 3.60 | 1.93 | 2.90 | 2.40 | 3.60 | 113.50 | 0.00 |
| 147.15 | 490.50 | 98.10 | 147.15 | 2.40 | 3.60 | 2.17 | 3.25 | 2.40 | 3.60 | 113.50 | 0.00 |
| 98.10 | 490.50 | 98.10 | 147.15 | 2.40 | 3.60 | 2.55 | 3.82 | 2.55 | 3.85 | 96.82 | 3.14 |

Table 2. Dimensioning of square footings

| Load capacity of the soil (kN/m$^2$) | Axial load in the footing $P$ (kN) | Moments (kN-m) | | Dimensions for zero minimum pressure (m) | Dimensions for the maximum pressure (m) | Dimensions proposed (m) | Stresses generated by the loads (kN/m$^2$) | |
|---|---|---|---|---|---|---|---|---|
| | | $M_y$ | $M_x$ | $L$ | $L$ | $L$ | $\sigma_{\max}$ | $\sigma_{\min}$ |
| Case 1 | | | | | | | | |
| 245.25 | 686.70 | 68.67 | 98.10 | 1.46 | 2.16 | 2.20 | 235.83 | 47.87 |
| 196.20 | 686.70 | 68.67 | 98.10 | 1.46 | 2.38 | 2.40 | 191.59 | 46.79 |
| 147.15 | 686.70 | 68.67 | 98.10 | 1.46 | 2.68 | 2.70 | 144.99 | 43.36 |
| 98.10 | 686.70 | 68.67 | 98.10 | 1.46 | 3.19 | 3.20 | 97.61 | 36.49 |
| Case 2 | | | | | | | | |
| 245.25 | 490.50 | 68.67 | 98.10 | 2.04 | 2.01 | 2.05 | 232.89 | 0.59 |
| 196.20 | 490.50 | 68.67 | 98.10 | 2.04 | 2.20 | 2.20 | 195.32 | 7.36 |
| 147.15 | 490.50 | 68.67 | 98.10 | 2.04 | 2.47 | 2.50 | 142.54 | 14.42 |
| 98.10 | 490.50 | 68.67 | 98.10 | 2.04 | 2.92 | 2.95 | 95.35 | 17.36 |
| Case 3 | | | | | | | | |
| 245.25 | 490.50 | 98.10 | 147.15 | 3.00 | 2.18 | 3.00 | 11.09 | 0.00 |
| 196.20 | 490.50 | 98.10 | 147.15 | 3.00 | 2.38 | 3.00 | 11.09 | 0.00 |
| 147.15 | 490.50 | 98.10 | 147.15 | 3.00 | 2.66 | 3.00 | 11.09 | 0.00 |
| 98.10 | 490.50 | 98.10 | 147.15 | 3.00 | 3.13 | 3.15 | 9.81 | 2.35 |

3.4. **Comparison between the three footings.** Table 4 presents the comparison between the rectangular footing, square and circular.

4. **Results and Discussion.** Table 1 shows the results for the model of the three cases of rectangular footings for 4 different types of load capacity of the soil. According to case 1 is observed that the second condition prevails. This means that the footing should be

