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DEVELOPING SUPPLY CHAINS INVENTORY MODELS THAT ACCOUNT FOR AN IMPERFECT ENVIRONMENT: WITH APPLICATION TO THE PCB MANUFACTURING INDUSTRY

Ping-Shun Chen[1] and Chien-Ming Lo[2]

[1]Department of Industrial and Systems Engineering
Chung Yuan Christian University
No. 200, Chung Pei Road, Chung Li District, Taoyuan City 32023, Taiwan
pingshun@cycu.edu.tw

[2]Department of Industrial Design
National Yunlin University of Science and Technology
No. 123, University Road, Section 3, Douliou, Yunlin 64002, Taiwan
jamin1215@gmail.com

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Abstract. *The impact of short product life cycle within the 3C industry means every few months price reductions are demanded for printed circuit board (PCB) products. As a result, a PCB inventory model must allow buyers and sellers to consider the impact of product quality, uncertainties in lead time, crashing cost and time value, in order to calculate the most effective inventory model. Previous work using this type of approach has not been applied to the PCB industry where three distinct manufacturing phases can be considered as three stages in a supply chain, thus providing the challenge of the calculation of an optimal joint supply chain inventory model. In this paper, to address the problem within the PCB industry of integration of a multi-step inventory, we propose a mathematical programming model to determine the best pairing mode for the overall supply chain. In the mathematical model, the paper industry must consider the quality of the PCB, delivery (lead time), the time value (present value) and feature the best integration costs. This study uses a model proposed by Jou et al. to attempt to find the best match between buyers and sellers, that is to reach a joint inventory cost minimization. The results of this study may provide a way for the PCB industry to facilitate evaluation and use of the overall management of product quality, delivery and cost in order to achieve the lowest cost optimal batch inventory management objectives.*

Keywords: Supply chain, Inventory management, Lead time, Present value, Crashing cost

1. **Introduction.** In recent times the rise in material prosperity has meant that much attention is now paid to the issue of quality of life. As a result, consumer electronic products, also referred to as “3C” are flourishing. “3C” is a collective way of referring to three types of products which in English begin with the letter C, namely, Computers, Communication devices and Consumer electronics. Examples of 3C products include computers, tablet computers, mobile phones, digital cameras, televisions, music players, electronic dictionaries, audio and video playback devices and digital audio players.

In order to meet the growing consumer demand, the 3C industry continues to develop new technology and product innovation, which results in fierce market competition and increasingly short product life cycle. Today’s mobile phones, for example, have a product life of around three months before being replaced by an updated model. For computers the product life is around six months [1]. As a result of this, the management of inventory within the 3C industry is extremely important. If not properly managed, the excess storing of out of date products will eat into corporate profits and lead to lack of competitiveness. Within the 3C industry the supply chain is very long, for example, the supply chain to a host computer contains a variety of electronic components, PCB, chassis frame, etc. So in order to facilitate each supplier in the chain achieving both profitability and long-term sustainability, this paper considers not only the efficient management of individual stock inventories, but also that those buyers and sellers should consider an integrated inventory model to minimize product storage.

Printed circuit board (PCB) production is a very important process, as the PCB is the key component of electronic products connecting together all the other electronic components. PCBs are present in all 3C products. Because 3C products have a short product cycle with new products quickly replacing old, the PCB industry needs to shorten the lead time to 3C products in response to rapidly changing markets. Therefore, the PCB industry attaches great importance to the order-oriented, real-time production (JIT: just in time) concept [2]. JIT was developed as a result of the desire to both "eliminate waste and emphasize simplicity so as to develop a flexible environment of continuous quality improvement". This definition sentence contains the terms "elimination of waste" and "continuous quality improvement". The key is to ensure that the entire staff take seriously the value of time, and the prevention of waste, and consider stockpiling of inventory as a 'cardinal sin', having a zero inventory as the key goal. So the importance of inventory monitoring is emphasized in this approach.

In the implementation of JIT in the PCB industry, the use of employee overtime is a strategy known as "crashing cost" [3] used for increasing production and reduction of lead time. This reduces the amount of reserve stock and prevents stored stock from the risk of becoming outdated and so eating into profits. In previous studies of traditional inventory models, delivery time is often assumed as a constant [4] or with insignificantly random variables [5,6] which are ruled out. In reality the PCB industry must consider changes to delivery time, as shorter delivery times [7-10] will increase the PCB industry 3C products ability face competition. This is also an important indicator of sustainable development.

In addition due to price competition resulting in rapid decrease of 3C product prices and leading to sharp reductions of product profitability the PCB industry must take account of a time influenced value of the inventory including the effect of inflation upon the current value [3,11-14]. In previous research concerning inventory models there are a lot of areas of potential further study.

In terms of quality, in previous literature the supply chain inventory model is often assumed to be functioning in a "perfect environment" [15-17]. The so-called "perfect environment" does not consider the issue of unreliable output within the production process leading to adverse delivery times. This is one of the factors leading to use of crashing time and crashing cost. In reality PCB industry inventory system must take into account that the environment is not perfect, that customer awareness of quality is rising and international quality control standards such as ISO 9001. Business attaches great importance to the impact of faulty products on lead delivery time causing increase in customer complaints and influencing the company's image. Since the application of statistical analysis to the PCB industrial technology continues to improve, the manufacturing process can be analyzed to find a fixed probability of product quality failure and on the production line can instantly carry out statistical product control (SPC). Therefore, in this work we assume a constant failure probability θ [10,18].

