

## ADAPTIVE FAULT TOLERANT ATTITUDE CONTROL FOR CUBE SATELLITE IN LOW EARTH ORBIT BASED ON DYNAMIC NEURAL NETWORK

MIN ZHANG<sup>1</sup>, LIPING YIN<sup>1</sup> AND LI QIAO<sup>2</sup>

<sup>1</sup>College of Information Science and Control  
Nanjing University of Information Science and Technology  
No. 219, Ningliu Road, Nanjing 210044, P. R. China  
anne\_zm@hotmail.com; lpyin@nuist.edu.cn

<sup>2</sup>Australian Center for Space Engineering Research  
University of New South Wales  
Sydney, N.S.W. 2052, Australia  
l.qiao@unsw.edu.cn

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**ABSTRACT.** *For the attitude control system of cube satellite in low earth orbit, a fault tolerant adaptive control method which combines  $H$ -infinity robust control and dynamic adaptive neural network is presented to deal with the case that magnetorquers as the satellite's actuators lose partial effectiveness after working long hours. Furthermore, the method can handle the negative influence from periodic variation of the earth magnetic field strength and uncertain external disturbances to keep the system stable within the tolerant error range. First, the  $H$ -infinity robust control algorithm is designed mainly to prevent the controlled system from external disturbances. Next, a kind of dynamic structure adaptive neural network is incorporated into the existed control loop to eliminate the negative influence caused by actuator faults and magnetic strength variation, so that it improves the ability of reconfiguration control to ensure the whole system being closed-loop stable ultimately in the faulty condition due to its better approximation for uncertain nonlinearities by updating not only network's weight parameters but also its topology structure adaptively on-line. Finally, the numerical simulation is performed for a two unit cube satellite and the results show that the control method presented is effective.*

**Keywords:** Fault-tolerant control, Satellite attitude control, Robust control, Dynamic neural network

**1. Introduction.** Now artificial satellite is in operation for various purposes such as communication, navigation, military service, weather forecasting, and astronomical observations. Since it takes much time and high payment to launch a satellite, there would be huge economy loss once the satellite fails to work normally due to some actuator faults. In view of this point, fault tolerant control (FTC) for satellite is a key practical issue on which much literature has proposed different methods, such as adaptive control, neural networks (NN), sliding mode control and fuzzy control [1-3]. In [4], a neural-network-based scheme is used for fault diagnosis of the reaction wheel in a satellite-attitude control system.  $H$ -infinity ( $H_\infty$ ) scheme is adopted in [5] to design fault diagnosis scheme for microscope satellite thrusters. And a dynamic inversion theory based fault tolerant controller was presented in [6] to achieve attitude control for a satellite with four reaction wheels. A sliding mode control method is now used intensively, for example, it is applied in [7] for the satellites with small actuator faults, and seems to be well done according to the simulation results. Though the FTC for satellite has been investigated extensively, it still lacks attention for cube satellite (CubeSat) with magnetorquer as its actuator

in low earth orbit [8], such as the satellites studied in famous QB50 project [9]. Since magnetorquer is seriously influenced from the periodic variation of the earth magnetic field strength on the orbit, there is nonlinear actuator output in kinetic equation of the CubeSat's attitude control system [10-12]. It brings more difficulties to FTC algorithm design for actuator faults such as effectiveness loss of magnetorquer.

In addition, another important issue about attitude control of CubeSats should be that how to prevent the system from uncertain external disturbances such as gravitational torque, solar radiation, aerodynamic torque and internal noise [13].  $H_\infty$  robust control has been proved in much research literature that it can restrain the system from uncertain external disturbances effectively and make the system achieve satisfactory control performance within the tolerant error range which is given as a norm bound [14]. However, traditional  $H_\infty$  robust control used solely is ineffective to keep the controlled system closed-loop stable if there are uncertain nonlinearities caused by actuators faults or earth magnetic field strength variation. In view of this point, a kind of dynamic adaptive neural network is adopted to  $H_\infty$  robust control algorithm in this paper, achieving the adaptive FTC which is of analytical redundancy to rearrange the control effect and make the controlled system work stably when some actuator faults occur. The primary advantage of the dynamic adaptive neural network presented in this paper is that it can eliminate the influence of uncertain nonlinearities excellently because both its weight parameters and topology structures can update on-line adaptively with approximation error altering. So far as we know, the neural-network-based control presented in most literature rarely applied this kind of dynamic neural network on control scheme due to its alterable topology structure whose altering rule is hard to design. However, the problem is solved in this paper by incorporating the dynamic NN into  $H_\infty$  robust control law to obtain the updating rules of weight parameters and topology structure based on system's stability analysis. In theory, the whole system can obtain the better dynamic control performance because the network's fixed approximation error can be reduced greatly with the on-line updating rule of its parameters and structure [15]. The adaptive FTC proposed in this paper is discussed mainly for the attitude control system of a 2-Unit (2U) CubeSat in LEO developed in QB50 project of Australia Center of Space Engineering and Research (ACSER) in UNSW [16] by theory analysis and numerical simulation, demonstrating its reconfiguration control ability for partial loss of magnetorquer's effectiveness when there are uncertain external disturbances and periodic variation of magnetic field strength in the satellite simultaneously.

