International Journal of Innovative Computing, Information and Control
Volume 10, Number 5, October 2014
 ISSN 1349-4198
pp. 1885–1896

OPTIMAL DESIGN OF FLEXIBLE KANBAN SYSTEM FOR A MULTI-STAGE MANUFACTURING LINE

Yunmei Fang[1] and Juntao Fei[2]

[1]College of Mechanical and Electrical Engineering
[2]College of Computer and Information
Hohai University
No. 200, Jinling North Road, Changzhou 213022, P. R. China
bigboyscn@yahoo.com

Received December 2013; revised April 2014

Abstract. *A multi-stage Kanban based production system (MKPS) with variable production rates is considered. This paper is to study and analyse the process of designing an effective flexible Kanban pull system and to design and develop a method for MKPS with varying production rate at different work-stage. Raw materials are replenished at the first work-stage and they are processed through MKPS to generate the finished products. Simultaneous releases of Kanbans are considered to determine the orders for raw parts or semi-finished parts from one stage to another. A study pursued is to determine the optimal number of raw material orders at the first work-stage for a varying (linear) demand pattern to minimize the total inventory cost incurred due to raw materials, finished goods and work-in-process (WIP) inventories. This decision eventually determines the number of Kanbans, the schedule for production, and the dispatching time intervals. Simulation results verify the effectiveness of the optimal operation planning mechanism of MKPS with managed Kanbans.*
Keywords: Optimal planning, Multi-stage production system, Kanban, Varying production rate

1. **Introduction.** Kanban controlled productions system has received much attention because of ease of implementation and wide application in the industry. Kanban controlled multi-stage just-in-time (JIT) production systems are the most popular systems among contemporary manufacturing companies because they can minimize the inventory build-up, reduce lead times, increase flexibility, and minimize waste of human and facilities which result in minimizing the total inventory cost. Most of existing researches consider the issues of raw material orders, the number of Kanbans and the finished products deliveries separately.

Hemamalini and Rajendran [1] presented a flow-shop controlled by Kanban and showed recursive equations for containers in a given sequence. Ramasesh [2], Pan and Liao [3], Parlar and Rempala [4], Yilmaz [5] and Hill [6] developed optimal orders and production quantity models for single-stage production system. Goyal [7], Goyal and Gupta [8], and Aderohunmu et al. [9] proposed models for joint vendor-buyer policy in a just-in-time manufacturing environment. Tardif and Maaseidvaag [10] introduced an adaptive Kanban-type pull control mechanism to determine release or reorder time for raw parts for the case of a single-stage, single-product Kanban system in which the demands occur according to a Poisson process and processing times are exponential random variables. Golhar and Sarker [11], Jamal and Sarker [12] and Sarker and Parija [13,14] presented a number of models for a single-product just-in-time production delivery system and incorporated the fixed quantity supply to a buyer at a fixed interval of time under an optimal ordering and production policy for a production system. Sarker and Parija [15] developed an ordering policy for raw materials and determined an economic batch size for a product at a manufacturing center which supplies finished products to multiple customers. Sarker and Balan [16] proposed an operations planning for Kanbans between two adjacent workstations. Diponegoro and Sarker [17] developed an ordering policy for raw materials and determined an economic batch size for a product in a manufacturing system that supplies finished products to customers for a finite planning horizon. Wang and Sarker [18] studied the assembly-type supply chain system controlled by Kanbans under an in time delivery policy. Matta et al. [19] proposed a new approximate analytical method for evaluating the performance of assembly system with independent release of Kanbans. Fang and Lin [20] developed optimal production policy for a multi-stage Kanban managed flexible production system.

Most of past works in modelling and optimization of supply chain manufacturing system have so far partially considered the aspects of JIT delivery, time varying demand, integrated inventory including raw materials, WIP and finished products and flexible production capacity separately. This research attempts to bridge this gap. The motivation of this research regarding inventory model for MKPS is to minimize the total inventory related cost in the system that integrates all ordering and holding cost of raw materials, WIP and finished product inventories.

1) The current models are limited to one type of shipment mechanism, which is level demand and infinite planning horizon. A more appropriate production model (MKPS) with time varying demand of the products and flexible production capacity of the production system is proposed which can respond to the market with significantly changing demand over the production cycle.
2) To achieve the goal of minimizing the inventory costs of MKPS, a performance evaluation is required for MKPS and the mathematical formulation of the problem is categorized as a mixed integer non-linear programming problem which is solved to find optimum raw material procurement rate, optimum finished product shipment, optimum Kanban number and its capacity at each Kanban stage, and the minimum system inventory cost.
3) The efficiency of the production process can be increased by studying the sensitivity of parameters on the integrated inventory cost of MKPS and finding operational plans.

This paper is organized as follows. In Section 2, the problem is described. In Section 3, the mathematical model and cost function of a multi-stage production system are derived. In Section 4 description and analysis of the numerical examples are demonstrated. Section 5 draws the conclusions.

