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PROFIT AND LOSS ANALYSIS ON A PRODUCTION BUSINESS USING LEAD TIME FUNCTION

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Abstract. *We propose a profit and loss analysis on the outlet side of a production flow process using a lead time function. This is the ongoing study, which analyzes a production process by using a lead time function. With respect to a production business, we need to secure operating revenue as a production company. To analyze the profit and loss on a production business, we introduce a system for the evaluation of revenue under the conditions of borrowing and capital repayments. We use the actual data obtained from a production flow process for evaluation of the break even point. With regard to a value after a repayment, whether the value is changed or not in the case where a guaranty by a company president is required is reported. In addition, how a value of manufacturing equipment (remaining value) changes after a repayment of a loan relative to a repayment period is reported. Finally, a degree (sensitivity) of influence of parameters, an initial plan money amount, a repaid money amount and a repayment period, on a remaining value and a result of risk analysis are also reported.*
Keywords: Break even point, Lead time function, Log-normal distribution, Black-Scholes equation, Stochastic differential equation

1. **Introduction.** Based on mathematical and physical understandings of production engineering, we are conducting research aimed at establishing an academic area called mathematical production engineering. As our business size is a small-to-medium-sized enterprise, human intervention constitutes a significant part of the production process, and revenue can sometimes be greatly affected by human behavior. Therefore, when considering human intervention from outside companies, a deep analysis of the production process and human collaboration is necessary to understand the potential negative effects of such intervention.

With respect to mathematical modeling of deterministic systems, a physical model of the production process was constructed using a one-dimensional diffusion equation in 2012 [1]. However, many concerns that occur in the supply chain are major problems facing production efficiency and business profitability. A stochastic partial bilinear differential equation with time delay was derived for outlet processes. The supply chain was modeled by considering as time delay [2]. With respect to the analysis of production processes in stochastic systems based on financial engineering, we have proposed that a production throughput rate can be estimated utilizing a Kalman filter based on a stochastic differential equation [3]. We have also proposed a stochastic differential equation (SDE) for the mathematical model describing production processes from the input of materials to the end. We utilized a risk-neutral principal in stochastic calculus based on the SDE [4].

With respect to the analysis of production processes based on physics, we have clarified the phenomena such as power-law distributions, self-similarity, phase transitions, and on-off intermittency can occur in production processes [5, 6, 7, 8, 9]. On the other hand, there is the famous theory of constraints (TOC) that describes the importance of avoiding bottlenecks in production processes [10]. Small fluctuations in an upstream subsystem appear as large fluctuations in the downstream (the so-called bullwhip effect) [13]. The bullwhip effect generates a large gap between the demand forecasts of the market and suppliers. Large fluctuations can be suppressed by the following mechanisms.

(1) Reducing the lead time, improving the throughput, and synchronizing the production process by the TOC.
(2) Sharing the demand information and performing mathematical evaluations.
(3) Analyzing the reduction and fluctuating demands of the subsystem (using nonlinear vibration theory).
(4) Basing the inventory management approach on stochastic demand.

When using manufacturing equipment, delays in one production step are propagated to the next. Hence, the use of manufacturing equipment itself may lead to delays. The improvement of production processes was presented that the “Synchronization with pre-process” method was the most desirable in practice using the actual data in production flow process based on the cash flow model by using the SDE of log-normal type [11]. In essence, we have proposed the best way, which is a synchronous method using the Vasicek model for mathematical finance [12]. Then, the supply chain theme, which was a time delay in the production processes, was proposed for the throughput improvement based on a stochastic differential equation of log-normal type [13].

In this study, to determine the break even point, we first construct a model for the guarantee expenses (GE), which is the present value of the total amount in the case of guaranteed repayment for all times. We also introduce an asset value of the investor, which is a sum of the present value of the remaining value (RV) that results from the subtraction of the payments from the cash flow. A break-even point refers to the point where GE and RV are equal. We validate the break-even point analysis and revenue evaluation based on the actual data obtained from a production flow process.