This paper is organized as follows. Section 2 summarizes the problem formulations. In Section 3, adaptive fault tolerant control is designed by embedding dynamic adaptive neural network into  $H_\infty$  robust control algorithm. Simulation results on a 2U CubeSat with the derived FTC controller are given in Section 4. Finally, conclusions are drawn in Section 5.

**2. Problem Formulations.** When you cite some references, consider a 2U CubeSat using magnetorquers in LEO [16], and its nonlinear equations of attitude motion are given by the kinematics and the dynamics [17]:

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = I_s^{-1} \begin{bmatrix} -\omega_y(I_s\omega)_z + \omega_z(I_s\omega)_y \\ \omega_x(I_s\omega)_z - \omega_z(I_s\omega)_x \\ -\omega_x(I_s\omega)_y + \omega_y(I_s\omega)_x \end{bmatrix} + I_s^{-1}B(t) \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} + I_s^{-1} \begin{bmatrix} w_{dx} \\ w_{dy} \\ w_{dz} \end{bmatrix} \quad (2)$$

where  $q_0, q_1, q_2, q_3$  are the attitude quaternion,  $\omega_x, \omega_y, \omega_z$  are the angle rates, the matrix  $I_s \in R^{3 \times 3}$  includes the elements of moment of inertia,  $B(t)$  is the local geomagnetic field matrix which is of periodic variation,  $m_x, m_y, m_z$  are magnetic dipole moments produced by 3-axis magnetorquer, i.e., the control input elements, and  $w_{dx}, w_{dy}, w_{dz}$  are the external disturbances. In (2),  $(I_s\omega)_x, (I_s\omega)_y$  and  $(I_s\omega)_z$  are defined as follows:

$$\begin{aligned} (I_s\omega)_x &= I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z \\ (I_s\omega)_y &= I_{yx}\omega_x + I_{yy}\omega_y + I_{yz}\omega_z \\ (I_s\omega)_z &= I_{zx}\omega_x + I_{zy}\omega_y + I_{zz}\omega_z \end{aligned} \quad (3)$$

Suppose that the attitude system's equilibrium point is  $[q_0 \ q_1 \ q_2 \ q_3]^T = [1 \ 0 \ 0 \ 0]^T$ , Equation (1) can be linearized around the equilibrium point, and then the following equation is derived by further simplifying Equation (2):

$$\dot{x}_1 = \begin{bmatrix} 0 \\ \frac{1}{2}I_3 \end{bmatrix} x_2 = T x_2 \quad (4)$$

$$\dot{x}_2 = f(x_2) + I_s^{-1}B(t)u + I_s^{-1}w_d \quad (5)$$

where  $x_1 = [q_0 \ q_1 \ q_2 \ q_3]^T$ ,  $x_2 = [\omega_x \ \omega_y \ \omega_z]^T$ ,  $u = [m_x \ m_y \ m_z]^T$  and  $w_d = [w_{dx} \ w_{dy} \ w_{dz}]^T$ .

Considering the case that each of the actuators may partially lose its actuation effectiveness, (4) and (5) can be rewritten as follows:

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} T x_2 \\ f(x_2, t) \end{bmatrix} + \begin{bmatrix} 0 \\ I_s^{-1}B_0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ I_s^{-1}(B(t) - B_0)u \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ \Delta g(x, u, t) \end{bmatrix} + \begin{bmatrix} 0 \\ I_s^{-1}w_d \end{bmatrix} \\ &= F(x(t), t) + BI_s^{-1}B_0u(t) + B\Delta f(x(t), u(t), t) + D(t) \end{aligned} \quad (6)$$

where  $x = [x_1 \ x_2]^T \in R^7$ ,  $B = [0 \ I_{3 \times 3}]^T \in R^{7 \times 3}$ ,  $B_0 \in R^{3 \times 3}$  is the mean value of  $B(t)$ , and  $\Delta f(x, u, t) = I_s^{-1}[B(t) - B_0]u(t) + \Delta g(x, u, t)$  is an uncertain nonlinear continuous function caused by actuator faults  $\Delta g(x, u, t)$  and local geomagnetic variation  $I_s^{-1}[B(t) - B_0]u(t)$ . If the attitude states are given as  $x_d = [x_{d1}^T \ x_{d2}^T]^T$ , then the attitude error is defined as  $e(t) = x - x_d$ , and accordingly, dynamic equation of the errors can be written as:

$$\dot{e}(t) = F(x(t), t) + BI_s^{-1}B_0u(t) + B\Delta f(x(t), u(t), t) + D(t) - \dot{x}_d \quad (7)$$

As the  $I_s^{-1}B_0$  is invertible, the control law is presented as follows:

$$u = (I_s^{-1}B_0)^{-1}[u_d + u_n + B^T \dot{x}_d - f(x_2, t)] \quad (8)$$

where  $u_d = Ke$  is the output of robust control, and  $u_n$  is the output of dynamic adaptive neural network (NN). The feedback gain matrix  $K$  will be designed in the next section.

Substituting (8) to (7), then

$$\begin{aligned}
 \dot{e}(t) &= F(x(t), t) + B\Delta f(x(t), u(t), t) + D(t) - \dot{x}_d + BKe(t) \\
 &\quad + Bu_n + BB^T\dot{x}_d - Bf(x_2, t) \\
 &= [F(x(t), t) - \dot{x}_d + BB^T\dot{x}_d - Bf(x_2, t)] + BKe(t) + D(t) \\
 &\quad + Bu_n + B\Delta f(x(t), u(t), t) \\
 &= Ae(t) + BKe(t) + D(t) + Bu_n + B\Delta f(x(t), u(t), t) \\
 &= (A + BK)e(t) + D(t) + Bu_n + B\Delta f(x(t), u(t), t)
 \end{aligned} \tag{9}$$

where  $A = \begin{bmatrix} \mathbf{0} & T \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \in R^{7 \times 7}$ . Equation (9) shows that the system can achieve ultimate closed-loop stability by carefully choosing  $u_n$  to eliminate the impact of  $\Delta f(x(t), u(t), t)$ . And choose appropriate  $K$  to eliminate disturbance element  $D(t)$ .

**3. Design of Adaptive FTC Based on Dynamic Neural Network.** First of all,  $K$  is obtained by use of  $H_\infty$  robust control algorithm. Here, assuming that  $\Delta f(x, u, t) = 0$  and  $u_n = 0$ , just design the  $H_\infty$  robust control law for the system:

$$\dot{e}(t) = (A + BK)e(t) + D(t) \tag{10}$$

to eliminate external disturbances  $D(t)$ . Set the following  $H$ -infinity performance index as:

$$\int_0^\infty e^T(t)Qe(t)dt \leq e^T(0)Pe(0) + \gamma^2 \int_0^\infty D^T(t)D(t)dt \tag{11}$$

where  $\gamma$  is a given positive value,  $Q$  is a given positive definite symmetric constant matrix, and  $P$  is a positive definite symmetric constant matrix to be calculated.

**Theorem 3.1.** *Given  $\gamma > 0$ ,  $Q = Q^T > 0$  to make the following Riccati equation have a solution of positive definite symmetric matrix  $P$ ,*

$$A^T P + PA - P \left( 2BB^T Q^{-1} BB^T - \frac{1}{\gamma^2} I \right) P + Q + I = 0 \tag{12}$$

the closed-loop gain is

$$K = -B^T Q^{-1} BB^T P \tag{13}$$

Then the system (10) is of bounded stability with the  $H$ -infinity norm bound  $\gamma$ , and satisfies the performance index (11).

**Proof:** Consider the following Lyapunov candidate function  $V(x) = e^T(t)Pe(t)$ . Supposing  $D(t) = 0$ , then we can get  $\dot{V}(x) = e^T(t)[(A + BK)^T P + P(A + BK)]e(t)$ . In view of (12) and (13), the following inequality can be derived:

$$\dot{V}(x) = e^T(t)[A^T P + PA - 2PB^T Q^{-1} BB^T P]e(t) < 0$$

Hence, the closed-loop system  $\dot{e}(t) = (A + BK)e(t)$  is asymptotically stable.

