International Journal of Innovative Computing, Information and Control
Volume **10**, Number **5**, October **2014**
 ISSN 1349-4198
pp. 1897–1913

GENERAL OBSERVER-BASED CONTROLLER DESIGN FOR SINGULAR MARKOVIAN JUMP SYSTEMS

Haiying Bo and Guoliang Wang
School of Information and Control Engineering
Liaoning Shihua University
No. 1, West Dandong Road, Wanghua District, Fushun 113001, P. R. China
bohaiying123@163.com

Received December 2013; revised April 2014

Abstract. *In this paper, a general kind of observer-based controller is developed for singular Markovian jump systems, which could bear perturbations under general transition rate matrices in terms of being known exactly, uncertain and partially unknown. In order to solve such problems, some new techniques in terms of introducing new inequalities are proposed, while the cross terms are handled by two different techniques. Sufficient conditions for such general observer-based controllers are developed as linear matrix inequalities. Finally, the effectiveness and superiority of the proposed methods are demonstrated by numerical examples.*
Keywords: Singular Markovian jump system, Observer-based controller, Linear matrix inequality

1. **Introduction.** Many practical systems such as power systems, solar receiver system and aircraft control, have their structures changed randomly, such as component failures or repairs, sudden environmental changes in their dynamics, sudden environmental disturbances, changes in subsystem interconnections, modification of the operating point of a nonlinear system. It is known that this kind of system is usually modeled into a kind of stochastic systems driven by the Markov chains and is always referred to be Markovian jump systems (MJSs). During the past decades, various kinds of MJSs have been extensively studied, see, e.g., [1, 2, 3] and the references therein.

Practically, we usually encounter physical systems that cannot be modeled by the normal systems in terms of ordinary differential equations. Instead, these systems are referred to be singular systems, implicit systems, descriptor systems, etc., which are described by coupled differential and algebraic equations. It is said that they are more complicated than the traditional state-space systems. It is because singular system includes three types of modes, where two additional modes named as impulsive modes and non-dynamic modes [4, 5] are included. When singular systems experience random abrupt changes, it is natural and convenient to model them into singular Markovian jump systems (SMJSs) [6, 7]. Over the past few years, a lot of attention has been paid to such systems, see, e.g., [8, 9, 10, 11, 12].

On the other hand, the state estimation is also a fundamental problem in control systems and signal processing. Hence, over the past decades, the problem about observer design has attracted a lot of attention. For more details, we refer to the references [13, 14, 15, 16]. Some work has been carried out on singular systems [17, 18, 19]. However, the observer design problem for SMJSs has not yet been fully investigated, and only few results [20, 21] are obtained. By investigating the listed references, it is seen that there are some limitations. For one thing, the desired observers cannot bear some disturbances, while the perturbations are impossible to be avoided in many applications. For another, the transition rate matrix (TRM) in the afore-cited references should be known exactly. As we know, due to many constraints of applications, it is impossible or higher cost to get the TRM exactly. Instead, only uncertain or partially unknown TRMs are accessible. It has been shown in [22, 23, 24] that such general TRMs can reduce the performance of a system or even make a system instability. Based on these facts, it is claimed that it is significant and necessary to study this kind of system under such general TRMs. In this sense, it is said that the obtained results have the application scope limited. Especially, in reference [20], it is known that the conditions for observer-based controllers are obtained as a series of coupled LMIs, which cannot be solved directly and easily. Moreover, the computation of the parameters of the desired observer-based controllers cannot be got simultaneously, which will bring more conservatism. To best of our knowledge, the stabilization problem of SMJSs by observer-based controllers has not yet been fully investigated, and there are still challenges, which needs further more investigations. All the observations motivate the current research.

In this paper, the main attention is focused on designing a kind of general observer-based controller with parameter perturbations, where the TRM of the underlying system can be known exactly, uncertain and partially unknown. Compared with the existing results mentioned above, the main contributions of this paper are as follows: 1) The desired observer-based controllers could bear some perturbations on the parameters, where the degrees of such perturbations can be computed; 2) The corresponding TRM in this paper is more general, which can be known exactly, uncertain and partially unknown respectively; 3) By the LMI approach, both gains of controller and observer are solved simultaneously, where two different techniques are used to deal with the cross terms; 4) In order to get LMI conditions ultimately, some new variables satisfying additional inequalities are introduced appropriately.

Notation: $\mathbb{R}^n$ denotes the n dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. In symmetric block matrices, we use " $*$ " as an ellipsis for the terms induced by symmetry, diag$\{\cdots\}$ for a block-diagonal matrix, and $(M)^* \triangleq M + M^T$.

2. Problem Statement and Preliminaries.

Consider a class of SMJS described as

$$\begin{cases} E\dot{x}(t) = A_{\eta_t}x(t) + B_{\eta_t}u(t) \\ \quad y(t) = C_{\eta_t}x(t) \\ \quad x(t) = x_0 \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, and $y(t) \in \mathbb{R}^p$ is measurement output. Matrix $E \in \mathbb{R}^{n\times n}$ may be singular and assumed rank$(E) = s < n$. $x(0) = x_0$ is the compatible initial condition, and η_0 is the initial mode. A_{η_t}, B_{η_t}, C_{η_t} are known matrices of appropriate dimensions. The operation mode $\{\eta_t, t \geq 0\}$ is a right-continuous Markov process taking values in a finite space $\mathbb{S} = \{1, 2, \cdots, N\}$ with TRM $\Pi = (\lambda_{ij}) \in \mathbb{R}^{N\times N}$ given by

$$\Pr\{\eta_{t+h} = j|\eta_t = i\} = \begin{cases} \lambda_{ij}h + o(h) & i \neq j \\ 1 + \lambda_{ii}h + o(h) & i = j \end{cases} \tag{2}$$

where $h > 0$, $\lim_{h\to 0^+}(o(h)/h) = 0$, and $\lambda_{ij} \geq 0$, if $i \neq j$, $\lambda_{ii} = -\sum_{j=1,j\neq i}^{N} \lambda_{ij}$. In this paper, we will design an observer with the following form:

$$E\dot{\hat{x}}(t) = A_{\eta_t}\hat{x}(t) + B_{\eta_t}u(t) - (L_{\eta_t} + \Delta L_{\eta_t})(y(t) - C_{\eta_t}\hat{x}(t)) \tag{3}$$

and the controller is

$$u(t) = (K_{\eta_t} + \Delta K_{\eta_t})\hat{x}(t) \tag{4}$$

where $\hat{x}(t)$ is the estimation of system state $x(t)$, L_{η_t} and K_{η_t} are observer parameter and control gain to be determined respectively. ΔL_{η_t} and ΔK_{η_t} are the corresponding fluctuations and are described as follows:

$$\Delta L_i^T \Delta L_i \leq \delta_1 I_m \tag{5}$$

$$\Delta K_i^T \Delta K_i \leq \delta_2 I_n \tag{6}$$

where δ_1 and δ_2 are named as the degrees of the perturbations and are to be determined. Letting $e(t) = x(t) - \hat{x}(t)$ and combining (1), (3) and (4), we get

$$E\dot{e}(t) = [A_{\eta_t} + (L_{\eta_t} + \Delta L_{\eta_t})C_{\eta_t}]e(t) \tag{7}$$

For notational simplicity, in the sequel, for each $\eta_t = i \in \mathbb{S}$, we write $A_{\eta_t} = A_i$, $B_{\eta_t} = B_i$, $C_{\eta_t} = C_i$, $L_{\eta_t} = L_i$, $K_{\eta_t} = K_i$ and so on. Especially, Π in this paper satisfies the following three cases:

Case 1: Π is known exactly and described by (2);

Case 2: Π is uncertain and has an admissible uncertainty

$$\Pi = \bar{\Pi} + \Delta\bar{\Pi}$$

in which $\bar{\Pi} = (\bar{\lambda}_{ij})$ is the estimation of Π, and $\Delta\bar{\Pi} = (\Delta\bar{\lambda}_{ij})$ with $\Delta\bar{\lambda}_{ij} = \lambda_{ij} - \bar{\lambda}_{ij}$ and $\alpha_{ij} = \bar{\lambda}_{ij} - \epsilon_{ij}$ is the estimated error with property (2) and $\Delta\bar{\lambda}_{ij}$, $j \neq i$, takes any value in $[-\epsilon_{ij}, \epsilon_{ij}]$. Moreover, it is obtained that $|\Delta\bar{\lambda}_{ii}| \leq -\epsilon_{ii}$ with $\epsilon_{ii} = -\sum_{j=1, j\neq i}^{N} \epsilon_{ij}$;