We also report that, in general, profit can be improved in a case where strategy to lead to an excessive production or an excessive order entry state is taken rather than a case where production is made to match the average order entry. In addition, because, also in the case of the manufacturing business that is the subject of the present research, a rate of return is distributed log-normally, a cash flow of a target company proportional to a rate of return will be also distributed log-normally, naturally. With regard to equipment manufacturing, a small-to-midsize firm is required company president’s guarantee for a borrowing inevitably. Therefore, whether a value of manufacturing equipment after a repayment of a loan varies relative to a repayment period or not is reported.

Also, how a value of manufacturing equipment after a repayment of a loan (hereinafter, referred to as a remaining value) varies relative to a repayment period is reported. Finally, a degree of influence (sensitivity) of parameters of an initial plan money amount, a repaid money amount and a repayment period on a remaining value, and also a risk analysis result are reported. To the best of our knowledge, this study represents the first use of the option pricing theory for a production evaluation method.

2. Production Systems in the Production Equipment Industry.

The production methods used in equipment are briefly covered in this paper (refer to Figure 1). Please see

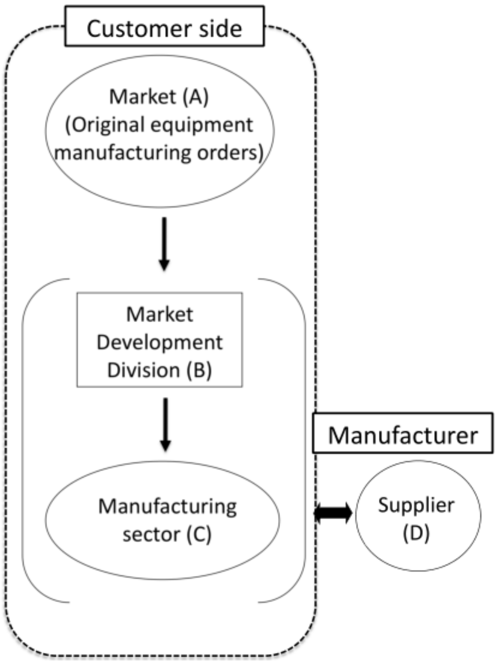

FIGURE 1. Business structure of company of research target

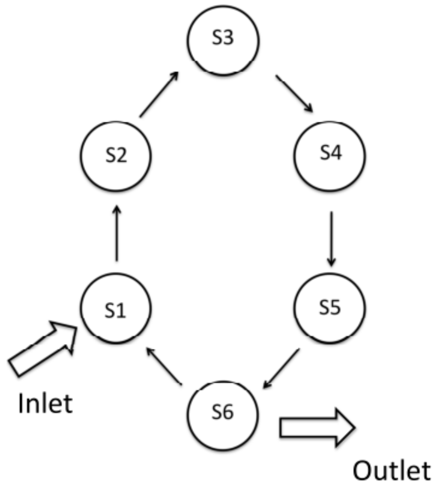

FIGURE 2. Production flow process

our research [5]. This system is considered to be a "Make-to-order system with version control", which enables manufacturing after orders are received from clients, resulting in "volatility" according to its delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

2.1. **Production flow process.** A manufacturing process that is termed as a production flow process is shown in Figure 2. The production flow process, which manufactures low volumes of a wide variety of products, is produced through several stages in the production process. In Figure 2, the process consists of six stages. In each step S1-S6 of the manufacturing process, materials are being produced.

The direction of the arrows represents the direction of the production flow. Production materials are supplied through the inlet and the end-product is shipped from the outlet [11].

2.2. **General production framework in manufacturing business.** Generally, the disclosed information of each other between the production side of the business management side is required to realize the steadily sales volume increasing in the manufacturing industry in Figure 3.

The management side strengthens the production capacity depending on demand state of the business and provides a certain cost for facilities management. The business sales is determined by the production capacity and product value.

With respect to revenue, the minimum amount of stock is best. Therefore, to improve the overall revenue based on observable amount of stock must be required. Of course, we implement during the management/production department for the information sharing about an inventory.