Note that  $A_d = A + BK$ , and then discuss robust stability of the system (10). First we obtain the following expression based on the expansion of left Equation (11):

$$\int_0^\infty e^T(t)Qe(t)dt = e^T(0)Pe(0) - e^T(\infty)Pe(\infty) + \int_0^\infty \left\{ e^T(t)Qe(t) + \frac{d}{dt} [e^T(t)Pe(t)] \right\} dt$$

As  $e^T(\infty)Pe(\infty) \geq 0$  and substitute (10) into the above expression, one has

$$\begin{aligned}
 & \int_0^\infty e^T(t)Qe(t)dt \\
 & \leq e^T(0)Pe(0) + \int_0^\infty \left\{ e^T(t)Qe(t) + \frac{d}{dt}[e^T(t)Pe(t)] \right\} dt \\
 & = e^T(0)Pe(0) + \int_0^\infty \left\{ e^T(t)Qe(t) + e^T(t)[PA_d + A_d^T P]e(t) \right. \\
 & \quad \left. - \left[ \frac{1}{\gamma}Pe(t) - \gamma D(t) \right]^T \left[ \frac{1}{\gamma}Pe(t) - \gamma D(t) \right] + \gamma^2 D^T(t)D(t) + \frac{1}{\gamma^2}e^T(t)PPe(t) \right\} dt \\
 & \leq e^T(0)Pe(0) + \int_0^\infty e^T(t) \left[ Q + PA_d + A_d^T P + \frac{1}{\gamma^2}PP \right] e(t)dt + \int_0^\infty \gamma^2 D^T(t)D(t)dt
 \end{aligned} \tag{14}$$

Here, the following inequality can be deduced

$$A_d^T P + PA_d + \frac{1}{\gamma^2}PP + Q = A^T P + PA - 2PBB^T Q^{-1}BB^T P + \frac{1}{\gamma^2}PP + Q < 0$$

Hence, (14) satisfies the performance index (11)

$$\int_0^\infty e^T(t)Qe(t)dt \leq e^T(0)Pe(0) + \int_0^\infty \gamma^2 D^T(t)D(t)dt$$

The proof is completed.

Next, a kind of dynamic adaptive neural network is designed to compensate the influence of the uncertain nonlinear function  $\Delta f(x, u, t)$  in (9). The output of the dynamic adaptive NN is constructed as follows:

$$u_n = u_{nn} + u_{ns} \tag{15}$$

where  $u_{nn}$  is the output of the neural network, and  $u_{ns}$  is its compensation for the approximation error. Modified from the fully tuned adaptive radial basis function neural network (RBF NN) [17] and dynamic structure neural network [18], a kind of dynamic adaptive RBF NN is presented in this paper with on-line variable topological structure and updating network's parameters to achieve more accurate approximation performance. The following equation can be obtained because dynamic adaptive RBF NN has the same approximation property with fully tuned RBF NN:

$$\Delta f(x, u, t) = W^{*T}G^*(X, \xi^*, \eta^*) + \varepsilon(X) \tag{16}$$

where  $X = [x^T, e^T, t]^T \in R^{11}$  is the input vector of dynamic adaptive RBF NN, and  $X \in \mathbf{A}_d$ , where,  $\mathbf{A}_d$  is a large enough compact set.  $W^*$ ,  $\xi^*$ ,  $\eta^*$  denote ideal constant weight, center and width separately, and  $\varepsilon(X)$  is the approximation error, satisfying  $\varepsilon_I = \sup_{\mathbf{X}_i \in \mathbf{A}_d} \|\varepsilon(X)\|$ .  $G^*$  is chosen as commonly used Gaussian functions. The output  $u_{nn}$  of dynamic adaptive RBF NN is defined as follows:

$$u_{nn} = -\hat{W}^T \hat{G}(X, \hat{\xi}, \hat{\eta}) \tag{17}$$

where  $\hat{W}$ ,  $\hat{\xi}$ ,  $\hat{\eta}$  are estimations. The Taylor's series of  $G^*(X, \xi^*, \eta^*)$  is taken at  $\xi^* = \hat{\xi}$ ,  $\eta^* = \hat{\eta}$ , and then substituting (16) and (17) into (9), we can get the following equation after simplification. Here, note that  $D(t) = 0$  since  $K$  is of robust capacity.