Case 3: The TRM is partially accessible. For example, a partially unknown Π may be expressed as

$$\Pi = \begin{bmatrix} \lambda_{11} & ? & \lambda_{13} & ? \\ \lambda_{21} & ? & ? & ? \\ ? & ? & ? & \lambda_{34} \\ ? & \lambda_{42} & ? & \lambda_{44} \end{bmatrix}$$

where '?' represents the unknown elements. Then, for any $i \in \mathbb{S}$, define $\mathbb{S}^i = \mathbb{S}_k^i \bigcup \bar{\mathbb{S}}_k^i$ with

$$\mathbb{S}_k^i = \{j : \lambda_{ij} \text{ is known}\} \text{ and } \bar{\mathbb{S}}_k^i = \{j : \lambda_{ij} \text{ is unknown}\}$$

which are further described as

$$\mathbb{S}_k^i = \{k_1^i, \cdots, k_m^i\} \text{ and } \bar{\mathbb{S}}_k^i = \{\bar{k}_1^i, \cdots, \bar{k}_{N-m}^i\}$$

where $k_j^i \in \mathbb{Z}^+$ represent the column index of the jth known element in the ith row of Π, and $\bar{k}_{N-j}^i \in \mathbb{Z}^+$ is the column index of the $(N-j)$th unknown element in the ith row of Π. In addition, it is assumed that $\tau = \min_{i \in \bar{\mathbb{S}}_k^i}\{\lambda_{ii}\}$ is known.

Lemma 2.1. [6, 7] *Unforced SMJS (1) is said to be stochastically admissible if there exists matrix P_i, such that the following LMIs hold for each $i \in \mathbb{S}$:*

$$E^T P_i = P_i^T E \geq 0 \tag{8}$$

$$(A_i^T P_i)^* + \sum_{j=1}^{N} \lambda_{ij} E^T P_j < 0 \tag{9}$$

3. Main Results.

Theorem 3.1. *Given system (1) with Case 1, there exists controller (4) based on observer (3) such that the resulting system is stochastically admissible if there exist matrices* $\hat{P}_i > 0$, $\hat{Q}_i$, $\bar{M}_i > 0$, $\bar{N}_i$, $Z_i > 0$, Y_i, $\bar{L}_i$, *scalars* $\bar{\delta}_1 > 0$ *and* $\bar{\delta}_2 > 0$ *satisfying*

$$\begin{bmatrix} \Phi_{i1} & \sqrt{2}B_i & X_i^T & \Psi_{i1} \\ * & -I & 0 & 0 \\ * & * & -\bar{\delta}_2 I_n & 0 \\ * & * & * & \Psi_{i2} \end{bmatrix} < 0 \tag{10}$$

$$\begin{bmatrix} \Phi_{i2} & \sqrt{2}I & Q_i^T & \sqrt{2}I & C_i^T \\ * & -(X_i)^* + Z_i & 0 & 0 & 0 \\ * & * & -\bar{\delta}_1 I_n & 0 & 0 \\ * & * & * & -\bar{\delta}_2 I_n & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix} -Z_i & Y_i^T \\ * & -I \end{bmatrix} \leq 0 \tag{12}$$

where

$$X_i = \hat{P}_i E^T + U\hat{Q}_i V, \ Q_i = \bar{M}_i E + V^T \bar{N}_i U^T$$

$$\Phi_{i1} = (A_i X_i + B_i Y_i)^* + \lambda_{ii} X_i^T E^T, \ \Phi_{i2} = (A_i^T Q_i + C_i^T \bar{L}_i)^* + \sum_{j=1}^{N} \lambda_{ij} E^T \bar{M}_j E$$

$$\Psi_{i1} = \left[\sqrt{\lambda_{i1}} X_i^T E_R \cdots \sqrt{\lambda_{i(i-1)}} X_i^T E_R \ \sqrt{\lambda_{i(i+1)}} X_i^T E_R \cdots \sqrt{\lambda_{iN}} X_i^T E_R\right]$$

$$\Psi_{i2} = -\mathrm{diag}\{E_R^T \hat{P}_1 E_R, \cdots, E_R^T \hat{P}_{i-1} E_R, E_R^T \hat{P}_{i+1} E_R, \cdots, E_R^T \hat{P}_N E_R\}$$

Then, the parameter of observer (3) is given by

$$L_i = Q_i^{-T} \bar{L}_i^T \tag{13}$$

and the gain of controller (4) is computed as

$$K_i = Y_i X_i^{-1} \tag{14}$$

Proof: Considering systems (1) and (7), we choose the following Lyapunov function:

$$V(x(t), e(t), \eta_t = i) = x^T(t) E^T P_i x(t) + e^T(t) E^T Q_i e(t) \tag{15}$$

where P_i and Q_i satisfy (8). Letting $\pounds$ be the weak infinitesimal operator of the stochastic process $\{(x(t), e(t), \eta_t), t \geq 0\}$, we have

$$\begin{aligned} \pounds V(x(t), e(t), \eta_t = i) = {} & x^T(t) \left[(A_i^T P_i + K_i^T B_i^T P_i + \Delta K_i^T B_i^T P_i)^* + \sum_{j=1}^{N} \lambda_{ij} E^T P_j\right] x(t) \\ & + e^T(t) \left[(A_i^T Q_i + C_i^T L_i^T Q_i + C_i^T \Delta L_i^T Q_i)^* + \sum_{j=1}^{N} \lambda_{ij} E^T Q_j\right] e(t) \\ & - 2x^T(t) P_i^T B_i (K_i + \Delta K_i) e(t) \end{aligned} \tag{16}$$

First, the cross term is dealt with as follows:

$$-2x^T(t) P_i^T B_i (K_i + \Delta K_i) e(t) \leq x^T(t) P_i^T B_i B_i^T P_i x(t) + e^T(t)(K_i + \Delta K_i)^T (K_i + \Delta K_i) e(t) \tag{17}$$

Taking into account (5), (6), (16) and (17), we have

$$
\begin{aligned}
&\pounds V(x(t), e(t), \eta_t = i) \\
&\leq x^T(t)\Bigg[(A_i^T P_i + K_i^T B_i^T P_i + \Delta K_i^T B_i^T P_i)^{*} \\
&\quad + P_i^T B_i B_i^T P_i + \sum_{j=1}^{N} \lambda_{ij} E^T P_j\Bigg] x(t) \\
&\quad + e^T(t)\Bigg[(A_i^T Q_i + C_i^T L_i^T Q_i + C_i^T \Delta L_i^T Q_i)^{*} \\
&\quad + (K_i + \Delta K_i)^T (K_i + \Delta K_i) + \sum_{j=1}^{N} \lambda_{ij} E^T Q_j\Bigg] e(t) \\
&\leq x^T(t)\left[(A_i^T P_i + K_i^T B_i^T P_i)^{*} + \delta_2 I_n + 2P_i^T B_i B_i^T P_i + \sum_{j=1}^{N} \lambda_{ij} E^T P_j\right] x(t) \\
&\quad + e^T(t)\Bigg[(A_i^T Q_i + C_i^T L_i^T Q_i)^{*} + \delta_1 Q_i^T Q_i \\
&\quad + C_i^T C_i + 2K_i^T K_i + 2\delta_2 I_n + \sum_{j=1}^{N} \lambda_{ij} E^T Q_j\Bigg] e(t) \\
&= \xi^T(t) J_i \xi(t)
\end{aligned} \tag{18}
$$

where

$$
\xi(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, J_i = \begin{bmatrix} J_i^P & 0 \\ 0 & J_i^Q \end{bmatrix}
$$

$$
J_i^P = (A_i^T P_i + K_i^T B_i^T P_i)^{*} + \delta_2 I_n + 2P_i^T B_i B_i^T P_i + \sum_{j=1}^{N} \lambda_{ij} E^T P_j
$$

$$
J_i^Q = (A_i^T Q_i + C_i^T L_i^T Q_i)^{*} + \delta_1 Q_i^T Q_i + C_i^T C_i + 2K_i^T K_i + 2\delta_2 I_n + \sum_{j=1}^{N} \lambda_{ij} E^T Q_j
$$