Within the PCB manufacturing process one of the main materials used in the circuit board manufacturing process is copper clad laminate (CCL) which is copper foil adhered to a polyimide film (the substrate) also known as copper foil substrate. This needs to be cut into sheets or working panels and then a complex etching process is used to produce the circuit. Multi layer circuit boards have inner and outer etched circuits with a film platen formed between the inner and outer circuits. Depending on the needs of the customer product specifications, the PCB manufacturing process can be divided into (1) the inner line/circuit production, (2) the outer line/circuit production, and (3) the post-production process.

These three stages run similar to a normal supply chain, and the middle and lower stages will have their own control of the inventory system and be integrated through a variety of information systems, such as ERP (Enterprise Resource Planning) systems, through the middle and lower reaches of the supply chain inventory model integration to achieve joint inventory cost optimization. From previous studies of multi-class model for the variation of inventory there is much room for further research [19,20]. The multi-class model is a variation of the inventory model including consideration of an imperfect environment [21], joint inventory cost and the failure rate of buyers and sellers to minimize these problems. There has not been an inventory model produced that is consistent with the three level characteristics of the PCB industry inventory model; that is, at the same time needing to meet the needs of cost considerations caused by shorten delivery time, failure in quality production and also consideration of the devaluation of the product due to inflation, market price competition and product short lifecycles.

Taking account of the facts mentioned above, this paper addresses the subject of supply chain inventory management in the PCB industry and proposes a recursive method in order to achieve the minimization of overall supply chain inventory costs. In this paper the proposed recursive method can be divided into two phases.

The first phase (phase I) utilizes the work of Jou et al. [11] who have proposed a simple single buyer-single seller inventory model. Further buyer seller transactions are analyzed separately following the model of the first transaction. Individual transactions are considered separately in estimating costs and potential supply situations.

The second phase (phase II) considers the results obtained from the first phase and the problems with inventory and cost estimates that are raised by the addition of multiple sellers and buyers to the model.

This paper presents a mathematical programming model to minimize the overall cost of the decision reached in the supply chain planning. The rest of the paper is organized as follows. In Section 2, the recursive model proposed in this paper for solving the problem of optimal cost and inventory with multiple buyers and sellers is described in detail. In Section 3, data derived from the PCB industry is used to verify the effectiveness of the model proposed in this paper. In Section 4, the results are analyzed and discussed and recommendations are made for future work.

2. Methodology.

Consideration is given to a three stage PCB supply chain-production process in which stage 1 is the inner line/circuit production, stage 2 the outer line/circuit production and stage 3 end user (buyer). It is assumed that in each of the various stages there are three sellers/buyers and at each stage the suppliers have the process capability to meet the requirements of the buyer (see Figure 1). In fact, in the PCB industry, as a result of much competitiveness in the production process and in order to provide consumers with better services, generally a one-stop shopping business model is utilized, that is, the industry moves toward vertical integration. In this case inner line/circuit production and outer line/circuit production systems are provided by the same company, so consumers only need to place orders with the outer line/circuit producers who manage the earlier stages of the production contract. To ensure that consumers receive a product that meets their requirements, for convenience in subsequent tracking of quality problems, production control will be limited to the use of a single PCB industry plant supply industrial model, that is, although there are probably at various stages multiple plants able to meet consumer demand customers still will each designate a single plant in each stage to meet production demand rather than multiple plants supplying only a portion of the demand.

For example, in the case of consumer node 31, although it has the process capability to receive the products of suppliers node 21, node 22 and node 23, consumer 31 will pick only one of the three sellers to supply its needs. The same is true for consumers 32 and 33. If a supplier is responsible for the supply of product to consumer node 33, then it can no longer supply other consumers. The final result shows as a match between supply node 21 and consumer node 33. Similarly node 22 supplies node 31 and node 23 supplies node 32.

Identical restrictions to those applied to the second stage are applied to the first stage. Figure 1 shows us that each consumer has a possibility of nine different supply routes through the two stages. In addition, when estimating the inventory cost, consideration must be given to the quantities of product demanded by each customer as production costs of the seller may vary according to order size between seller and buyer.

Considering the supply chain relationships in Figure 1 and viewing the relationship between stage 1 and stage 2 from a simplistic point of view, stage 1 can be seen as the supplier to the needs of stage 2. Similarly looking at stage 2 from the point of view of stage 3 it is seen as the supplier to stage 3's needs. In fact when a customer places an order to the PCB industry, it will be a demand for an annual production. The demand is placed at stage 3 and so it is at this stage that the annual production demands are determined. It is not possible to consider the matching of stages 1 and 2 before the demands of stage 3 have been used to match the relationships between stages 2 and 3 as until this match has been made the demands of stage 2 will be unknown and so optimal cost matching with stage 1 sellers may not be made.