$$\dot{e}(t) = (A + BK)e(t) + B\tilde{W}^T \hat{G} + B\hat{W}^T (\hat{G}'_\xi \tilde{\xi} + \hat{G}'_\eta \tilde{\eta}) + BE - Bu_{ns} \tag{18}$$

where  $\tilde{W} = W^* - \hat{W}$ ,  $\tilde{\xi} = \xi^* - \hat{\xi}$ ,  $\tilde{\eta} = \eta^* - \hat{\eta}$ ,  $\hat{G}'_{\xi} = \text{diag}(g_{\xi l}) \in R^{L \times KL}$ ,  $\hat{G}'_{\eta} = \text{diag}(g_{\eta l}) \in R^{L \times L}$ ,  $E = \tilde{W}^T(\hat{G}'_{\xi}\tilde{\xi} + \hat{G}'_{\eta}\tilde{\eta}) + W^{*T}O(X, \tilde{\xi}, \tilde{\eta}) + \varepsilon(X)$ ,  $O(X, \tilde{\xi}, \tilde{\eta})$  is the sum of high-order arguments in Taylor's series expansion. Based on the property of RBF NN, it is supposed that  $\|E\| \leq \varphi(t)$ , where  $\varphi(t)$  is nonnegative function.

The compensator of dynamic adaptive RBF NN is designed as:

$$u_{ns} = -\text{sgn}(B^T P e)\hat{\varphi}(t) \tag{19}$$

where  $\hat{\varphi}(t)$  is the estimation of  $\varphi(t)$ ,  $P$  is the given symmetrical positive definite matrix, and  $\text{sgn}(B^T P e)$  is the symbol matrix of  $B^T P e$ .

The dynamic adaptive RBF NN adopted can increase its hidden units on line from an original given number to an appropriate scalar with the approximation error growing until it is within the tolerant range. The threshold logic unit (TLU) is designed to avoid the case that the on-line altering of the topologic structure influences the real-time property of the system. TLU has nothing to do with the existed feedback control loop, only being used to judge whether to add a new hidden unit. The input of TLU is the sampling value of current attitude error, and its sampling frequency is the same as the NN's parameters updating frequency. The output of TLU determines whether it is necessary to add a new hidden unit according to the given growth criterion as follows. Therefore, TLU is defined as the following form with two main components.

One is operation rule:  $\rho = \alpha \exp(e_{tra} - E_1) + (1 - \alpha) \exp(e_{rms} - E_2)$  and the other is growth criterion of hidden units formed by logical compare:

$$\begin{cases} \rho > 1, & L = L + 1 \\ \rho \leq 1, & L = L \end{cases} \tag{20}$$

where  $L$  denotes the number of hidden units,  $e_{tra} = \|e(n)\|$  denotes the approximation error at the sampling time  $n$ ;  $e_{rms} = \sqrt{\sum_{i=n-(M-1)}^n \|e(i)\|^2} / M$  denotes the accumulative error covering a time sliding window  $M$ .  $E_1$  and  $E_2$  are the given bound values, and  $0 < \alpha < 1$  is the influence factor. The parameters associated with the new hidden unit are given initially:  $\xi_{L+1} = x(n)$ ,  $\eta_{L+1} = \lambda e_{tra}$ , where  $\lambda$  is a regulatory factor.

**Theorem 3.2.** *For the system (9), if there is a symmetrical positive definite matrix  $P$  satisfying the inequality (12), it can achieve closed-loop bounded stability ultimately under the control of the adaptive FTC (8). Supposing that the number of dynamic adaptive RBF NN's hidden units grows as (20), its parameters such as weights, centers and widths update by the following expressions:*

$$\dot{\hat{W}} = \sigma_1 \hat{G} e^T P B \tag{21}$$

$$\dot{\hat{\xi}} = \sigma_2 (e^T P B \hat{W}^T \hat{G}'_{\xi})^T \tag{22}$$

$$\dot{\hat{\eta}} = \sigma_3 (e^T P B \hat{W}^T \hat{G}'_{\eta})^T \tag{23}$$

$$\dot{\hat{\varphi}}_i = \sigma_4 \|e^T P B\| \tag{24}$$

where  $\sigma_1, \sigma_2, \sigma_3, \sigma_4$  are all positive constants as the regulatory factors of the dynamic adaptive RBF NN.