2. **Problem Description.** A multi-work-stage Kanban controlled production system (MKPS) receives raw materials from outside suppliers and processes them into single type of finished products through Kanban managed N-stage with varying production rate at different work-stage, and finally delivers them to customers.

For this study, a hypothetical JIT production system is assumed, which consists of a general serial system with N work stages controlled with Kanbans. The system is modelled with the queuing network shown in Figure 1. The products have a demand pattern that increases linearly with time. Raw material flows entering the production system and semi-finished products of each work-stage must receive an authorization order before they can enter the next work-stage. In MKPS system, the number of Kanbans represents the authorization order and controls the part flows in the system, so that the amount of WIP in the system is limited. The part flows follow a linear path starting from the first stage and entering the following production stages to be processed until the last

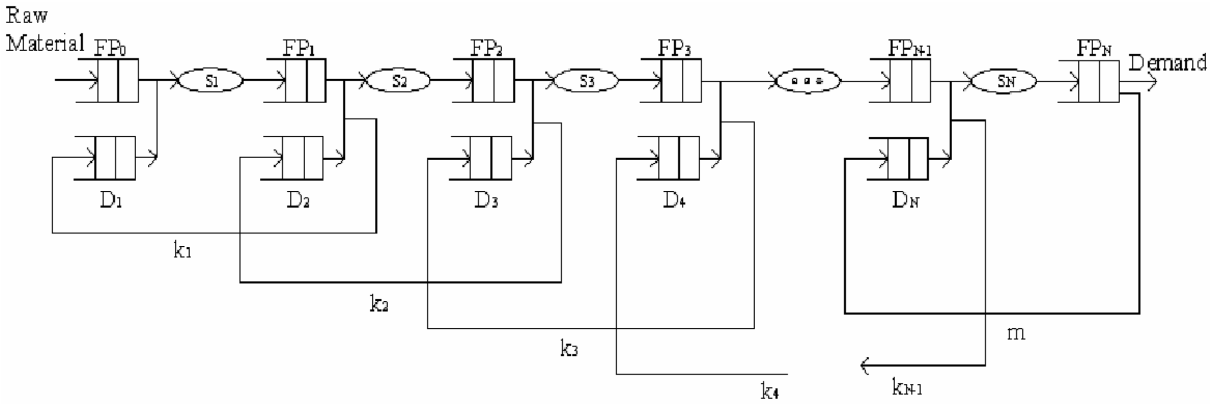

Figure 1. Architecture of MKPS

stage, and then finished products wait in the last buffer (i.e., the buffer FP_N in Figure 1) and are delivered at a fixed-time interval and linear increase quantities during the cycle time period. Buffers separate production phases in order to decrease interdependencies among different stages. The assumption is made that enough inventories exist in each stage buffer so that shortages never occur in the system.

In order to minimize the total inventory costs of raw materials, WIP, and finished products in MKPS, a general cost function is developed to determine the optimal ordering and production policy which will minimize the total inventory cost of the JIT production system.

A multi-order policy for processing the raw parts supplied to work-stage S_i ($i = 1, \ldots, N$) is used in the system, because this policy encourages the timely use of raw materials or raw parts for each stage, resulting in a lower inventory carrying cost. Since the production rate p_i is assumed to be higher than the consumption rate ω, the WIP inventories are built up at the Kanban stage of each work-stage. At time T_{pi}, the production of stage S_i stops, and pure demand without any production occurs during the rest of the cycle time $T_c - T_{pi}$. As shown in Figure 1, raw materials for stage S_1 are ordered n times during the uptime T_{p1}, and similarly raw parts for stage S_{i+1} in this model are ordered k_i times during the uptime $T_{p(i+1)}$.

The problem here is to determine an optimal ordering policy for raw materials as to how frequently and how much of raw materials should be ordered per request during the uptime T_{p1}, the corresponding optimal Kanban numbers for each work-stage, and the corresponding economic number of deliveries of finished product which result in minimizing the total inventory costs of raw materials, WIP, and finished products in MKPS.

3. **Mathematical Model and Cost Function.** To model the interactions of the order of raw materials, the WIP, and the deliveries of finished products, the following notations are used:

Raw materials related: Q_R, H_{R1}, K_{R1}, n, and P_{01}.

WIP materials related: W_{ki}, H_{ki}, H_{Ri}, K_{Ri}, P_{oi}, α_i, β_i, p_i, f_i and k_i.

Finished products related: D_F, H_F, K_F, D_0, m, L, ω.

Cycle time related: T_c, T_{pi}.

D_0: the inventory level of finished products at the beginning of a cycle.

D_F: total demand for finished products by customers, units/cycle.

f_i: conversion factor of raw materials or raw parts to semi-finished or finished products between adjacent work-stage.

H_F: holding cost of finished products, \$/units*day.