Then, if

$$
J_i^P = (A_i^T P_i + K_i^T B_i^T P_i)^{*} + \delta_2 I_n + 2P_i^T B_i B_i^T P_i + \sum_{j=1}^{N} \lambda_{ij} E^T P_j < 0 \tag{19}
$$

$$
J_i^Q = (A_i^T Q_i + C_i^T L_i^T Q_i)^{*} + \delta_1 Q_i^T Q_i + C_i^T C_i + 2K_i^T K_i + 2\delta_2 I_n + \sum_{j=1}^{N} \lambda_{ij} E^T Q_j < 0 \tag{20}
$$

one has $J_i < 0$ for all $i \in \mathbb{S}$. Letting $Y_i = K_i X_i$, it is obtained that (19) is equivalent to

$$
(A_i X_i + B_i Y_i)^{*} + \delta_2 X_i^T X_i + 2B_i B_i^T + \sum_{j=1}^{N} \lambda_{ij} X_i^T E^T P_j X_i < 0 \tag{21}
$$

with $X_i = P_i^{-1}$. Letting $\bar{\delta}_2 = \delta_2^{-1}$, by the Schur's complement, inequality (21) implies that

$$\begin{bmatrix} \bar{\Phi}_{i1} & \sqrt{2}B_i & X_i^T \\ * & -I & 0 \\ * & * & -\bar{\delta}_2 I_n \end{bmatrix} < 0 \tag{22}$$

where

$$\bar{\Phi}_{i1} = (A_i X_i + B_i Y_i)^* + \sum_{j=1}^{N} \lambda_{ij} X_i^T E^T P_j X_i$$

Letting

$$P_i = \bar{P}_i E + V^T \bar{Q}_i U^T \tag{23}$$

where $\bar{P}_i > 0$, $|\bar{Q}_i| \neq 0$, $V \in \mathbb{R}^{(n-s)\times n}$ is any matrix with full row rank and $U \in \mathbb{R}^{n\times(n-s)}$ is any matrix with full column rank, $VE = 0$ and $EU = 0$ are satisfied. Then, one has $E_L^T \bar{P}_i E_L > 0$, where matrix E is decomposed as $E = E_L E_R^T$ with $E_L \in \mathbb{R}^{n\times s}$ and $E_R \in \mathbb{R}^{n\times s}$ are of full column rank. Moreover, it is obvious that matrix P_i satisfies (8). Via the method in [25], it is known that

$$X_i \triangleq \hat{P}_i E^T + U\hat{Q}_i V = P_i^{-1} \tag{24}$$

where $\hat{P}_i = \hat{P}_i^T$ and $|\hat{Q}_i| \neq 0$. Meanwhile, we have $E_L^T \bar{P}_i E_L = (E_R^T \hat{P}_i E_R)^{-1}$. Then, we get

$$\begin{aligned} \sum_{j=1}^{N} \lambda_{ij} X_i^T E^T P_j X_i &= \lambda_{ii} X_i^T E^T + \sum_{j=1, j\neq i}^{N} \lambda_{ij} X_i^T E_R (E_L^T \bar{P}_j E_L) E_R^T X_i \\ &= \lambda_{ii} X_i^T E^T + \sum_{j=1, j\neq i}^{N} \lambda_{ij} X_i^T E_R (E_R^T \hat{P}_i E_R)^{-1} E_R^T X_i \end{aligned} \tag{25}$$

Taking into account (23)-(25), it is known that (22) is equivalent to (10). Now, we consider how to transform the condition $J_i^Q < 0$ into the LMI form. Letting $Q_i = \bar{M}_i E + V^T \bar{N}_i U^T$, with $\bar{M}_i > 0$ and $|\bar{N}_i| \neq 0$, one has $E^T Q_i = Q_i^T E \geq 0$. To solve such a problem, we introduce an inequality

$$K_i^T K_i \leq X_i^{-T} Z_i X_i^{-1} \tag{26}$$

for any $Z_i > 0$. Letting $\bar{L}_i = L_i^T Q_i$, we have

$$\begin{bmatrix} (A_i^T Q_i + C_i^T L_i^T Q_i)^* + \delta_1 Q_i^T I_n Q_i + C_i^T C_i + 2\delta_2 I_n + \sum_{j=1}^{N} \lambda_{ij} E^T \bar{M}_j E & \sqrt{2} X_i^{-T} \\ * & -Z_i^{-1} \end{bmatrix} < 0 \tag{27}$$

It is known that $-X_i Z_i^{-1} X_i^T \leq -X_i - X_i^T + Z_i$ with any $Z_i > 0$. Simultaneously, letting $\delta_1^{-1} = \bar{\delta}_1$, by the Schur's complement, inequality (27) implies (11). On the other hand, it is seen that (26) plays an important role, which is equivalent to (12). This completes the proof.

In the proof of Theorem 3.1, we deal with the cross term $-2x^T(t) P_i^T B_i (K_i + \Delta K_i) e(t)$ by (17). Moreover, it can be done by another method. Then, one has the following theorem.

Theorem 3.2. *Given system (1) with Case 1, there exists a controller (4) based on observer (3) such that the resulting system is stochastically admissible if there exist matrices*

$\hat{P}_i > 0$, $\hat{Q}_i$, $\bar{M}_i > 0$, $\bar{N}_i$, Y_i, $\bar{L}_i$, *scalars* $\bar{\delta}_1 > 0$ *and* $\bar{\delta}_2 > 0$ *satisfying*

$$\begin{bmatrix} \Phi_{i1} & \sqrt{2}B_iY_i & B_i & \sqrt{2}B_i & X_i^T & \Psi_{i1} \\ * & -(X_i)^* + I & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\bar{\delta}_2 I_m & 0 & 0 \\ * & * & * & * & -\bar{\delta}_2 I_n & 0 \\ * & * & * & * & * & \Psi_{i2} \end{bmatrix} < 0 \tag{28}$$

$$\begin{bmatrix} \Phi_{i3} & Q_i^T & C_i^T \\ * & -\bar{\delta}_1 I_n & 0 \\ * & * & -I \end{bmatrix} \tag{29}$$

where

$$\Phi_{i3} = (A_i^T Q_i + C_i^T \bar{L}_i)^* + I + \sum_{j=1}^{N} \lambda_{ij} E^T \bar{M}_j E < 0$$

The others are defined in Theorem 3.1. Then, the parameters of L_i *and* K_i *can be computed by (13) and (14) respectively.*

Proof: Similar to the proof of Theorem 3.1, another technique is used to deal with the cross term. That is

$$-2x^T(t)P_i^T B_i(K_i + \Delta K_i)e(t) \le x^T(t)P_i^T B_i(K_i + \Delta K_i)(K_i + \Delta K_i)^T B_i^T P_i x(t) + e^T(t)Ie(t) \tag{30}$$

Based on (5), we also have

$$\Delta K_i \Delta K_i^T \le \delta_2 I_m \tag{31}$$

Taking into account (5), (30) and (31), we obtain

$$\begin{aligned} &\pounds V(e(t), \eta_t = i) \\ &\le x^T(t)\Bigg[(A_i^T P_i + K_i^T B_i^T P_i)^* + P_i^T B_i B_i^T P_i + \delta_2 I_n \\ &\quad + 2P_i^T B_i K_i K_i^T B_i^T P_i + 2\delta_2 P_i^T B_i B_i^T P_i + \sum_{j=1}^{N} \lambda_{ij} E^T P_j\Bigg] x(t) \\ &\quad + e^T(t)\left[(A_i^T Q_i + C_i^T L_i^T Q_i)^* + \delta_1 Q_i^T Q_i + C_i^T C_i + I + \sum_{j=1}^{N} \lambda_{ij} E^T \bar{M}_j E\right] e(t) \\ &= \xi^T(t) J_i \xi(t) \end{aligned} \tag{32}$$

Then, we conclude that $J_i < 0$ for all $i \in \mathbb{S}$, is guaranteed by

$$\begin{aligned} J_i^P &= (A_i^T P_i + K_i^T B_i^T P_i)^* + P_i^T B_i B_i^T P_i + \delta_2 I_n \\ &\quad + 2P_i^T B_i K_i K_i^T B_i^T P_i + 2\delta_2 P_i^T B_i B_i^T P_i + \sum_{j=1}^{N} \lambda_{ij} E^T P_j < 0 \end{aligned} \tag{33}$$

$$J_i^Q = (A_i^T Q_i + C_i^T L_i^T Q_i)^* + \delta_1 Q_i^T Q_i + C_i^T C_i + I + \sum_{j=1}^{N} \lambda_{ij} E^T \bar{M}_j E < 0 \tag{34}$$

It is known that (33) and (34) imply (28) and (29) respectively. The next process is similar to the proof of Theorem 3.1, which is omitted here. This completes the proof.