**Proof:** Choose the following Lyapunov candidate function

$$V = \frac{1}{2} e^T P e + \frac{1}{2\sigma_1} \text{tr}(\tilde{W}^T \tilde{W}) + \frac{1}{2\sigma_2} \tilde{\xi}^T \tilde{\xi} + \frac{1}{2\sigma_3} \tilde{\eta}^T \tilde{\eta} + \frac{1}{2\sigma_4} \tilde{\varphi}^T \tilde{\varphi}$$

where  $\tilde{\varphi} = \varphi - \hat{\varphi}$  is the compensation estimation error of dynamic adaptive RBF NN. Differentiate the above function and result in

$$\dot{V} = \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} + \frac{1}{\sigma_1}tr(\tilde{W}^T \dot{\tilde{W}}) + \frac{1}{\sigma_2}\tilde{\xi}^T \dot{\tilde{\xi}} + \frac{1}{\sigma_3}\tilde{\eta}^T \dot{\tilde{\eta}} + \frac{1}{\sigma_4}\tilde{\varphi}^T \dot{\tilde{\varphi}}$$

According to  $\dot{\tilde{W}} = -\dot{\tilde{W}}$ ,  $\dot{\tilde{\xi}} = -\dot{\tilde{\xi}}$ ,  $\dot{\tilde{\eta}} = -\dot{\tilde{\eta}}$ ,  $\dot{\tilde{\varphi}} = -\dot{\tilde{\varphi}}$ , the following function is derived after inserting (9), (21)-(24).

$$\dot{V} = \frac{1}{2}e^T [(A + BK)^T P + P(A + BK)]e + e^T P B E - e^T P B \text{sgn}(e^T P B) \hat{\varphi}(t) - \tilde{\varphi}^T \|e^T P B\|$$

Obviously,

$$\begin{aligned} \dot{V} &\leq \frac{1}{2}e^T [(A + BK)^T P + P(A + BK)]e + \|e^T P B\| \|E\| \\ &\quad - \|e^T P B\| \hat{\varphi}(t) - (\varphi - \hat{\varphi})^T \|e^T P B\| \\ &\leq \frac{1}{2}e^T [(A + BK)^T P + P(A + BK)]e \end{aligned}$$

Based on the proof of Theorem 3.1, it is derived that  $\dot{V} < 0$ , and the system controlled by adaptive FTC (8) can be closed-loop bounded stable. The proof is completed.

**4. Simulation and Comparison Results.** To verify the effectiveness of the proposed FTC (8), numerical simulations are done for a 2U CubeSat of QB50 project in ACSER of UNSW. The parameters of this 2U CubeSat are listed in Table 1.

TABLE 1. Main parameters of satellite

Mass (kg)	1.92
Inertia moments (kg · m <sup>2</sup> ): Principal moments of inertia Products of inertia	$I_x = 0.0115, I_y = 0.0115, I_z = 0.00369$ Can be neglected
Orbit Altitude (km)	320
Attitude control type	Three axis control by three magnetorquers

The initial attitude orientation is set to be  $[\phi \ \theta \ \psi] = [1 \ 2 \ 1]^\circ$  with initial angle rate  $\omega = [0.0001 \ 0.0006 \ -0.0003]$  rad/s.

The external disturbance is considered to be the gravitational torque as follows [19]:

$$T_g = 3.999 \times 10^{-6} \begin{bmatrix} (I_z - I_y)q_1 \\ (I_z - I_x)q_2 \\ 0 \end{bmatrix}$$

According to [19], the mean value of the local geomagnetic field matrix  $B(t)$  can be obtained that

$$B_0 = \begin{bmatrix} 0 & 1.1407 \times 10^{-6} & -5.0828 \times 10^{-12} \\ -1.1407 \times 10^{-6} & 0 & 9.8757 \times 10^{-8} \\ 5.0828 \times 10^{-12} & -9.8757 \times 10^{-8} & 0 \end{bmatrix}$$

As proposed in Section 3, the regulatory factors are chosen as:

$$\gamma = 1, L = 3, \lambda = 1, \alpha = 0.6, \sigma_1 = \sigma_2 = \sigma_3 = 1, \sigma_4 = 0.8$$

and  $Q = \text{diag} [1.5 \ 1.5 \ 1.5 \ 1.5 \ 1.5 \ 1.5 \ 1.5]$ .

So the feedback gain  $K = \begin{bmatrix} 0 & -0.47 & 0 & 0 & -2.01 & 0 & 0 \\ 0 & 0 & -0.47 & 0 & 0 & -2.01 & 0 \\ 0 & 0 & 0 & -0.47 & 0 & 0 & -2.01 \end{bmatrix}$  is obtained according to (12) and (13).