H_{ki}: the WIP inventory holding cost of the production stage S_i, \$/units*day.
H_{R1}: raw material holding cost, \$/units*day.
H_{Ri}: raw parts of production stage S_i holding cost, \$/units*day.
K_F: finished product ordering cost, \$/order.
K_{R1}: raw material ordering cost, \$/order.
K_{Ri}: ordering cost of raw part for work-stage S_i, \$/order.
k_i: the number of Kanbans of the work-stage S_i.
L: the interval between successive shipments of finished products delivered to customer.
m: the number of full shipments of finished products per cycle time.
n: the raw material order numbers during the T_{p1}.
P_{01}: the inventory level of raw material at the beginning of a cycle.
P_{oi}: the inventory level of raw part of work-stage S_i at the beginning of a cycle.
p_i: production increase rate of the work-stage S_i, units/day.
Q_R: total demand of raw materials, units/day.
T_c: cycle time, $T_c = m \times L$.
T_{pi}: manufacturing period (uptime) of the work-stage S_i.
W_{ki}: the Kanban withdrawal cost for the work-stage S_i.
ω: demand increase rate of finished products units/day.

3.1. **Mathematical model of MKPS.** In an inventory situation, work-stage S_i, is assumed that the quantity of raw materials or raw parts consumed at an increased rate of p_i during the production period (i.e., T_{pi}) is equal to that are supplied during the time. Therefore,

$$Q_R = \int_0^{T_{p1}} (P_{01} + p_1 t)\, dt \tag{1}$$

$$Q_{Ri} = \int_0^{T_{pi}} (P_{0i} + p_i t)\, dt \tag{2}$$

Assuming a one-to-one conversion factor f_i of the raw parts to semi-finished products for work-stage S_i:

$$Q_R = Q_{Ri} = D_F \tag{3}$$

which yields,

$$T_{pi} = \frac{-P_{0i} + \sqrt{P_{0i}^2 + 2p_i Q_R}}{p_i} \tag{4}$$

The total quantity of raw parts of work-stage S_N consumed during the cycle time T_c at an increased rate of ω is equal to the finished products supplied to the customer at a linear rate of ω at an interval of L time units. Assuming there are m full shipments per cycle:

$$D_F = (m+1)(D_0 + \omega T_c/2) \tag{5}$$

$$Q_R = \int_0^{T_c} (D_0 + \omega t)\, dt \tag{6}$$

which yields,

$$T_c = \frac{-D_0 + \sqrt{D_0^2 + 2\omega Q_R}}{\omega} \tag{7}$$

There are n replenishments of raw materials during the production time T_{p1} at an interval of T_{p1}/n time units.

$$Q_R = nP_{01} + \frac{(n-1)p_1 T_{p1}}{2} \tag{8}$$

Now let us derive the relationship between n, m and k_i. According to the relationship in Equations (5) and (8), the number of raw material orders n and the number of finished products deliveries can be presented as

$$m = \frac{2nP_{01} + (n-1)p_1 T_{p1}}{2D_0 + \omega T_c} - 1 \tag{9}$$

The relationship between the number of Kanbans k and the number of raw material orders n for two adjacent work stages controlled by one Kanban stage, as formulated by Sarker and Balan [16], is given by

$$k = \frac{\beta}{\alpha} n \tag{10}$$

where $\alpha = \frac{H_k}{H_R}$, $\beta = \frac{W_k}{K_R}$.

Now, if there are N production stages managed with $N-1$ Kanban stages, and the number of Kanbans of the work-stage S_i is k_i, and the number of Kanbans of the work-stage S_{i-1} is k_{i-1} which represents the authorized raw parts order of work-stage S_i is k_{i-1}. Hence, equation above can be modified as:

$$k_i = \frac{\beta_i}{\alpha_i} k_{i-1} \ (\forall i = 1, \ldots, N-1) \tag{11}$$

$$k_0 = n,\ k_1 = n\frac{\beta_1}{\alpha_1},\ k_2 = n\frac{\beta_1\beta_2}{\alpha_1\alpha_2}, \ldots,\ k_{N-1} = n\frac{\prod_{i=1}^{N-1}\beta_i}{\prod_{i=1}^{N-1}\alpha_i} \tag{12}$$

where $\alpha_i = \frac{H_{ki}}{H_{Ri}}$, $\beta_i = \frac{W_{ki}}{K_{Ri}}$.

3.2. **Total cost functions.** Considering a hypothetical JIT production system, the total cost of inventory is including three aspects:

1) Raw material related: raw material ordering cost and holding cost.
2) WIP inventory related: Kanban withdrawal cost and WIP holding cost.
3) Finished products related: finished products delivery costs and holding cost.