2.3. **Rate of return.** For a small-to-midsize firm, it is of the upmost importance not to cause default in a cash flow, and it is necessary for business continuity. As is the case

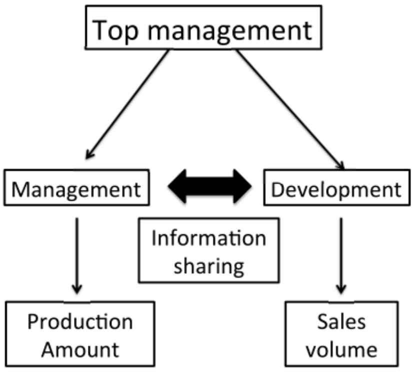

FIGURE 3. Information sharing between a management div. and a development div.

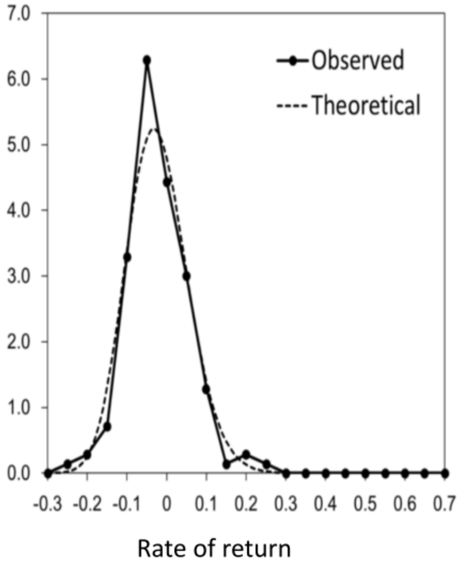

FIGURE 4. Probability density function of rate-of-return deviation: actual data (solid line) and data based on theoretical equation (dotted line)

with rate-of-return deviation described in the previous half, we also analyzed a return acquisition rate defined by Equation (1). The result is shown in Figure 4.

We place the net sales to X_t and the production capacity to C_t. Then we are able to obtain as follows [5]: From the data of monthly rate of return (RoR) observed, its probability density function was calculated (Figure 4). As a result, it was found that the probability density function conforms to log-normal distribution (PDF, Figure 4, Theoretical).

Theoretical curve was calculated using EasyFit software (http://www.mathwave.com/), and as a result of Kolmogorov and Smirnov test, the observed values conformed to a log-normal type probability density function. Because, in the goodness-of-fit test of Kolmogorov-Smirnov, a null hypothesis that it is "log-normal" was not rejected with rejection rate 0.2, this data conforms to "log-normal" distribution. P-value was 0.588. The parameters of a theoretical curve were: $\mu_p = -0.134$ (average), $\sigma_p = 0.0873$ (standard deviation), $\gamma_p = -0.900$. The theoretical curve is given by the following equation.

$$f(x) = \frac{1}{\sqrt{2\pi}(x - \gamma_p)\sigma_p} \times \exp\left\{ -\frac{1}{2}\left(\frac{(\ln x - \gamma_p) - \mu}{\sigma_p}\right)^2 \right\} \tag{1}$$

3. **Description of Cash Flow.** In a small-to-midsize firm, because it does not have ample working capital for the company, in order to continue the company operation by any means, it needs to raise working capital from financial institutions. Let this be called a cash flow. In essence, a lead time is at least proportional to a manufacturing cost.

3.1. **The introduction of lead-time function.** Figure 5 shows that a throughput is proportional to a rate of return in production processes. Then, we introduce the lead-time function so that we can analyze a production process [14]. The lead time of production equipment is proportional to the RoR. Therefore, we determined that the lead time PDF was also the same PDF of RoR. Thus, the lead-time function $f(y)$ is assumed