Based on the considerations above, in order for the entire PCB supply chain to operate under optimal environment (i.e., to achieve overall inventory cost minimization), this paper considers supply chain inventory management within the PCB industry environment and proposes a recursive solving method. The proposed recursive method can be divided into two phases.

Phase I uses the simple single buyer-single seller inventory model of Jou et al. [11]. In this model buyer seller transactions are analyzed as separate transaction which are used together in estimating costs and potential supply situations.

In Phase II with regard to the supply chain environment, in Figure 1 we first estimate the cost between stage 2 and stage 3 for all possible routes (arc) of the stock. This second phase then considers the results obtained from the first phase and the problems of inventory and cost estimates caused by the addition of multiple sellers and buyers to the model. This paper presents a mathematical programming model which can be used as an aid in considering overall cost minimization supply decisions. In Figure 1 between stage 2 and stage 3 for example, the model will determine the configuration of blue arcs representing the optimal stock route. Having achieved this consideration is made of phase I results to determine the cost of the inventory of all arcs between stage 1 and stage 2. Phase II uses a mathematical programming model to determine the optimal configuration. The process is continued until all the possible routes have been considered. The overall calculation process is shown in Figure 2.

2.1. **Notations.** The following notations are used throughout One Seller – One Buyer Inventory Management Model in Two Stages Environment.

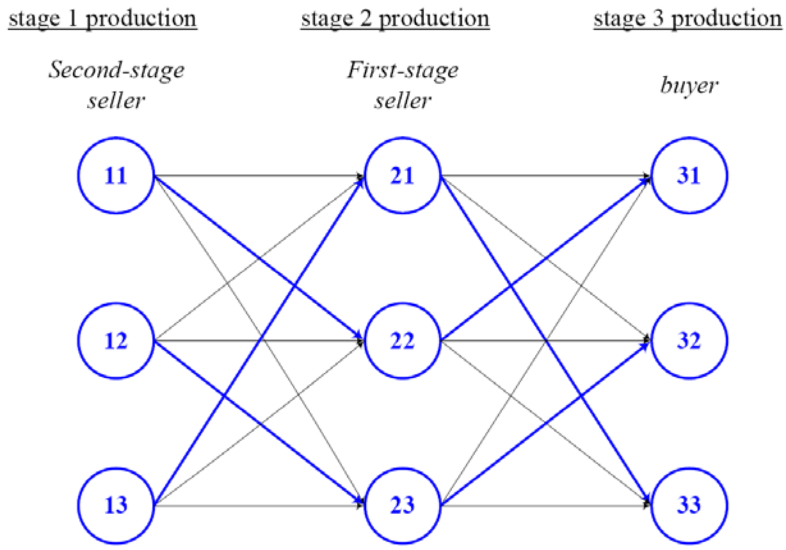

FIGURE 1. An example of the supply chain production flow in PCB industries

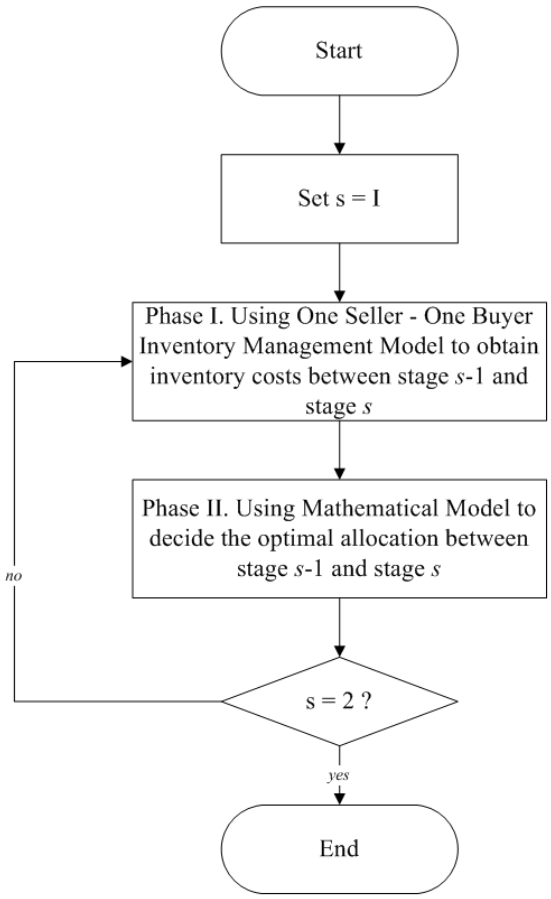

FIGURE 2. The flowchart of the proposed recursive solving method

Notations:

$D_{s,j}$ = The demand of purchaser (j) per year in the stage s of supply chain.
$P_{s,j}$ = The production of vendor (j) per year in the stage s of supply chain.
$Q_{s,j,s+1,k}$ = Order quantity of the purchaser (k) in stage $s+1$ buy to vendor (j) in stage s of supply chain.

$LT_{s,j,s+1,k}$ = Length of lead time from the vendor (j) in stage s to the purchaser (k) in stage $s+1$ of supply chain.

$CV_{s,j}$ = Unit production cost paid by the vendor (j) in stage s of supply chain.

$NL_{s,j,s+1,k}$ = An integer representing the number of lots in which the items are delivered from the vendor (j) in stage s to the purchaser (k) in stage $s+1$ of supply chain; m is positive integer value.