The raw materials are ordered by the first work-stage S_1 of the N-stage production system from the supplier. The total cost of raw materials $TC_R(n)$ can be expressed as

$$TC_R(n) = nK_{R1} + \left(\frac{H_{R1}}{T_c}\right)\left(\frac{T_{p1}^2 P_{01}}{2n^2}\right) + \left(\frac{H_{R1}}{T_c}\right)\left(\frac{p_1 T_{p1}^3}{3}\right)\left(\frac{1}{n} + \frac{1}{n^2} - \frac{1}{2n^3}\right) \tag{13}$$

During the production period of each work-stage, the WIP is built up and sufficient for the demand of the following work-stage in the total cycle time. The cost of WIP inventory at each work-stage is assumed to be independent. The total cost of WIP inventory $\sum_{i=1}^{N-1} TC_w(k_i)$ can be obtained using the WIP inventory cost of single work-stage $TC_w(k_i)$ and the principle of linear superposition.

$$TC_w(k_i) = k_i W_{ki} + \frac{1}{k_i}\frac{H_{ki} p_i T_{pi}^3}{3T_c} + \frac{1}{k_i^2}\frac{H_{ki}}{T_c}\left(\frac{P_{0i} T_{pi}^2}{2} + \frac{p_i T_{pi}^3}{3}\right) - \frac{1}{k_i^3}\frac{H_{ki} p_i T_{pi}^3}{6T_c} \tag{14}$$

$$\sum_{i=1}^{N-1} TC_w(k_i) = \sum_{i=1}^{N-1}\left[k_i W_{ki} + \frac{1}{k_i}\frac{H_{ki}p_i T_{pi}^3}{3T_c}\right] + \sum_{i=1}^{N-1}\left[\frac{1}{k_i^2}\frac{H_{ki}}{T_c}\left(\frac{P_{0i}T_{pi}^2}{2} + \frac{p_i T_{pi}^3}{3}\right) - \frac{1}{k_i^3}\frac{H_{ki}p_i T_{pi}^3}{6T_c}\right] \tag{15}$$

The finished products wait in the buffer FP_N and are delivered at a fixed-time interval and linear increase quantities during the total cycle time period. The total finished product inventory cost $TC_F(m)$ can be written as

$$TC_F(m) = mK_F + \left(\frac{H_F}{T_c}\right)\left[\frac{P_{0N}T_{pN}^2}{2} + \frac{p_N T_{pN}^3}{6} - Q_R\left(T_c - T_{pN}\right)\right] - \left(\frac{H_F}{T_c}\right)\left[k_{N-1}P_{0N} + \left(\frac{k_{N-1}-1}{2}\right)p_N T_{pN}\right] \tag{16}$$

Now let us derive the total cost of the N-stage JIT production system. The total cost of the MKPS model $TC^M(m,n,k_i)$ is the sum of the raw material inventory with orders n, the WIP inventory with the Kanban numbers of k_i for each work-stage S_i, and the finished products inventory with shipments m. It can be expressed as

$$TC^M(m,n,k_i) = TC_R(n) + \sum_{i=1}^{N-1} TC_w(k_i) + TC_F(m) \tag{17}$$

$$\begin{aligned}
TC^M(n) = {} & n\left[K_{R1}\left(1+\frac{\beta_1^2}{\alpha_1}\right) + K_{R2}\frac{\beta_1\beta_2^2}{\alpha_1\alpha_2} + K_{R3}\left(\prod_{i=1}^{3}\frac{\beta_i}{\alpha_i}\right)\beta_3 + ...\right] \\
& + n\left[K_{R(N-1)}\left(\prod_{i=1}^{N-1}\frac{\beta_i}{\alpha_i}\right)\beta_{N-1} + \frac{2P_{01}+p_1T_{p1}}{2D_0+\omega T_c}K_F \right. \\
& \left. - \frac{H_F(2P_{0N}+p_N T_{pN})}{2T_c}\left(\prod_{i=1}^{N-1}\frac{\beta_i}{\alpha_i}\right)\right] \\
& + \frac{1}{n}\left[\left(\frac{H_{R1}p_1T_{p1}^3}{3T_c}\right)\left(1+\frac{\alpha_1^2}{\beta_1}\right) + \left(\frac{H_{R2}p_2T_{p2}^3}{3T_c}\right)\left(\frac{\alpha_1\alpha_2^2}{\beta_1\beta_2}\right) + \dots \right. \\
& \left. + \left(\frac{H_{R(N-1)}p_{N-1}T_{p(N-1)}^3}{3T_c}\right)\left(\prod_{i=1}^{N-1}\frac{\alpha_i}{\beta_i}\right)\alpha_{N-1}\right] \\
& + \frac{1}{n^2}\left[\left(\frac{H_{R1}}{T_c}\right)\left(\frac{P_{01}T_{p1}^2}{2} + \frac{p_1T_{p1}^3}{3}\right)\left(1+\frac{\alpha_1^3}{\beta_1^2}\right) \right. \\
& \left. + \left(\frac{H_{R2}}{T_c}\right)\left(\frac{P_{02}T_{p2}^2}{2} + \frac{p_2T_{p2}^3}{3}\right)\left(\frac{\alpha_1^2\alpha_2^3}{\beta_1^2\beta_2^2}\right) + \dots\right] \\
& + \frac{1}{n^2}\left[\left(\frac{H_{R(N-1)}}{T_c}\right)\left(\frac{P_{0(N-1)}T_{p(N-1)}^2}{2} + \frac{p_{N-1}T_{p(N-1)}^3}{3}\right)\left(\prod_{i=1}^{N-1}\frac{\alpha_i^2}{\beta_i^2}\right)\alpha_{N-1}\right] \\
& - \frac{1}{n^3}\left[\frac{H_{R1}p_1T_{p1}^3}{6T_c}\left(1+\frac{\alpha_1^4}{\beta_1^3}\right) + \frac{H_{R2}p_2T_{p2}^3}{6T_c}\left(\frac{\alpha_1^3\alpha_2^4}{\beta_1^3\beta_2^3}\right) + \dots \right. \\
& \left. + \frac{H_{R(N-1)}p_{N-1}T_{p(N-1)}^3}{6T_c}\left(\prod_{i=1}^{N-1}\frac{\alpha_i^3}{\beta_i^3}\right)\alpha_{N-1}\right]
\end{aligned}$$