The aim of simulations is to demonstrate the reconfiguration control performance for magnetorquers' effectiveness loss by use of the adaptive FTC algorithm (8). Suppose that the magnetorquers in the roll, pitch and yaw axis decrease 20% of their normal values from 30000 s respectively. Figure 1 shows the time responses of the attitude control system under the  $H_\infty$  robust control without NN. And Figure 2 shows the corresponding responses under the control of the proposed FTC (8). From the simulation results, it is shown that the  $H_\infty$  robust control is not of capability for the magnetorquer faults, and could not control the faulty system to achieve the stability requirement, though it seems that it can deal with the periodic variation of magnetic strength to a certain extent. And the results from Figure 2 show that the attitude angle in the roll, pitch and yaw axis of the satellite can be controlled in desired orientation stably, quickly and smoothly with negligible error when the effectiveness of the magnetorquers decreases under the control of the FTC (8) presented in this paper, which does not bring too much negative influence into the attitude control system, just having some tiny vibrations in the related curves at the beginning of faults.

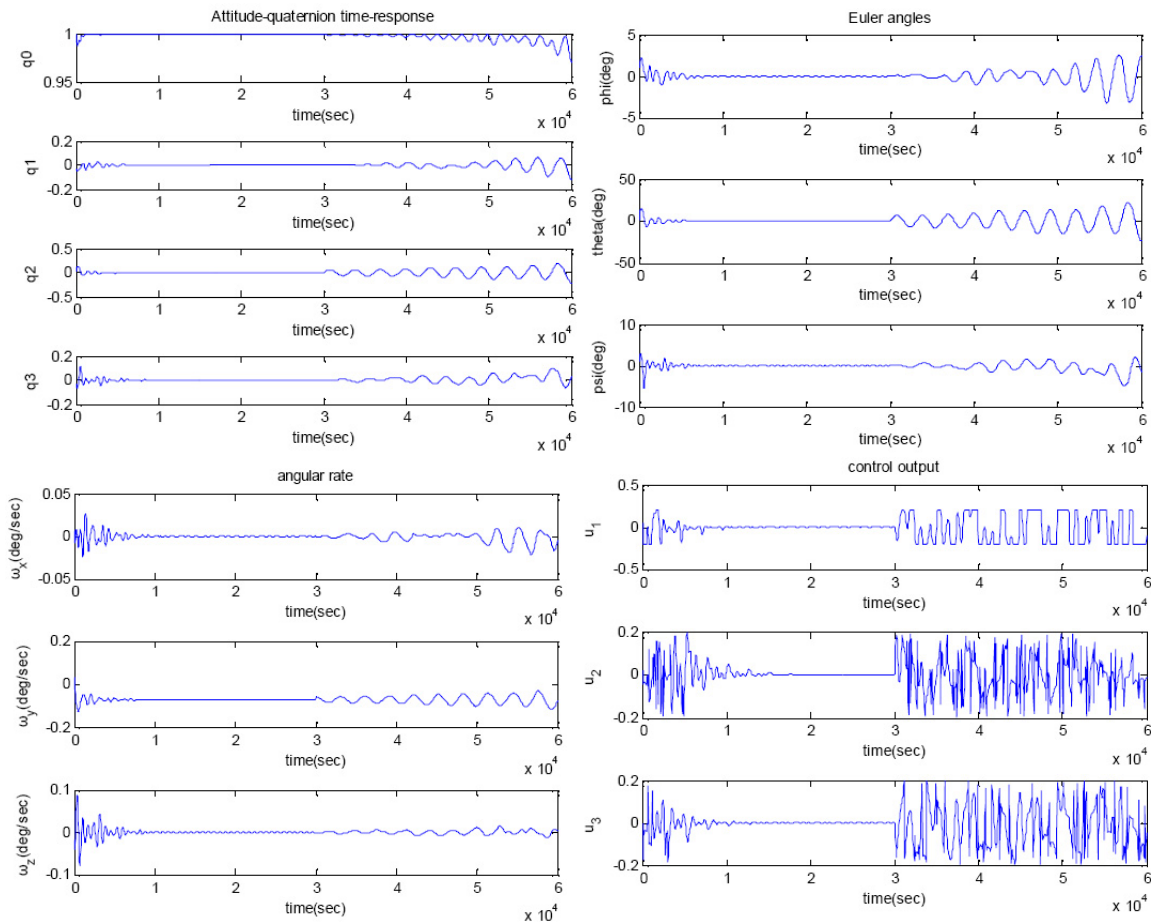


FIGURE 1. State response under  $H_\infty$  robust control (the effectiveness of actuators loses 20%)