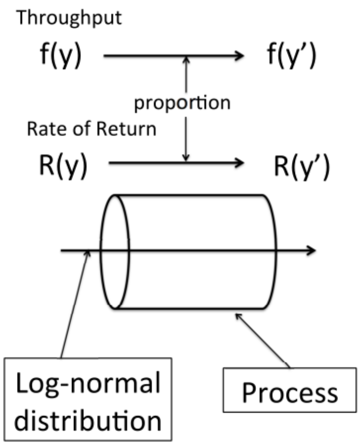

FIGURE 5. Throughput fluctuation in a process distribution amount

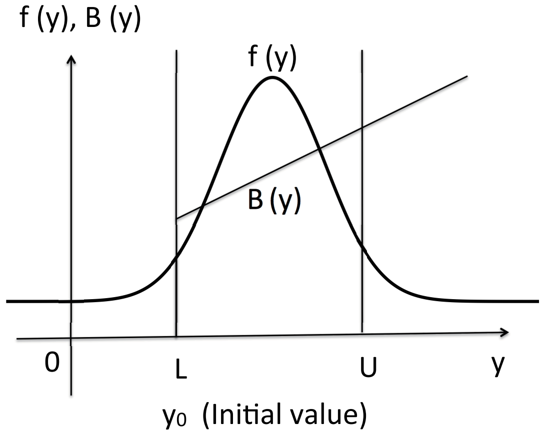

FIGURE 6. Lead time function $f(y)$ and loss function $B(y)$

as a log-normal probability density function so that we can calculate the lead time using a continuous expected value calculation as shown in Figure 6.

Assumption 3.1. *Lead time function of a probability density function with log-normal type.*

$$f(y) \equiv \frac{1}{\sqrt{2\pi}\sigma(y/y_0)} \exp\left\{ -\frac{(\ln(y/y_0)-\mu)^2}{2\sigma^2} \right\} \tag{2}$$

where, μ is an average value, σ is a volatility and y_0 is an initial lead time.

Now, let C_0 as a fixed cost, and we calculate a continuous expected loss value F as a cash-in flow.

$$\begin{aligned} F &= \int_{-\infty}^{\infty} f(y)B(y)dy + C_0 \\ &= \int_{-\infty}^{L} B(y)f(y)dy + \int_{L}^{U} B(y)f(y)dy + \int_{U}^{\infty} B(y)f(y)dy + C_0 \end{aligned} \tag{3}$$

where,

$$B(y) = py + q, \quad p \geq 0 \tag{4}$$

where q is a constant parameter. L is a minimal lead time. $U = kL$ and k (> 1) is a constant parameter. p is as diminishing increasing function; for example, believe the following function. $p = \alpha\sqrt{y - kU}$, α is a constant value. U is a maximum lead time.

When $y < L$, production activities are not running. When $y > U$, the quantity ordered exceeds the physical limits of the production. Therefore, we must reduce the demand, and the problem becomes an analysis of $L \leq y \leq U$. Thus, the expected loss value F is derived as follows:

$$F = \int_{L}^{U} (py + q)f(y)dy + C_0 \tag{5}$$

The calculation process by obtaining Equation (6), please refer to the Appendix A.

$$F = +py_0^2 e^{\left(\mu + \frac{1}{2}\sigma^2\right)} \Phi(d_1) - q\Phi(d_2) + C_0 \tag{6}$$

where,

$$d_1 = \frac{\ln(L/y_0) - (\mu + \sigma^2)}{\sigma}, \quad d_2 = \frac{\ln(U/y_0) - \mu}{\sigma}$$

3.2. **Cash flow analysis.** With respect to a profit and loss analysis in the manufacturing industry, we have researched from a mathematical point of view in order to develop a strategy for the allocation balanced revenue by integrating both of management and production division. A corporate revenue is proportional to the production lead time and the order rates. The order rates can be regarded as a demand distribution by analyzing from the viewpoint of quantitative. Order rate was difficult to order at a constant rate throughout the year. In other words, the order rate varies every month. Considering our experience, the product value sometimes becomes to decrease in inverse proportion to the volume of orders.

A corporate management is for an investment in production business about a product volume efficiency. Therefore, we need to quantify the product size. When investing, its limit exists in the company capability for business sales. Therefore, the corporate management must determine an investment to get the product value in consideration of the demand distribution.

Consequently, it can be said that it is realistic to assume that a cash flow will be also the same log-normal distribution. Therefore, a cash flow model $S_i(t)$ is defined as follows:

Definition 3.1. *Definition of a cash flow model*

$$dS_i(t) = \mu S_i(t)dt + \sigma_i S_i(t)dW^{S_i}(t) \tag{7}$$

where, $S_i(0)$ is an initial plan expense that is considered to be needed at the time of manufacturing, the left-hand side is a monthly rate of return, and a rate of return varies with expected value μ_s and $\sigma_i^2 t$. Further, σ_i^2 represents variance, and $W^{S_i}(t)$ standard Brownian motion. Equation of repayment guarantee money will be described.