HC = Annual inventory holding cost per dollar invested in stocks.

i = Interest rate per year that is compounded continuously.

θ = The out-of-control probability.

DC = The cost of replacing a defective unit.

2.2. **Mathematical model.** In order to comply with the characteristics of the PCB industry, we use Jou et al. [11] integrated inventory vender-buyer controllable lead time model. In addition we consider the short product life of 3C products and the effect on product value. And so by necessity also refer to Yang et al. [10] who propose a time value concept for amending the purchaser's inventory model with variable lead time. In addition it can often be observed that there are defective items being produced through unreliable production process. Here we derive a model in the same way as proposed in Yang and Pan [2] and Yang et al. [10]. In this model we assumed the lead time crashing cost which is determined by the length of lead time is polynomial.

Based on the above notations and assumptions the total expected cost for the purchaser is given by:

$TRCP_{s,j}$ = Ordering cost + holding cost + crashing cost

$OC_{s,j}$ = Ordering cost per order of purchaser j in stage s of supply chain

$SC_{s,j}$ = Set-up cost per order of seller j in stage s of supply chain

$CP_{s,j}$ = Unit purchase cost paid by the purchaser (j) in stage s of supply chain

$$\begin{aligned} & TRCP_{s,j}\left(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k}\right) \\ &= NL_{s,j,s+1,k}OC_{s,j} + NL_{s,j,s+1,k}CLT_{s,j,s+1,k}^{-a} \\ &\quad + \frac{NL_{s,j,s+1,k}HCCP_{s,j}}{i}\left[\left(Q_{s,j,s+1,k}\right.\right. \\ &\quad \left. + k\sigma\sqrt{LT_{s,j,s+1,k}}\right)\left(1 - e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right) \\ &\quad \left. + Q_{s,j,s+1,k}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}} + \frac{D_{s,j}}{i}\left(e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}} - 1\right)\right] \end{aligned} \tag{1}$$

The vender's total expected cost can be represented by Yang and Pan [2]:

TRC_V = set-up cost + holding cost + defective cost

$$\begin{aligned} & TRCV_{s,j}\left(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}\right) \\ &= SC_{s,j} + \frac{HCC_V}{i}\left(1 - e^{-\frac{NL_{s,j,s+1,k}Q_{s,j,s+1,k}}{D_{s,j}}}\right)\frac{Q_{s,j,s+1,k}}{2}\left[NL_{s,j,s+1,k}\left(1 - \frac{P_{s,j}}{D_{s,j}}\right)\right. \\ &\quad \left. - 1 + \frac{2P_{s,j}}{D_{s,j}}\right] + \frac{DCNL_{s,j,s+1,k}^2 Q_{s,j,s+1,k}^2\theta}{2} \end{aligned} \tag{2}$$

Therefore, the joint expected total relevant cost for the first cycle $JTRC(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k})$ is presented as follows.

$$\begin{aligned} & JTRC\left(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k}\right) \\ &= TRCP_{s,j}\left(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k}\right) + TRC_V\left(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}\right) \end{aligned}$$

We adopt the time value concepts approach of Yang et al. [10]. Therefore, the present value of the joint expected total relevant cost over infinite time horizon, $PVC(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k})$, is given by

$$
\begin{aligned}
&PVC\left(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k}\right) \\
&= \frac{1}{1-e^{-\frac{NL_{s,j,s+1,k}Q_{i,j,k}}{D_{s,j}}}}\left[JTRC\left(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k}\right)\right] \\
&= \frac{1}{1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}}\Bigg\{NL_{s,j,s+1,k}OC_{s,j} + NL_{s,j,s+1,k}CLT_{s,j,s+1,k}^{-a} + SC_{s,j} \\
&+ \frac{DCNL_{s,j,s+1,k}^{2}Q_{s,j,s+1,k}^{2}\theta}{2} + \frac{NL_{s,j,s+1,k}HCCP_{s,j}}{i}\Bigg[\Bigg(Q_{s,j,s+1,k} \\
&+k\sigma\sqrt{L_{i,j,s+1}}\Bigg)\left(1-e^{-\frac{NL_{s,j,s+1,k}Q_{s,j,s+1,k}}{D_{s,j}}}\right) + Q_{s,j,s+1,k}\exp\left(-\frac{Q_{s,j,s+1,k}}{D_{s,j}}\right) \\
&+ \frac{D_{s,j}}{i}\left(e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}-1\right)\Bigg] + \frac{HCCV_{s,j}}{i}\Bigg(1 \\
&- e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}\Bigg)\frac{Q_{s,j,s+1,k}}{2}\left[NL_{s,j,s+1,k}\left(1-\frac{D_{s,j}}{P_{s,j}}\right)-1+\frac{2P_{s,j}}{D_{s,j}}\right]\Bigg\}
\end{aligned} \tag{3}
$$