$$-\left(\frac{p_1T_{p1}+2D_0+\omega T_c}{2D_0+\omega T_c}\right)K_F+\frac{H_F}{T_c}\left[\frac{P_{0N}T_{pN}^2}{2}+\frac{p_NT_{pN}^3}{6}\right.$$
$$\left.-Q_R(T_c-T_{pN})+\frac{p_NT_{pN}}{2}\right] \tag{18}$$

Thus, the total inventory cost of the N-stage Kanban controlled production system has an extreme value by setting $\frac{dTC^M(n)}{dn}=0$. After solving this equation, we can find the optimal number of raw material orders corresponding to the minimum value of total inventory cost $TC^M(n^*)$.

4. **Numerical Analysis.** The research issue considered here is to give an accurate and convenient methodology for the problems faced by the companies which apply Kanban-based production systems. Our goal in this section is to verify the effectiveness of the proposed methodology. To do this, we illustrate the proposed methodology by two examples. In the first example, the production system is made up of two work-stages controlled with one Kanban stage. Our MKPS model indicates that the ordering policy of raw materials, the WIP inventory, and the finished product deliveries are dependent on one another.

Using the cost and production parameters and the proposed methodology, we can decide the optimal ordering policy of raw materials, the optimal Kanban number and the economic number of the finished product delivery accurately and quickly.

In the example, we expand a production system to be three-work-stages controlled with two Kanban stages, and a production system made up of four-work-stages controlled with three Kanban stages. Through similar derivation and calculation, we found the optimal results for them which are summarized in Table 2. For the purpose of simplicity and comparability we assume that the cost and production parameters are the same for the same work-stage of the three production systems.

Example 4.1. *Three-production stages with two Kanban stages and four-production stage with three-Kanban stages. Cost and production parameters are shown in Table 1.*

Using similar procedure, we can calculate the optimal raw material order number n^*, the number of Kanbans of each work-stage k_i^*, the economic number of finished products deliveries m^*, and the corresponding minimum $TC(n^*)$ per cycle time for three-work-stage production system and four-work-stage production system. The results are summarized in Table 2.

Examples were used to explain the developed MKPS model by deriving minimum total inventory cost $TC(n^*)$ including raw materials, WIP, and finished products cost for two-work-stage, three-work-stage and four-work-stage production lines individually. It can be shown from Table 1 and Table 2 that work-stages of these production systems are independent.

Table 2 compiles minimum total inventory cost $TC(n^*)$ and corresponding optimal number of Kanban k_i^*, economic delivery number of finished products m^* for two-work-stage, three-work-stage, and four-work-stage production line individually. It is observed that, given the other parameters constant, the minimum total cost is obtained as $TC(n^*)=$ \$828.6 for two-work-stages, $TC(n^*)=\$1290$ for three-work-stages and $TC(n^*)=\$1527$ with four-work-stages. The reason for varying the number of Kanban at different stages underlies in the differences of Kanban ordering and inventory holding cost at each stage in the system.