Now, repayment guarantee money for a loan from financial institutions can be defined by the following equation.

Definition 3.2. *H: Repayment guarantee money for a loan*

$$H = E\left[\max(P - S_i(t), 0) \cdot \frac{1}{(1+r)^i}\right] \tag{8}$$

Here, because $1/(1+r)^i$ of Equation (8) means that it is a case where equipment manufacturing is performed within a year, $E[\cdot]$ is an expected value under a risk neutral probability [15]. This expense H can be represented as European Put Option [15]. In other words, regarding repayment guarantee expense, from *Black* $\cdot$ *Scholes* model, an appraisal value of guarantee at the time of each repayment can be indicated as follows [5]:

$$\begin{aligned} H &= E\left[\max(P - S_i(t), 0) \cdot \frac{1}{(1+r)^i}\right] \\ &= \frac{P}{(1+r)^i} \cdot \Phi\left(-h + \sigma_i\sqrt{i}\right) - S_i \cdot \Phi(-h) \end{aligned} \tag{9}$$

where, i as it is of single-year, and h is indicated by the following equation:

$$h = \frac{\ln(S_i/P) + (r + (1/2)\sigma_i^2 i}{\sigma_i\sqrt{i}} \tag{10}$$

Also, $\Phi(\cdot)$ indicates a probability value of standard normal distribution, and is indicated by the following equation:

$$\Phi(h) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{h} \exp\left(-\frac{1}{2}x^2\right)dx \tag{11}$$

An appraisal value of equipment manufacturing will be described. If financial institutions guarantee repayment P of a small-to-midsize firm with an equipment manufacturing period, the company can avoid a default risk completely if it pays H. At that time, because all money is paid back to the debt guarantor of a company (the company president in the case of a small-to-midsize firm), the appraisal value of such credit obligation is equal to the amount of the loan. On the other hand, because asset value J of manufacturing equipment is a remaining value after subtracting a repaid money amount from a cash flow, it can be represented [5].

$$\begin{aligned} J &= E\left[\max(S_i(t) - P, 0) \cdot \frac{1}{(1+r)^i}\right] \\ &= S_i \cdot \Phi(+h) - \frac{P}{(1+r)^i} \cdot \Phi\left(h - \sigma_i\sqrt{i}\right) \end{aligned} \tag{12}$$

Then, a cash flow model is rewritten (the subscript i is omitted)

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \tag{13}$$

At this time, an evaluation of the end of the year,

$$J(S,T) = E\left[\max(S(T) - P, 0)\right] \tag{14}$$

Therefore, an evaluation equation is [15]

$$J(S,T) = S \cdot \Phi(d_3) - P \cdot e^{-ri} \cdot \Phi(d_4) \tag{15}$$

where, d_3 and d_4 are

$$d_3 = \frac{\ln(S/P) + (r + (1/2)\sigma^2)\, i}{\sigma\sqrt{i}} \tag{16}$$

$$d_4 = \frac{\ln(S/P) + (r - (1/2)\sigma^2)\, i}{\sigma\sqrt{i}} \tag{17}$$

4. **Numerical Example.**

4.1. **Numerical simulation.** Figure 7 shows results of simulation based on Equation (12). Type 1 through Type 4 in Table 1, represent the possible values when changing the parameters. Referring to Figure 7, Type 1 is a setting that is not problematic to a manufacturing plan at all. In Type 2, although a repaid money amount was set high, because a cash flow initial value is not low, a remaining value becomes high if repayment is performed early. In Type 3, even when variance had been made lowered, no effect was observed. In Type 4, if a cash flow initial value is low, as a matter of course, due to delayed repayment date, a remaining value becomes low.