2.3. **Solution procedure.** It is necessary to find the minimum value of the present value of the expected total relevant cost $PVC(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k})$. For fixed $Q_{s,j,s+1,k}$ and $NL_{s,j,s+1,k}$, $PVC(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k})$ is a convex function in $LT_{s,j,s+1,k}$. Thus, there exists a unique value of L which minimizes $PVC(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k})$ can be obtained by solving the equation $\partial PVC(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}, LT_{s,j,s+1,k})/\partial LT_{s,j,s+1,k} = 0$ and is given by

$$
LT_{s,j,s+1,k} = \left(\frac{2iac}{HCCP_{s,j}k\sigma\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)}\right)^{\frac{1}{a+\frac{1}{2}}} \tag{4}
$$

Let $\left(\frac{1}{HCCP_{s,j}}\frac{2iac}{k\sigma}\right)^{\frac{1}{a+\frac{1}{2}}} = B$. Substituting (4) into (3), we obtain

$$
\begin{aligned}
&PVC\left(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}\right) \\
&= \frac{1}{1-e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}}\Bigg\{\Bigg(NL_{s,j,s+1,k}OC_{s,j} \\
&+ NL_{s,j,s+1,k}CB^{-a}\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{2a}{2a+1}} \\
&+ SC_{s,j}\frac{NL_{s,j,s+1,k}HCCP_{s,j}}{i}\Bigg[\Bigg(Q_{s,j,s+1,k} \\
&+ k\sigma B^{\frac{1}{2}}\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{-1}{2a+1}}\Bigg)\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right) \\
&+ Q_{s,j,s+1,k}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}} + \frac{D_{s,j}}{i}\left(e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}-1\right)\Bigg]
\end{aligned}
$$

$$+\frac{\frac{HCCV_{s,j}}{i}\left(1-e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}\right)Q_{s,j,s+1,k}}{2}\left[NL_{s,j,s+1,k}\left(1-\frac{D_{s,j}}{P_{s,j}}\right)\right.$$
$$\left.\left.\left.-1+\frac{2P_{s,j}}{D_{s,j}}\right]+\frac{DCNL^2_{s,j,s+1,k}Q^2_{s,j,s+1,k}\theta}{2}\right)\right\} \tag{5}$$

For fixed $NL_{s,j,s+1,k}$, to obtain the optimal order quantity of the purchaser $Q_{s,j,s+1,k}$, by taking the first partial derivation of $PVC(NL_{s,j,s+1,k}, Q_{s,j,s+1,k})$ in (5) with respect to $Q_{s,j,s+1,k}$ and setting the result to be zero, we have

$$\begin{aligned}
&\frac{\partial PVC\left(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}\right)}{\partial Q_{s,j,s+1,k}}\\
&=\frac{1}{1-e^{-\frac{NL_{s,j,s+1,k}Q_{i,j,k}}{D_{s,j}}}}\left\{\frac{NL_{s,j,s+1,k}rCP_{s,j}}{i}\left[\left(\frac{2a}{2a+1}k\sigma B^{\frac{1}{2}}\left(1\right.\right.\right.\right.\\
&\left.\left.\left.-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{-1}{2a+1}}\right)\left(\frac{i}{D_{s1}}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)+\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)\right]\\
&+\frac{2aNL_{s,j,s+1,k}}{2a+1}CB^{-a}\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{-1}{2a+1}}\left(\frac{i}{D_{s,j}}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)\\
&+\frac{HCCV_{s,j}}{2i}\left(1-e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}\right)\left[NL_{s,j,s+1,k}\left(1-\frac{D_{s,j}}{P_{s,j}}\right)\right.\\
&\left.\left.-1+\frac{2P_{s,j}}{D_{s,j}}\right]+DCNL^2_{s,j,s+1,k}Q^2_{s,j,s+1,k}\theta\right\}\\
&-\frac{1}{1-e^{-\frac{NL_{s,j,s+1,k}Q_{s,j,s+1,k}}{D_{s,j}}}}\left(e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}\right)\left\{NL_{s,j,s+1,k}OC_{s,j}\right.\\
&+\frac{NL_{s,j,s+1,k}HCCP_{s,j}}{i}\left[Q_{s,j,s+1,k}+k\sigma B^{\frac{1}{2}}\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{2a}{2a+1}}\right.\\
&+\frac{D_{s,j}}{i}\left(e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}-1\right)+CB^{-a}\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{2a}{2a+1}}\\
&\left.+SC_{s,j}+\frac{DCNL^2_{s,j,s+1,k}Q^2_{s,j,s+1,k}\theta}{2}\right\}=0
\end{aligned} \tag{6}$$

Next the second-order condition for concavity needs to be checked, that is:

$$\begin{aligned}
&\frac{\partial^2 PVC\left(NL_{s,j,s+1,k}, Q_{s,j,s+1,k}\right)}{\partial^2 Q_{s,j,s+1,k}}\\
&=\frac{1}{1-e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}}\left\{\frac{NL_{s,j,s+1,k}HCCP_{s,j}}{i}\right.\\
&+DCNL^2_{s,j,s+1,k}\theta\left\{\frac{2a}{2a+1}kaB^{\frac{1}{2}}\left[\frac{-1}{2a+1}\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{-2a-2}{2a+1}}\left(\frac{i}{D_{s,j}}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^2\right.\right.\\
&\left.-\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{-1}{2a+1}}\left(\frac{i^2}{D^2_{s,j}}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)\right]
\end{aligned}$$