Figure 2 shows the relationship between the total inventory cost versus the raw material order numbers for the three production systems with two-work-stages, three-work-stages and four-work-stages respectively. The optimal raw material orders can be decided by figuring out the lowest value of the total inventory cost curves. It can be observed that the total inventory costs versus raw material orders follow the similar pattern for these three different production systems with the same cost and production parameters at the

TABLE 1. Parameters of simulation models

Production line with N work-stage	N = 2	N = 3	N = 4
Total demand (unit/cycle)	$D_F = 2500$		
Demand increase rate	$\omega = 8$		
Production increase rate	$p_1 = 10$ $p_2 = 10$	$p_1 = 10$ $p_2 = 10$ $p_3 = 10$	$p_1 = 10$ $p_2 = 10$ $p_3 = 10$ $p_4 = 8$
Initial inventory (unit/cycle)	$P_{01} = 100$ $P_{02} = 110$ $D_0 = 80$	$P_{01} = 100$ $P_{02} = 110$ $P_{03} = 80$ $D_0 = 80$	$P_{01} = 100$ $P_{02} = 110$ $P_{03} = 80$ $P_{04} = 80$ $D_0 = 80$
Kanban withdrawal cost ($/Kanban)	$W_{k1} = 10$	$W_{k1} = 10$ $W_{k2} = 8$	$W_{k1} = 10$ $W_{k2} = 8$ $W_{k3} = 8$
Ordering cost ($/unit*order)	$K_{R1} = 10$ $K_F = 50$	$K_{R1} = 10$ $K_{R2} = 9$ $K_F = 50$	$K_{R1} = 10$ $K_{R2} = 9$ $K_{R3} = 8$ $K_F = 50$
Holding cost ($/unit*day)	$H_{R1} = 0.5$ $H_{k1} = 0.4$ $H_F = 1$	$H_{R1} = 0.5$ $H_{R2} = 0.6$ $H_{k1} = 0.4$ $H_{k2} = 0.5$ $H_F = 1$	$H_{R1} = 0.5$ $H_{R2} = 0.6$ $H_{R3} = 0.8$ $H_{k1} = 0.4$ $H_{k2} = 0.5$ $H_{k3} = 0.6$ $H_F = 1$

TABLE 2. Optimal results of simulation models

Production line with N work-stage	$N = 2$	$N = 3$	$N = 4$
Production time T_{pi}	$T_{p1} = 14.495$ $T_{p2} = 13.92$	$T_{p1} = 14.495$ $T_{p2} = 13.92$ $T_{p3} = 15.749$	$T_{p1} = 14.495$ $T_{p2} = 13.92$ $T_{p3} = 15.749$ $T_{p4} = 16.165$
Cycle time T_c	$T_c = 16.926$	$T_c = 16.926$	$T_c = 16.926$
Optimal raw material orders n^*	$n^* = 4$	$n^* = 4$	$n^* = 4$
Minimum total inventory cost $TC(n^*)$	$TC(n^*) = \$828.6$	$TC(n*) = \$1290$	$TC(n^*) = \$1527$
Optimal number of Kanban k_i^*	$k_1^* = 5$	$k_1^* = 5$ $k_2^* = 6$	$k_1^* = 5$ $k_2^* = 6$ $k_3^* = 9$
Economic delivery number of finished products m^*	$m^* = 4$	$m^* = 4$	$m^* = 4$