Figure 8 is a diagram in which Equation (9) is graphed. Type 1 is an example in which a repayment guaranteed amount is low, a risk is relatively low, and profit is not oppressed. Type 2 is an example in which a repaid money amount is set high, and, naturally, a repaid money amount is inversely proportional to a repayment period. It can be said that a risk

Table 1. Set parameter values

	Type 1	Type 2	Type 3	Type 4
Repayment	2.5	5	2.5	2.5
Initial value of cash flow	3	3	3	1
Volatility (σ)	0.8	0.8	0.3	0.8
Risk-free rate	0.2	0.2	0.2	0.2

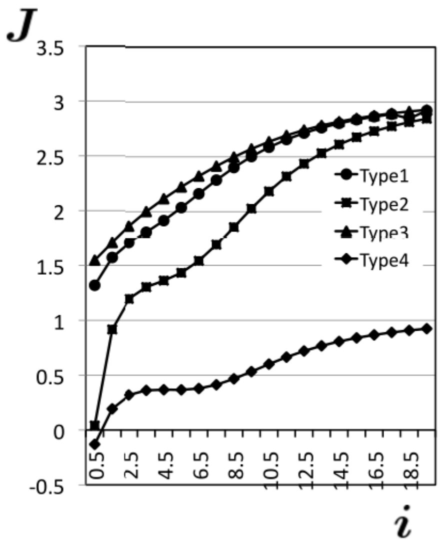

FIGURE 7. Remaining value obtained by subtracting repaid money from cash flow

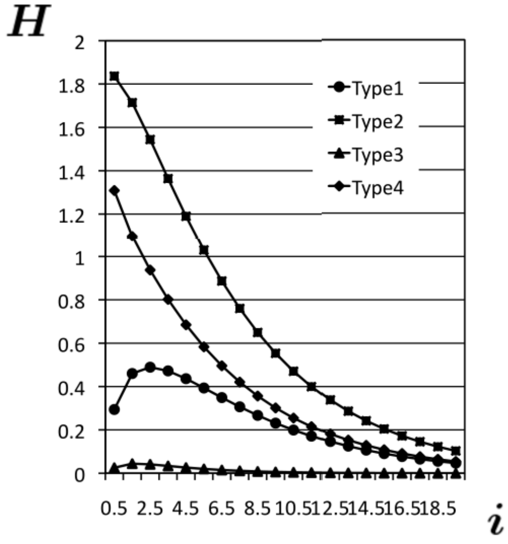

FIGURE 8. Appraisal value of guarantee at the time of repayment

is high. Type 3 is an example where there is a little variation in a cash flow, and, because a repayment guaranteed amount is not high, it is an example of the lowest risk. Type 4 has the same tendency with Type 2. However, because a cash flow initial value is the lowest, and thus a repayment guaranteed amount is inversely proportional to repayment date, it can be said that a risk is high.

Sensitivity analysis and risk analysis of remaining value J is described. Figure 9 through Figure 11 show results of utilizing a simulation tool DECISION SHARE (http=//www.integratto.co.jp/) for quantitative evaluation.

Figure 9 is referred to as a "tornado chart", and, on the longitudinal axis, indication is made in order of a degree of influence from highest to lowest starting from the upper side. Horizontal axis shows a value that J, which is a target value, can take. This indicates that, when J is set to a target value, sensitivity of planned value P_0, repaid money amount L, standard deviation of cash flow σ and a repayment period to the target value. Figure 10 is a diagram referred to as a "spider chart", and its longitudinal axis indicates a range that the three parameters can take, and the horizontal axis indicates a value that J can take, and, in essence, it represents the same things as Figure 9. The most influential parameter is a planned value, the next is a repaid money amount followed by standard deviation, and, finally, a repayment period. Figure 11 indicates that, when target value $J \approx 2.0$, a risk is about 60%. In Figure 11, although simulation was performed with a reference value 3.0, it can be found that this target value itself has a high risk. As a target value recommended by the simulation, in the case of the parameter values set here, $J \approx 2.0$ is recommended. About whether this has a high feasibility as an execution plan or not needs to be reviewed once again in the research project. When promoting equipment manufacturing, it will be beneficial information that an uncertainty element has been made clear as above.