Remark 3.1. *It is remarked that, at the current formats of Theorems 3.1 and 3.2, it is difficult to make a conclusion that which one is less conservative. Inequalities (17) and (30) are seen as two respective techniques for dealing with the cross terms. From this fact, it is said that both of them are effective to design an observer-based controller for SMJSs. Especially, different from the existing methods such as in* [20]*, both parameters of the observer-based controller can be computed simultaneously. Instead of solving them one by one, it is said that Theorems 3.1 and 3.2 are less conservative, whose conservatism is demonstrated by a numerical example. Moreover, all the conditions for the existence of the desired observer-based controller are established within LMI framework, which could be solved directly and easily.*

When TRM satisfies more general conditions such as in Cases 2 and 3, the established results in [20, 21] will be failed. Up to now, very few results report this problem. Based on Theorems 3.1 and 3.2, we can obtain the corresponding results whose conditions for an observer-based controller of SMJS are established within LMI frameworks.

Theorem 3.3. *Given system (1) with Case 2, there exists a controller (4) based on observer (3) such that the resulting system is stochastically admissible if there exist matrices $\hat{P}_i > 0$, $\hat{Q}_i$, $\bar{M}_i > 0$, ,$\bar{N}_i$, $Z_i > 0$, $M_i > 0$, $N_i > 0$, $T_i > 0$, $S_i > 0$, Y_i, $\bar{L}_i$, scalars $\bar{\delta}_1 > 0$ and $\bar{\delta}_2 > 0$ satisfying*

$$\begin{bmatrix} \Phi_{i4} & \sqrt{2}B_i & X_i^T & M_i & \Psi_{i3} \\ * & -I & 0 & 0 & 0 \\ * & * & -\bar{\delta}_2 I_n & 0 & 0 \\ * & * & * & -T_i & 0 \\ * & * & * & * & \Psi_{i2} \end{bmatrix} < 0 \tag{35}$$

$$\begin{bmatrix} \Phi_{i5} & \sqrt{2}I & Q_i^T & \sqrt{2}I & C_i^T & N_i \\ * & -(X_i)^* + Z_i & 0 & 0 & 0 & 0 \\ * & * & -\bar{\delta}_1 I_n & 0 & 0 & 0 \\ * & * & * & -\bar{\delta}_2 I_n & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -S_i \end{bmatrix} < 0 \tag{36}$$

$$\begin{bmatrix} -X_i^T E^T - M_i & X_i^T E_R \\ * & -E_R^T \hat{P}_j E_R \end{bmatrix} \le 0 \ , \ j \neq i \tag{37}$$

$$E^T \bar{M}_j E - E^T \bar{M}_i E - N_i \le 0 \ , \ j \neq i \tag{38}$$

$$\begin{bmatrix} -Z_i & Y_i^T \\ * & -I \end{bmatrix} \le 0 \tag{39}$$

where

$$\Phi_{i4} = (A_i X_i + B_i Y_i)^* + \alpha_{ii} X_i^T E^T - \epsilon_{ii} M_i + \frac{1}{4}\epsilon_{ii}^2 T_i$$

$$\Phi_{i5} = (A_i^T Q_i + C_i^T \bar{L}_i)^* + \sum_{j=1}^{N} \alpha_{ij} E^T \bar{M}_j E - \epsilon_{ii} N_i + \frac{1}{4}\epsilon_{ii}^2 S_i$$

$$\Psi_{i3} = [\sqrt{\alpha_{i1}} X_i^T E_R \cdots \sqrt{\alpha_{i(i-1)}} X_i^T E_R \ \sqrt{\alpha_{i(i+1)}} X_i^T E_R \cdots \sqrt{\alpha_{iN}} X_i^T E_R]$$

Then, (13) and (14) are used to get the parameters of observer (3) and controller (4) respectively.

Proof: Based on the proof of Theorem 3.1 and Case 2, $\bar{\Phi}_{i1} < 0$ is rewritten to

$$\begin{aligned}&(A_iX_i + B_iY_i)^* + \sum_{j=1}^{N} \alpha_{ij} X_i^T E^T P_j X_i - \epsilon_{ii} M_i + \frac{1}{4}\epsilon_{ii}^2 T_i + M_i T_i^{-1} M_i \\ &+ \sum_{j=1, j\neq i}^{N} (\Delta\bar{\lambda}_{ij} + \epsilon_{ij})(X_i^T E^T P_j X_i - X_i^T E^T - M_i) < 0\end{aligned} \tag{40}$$

It is known that (40) is guaranteed by

$$(A_iX_i + B_iY_i)^* + \sum_{j=1}^{N} \alpha_{ij} X_i^T E^T P_j X_i - \epsilon_{ii} M_i + \frac{1}{4}\epsilon_{ii}^2 T_i + M_i T_i^{-1} M_i < 0 \tag{41}$$

$$X_i^T E^T P_j X_i - X_i^T E^T - M_i < 0 \ , \ j \neq i \tag{42}$$

Based on this, one has (41) equivalent to (35). In addition, by (23), it is obtained that (42) implies (37). Analogously, we get (36) and (38). The next process of the proof is similar to Theorem 3.1, which is omitted here. This completes the proof.

Theorem 3.4. *Given system (1) with Case 2, there exists a controller (4) based on observer (3) such that the resulting system is stochastically admissible if there exist matrices* $\hat{P}_i > 0$, $\hat{Q}_i$, $\bar{M}_i > 0$, $\bar{N}_i$, $M_i > 0$, $N_i > 0$, $T_i > 0$, $S_i > 0$, Y_i, $\bar{L}_i$, *scalars* $\bar{\delta}_1 > 0$ *and* $\bar{\delta}_2 > 0$ *satisfying*

$$\begin{bmatrix} \Phi_{i4} & \sqrt{2}B_iY_i & B_i & \sqrt{2}B_i & X_i^T & M_i & \Psi_{i3} \\ * & -(X_i)^* + I & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{\delta}_2 I_m & 0 & 0 & 0 \\ * & * & * & * & -\bar{\delta}_2 I_n & 0 & 0 \\ * & * & * & * & * & -T_i & 0 \\ * & * & * & * & * & * & \Psi_{i2} \end{bmatrix} < 0 \tag{43}$$

$$\begin{bmatrix} \Phi_{i6} & Q_i^T & C_i^T & N_i \\ * & -\bar{\delta}_1 I_n & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -S_i \end{bmatrix} < 0 \tag{44}$$

$$\begin{bmatrix} -X_i^T E^T - M_i & X_i^T E_R \\ * & -E_R^T \hat{P}_j E_R \end{bmatrix} \leq 0 \ , \ j \neq i \tag{45}$$

$$E^T \bar{M}_j E - E^T \bar{M}_i E - N_i \leq 0 \ , \ j \neq i \tag{46}$$

where

$$\Phi_{i6} = (A_i^T Q_i + C_i^T \bar{L}_i)^* + I + \sum_{j=1}^{N} \alpha_{ij} E^T \bar{M}_j E - \epsilon_{ii} N_i + \frac{1}{4}\epsilon_{ii}^2 S_i$$

Then the corresponding parameters can be computed by (13) and (14).

Proof: Similar to the proofs of Theorems 3.2 and 3.3, one can get Theorem 3.4 directly. The proof is omitted here. This completes the proof.