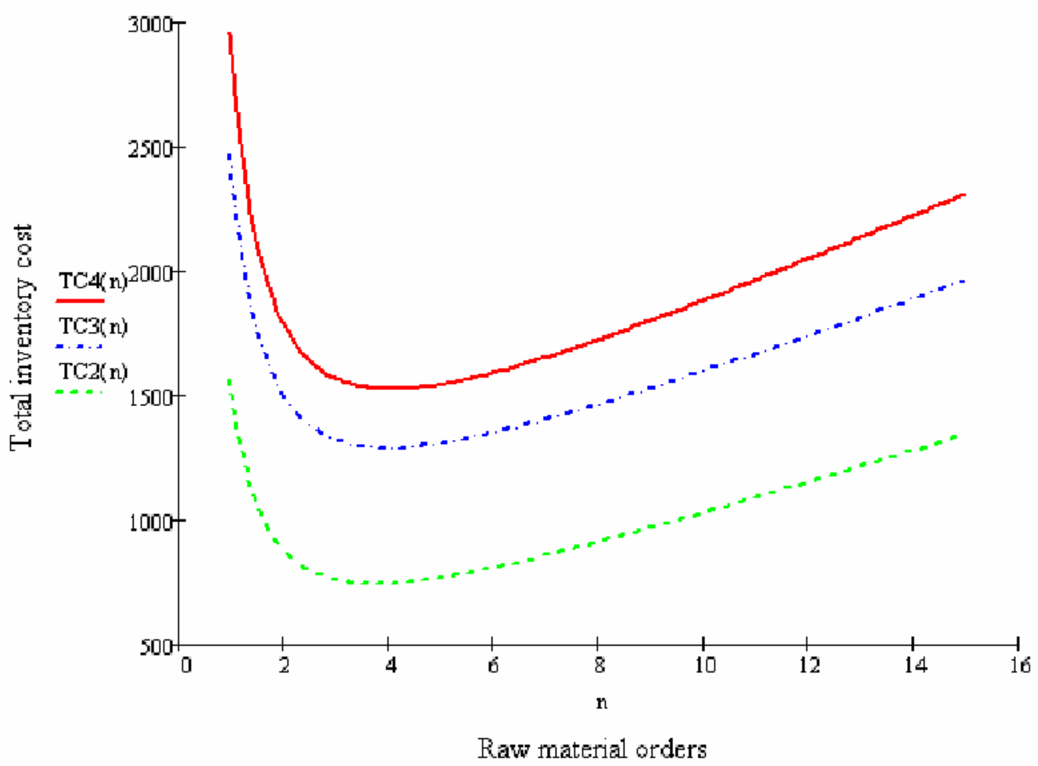

FIGURE 2. Total inventory of MKPS

same work-stages. For the constant cost parameters of MKPS, the optimal ordering policy of raw materials, the optimal Kanban number of each work-stage and the economic number of finished products are dictated by the customers demand patterns. Increasing the production stage will not affect the performance of the proceeding work-stages. It can be found that the optimal results obtained by using this figure are matched with the computational results listed in Table 3.

This research reveals the insights into the raw material order policy and production capacity changing rate for a demand-volume-flexible supply chain and production system. Using a rigorous mathematical approach, the optimal solution of the problem is accomplished. This table shows that the effects of varying production could increase rate p_i at 8, 10, 12, and 14 for $\forall i = 1, 2, 3, 4$ individually and keep the rest of the parameters fixed as shown in Table 1 for the four-work-stage production system.

It is observed that though the minimum total cost is different, the optimal raw material orders is unchanged that is $n^* = 4$ with the different configuration of production increase rate. So the changes in configuration for each work-stage in a production line are ineffective in decision process. That is the optimal raw material orders and corresponding optimal Kanban numbers k_i^* and economic delivery number of finished products m^* remain the same. On the other hand, the last work-stage's production increase rate p_4 is more sensitive to minimum $TC(n^*)$ than those of proceeding work-stage: p_1, p_2 and p_3. As the p_4 increases, the minimum $TC(n^*)$ decreases greatly. Numerical examples demonstrate that the proposed methodology provides an easy way for us to determine the optimal ordering policy of raw materials, the corresponding Kanban number of each work-stage and the economic finished product deliveries and simultaneously minimize the total inventory cost for MKPS.

Table 3. Total inventory cost for different raw material order at different configuration of production increase rate

Total inventory $TC(n^*)$ / Production increase rate p_i / Raw material orders n	p_2 = 10, p_3 = 10, p_4 = 8				p_1 = 10, p_3 = 10, p_4 = 8			
	p_1				p_2			
	8	10	12	8	8	10	12	14
$n=1$	3033	2962	2965	2972	2972	2962	2952	2942
$n=2$	1800	1798	1794	1798	1798	1798	1797	1795
$n=3$	1564	1571	1575	1570	1570	1571	1571	1570
$n=4$	1515	1527	1536	1526	1526	1527	1527	1527
$n=5$	1530	1546	1559	1545	1545	1546	1546	1546
$n=6$	1574	1593	1610	1592	1592	1593	1593	1593
Total inventory $TC(n^*)$ / Production increase rate p_i / Raw material orders n	p_1 = 10, p_2 = 10, p_4 = 8				p_1 = 10, p_2 = 10, p_3 = 8			
	p_3				p_4			
	8	10	12	14	8	10	12	14
$n=1$	2971	2962	2953	2944	2962	2723	2531	2372
$n=2$	1800	1798	1796	1793	1798	1577	1364	1204
$n=3$	1571	1571	1570	1568	1571	1329	1134	973.1
$n=4$	1527	1527	1526	1525	1527	1283	1088	925.2
$n=5$	1546	1546	1545	1545	1546	1301	1104	940.1
$n=6$	1593	1593	1592	1592	1593	1347	1148	983.4

5. **Conclusion.** This paper proposed a solution methodology about finding the minimum total inventory cost of single model multi-stage Kanban controlled production system with varying production rates at different work-stage and a linearly increase demand. For the issues of raw material orders, WIP inventories and finished product deliveries which influence the total inventory cost of the system simultaneously, these three components should be considered all together. According to the numerical simulation examples, we find that the solution of MKPS model is satisfactory, and we can make following conclusions:

1) The MKPS JIT model developed here consists of a general serial system with multi-work stages controlled with Kanbans, and the production rates at different stages are varying which is more suitable to be used into manufacturing system.

2) The total inventory cost calculation model developed here can help a manufacturing system quickly determine the schedule of production, the optimal number of Kanbans of each work-stage, and dispatching time intervals and economic delivery number of finished products according to the demand of costumers.

3) Varying the production variables p_i at each stage does not lead to any changes in the decision of the optimal raw material orders n^* and corresponding optimal Kanban numbers k^*, and the economic delivery number of finished products m^*.