Figure 12 is a diagram obtained by making initial plan money amount (P_0), repayment (L), standard deviation of a cash flow (σ) and repayment period ($Period$), which have been adopted as a parameter, and vary with the probability distribution. The ranges are $P_0 = 2.4 \rightarrow 3.6$, $L = 1.6 \rightarrow 2.4$, $\sigma = 0.4 \rightarrow 0.9$, $Period = 0.1 \rightarrow 0.4$.

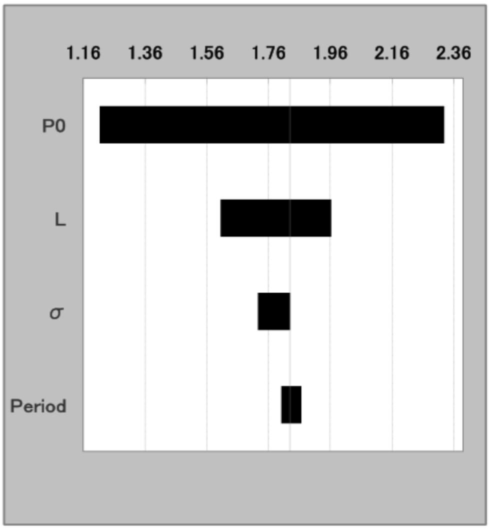

FIGURE 9. Sensitivity of parameters to target value J (tornado chart)

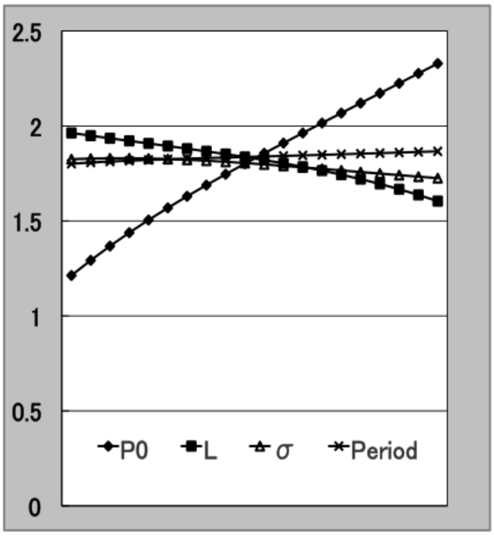

FIGURE 10. Sensitivity of parameters to target value J (spider chart)

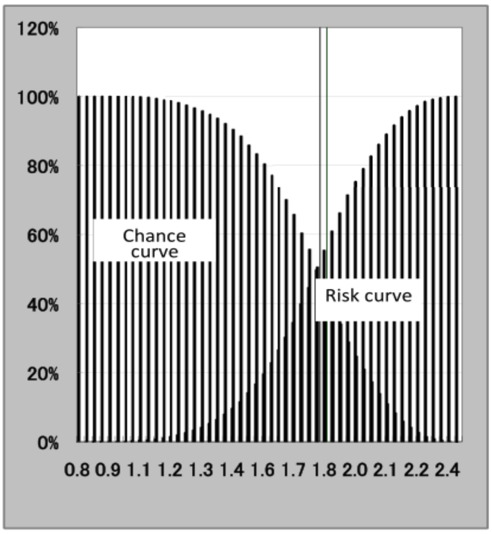

FIGURE 11. Measurement of risk when each parameter is made to vary relative to target value J

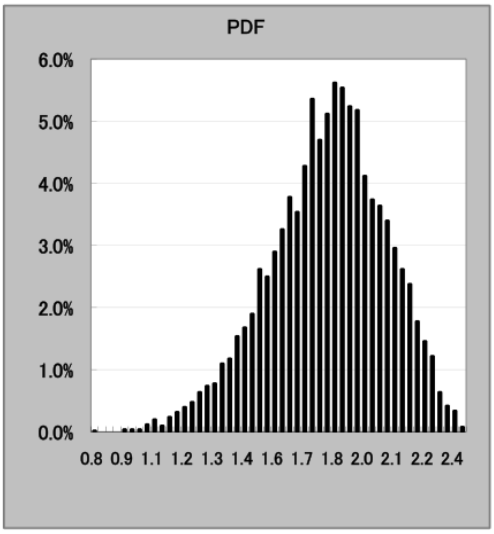

FIGURE 12. Probability distribution chart when each parameter is made to vary randomly within setting range