Theorem 3.5. *Given system (1) with Case 3, there exists a controller (4) based on observer (3) such that the resulting system is stochastically admissible if there exist matrices*

$\hat{P}_i > 0$, $\hat{Q}_i$, $\bar{M}_i > 0$, $\bar{N}_i$, $Z_i > 0$, $M_i > 0$, $N_i > 0$, Y_i, $\bar{L}_i$, scalars $\bar{\delta}_1 > 0$ and $\bar{\delta}_2 > 0$ satisfying

$$\begin{bmatrix} \Phi_{i7} & \sqrt{2}B_i & X_i^T & \Psi_{i1} \\ * & -I & 0 & 0 \\ * & * & -\bar{\delta}_2 I_n & 0 \\ * & * & * & \Psi_{i2} \end{bmatrix} < 0, \ i \in \mathbb{S}_k^i \tag{47}$$

$$\begin{bmatrix} \Phi_{i8} & \sqrt{2}B_i & X_i^T & \Psi_{i1} \\ * & -I & 0 & 0 \\ * & * & -\bar{\delta}_2 I_n & 0 \\ * & * & * & \Psi_{i2} \end{bmatrix} < 0, \ i \in \bar{\mathbb{S}}_k^i \tag{48}$$

$$\begin{bmatrix} \Phi_{i9} & \sqrt{2}I & Q_i^T & \sqrt{2}I & C_i^T \\ * & -(X_i)^* + Z_i & 0 & 0 & 0 \\ * & * & -\bar{\delta}_1 I_n & 0 & 0 \\ * & * & * & -\bar{\delta}_2 I_n & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \ i \in \mathbb{S}_k^i \tag{49}$$

$$\begin{bmatrix} \Phi_{i10} & \sqrt{2}I & Q_i^T & \sqrt{2}I & C_i^T \\ * & -(X_i)^* + Z_i & 0 & 0 & 0 \\ * & * & -\bar{\delta}_1 I_n & 0 & 0 \\ * & * & * & -\bar{\delta}_2 I_n & 0 \\ * & * & * & 0 & -I \end{bmatrix} < 0, \ i \in \bar{\mathbb{S}}_k^i \tag{50}$$

$$\begin{bmatrix} -X_i^T E^T - M_i & X_i^T E_R \\ * & -E_R^T \hat{P}_j E_R \end{bmatrix} \le 0, \ i \in \mathbb{S}, \ j \in \bar{\mathbb{S}}_k^i, \ j \neq i \tag{51}$$

$$E^T \bar{M}_j E - E^T \bar{M}_i E - N_i \le 0, \ i \in \mathbb{S}, \ j \in \bar{\mathbb{S}}_k^i, \ j \neq i \tag{52}$$

$$\begin{bmatrix} -Z_i & Y_i^T \\ * & -I \end{bmatrix} \le 0 \tag{53}$$

where

$$\Phi_{i7} = (A_i X_i + B_i Y_i)^* - \sum_{j \in \mathbb{S}_k^i, j \neq i} \lambda_{ij} X_i^T E^T - \sum_{j \in \mathbb{S}_k^i} \lambda_{ij} M_i$$

$$\Phi_{i8} = (A_i X_i + B_i Y_i)^* - \sum_{j \in \mathbb{S}_k^i, j \neq i} \lambda_{ij} X_i^T E^T - \sum_{j \in \mathbb{S}_k^i, j \neq i} \lambda_{ij} M_i - \tau M_i$$

$$\Phi_{i9} = (A_i^T Q_i + C_i^T \bar{L}_i)^* + \sum_{j \in \mathbb{S}_k^i, j \neq i} \lambda_{ij} (E^T \bar{M}_j E - E^T Q_i) - \sum_{j \in \mathbb{S}_k^i} \lambda_{ij} N_i$$

$$\Phi_{i10} = (A_i^T Q_i + C_i^T \bar{L}_i)^* + \sum_{j \in \mathbb{S}_k^i, j \neq i} \lambda_{ij} (E^T \bar{M}_j E - E^T Q_i) - \sum_{j \in \mathbb{S}_k^i, j \neq i} \lambda_{ij} N_i - \tau N_i$$

The parameters of L_i and K_i are computed by (13) and (14) respectively.

Proof: Similar to Theorem 3.3, for any $M_i > 0$, it is obtained that $\bar{\Phi}_{i1} < 0$ is equivalent to

$$\begin{aligned} &(A_i X_i + B_i Y_i)^* + \sum_{j \in \mathbb{S}_k^i, j \neq i} \lambda_{ij} (X_i^T E^T P_j X_i - X_i^T E^T - M_i) \\ &+ \sum_{j \in \bar{\mathbb{S}}_k^i, j \neq i} \lambda_{ij} (X_i^T E^T P_j X_i - X_i^T E^T - M_i) - \lambda_{ii} M_i < 0 \end{aligned} \tag{54}$$

which is guaranteed by

$$(A_iX_i + B_iY_i)^* + \sum_{j\in\mathbb{S}_k^i, j\neq i} \lambda_{ij}(X_i^TE^TP_jX_i - X_i^TE^T - M_i) - \lambda_{ii}M_i < 0 \tag{55}$$

and

$$X_i^TE^TP_jX_i - X_i^TE^T - M_i < 0\ ,\ i \in \mathbb{S},\ j \in \bar{\mathbb{S}}_k^i,\ j \neq i \tag{56}$$

Based on this, we know that (56) implies (51), and (55) is guaranteed by

$$(A_iX_i + B_iY_i)^* + \sum_{j\in\mathbb{S}_k^i, j\neq i} \lambda_{ij}(X_i^TE^TP_jX_i - X_i^TE^T) - \sum_{j\in\mathbb{S}_k^i} \lambda_{ij}M_i < 0,\ i \in \mathbb{S}_k^i \tag{57}$$

$$(A_iX_i + B_iY_i)^* + \sum_{j\in\mathbb{S}_k^i, j\neq i} \lambda_{ij}(X_i^TE^TP_jX_i - X_i^TE^T - M_i) - \tau M_i < 0,\ i \in \bar{\mathbb{S}}_k^i \tag{58}$$

Then, it is known that (57) and (58) imply (47) and (48) respectively. Similar to the above proof, (49), (50) and (52) are obtained, and the next process is omitted here. This completes the proof.

Theorem 3.6. *Given system (1) with Case 3, there exists a controller (4) based on observer (3) such that the resulting system is stochastically admissible if there exist matrices* $\hat{P}_i > 0$, $\hat{Q}_i$, $\bar{M}_i > 0$, $\bar{N}_i$, $M_i > 0$, $N_i > 0$, Y_i, $\bar{L}_i$, *scalars* $\bar{\delta}_1 > 0$ *and* $\bar{\delta}_2 > 0$ *satisfying*

$$\begin{bmatrix} \Phi_{i7} & \sqrt{2}B_iY_i & B_i & \sqrt{2}B_i & X_i^T & \Psi_{i1} \\ * & -(X_i)^* + I & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\bar{\delta}_2 I_m & 0 & 0 \\ * & * & * & * & -\bar{\delta}_2 I_n & 0 \\ * & * & * & * & * & \Psi_{i2} \end{bmatrix} < 0,\ i \in \mathbb{S}_k^i \tag{59}$$

$$\begin{bmatrix} \Phi_{i8} & \sqrt{2}B_iY_i & B_i & \sqrt{2}B_i & X_i^T & \Psi_{i1} \\ * & -(X_i)^* + I & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\bar{\delta}_2 I_m & 0 & 0 \\ * & * & * & * & -\bar{\delta}_2 I_n & 0 \\ * & * & * & * & * & \Psi_{i2} \end{bmatrix} < 0,\ i \in \bar{\mathbb{S}}_k^i \tag{60}$$

$$\begin{bmatrix} \Phi_{i11} & Q_i^T & C_i^T \\ * & -\bar{\delta}_1 I_n & 0 \\ * & * & -I \end{bmatrix} < 0,\ i \in \mathbb{S}_k^i \tag{61}$$

$$\begin{bmatrix} \Phi_{i12} & Q_i^T & C_i^T \\ * & -\bar{\delta}_1 I_n & 0 \\ * & * & -I \end{bmatrix} < 0,\ i \in \bar{\mathbb{S}}_k^i \tag{62}$$