4) Using a rigorous mathematical approach, the optimal solution of the problem is accomplished. In the practical situation, the proposed model can be implemented in industries with flexible production capacity and produce short-lift cycle products where product supply is in the JIT and demand-volume-flexible environment.

It is worthwhile for the future research to apply the proposed solution method to the production systems listed as the following:

1) Multi-product production system, which is a common operation characteristic in many industrial situations. The Kanban systems used to multi-product assembly manufacturing systems are more complicated than single product production system and its performance needs accurately evaluating.

2) Varying cost-parameters production system. The sensitivity analysis should be given on the dynamic nature of the system parameters and their influences on model costs. It is observed that finished product demand D_F, finished product demand changing rate ω, the ratio of α_j, β_j have significant effect on total inventory cost and are interactive in many cases. So it is required to develop a production evaluating model with varying cost parameter at different instant.

Acknowledgement. This work is partially supported by National Science Foundation of China under Grant No. 61374100, the Fundamental Research Funds for the Central Universities under Grant No. 2013B19314.

REFERENCES

[1] B. Hemamalini and C. Rajendran, Determination of the number of containers, production Kanbans; and withdrawal Kanban and scheduling in Kanban flowshops – Part 2, *International Journal of Production Research*, vol.38, pp.2549-2572, 2000.

[2] R. V. Ramasesh, Recasting the traditional inventory model to implement just-in-time purchasing, *Production and Inventory Management*, vol.31, pp.71-75, 1990.

[3] A. C. Pan and C. J. Liao, An inventory model under just-in-time purchasing agreements, *Production and Inventory Management*, vol.30, pp.49-52, 1989.

[4] M. Parlar and R. Rempala, A stochastic inventory problem with piecewise quadric costs, *International Journal of Production Economics*, vol.26, pp.327-332, 1992.

[5] C. Yilmaz, Incremental order quantity for the case of very lumpy demand, *International Journal of Production Economics*, vol.26, pp.367-371, 1992.

[6] R. M. Hill, Optimizing a production system with a fixed delivery schedule, *Journal of the Operational Research Society*, vol.47, pp.954-960, 1996.

[7] S. K. Goyal, A one-vendor multi-buyer integrated inventory model: A comment, *European Journal of Operational Research*, vol.82, pp.209-210, 1995.

[8] S. K. Goyal and Y. P. Gupta, Integrated inventory models: The buyer-vendor coordination, *European Journal of Operational Research*, vol.41, pp.38-42, 1989.

[9] R. Aderohunmu, A. Mobolurin and N. Bryson, Joint vendor-buyer policy in JIT manufacturing, *Journal of the Operational Research Society*, vol.46, pp.375-385, 1995.

[10] V. Tardif and L. Maaseidvaag, An adaptive approach to controlling Kanban systems, *European Journal of Operation Research*, vol.132, pp.411-424, 2001.

[11] D. Y. Golhar and B. R. Sarker, Economic manufacturing quantity in a just-in-time delivery system, *International Journal of Production Research*, vol.30, pp.961-972, 1992.

[12] A. M. Jamal and B. R. Sarker, An optimal batch size for a production system operating under a just-in-time delivery system, *International Journal of Production Economics*, vol.32, pp.255-260, 1993.

[13] B. R. Sarker and G. R. Parija, An optimal batch size for a production system operating under fixed-quantity, periodic delivery policy, *Journal of the Operational Research Society*, vol.45, pp.891-900, 1994.

[14] B. R. Sarker and G. R. Parija, Optimal batch size and raw material ordering policy for a production system with a fixed-interval, lumpy demand delivery system, *European Journal of Operation Research*, vol.89, pp.593-608, 1996.

[15] G. R. Parija and B. R. Sarker, Operation planning in a supply chain system with fixed-interval deliveries of finished goods to multiple customers, *IIE Transaction*, vol.31, pp.1075-1082, 1999.

[16] B. R. Sarker and C. V. Balan, Operation planning for Kanbans between two adjacent workstations, *Proc. of the 19th International Conference on Computers and Industrial Engineering*, vol.33, pp.221-224, 1996.

[17] A. Diponegoro and B. R. Sarker, Finite horizon planning for a production system with permitted shortage and fixed-interval deliveries, *Computer and Operation Research*, vol.33, pp.2387-2404, 2006.

[18] S. Wang and B. R. Sarker, An assembly-type supply chain system controlled by Kanbans under a just-in-time delivery policy, *European Journal of Operation Research*, vol.162, pp.153-172, 2005.

[19] A. Matta, Y. Dallery and M. D. Mascolo, Analysis of assembly systems controlled with Kanbans, *European Journal of Operation Research*, vol.166, pp.310-336, 2005.

[20] Y. Fang and Y. Lin, Optimal production policy for a multi-stage Kanban managed flexible production system, *ICIC Express Letters*, vol.4, no.4, pp.1337-1343, 2010.