Table 3. Dimensioning of circular footings

| Load capacity of the soil (kN/m$^2$) | Axial load in the footing $P$ (kN) | Moments (kN-m) | | Dimensions for zero minimum pressure (m) | Dimensions for the maximum pressure (m) | Dimensions proposed (m) | Stresses generated by the loads (kN/m$^2$) | |
|---|---|---|---|---|---|---|---|---|
| | | $M_y$ | $M_x$ | $d$ | $d$ | $d$ | $\sigma_{\max}$ | $\sigma_{\min}$ |
| Case 1 | | | | | | | | |
| 245.25 | 686.70 | 68.67 | 98.10 | 1.40 | 2.38 | 2.40 | 240.05 | 63.57 |
| 196.20 | 686.70 | 68.67 | 98.10 | 1.40 | 2.61 | 2.65 | 190.02 | 58.96 |
| 147.15 | 686.70 | 68.67 | 98.10 | 1.40 | 2.96 | 3.00 | 142.34 | 51.99 |
| 98.10 | 686.70 | 68.67 | 98.10 | 1.40 | 3.53 | 3.55 | 96.63 | 42.08 |
| Case 2 | | | | | | | | |
| 245.25 | 490.50 | 68.67 | 98.10 | 1.95 | 2.19 | 2.20 | 243.58 | 14.52 |
| 196.20 | 490.50 | 68.67 | 98.10 | 1.95 | 2.40 | 2.45 | 186.98 | 21.09 |
| 147.15 | 490.50 | 68.67 | 98.10 | 1.95 | 2.70 | 2.75 | 141.26 | 23.94 |
| 98.10 | 490.50 | 68.67 | 98.10 | 1.95 | 3.20 | 3.25 | 94.67 | 23.64 |
| Case 3 | | | | | | | | |
| 245.25 | 490.50 | 98.10 | 147.15 | 2.88 | 2.37 | 2.90 | 148.13 | 0.39 |
| 196.20 | 490.50 | 98.10 | 147.15 | 2.88 | 2.59 | 2.90 | 148.13 | 0.39 |
| 147.15 | 490.50 | 98.10 | 147.15 | 2.88 | 2.91 | 2.95 | 141.95 | 1.57 |
| 98.10 | 490.50 | 98.10 | 147.15 | 2.88 | 3.42 | 3.45 | 96.33 | 8.63 |

dimensioned on the basis of load capacity of the soil. As regards the case 2, also the second condition is dominant. Finally, we analyze the case 3, in which the first three types of load capacities of the soil are dominant to the first condition. This means that the footing should be dimensioned on the basis of the minimum pressure where this is zero, because the soil cannot support tensile stresses. The fourth type is dominant in this case to the second condition.

Table 2 presents the results for the model of the three cases of square footings for 4 different types of load capacity of the soil. According to case 1 shows that the second condition is dominant. This means that the footing should be dimensioned on the basis of load capacity of the soil. As regards the case 2, also the second condition prevails with the exception of the first type which is dominant to the first condition. Finally, we analyze the case 3, in which the first three types of load capacities of the soil are dominant to the first condition. This means that the footing should be dimensioned on the basis of the minimum pressure where this is zero, because the soil cannot support tensile stresses. The fourth type is dominant in this case to the second condition.

Table 3 shows the results for the model of the three cases of circular footings for 4 different types of load capacity of the soil. According to case 1 shows that the second condition prevails. This means that the footing should be dimensioned on the basis of load capacity of the soil. As regards the case 2, also the second condition is dominant. Finally, we analyze case 3, in which the first two types of load capacities of the soil are dominant the first condition. This means that the footing should be dimensioned on the basis of the minimum pressure where this is zero, because the soil cannot support tensile stresses. The third type and fourth are dominant in this case to the second condition.

Table 4 presents the comparison among the rectangular footing, square and circular. According to the results tables show that the circular footings are more economic, subsequently the rectangular footings and last the square footings. Hence, the comparison is made between the square and rectangular footings with respect to the circular footings.