$$\begin{bmatrix} -X_i^TE^T - M_i & X_i^TE_R \\ * & -E_R^T\hat{P}_jE_R \end{bmatrix} \leq 0,\ i \in \mathbb{S},\ j \in \bar{\mathbb{S}}_k^i,\ j \neq i \tag{63}$$

$$E^T\bar{M}_jE - E^T\bar{M}_iE - N_i \leq 0,\ i \in \mathbb{S},\ j \in \bar{\mathbb{S}}_k^i,\ j \neq i \tag{64}$$

where

$$\Phi_{i11} = (A_i^TQ_i + C_i^T\bar{L}_i)^* + I + \sum_{j\in\mathbb{S}_k^i, j\neq i} \lambda_{ij}(E^T\bar{M}_jE - E^TQ_i) - \sum_{j\in\mathbb{S}_k^i} \lambda_{ij}N_i$$

$$\Phi_{i12} = (A_i^TQ_i + C_i^T\bar{L}_i)^* + I + \sum_{j\in\mathbb{S}_k^i, j\neq i} \lambda_{ij}(E^T\bar{M}_jE - E^TQ_i) - \sum_{j\in\mathbb{S}_k^i, j\neq i} \lambda_{ij}N_i - \tau N_i$$

Then the corresponding parameters of (3) and (4) are computed by (13) and (14) respectively.

Proof: Similar to the proofs of Theorems 3.2 and 3.5, one gets Theorem 3.6 directly. The proof is omitted here.

4. Numerical Example.

Example 4.1. *Considering an SMJS with three modes, the detailed matrices are given as follows:*

$$A_1 = \begin{bmatrix} -2.5 & -1+d_{12} \\ 1 & -1.5 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.3 \\ -0.1 \end{bmatrix},\ C_1 = \begin{bmatrix} 1 & 0.5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1.7 & 0.6 \\ -1 & -1.5 \end{bmatrix},\ B_2 = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix},\ C_2 = \begin{bmatrix} -0.3 & -1.2 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1 & 0.5 \\ 1 & -1.5 \end{bmatrix},\ B_3 = \begin{bmatrix} -0.4 \\ 0.1 \end{bmatrix},\ C_3 = \begin{bmatrix} 0.1 & -0.2 \end{bmatrix}$$

where d_{12} is time-varying, and d_{12}^* is the maximum value of d_{12}. Singular matrix E is given as

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

First, the TRM is assumed to be known exactly and is given by

$$\Pi = \begin{bmatrix} -1 & 0.6 & 0.4 \\ 0.5 & -2.2 & 1.7 \\ 0.9 & 0.8 & -1.7 \end{bmatrix}$$

For this ideal case, based on [20], it is known that there is no solution to a controller (4) based on observer (3) no matter what value d_{12} takes. By Theorem 3.1, the corresponding parameters are obtained as

$$K_1 = \begin{bmatrix} -6.7481 & -7.8615 \end{bmatrix},\ K_2 = \begin{bmatrix} 2.7764 & 0.5298 \end{bmatrix},\ K_3 = \begin{bmatrix} 4.6785 & 0.4934 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -7.2306 \\ 52.5806 \end{bmatrix},\ L_2 = \begin{bmatrix} 0.1855 \\ 2.3726 \end{bmatrix},\ L_3 = \begin{bmatrix} 0.1569 \\ 0.3056 \end{bmatrix}$$

with $d_{12}^* = 5.456$. Letting the initial condition $x_0 = \begin{bmatrix} 0.5 & -1 & 0.2 & 0.5 \end{bmatrix}^T$, the state response of the closed-loop system is shown in Figure 1, where the simulations of estimation $\hat{x}(t)$, system mode η_t and error $e(t)$ are illustrated in Figure 2, Figure 3 and Figure 4. From such simulations, it is seen that the resulting system is stable, which also demonstrates the proposed results in this paper having less conservatism.

Similarly, by Theorem 3.2, one has the corresponding parameters

$$K_1 = \begin{bmatrix} -16.9921 & -9.9870 \end{bmatrix},\ K_2 = \begin{bmatrix} 22.6551 & 3.3636 \end{bmatrix},\ K_3 = \begin{bmatrix} 2.8302 & 0.3533 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0.8828 \\ 9.6772 \end{bmatrix},\quad L_2 = \begin{bmatrix} -0.5862 \\ -31.4084 \end{bmatrix},\quad L_3 = \begin{bmatrix} 9.4539 \\ 71.5967 \end{bmatrix}$$

with $d_{12}^* = 5.483$.

When Π is assumed to satisfy Case 2, the uncertainties are given as

$$\Delta\bar{\lambda}_{ij} \le \epsilon_{ij} \triangleq 0.1\bar{\lambda}_{ij}, \forall j \neq i \in \mathbb{S}, \bar{\Pi} \triangleq (\bar{\lambda}_{ij}) = \begin{bmatrix} -1 & 0.6 & 0.4 \\ 0.5 & -2.2 & 1.7 \\ 0.9 & 0.8 & -1.7 \end{bmatrix}$$

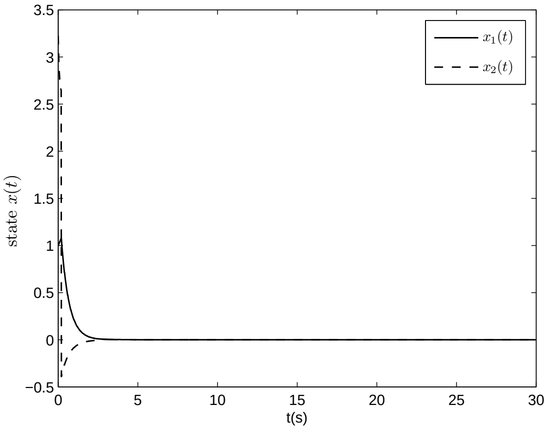

FIGURE 1. The response of state $x(t)$

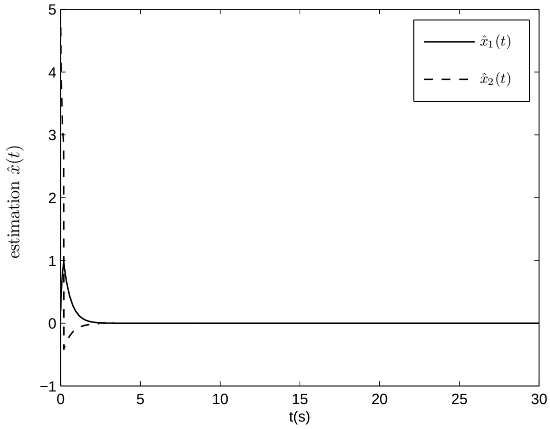

FIGURE 2. The response of estimation $\hat{x}(t)$

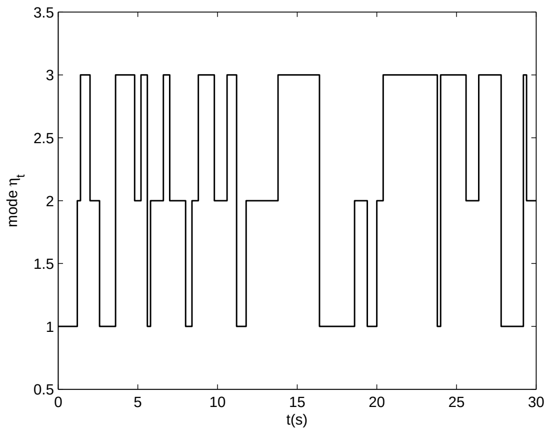

FIGURE 3. The simulation of mode η_t

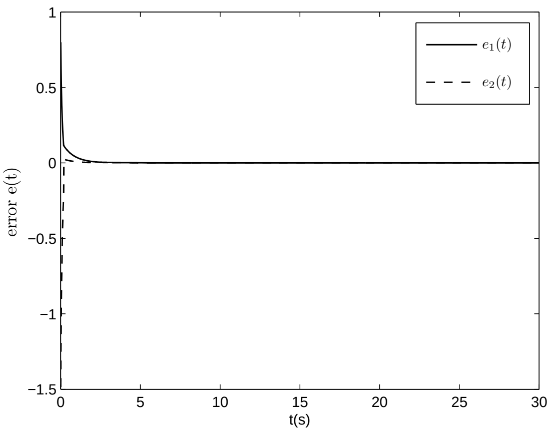

FIGURE 4. The response of error $e(t)$

By Theorem 3.3, the corresponding parameters of the observer-based controller are given as