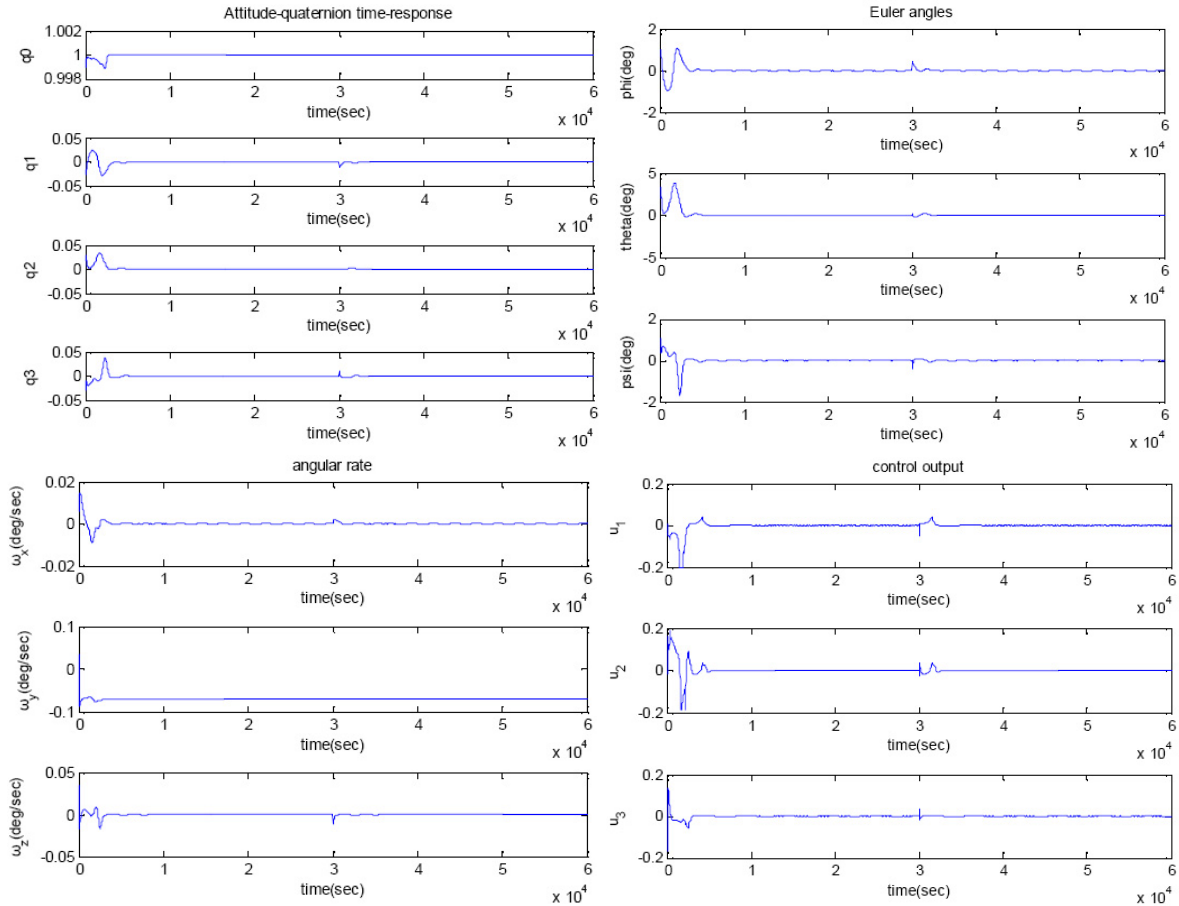


FIGURE 2. State response under the adaptive FTC control presented (the effectiveness of actuators loses 20%)

**5. Conclusions.** For 2U CubeSats in LEO using 3-axis magnetorquer as its actuator, adaptive attitude FTC is designed in this paper based on dynamic adaptive RBF NN. This FTC scheme could ensure ultimate bounded convergence of the attitude angle error when there are partial loss of magnetorquers' effectiveness, periodic variation of magnetic field strength and uncertain external disturbances in the satellite simultaneously. Numerical simulations are carried out to verify the effectiveness of the presented FTC scheme by using the 2U CubeSat of QB50 project in ACSER as the plant. From comparing the results of  $H_\infty$  robust control and those of the FTC presented, it is shown that the presented FTC has satisfactory reconfiguration ability for partial loss of actuators' effectiveness, making the attitude system with modeling nonlinearities and external disturbances to be ultimate bounded closed-loop stable.

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