TABLE 4. Comparison of results between the three types of footings

| Load capacity of the soil (kN/m$^2$) | Axial load in the footing $P$ (ton) | Moments (kN-m) | | Dimensions of rectangular footings | | | Dimensions of square footings | | Dimensions of circular footings | | Comparison of results | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $M_y$ | $M_x$ | $b$ (m) | $h$ (m) | $A$ (m$^2$) | $L$ (m) | $A$ (m$^2$) | $d$ (m) | $A$ (m$^2$) | RF/CF | SF/CF |
| Case 1 | | | | | | | | | | | | |
| 245.25 | 686.70 | 68.67 | 98.10 | 1.85 | 2.60 | 4.81 | 2.20 | 4.84 | 2.40 | 4.52 | 1.064 | 1.071 |
| 196.20 | 686.70 | 68.67 | 98.10 | 2.00 | 2.85 | 5.70 | 2.40 | 5.76 | 2.65 | 5.52 | 1.033 | 1.043 |
| 147.15 | 686.70 | 68.67 | 98.10 | 2.25 | 3.20 | 7.20 | 2.70 | 7.29 | 3.00 | 7.07 | 1.018 | 1.031 |
| 98.10 | 686.70 | 68.67 | 98.10 | 2.70 | 3.85 | 10.40 | 3.20 | 10.24 | 3.55 | 9.90 | 1.051 | 1.034 |
| Case 2 | | | | | | | | | | | | |
| 245.25 | 490.50 | 68.67 | 98.10 | 1.70 | 2.40 | 4.08 | 2.05 | 4.20 | 2.20 | 3.80 | 1.074 | 1.105 |
| 196.20 | 490.50 | 68.67 | 98.10 | 1.85 | 2.65 | 4.90 | 2.20 | 4.84 | 2.40 | 4.52 | 1.084 | 1.071 |
| 147.15 | 490.50 | 68.67 | 98.10 | 2.10 | 2.95 | 6.20 | 2.50 | 6.25 | 2.70 | 5.73 | 1.082 | 1.091 |
| 98.10 | 490.50 | 68.67 | 98.10 | 2.45 | 3.50 | 8.58 | 2.95 | 8.70 | 3.20 | 8.04 | 1.067 | 1.082 |
| Case 3 | | | | | | | | | | | | |
| 245.25 | 490.50 | 98.10 | 147.15 | 2.40 | 3.60 | 8.64 | 3.00 | 9.00 | 2.90 | 6.61 | 1.307 | 1.362 |
| 196.20 | 490.50 | 98.10 | 147.15 | 2.40 | 3.60 | 8.64 | 3.00 | 9.00 | 2.90 | 6.61 | 1.307 | 1.362 |
| 147.15 | 490.50 | 98.10 | 147.15 | 2.40 | 3.60 | 8.64 | 3.00 | 9.00 | 2.95 | 6.83 | 1.265 | 1.318 |
| 98.10 | 490.50 | 98.10 | 147.15 | 2.55 | 3.85 | 9.82 | 3.15 | 9.92 | 3.45 | 9.35 | 1.050 | 1.061 |

RF is the rectangular footing
SF is the square footing
CF is the circular footing

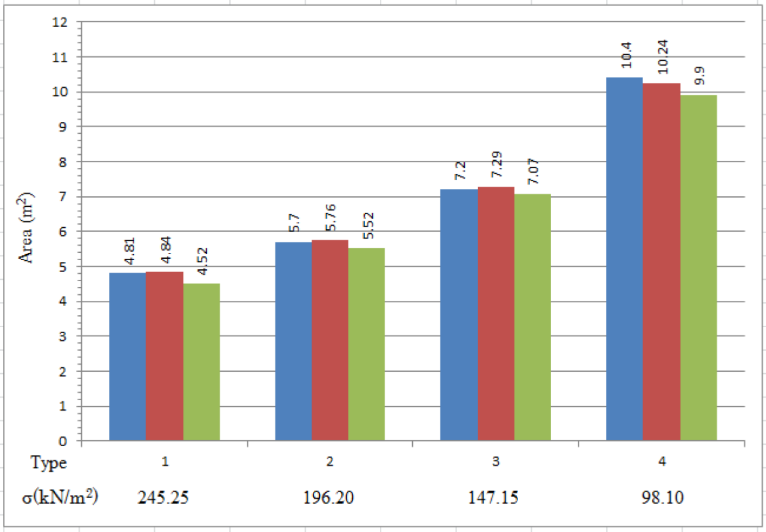


FIGURE 8. Case 1

With regard to case 1 presented in Figure 8 shows that the largest difference is in the type 1 of a 7.1%, this appears in the square footing and the smaller difference is shown of 1.8% in the type 3 for the rectangular footing. According to case 2 shown in Figure 9 presents that the largest difference is in the type 1 of a 10.5%, this is observed in the square footing and the smaller difference is of 6.7% in the type 4 for the rectangular footing. In terms of the case 3 presenting Figure 10 shows that the largest difference is in the type 1 and 2 of a 36.2%, this is presented in the square footing and the smaller difference is of 5.0% in the type 4 for the rectangular footing.