$$K_1 = \begin{bmatrix} -7.5334 & -7.0483 \end{bmatrix},\ K_2 = \begin{bmatrix} 3.5214 & 0.6709 \end{bmatrix},\ K_3 = \begin{bmatrix} 5.0032 & 0.5107 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0.2457 \\ 20.1704 \end{bmatrix},\ L_2 = \begin{bmatrix} -0.8916 \\ 23.6634 \end{bmatrix},\ L_3 = \begin{bmatrix} 11.9037 \\ 33.5873 \end{bmatrix}$$

with $d_{12}^* = 5.420$. *By Theorem 3.4, it is known that*

$$K_1 = \begin{bmatrix} -16.8861 & -9.9885 \end{bmatrix},\ K_2 = \begin{bmatrix} 22.8399 & 3.4658 \end{bmatrix},\ K_3 = \begin{bmatrix} 2.8275 & 0.3521 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0.1442 \\ 19.6918 \end{bmatrix},\ L_2 = \begin{bmatrix} -0.7771 \\ 23.4880 \end{bmatrix},\ L_3 = \begin{bmatrix} 12.0257 \\ 34.0764 \end{bmatrix}$$

with $d_{12}^* = 5.470$.

When TRM is partially unknown, it is given as

$$\Pi = \begin{bmatrix} -1 & ? & ? \\ 0.5 & ? & ? \\ 0.9 & 0.8 & -1.7 \end{bmatrix}$$

with $\tau = -2.2$. *By Theorem 3.5, we have*

$$K_1 = \begin{bmatrix} -7.0736 & -6.8690 \end{bmatrix}, \ K_2 = \begin{bmatrix} 3.2499 & 0.6156 \end{bmatrix}, \ K_3 = \begin{bmatrix} 5.8479 & 0.5611 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 1.9734 \\ -140.6200 \end{bmatrix}, \ L_2 = \begin{bmatrix} -1.0503 \\ 10.9990 \end{bmatrix}, \ L_3 = \begin{bmatrix} 2.4279 \\ 3.8006 \end{bmatrix}$$

where $d_{12}^* = 5.245$. *By Theorem 3.6, it is obtained that*

$$K_1 = \begin{bmatrix} -16.2601 & -9.9977 \end{bmatrix}, \ K_2 = \begin{bmatrix} 19.4350 & 2.2398 \end{bmatrix}, \ K_3 = \begin{bmatrix} 3.0039 & 0.3452 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -0.2628 \\ 337.5767 \end{bmatrix}, \ L_2 = \begin{bmatrix} -1.5766 \\ 15.0741 \end{bmatrix}, \ L_3 = \begin{bmatrix} 8.7106 \\ 40.2816 \end{bmatrix}$$

with $d_{12}^* = 5.393$. *In addition, when matrix* A_1 *is assumed to be*

$$A_1 = \begin{bmatrix} -2.5 & -1 \\ 1 + d_{21} & -1.5 \end{bmatrix}$$

we could get the corresponding parameters with some index d_{21}^* *by Theorems 3.1-3.6. The detailed comparison results are listed in the following tables.*

TABLE 1. Result comparison on the first method

	Theorem 3.1	*Theorem 3.3*	*Theorem 3.5*
d_{12}^*	*5.456*	*5.420*	*5.245*
d_{21}^*	*−7.817*	*−7.684*	*−6.999*

TABLE 2. Result comparison on the second method

	Theorem 3.2	*Theorem 3.4*	*Theorem 3.6*
d_{12}^*	*5.483*	*5.470*	*5.393*
d_{21}^*	*−7.829*	*−7.671*	*−6.999*

From the comparison results in such tables, one cannot conclude which method is less conservative. It means that both methods are effective to deal with such problems respectively.

Example 4.2. *Consider the DC motor with a random load generated from* [7] *and given in Figure 5. The switching is driven by a continuous-time Markov process* $\{\eta_t, t > 0\}$ *taking values in a finite set* $N = \{1, 2\}$. *That is, for any time* $t \geq 0$, *if the switching is on 1, we have* $\eta_t = 1$, *otherwise one has* $\eta_t = 2$. *When the DC motor inductance* L_m *is neglected, let* $i(t), \varpi(t)$ *represent electric current, and the speed of the shaft at time t and* $u(t)$ *denote the voltage, respectively, based on the basic electrical and mechanic laws:*

$$\begin{cases} \dot{\varpi}(t) = -\dfrac{b_i}{J_i}\varpi(t) + \dfrac{K_t}{J_i}i(t) \\ u(t) = K_{\varpi}\varpi(t) + Ri(t) \end{cases} \tag{65}$$

where R is the resistor and K_ϖ, K_t respectively denote the electromotive force constant and the torque constant, thereinto, J_i and b_i are defined by:

$$\begin{cases} J_i = J_m + \dfrac{J_{ci}}{n^2} \\ b_i = b_m + \dfrac{b_{ci}}{n^2} \end{cases} \tag{66}$$

where J_m and J_{ci} represent the moments of the motor and the load. b_m and b_{ci} are the damping ratios with gear ratio n. Now we let $x_1(t) = \varpi(t)$, $x_2(t) = i(t)$, and one has

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \dot{x}(t) = \begin{bmatrix} -\frac{b_i}{J_i} & \frac{K_t}{J_i} \\ K_\varpi & R \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{67}$$

where the referred parameters are selected as $J_m = 0.3kg \cdot m$, $J_{c1} = 100kg \cdot m$, $J_{c2} = 50kg \cdot m$, $b_{c1} = 200$, $b_{c2} = 150$, $R = 0.8\Omega$, $b_m = 1$, $K_t = 2Nm/A$, $K_\varpi = 1Vs/rad$ and $n = 15$. Without loss of generality, the TRM is assumed to be

$$\Pi = \begin{bmatrix} -0.53 & 0.53 \\ 0.31 & -0.31 \end{bmatrix}$$

The related matrix C_{η_t} of the measurement output is given as $C_1 = \begin{bmatrix} 1.2 & 0.5 \end{bmatrix}$ and $C_2 = \begin{bmatrix} -0.6 & -1.0 \end{bmatrix}$. By Theorem 3.1, we can design an observer-based controller whose parameters are given by

$$K_1 = \begin{bmatrix} 0.0772 & -0.3206 \end{bmatrix}, \; K_2 = \begin{bmatrix} 0.0246 & -0.4720 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0.5907 \\ -0.2892 \end{bmatrix}, \; L_2 = \begin{bmatrix} -1.1084 \\ -20.2085 \end{bmatrix}$$

Letting the initial condition $x_0 = \begin{bmatrix} 2.5 & -3 \end{bmatrix}^T$, the simulations of state response, estimated state and estimated error of the resulting system are illustrated in Figure 6. Based on such simulations, it shows that the desired observer-based controller is effective.