$$+\frac{h}{D_{s,j}}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\Bigg\}+\frac{2aNL_{s,j,s+1,k}}{2a+1}CB^{-a}\left[\frac{-1}{2a+1}\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{-2a-2}{2a+1}}\right.$$
$$\left.\left(\frac{h}{D_{s,j}}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^2-\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{-1}{2a+1}}\left(\frac{h^2}{D_{s,j}^2}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)\right]\Bigg\}$$
$$-\frac{2}{\left(1-e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}\right)^2}\left(\frac{NL_{s,j,s+1,k}i}{D_{s,j}}e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}\right)$$
$$\Bigg\{\frac{NL_{s,j,s+1,k}HCCP_{s,j}}{i}\left[\frac{2a}{2a+1}kaB^{\frac{1}{2}}\left(\frac{i}{D_{s,j}}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{-1}{2a+1}}\right.$$
$$\left.+\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)\right]+\frac{2aNL_{s,j,s+1,k}}{2a+1}CB^{-a}\left(\frac{i}{D_{s,j}}e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)\Bigg(1$$
$$-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\Bigg)^{\frac{-1}{2a+1}}\theta DCNL_{s,j,s+1,k}^2Q_{s,j,s+1,k}\Bigg\} \tag{7}$$
$$+\frac{1+e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}}{\left(1-e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}\right)^2}\left(\frac{NL_{s,j,s+1,k}^2i^2}{D_{s,j}^2}e^{-\frac{Q_{s,j,s+1,k}NL_{s,j,s+1,k}}{D_{s,j}}}\right)$$
$$\Bigg\{NL_{s,j,s+1,k}OC_{s,j}+\frac{NL_{s,j,s+1,k}rCP_{s,j}}{i}\left[Q_{s,j,s+1,k}+kaB^{\frac{1}{2}}\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{-1}{2a+1}}\right.$$
$$\left.+\frac{D_{s,j}}{i}\left(e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}-1\right)\right]+NL_{s,j,s+1,k}CB^{-a}\left(1-e^{-\frac{Q_{s,j,s+1,k}}{D_{s,j}}}\right)^{\frac{2a}{2a+1}}$$
$$+SC_{s,j}+\frac{DCNL_{s,j,s+1,k}^2Q_{s,j,s+1,k}^2\theta}{2}\Bigg\}$$

The results identify $PVC\left(NL_{s,j,s+1,k},Q_{s,j,s+1,k}\right)$ is a convex function in $Q_{s,j,s+1,k}$ for fixed $NL_{s,j,s+1,k}$. Therefore, it is reduced to find a local optimal solution in local minimum.

Summarizing the above arguments, we establish the following algorithm to obtain the optimal values of $NL_{s,j,s+1,k}$, $Q_{s,j,s+1,k}$, $LT_{s,j,s+1,k}$ (Jou et al. [11]).

Step 1. Set $NL_{s,j,s+1,k}=1$.

Step 2. Determine $Q_{s,j,s+1,k}$ by solving (6).

Step 3. If there exists a $Q_{s,j,s+1,k}$ satisfying (7), we could determine $LT_{s,j,s+1,k}$ by (4). Then $(NL_{s,j,s+1,k},Q_{s,j,s+1,k})$ is the optimal solution for given $NL_{s,j,s+1,k}$.

Step 4. Get $PVC(NL_{s,j,s+1,k},Q_{s,j,s+1,k},LT_{s,j,s+1,k})$ by (3).

Step 5. If $PVC(NL_{s,j,s+1,k}-1,Q_{s,j,s+1,k}-1,LT_{s,j,s+1,k}-1)\le PVC(NL_{s,j,s+1,k},Q_{s,j,s+1,k},LT_{s,j,s+1,k})$ then set $NL_{s,j,s+1,k}=NL_{s,j,s+1,k}+1$ and repeat Steps 2-4; otherwise, go to Step 6.

Step 6. Set $PVC(NL_{s,j,s+1,k},Q_{s,j,s+1,k},LT_{s,j,s+1,k})=PVC(NL_{s,j,s+1,k}-1,Q_{s,j,s+1,k}-1,LT_{s,j,s+1,k}-1)$. Then $NL_{s,j,s+1,k}$, $Q_{s,j,s+1,k}$, $LT_{s,j,s+1,k}$ is the optimal solution.

2.4. **Mathematical model.**

Explanation of symbols

s: Stage no.; $s=1,2,\ldots,S$

j: factory no. in stage s; $j=1,\ldots,J_s$

Parameters

$PVC_{s,j,s+1,k}$: on stage s, seller j seller to next stage $s+1$ purchaser k production

$H_{s,j,s+1,k}$: on stage s, seller j is the production capability suitable for the next $s+1$ purchaser k demands; if yes $H_{s,j,s+1,k}=1$; if no $H_{s,j,s+1,k}=0$.

Decision Variables

$y_{s,j,s+1,k}$: stage s vendor j is able to supply demands of stage $s+1$ purchaser k?; if yes $y_{s,j,s+1,k}=1$; if no $y_{s,j,s+1,k}=0$.