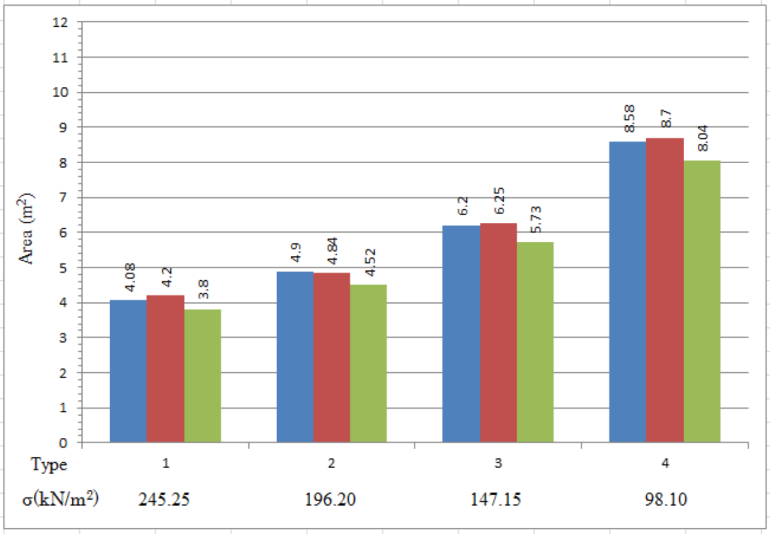


FIGURE 9. Case 2

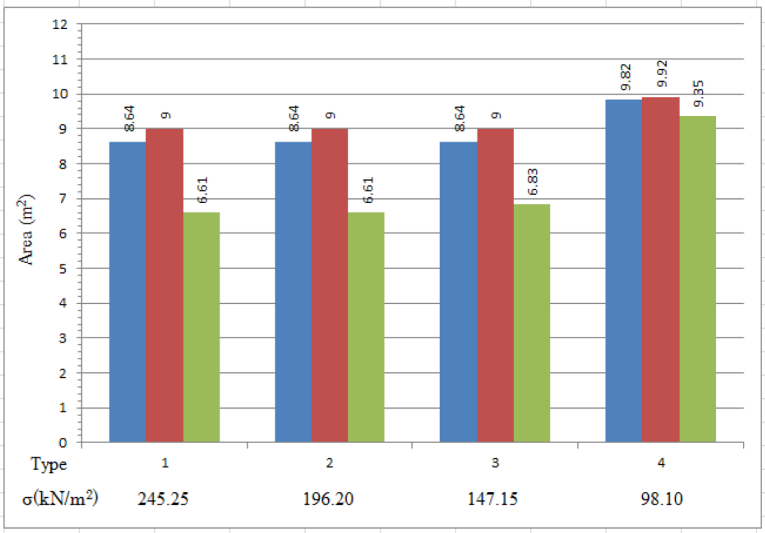


FIGURE 10. Case 3

5. **Conclusions.** The foundation is the member of the structure which is the due thereof essential part to allow the transmission of loads from the structure to the soil, such member helps the soil to resist these loads for that the same it will not suffer and its behavior to ideal for conditions to which will be submitted. Therefore, the foundation comes to form basis of the structure and of hence that the behavior of building or the civil work is presented correctly.

Due to the importance of the foundation, this is forced to meet certain geometrical parameters, pressure, conformation that respond to the characteristics of soil and loads of the structure. Therefore, the design of a foundation is not something that is performed in an intuitive manner, but which must satisfy with a design methodology to evaluate from the shape of the foundation to the depth to that is desired to construct the structural member, as well as the natural characteristics of the soil.

The mathematical models presented in this paper produce results having tangible accuracy for all problems under investigation for finding the more economical solution.

The models presented in this paper may be used in terms of the application of the loads. 1) Footings subject to concentric axial load. 2) Footings subject to axial load and moment in one direction (unidirectional bending). 3) Footings subject to axial load and moment in two directions (bidirectional bending).

Then we recommend the model of circular footing for the structural design of isolated footings subjected to axial load and bidirectional bending, because this is more economical. Furthermore, this more adheres to the real conditions of the soil pressures that are applied to the foundation.

The mathematical models presented in this paper are applied only to rigid soils that meet expression of the bidirectional bending, i.e., the variation of pressures is linear. The suggestions for future research may be, when is presented another type of soil, by example in cohesive soils and granular soils the pressures diagram is not linear and should be treated differently. Also another type of research that could be presented is the design of circular footings according to the rules of construction of the ACI (American Concrete Institute) for being the one that produced the most economical dimension.

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