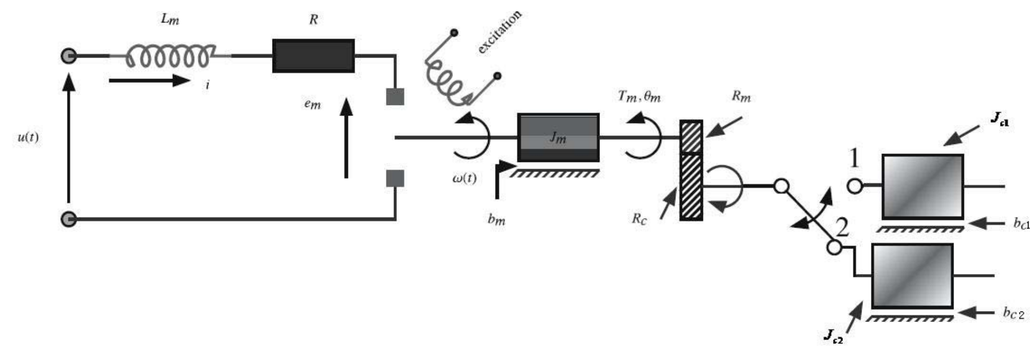

FIGURE 5. The block diagram of a DC motor

5. **Conclusions.** In this paper, the stabilization problem of SMJSs via a general observer-based controller has been studied, where the observer-based controller could bear perturbations. Sufficient conditions for the existence of such a controller are formulated in terms of LMIs, where two different techniques for dealing with some cross terms in addition to new inequalities are introduced to get the LMI conditions ultimately. Moreover, more general cases such as TRM being uncertain and partially unknown are considered, whose

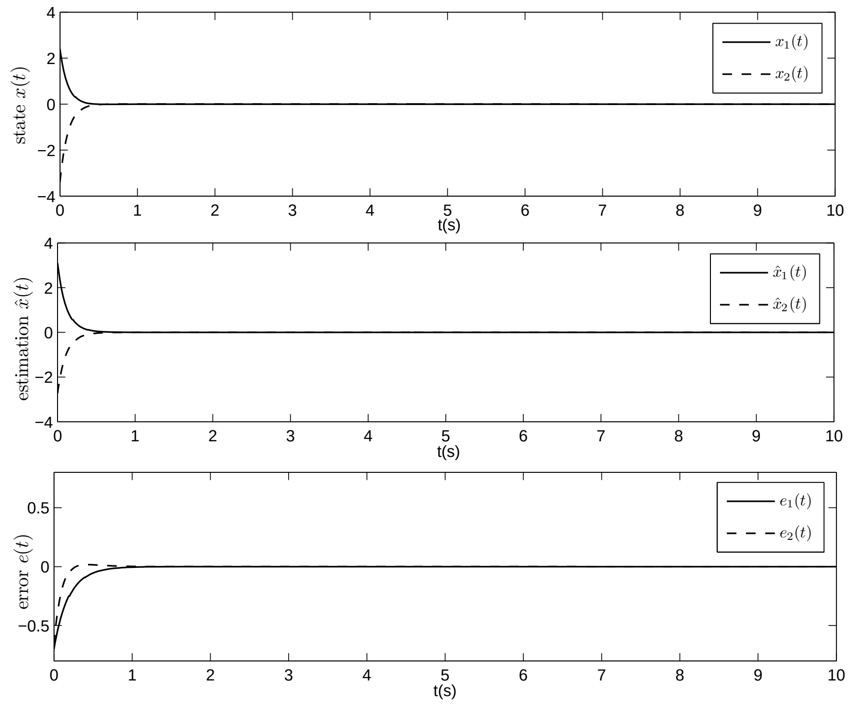

FIGURE 6. The simulations of the resulting system

conditions are also given by the LMI approach. Finally, two numerical examples are used to illustrate the effectiveness of the results in this paper.

Acknowledgment. The authors would like to thank the National Natural Science Foundation of China under Grants 61104066 and 61374043, the China Postdoctoral Science Foundation funded project under Grant 2012M521086, the Program for Liaoning Excellent Talents in University under Grant LJQ2013040.

REFERENCES

[1] E. K. Boukas and Z. K. Liu, *Deterministic and Stochastic Time-Delay Systems*, Birkhauser, Boston, 2002.
[2] M. Mariton, Control of nonlinear systems with Markovian parameter changes, *IEEE Transactions on Automatic Control*, vol.36, no.2, pp.233-238, 1991.
[3] Y. Y. Wang, P. Shi, Q. B. Wang and D. P. Duan, Exponential H_∞ filtering for singular Markovian jump systems with mixed mode-dependent time-varying delay, *IEEE Transactions on Circuits and Systems – I*, vol.60, no.9, pp.2440-2452, 2013.
[4] J. N. Li, H. Y. Su, Y. B. Zhang, Z. G. Wu and J. Chu, Chattering free sliding mode control for uncertain discrete time-delay singular systems, *Asian Journal of Control*, vol.15, no.1, pp.260-269, 2013.
[5] P. C. Muller, *Linear Mechanical Descriptor Systems: Identification, Analysis and Design*, Pitman, London, 1980.
[6] S. Y. Xu and J. Lam, *Control and Filtering of Singular Systems*, Springer-Verlag, Berlin, 2006.
[7] E. K. Boukas, *Control of Singular Systems with Random Abrupt Changes*, Springer-Verlag, Berlin, 2008.
[8] L. G. Wu, P. Shi and H. J. Gao, State estimation and sliding-mode control of Markovian jump singular systems, *IEEE Transactions on Automatic Control*, vol.55, no.5, pp.1213-1219, 2010.
[9] G. L. Wang, Q. L. Zhang and C. Y. Yang, Dissipative control for singular Markovian jump systems with time delay, *Optimal Control Applications and Methods*, vol.33, no.4, pp.415-432, 2012.
[10] G. L. Wang and Q. L. Zhang, Robust control of uncertain singular stochastic systems with Markovian switching via proportional-derivative state feedback, *IET Control Theory and Applications*, vol.6, no.8, pp.1089-1096, 2012.

[11] L. G. Wu, X. J. Su and P. Shi, Sliding mode control with bounded L_2 gain performance of Markovian jump singular time-delay systems, *Automatica*, vol.48, no.8, pp.1929-1933, 2012.
[12] Z. G. Wu, P. Shi, H. Y. Su and J. Chu, Delay-dependent stability analysis for discrete-time singular Markovian jump systems with time-varying delay, *International Journal of Systems Science*, vol.43, no.11, pp.2095-2106, 2012.
[13] A. Gelb, *Applied Optimal Estimation*, MIT Press, Cambridge, 1994.
[14] G. P. Lu and D. W. C. Ho, Robust H_∞ observer for nonlinear discrete systems with time delay and parameter uncertainties, *IEE Proceedings, Control Theory and Applications*, vol.151, no.4, pp.439-444, 2004.
[15] J. O. Reilly, *Observers for Linear Systems*, Academic, New York, 1983.
[16] Z. Wang, D. P. Goodall and K. J. Burnham, On designing observers for time-delay systems with nonlinear disturbances, *International Journal of Control*, vol.75, no.11, pp.803-811, 2002.
[17] B. Shafai and R. L. Carroll, Design of a minimal-order observer for singular systems, *International Journal of Control*, vol.45, no.3, pp.1075-1081, 1987.
[18] D. W. C. Ho and Z. W. Gao, Bezout identity related to reduced-order observer-based controllers for singular systems, *Automatica*, vol.37, no.10, pp.1655-1662, 2001.
[19] Y. Uetake, Adaptive observer for continuous descriptor systems, *IEEE Transactions on Automatic Control*, vol.39, no.10, pp.2095-2100, 1994.
[20] J. Raouf and E. K. Boukas, Observer-based controller design for linear singular systems with markovian switching, *Proc. of the 43rd IEEE Conference on Decision and Control*, Atlantis, Paradise Island, Bahamas, pp.3619-3624, 2004.
[21] X. M. Zhang, G. P. Lu and Y. F. Zheng, Observer design for descriptor Markovian jumping systems with nonlinear perturbations, *Circuits, Systems and Signal Processing*, vol.27, no.1, pp.95-112, 2008.
[22] M. S. Mahmound and P. Shi, Robust stability, stabilization and H_∞ control of time-delay systems with Markovian jump parameters, *International Journal of Robust and Nonlinear Control*, vol.13, no.8, pp.755-784, 2003.
[23] J. L. Xiong and J. Lam, Fixed-order robust H_∞ filter design for Markovian jump systems with uncertain switching probabilities, *IEEE Transactions on Signal Processing*, vol.54, no.4, pp.1421-1430, 2006.
[24] L. X. Zhang and J. Lam, Necessary and sufficient conditions for analysis and synthesis of Markov jump linear systems with incomplete transition descriptions, *IEEE Transactions on Automatic Control*, vol.55, no.7, pp.1695-1701, 2010.
[25] E. Uezato and M. Ikeda, Strict LMI conditions for stability, robust stabilization and H_∞ control of descriptor systems, *Proc. of the 38th Conference on Decision and Control*, Phoenix, Arizona USA, pp.4092-4097, 1999.