Mixed Integer Model

$$\text{Minimize} \sum_{s=1}^{S-1}\sum_{j=1}^{J_s}\sum_{k=1}^{K_{s+1}} PVC_{s,j,s+1,k}\times y_{s,j,s+1,k} \tag{8}$$

Subject to

$$\sum_{j=1}^{J_i} H_{s,j,s+1,k}\times y_{s,j,s+1,k}=1, \quad \text{for } s=1,2, \quad k=1,2,\dots,k_{s+1} \tag{9}$$

$$\sum_{k=1}^{L_{i+1}} H_{s,j,s+1,k}\times y_{s,j,s+1,k}=1, \quad \text{for } s=1,2, \quad j=1,2,\dots,J_s \tag{10}$$

$$y_{s,j,s+1,k}\in\{0,1\}, \quad \text{for all } s,j,k \tag{11}$$

Equation (8) is a targeted integer model. Decision making variables $y_{s,j,s+1,k}$ for stage s vendor j supply demands for $s+1$ for manufacturer l production. Therefore, by minimizing the associated product $PVC_{s,j,s+1,k}$ and $y_{s,j,s+1,k}$, allocation of results can be achieved in order to minimize the overall cost of inventory integration of the supply chain.

Limiting Formula (9): It is necessary to confirm the relationship between the demands at the various stages, i.e., stage $s+1$ if purchaser l demands are in line with the previous stage s technological capability to meet the purchasers requirements. Seller j is responsible for supplying the demand.

Limiting Formula (10): It is necessary to confirm the relationship between the various stages of the supply, i.e., the capacity of all stage s j vendors to meet later stage $s+1$ vendors l production requirements.

3. **Numerical Experiment.** This study was divided into two phases. In the first phase the mathematical model of Jou et al. [11] was adjusted to extend to two classes of supply chain inventory model to obtain a better variation of lead time and varying delivery schedules with increased but smaller deliveries of partial orders. The appropriateness of the selection of this model is explained in the methodology section above. In the second phase it was assumed that these factors could be considered as a part of supply chain. A second-order degenerate inventory model was used to calculate paired stock buyers and sellers to find the minimum cost approach. The simplex method was utilized to find the best configuration for the entire supply chain. This study attempted to find the best match between buyers and sellers that is to reach a joint inventory cost minimization. Described below steps 1-5 are the various steps involved in the second phase.

Step 1: Supposing that PCB follows a three-stage production process before delivery to customers. The second stage of outer line circuit production was treated as a first stage seller stage. There are, therefore, three levels to customer demand. The first is a need for a trial sample production. This is followed by pilot/trial production and finally mass production. So following Jou et al. [11] parameters were assumed to be as Table 1.

TABLE 1. Parameters applied to seller and buyer

buyer	First stage seller	D Unit/ year	P Unit/ year	OC/ order	SC/ set up	C_P/ unit	C_V/ unit	HC	k	σ Unit/ week	Θ %	DC/ defective unit
1	1	500	1000	8	130	25	20	0.2	2.33	5	0.02	8
1	2	500	3000	24	390	28	20	0.2	2.33	3	0.02	5
1	3	500	600	4	70	20	20	0.2	2.33	8	0.02	11
2	1	300	1000	8	130	25	20	0.2	2.33	5	0.02	8
2	2	300	3000	24	390	28	20	0.2	2.33	3	0.02	5
2	3	300	600	4	70	20	20	0.2	2.33	8	0.02	11
3	1	100	1000	8	130	25	20	0.2	2.33	5	0.02	8
3	2	100	3000	24	390	28	20	0.2	2.33	3	0.02	5
3	3	100	600	4	70	20	20	0.2	2.33	8	0.02	11

Step 2: Using the parameters applied to seller and buyer in Table 1, the PCB industry buyer and first stage seller integrated joint inventory model developed above was applied, and the results of the calculation are shown in Table 2.

TABLE 2. Summary of the computation results

$NL_{s,j,s+1,k}$	$Q_{s,j,s+1,k}$	$LT_{s,j,s+1,k}$ (days)	*First-stage seller*	*Buyer*	*PVC*
			level		Integrated
2	20	9.43	1	1	384512
4	22	10.28	1	2	580457
1	25	8.25	1	3	323384
3	13	9.22	2	1	256286
6	14	10.10	2	2	403940
2	13	8.59	2	3	214377
7	5	8.85	3	1	128082
15	5	9.91	3	2	227350
5	5	8.25	3	3	104751

Step 3: From Table 2, the PVC of buyer and first stage seller were analyzed using the integer programming model to obtain Table 3.

TABLE 3. The optimal from buyer to first stage seller path utilizing minimal production

The objective value and the required solving time			
Node limit, integer feasible, Objective = 807020, Solution time = 0.00 sec.			
Constraints = 6 Variables = 9			
The values for all variables			
Name	Value	Name	Value
y_{2131}	0.0000	y_{2233}	0.0000
y_{2132}	0.0000	y_{2331}	0.0000
y_{2133}	1.0000	y_{2332}	1.0000
y_{2231}	1.0000	y_{2333}	0.0000
y_{2232}	0.0000	y_{2233}	0.0000

Step 4: From Step 3 the buyer is able to obtain the best path and primary sellers, and can determine the best allocation of orders. This pathway confirms that the sellers at stage 1 are able to meet the demands of buyers at stage 2. Given the assumption made in Step 1 where it was stated that PCB follows a three-stage production process, so following Jou et al. [2] the assumption was made that parameters are as Table 4.

Table 4. Parameters of first and second stage sellers

First stage seller	Second stage seller	D Unit /year	P Unit /year	OC/ order	SC/ set up	C_P /unit	C_V /unit	HC	k	σ Unit/ week	Θ %	DC/ defective unit
1	1	100	1000	8	130	30	25	0.2	2.33	2	0.002	8
1	2	100	3000	24	390	33	28	0.2	2.33	5	0.002	5
1	3	100	600	4	70	20	20	0.2	2.33	1	0.002	11
2	1	500	1000	8	130	30	25	0.2	2.33	2	0.002	8
2	2	500	3000	24	390	33	28	0.2	2.33	5	0.002	5
2	3	500	600	4	70	20	20	0.2	2.33	1	0.002	11
3	1	300	1000	8	130	30	25	0.2	2.33	2	0.002	8
3	2	300	3000	24	390	33	28	0.2	2.33	5	0.002	5
3	3	300	600	4	70	20	20	0.2	2.33	1	0.002	11

Step 5: The parameters of first and second stage seller from Table 4 above were applied to the PCB industry buyers and sellers integrated joint inventory model developed in Section 2 above. The results are shown in Table 5.

Table 5. Summary of the computation results

$NL_{s,j,s+1,k}$	$Q_{s,j,s+1,k}$	$LT_{s,j,s+1,k}$ (days)	*Second-stage seller*	*First-stage seller*	PVC
			level		Integrated
2	20	9.43	1	1	384512
4	22	10.28	1	2	580457
1	25	8.25	1	3	323384
3	13	9.22	2	1	256286
6	14	10.10	2	2	403940
2	13	8.59	2	3	214377
7	5	8.85	3	1	128082
15	5	9.91	3	2	227350
5	5	8.25	3	3	104751

Step 6: Utilize the simplex method to find the best configuration for the entire supply chain. From Table 5, the PVC of the first and second stage seller were analyzed using the integer programming model to obtain Table 6.

Discussion of Results

Table two reveals the optimal match between buyer and first stage seller (shown by the lowest figure obtained for PVC). The optimal matches are 1-3, 2-3, 3-3. However, buyer 1 has already been matched with the first stage seller 3 so buyer 2 should be matched with the next best option which is seller 1. Following this process we obtain the following matches: 1-2, 2-3, 3-1. These choices are displayed in Table 3. Table 5 displays the results of analyzing the parameters of the first and second stage sellers. Again the

TABLE 6. The optimal buyer to one-level seller path utilizing minimal production

The objective value and the required solving time			
Node limit, integer feasible: Objective = 500956		Solution time =0.00 sec.	
Constraints = 6 Variables = 9			
The values for all variables			
Name	Value	Name	Value
y_{1121}	0.0000	y_{1223}	1.0000
y_{1122}	1.0000	y_{1321}	1.0000
y_{1123}	0.0000	y_{1322}	0.0000
y_{1221}	0.0000	y_{1323}	0.0000
y_{1222}	0.0000	y_{1223}	0.0000

optimal matches can be seen by the lowest figure for PVC. Following the same procedure as described above the following matches between the first and second stage sellers are obtained: 1-3, 2-1, 3-2. Table 6 displays these choices. The results are summarized in Figure 1. This paper addresses the problem of a three stage production process with three potential suppliers/customers at each stage. The alogarithm proposed in this paper incorporates the work of Jou et al. [2] which is a key foundation for the solution proposed here. Therefore, in this three supplier/customer at three stages example the best solution can be identified. Future work may allow the extension of this model to incorporate four or more suppliers/customers over a four stage or greater supply chain to even more accurately model the supply chain process.

4. **Conclusions.** In today's PCP industry, in terms of quality, due to statistical product control (SPC) it is possible to calculate the out-of-control probability θ within a line production process with a normal distribution. On the production side most of the use of triple shift 24 hour production methods are carried out in a controllable way. Because of this the lead time crashing cost is in compliance with a polynomial. In this paper we utilized Yang and Pan [2] integrated inventory seller-buyer controllable lead time model. We also consider the impact of the time value of Yang et al. [10].

Having determined the suitability of this model this study was carried out in three phases. The first phase utilized the work of Jou et al. [11] extending it to apply to two classes of supply chain inventory model and so obtain a better variation of lead time and varying delivery schedules with increased but smaller deliveries of partial orders. In the second phase these factors were considered as a part of supply chain and a second-order degenerate inventory model was used to calculate paired stock buyers and sellers to find the minimum cost approach. The simplex method was then used to find the best configuration for the entire supply chain. This study attempted to reach a joint inventory cost minimization by finding the optimal match between buyers and sellers.

In the past within the PCB industry from the point of view of supplier assessment each project, such as quality, delivery (lead time) and cost, has been handled separately. The results of this study may provide a way for the PCB industry to integrate considerations in these areas to all qualified suppliers. It could facilitate evaluation and use of the overall management of product quality, delivery and cost in order to achieve the lowest cost optimal batch inventory management objectives.

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