4.2. **Verification of a break-even point by actual data using a production flow process.** Here, the trend coefficient, which is the ratio of actual number to the target number of pieces of equipment, represents a factor that indicates the trend for the number of pieces of production equipment, as shown in Table 2. We obtained the data of Table 2 from actual data of Test run1 through Test run3. Test run1 is an asynchronous process and Test run2/Test run3 are synchronous processes in Appendix B.

Since the point of a break-even status is derived as $H = J$ in Equations (9) and (15), the inspection of Figures 13, 15, and 17 shows that the values of the guaranteed premium vary with the revenue values. In other words, as $S < P$, the risk for production business is high, and for $S > P$, the risk can be eliminated. Namely, the magnitude of the risk can be evaluated from Test run1 > Test run2 > Test run3. Moreover, Figures 14, 16, and

TABLE 2. Parameter settings in Test run1 through Test run3

Test Type	Average μ	Volatility σ
Test run1	0.73	0.29
Test run2	0.92	0.06
Test run3	0.92	0.03

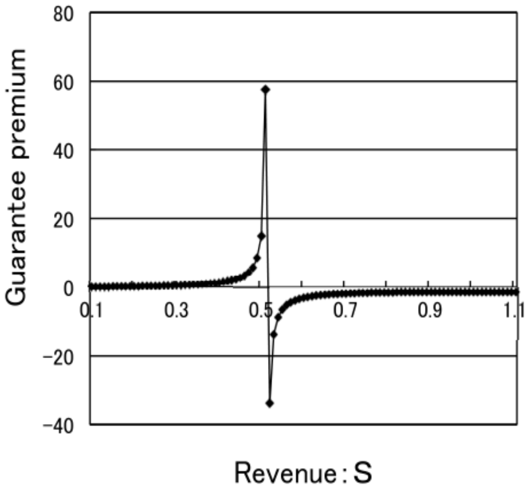

FIGURE 13. Calculation of break-even point guarantee premium in Test run1

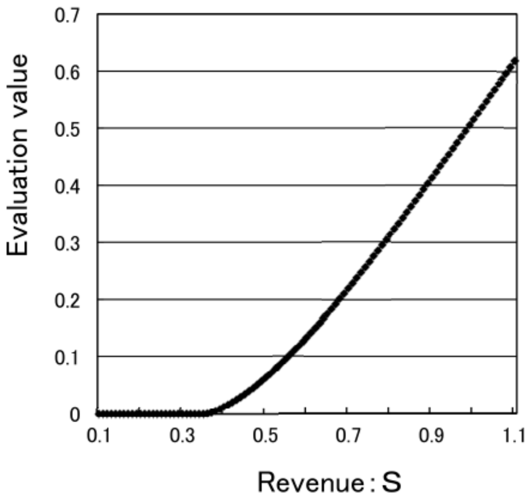

FIGURE 14. Calculation of revenue evaluation in Test run1

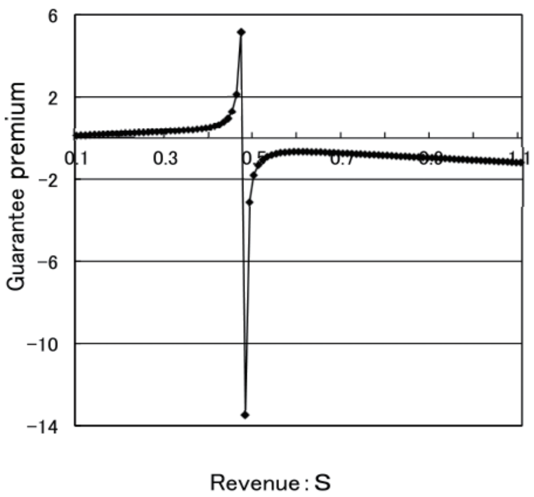

FIGURE 15. Calculation of break-even point guarantee premium in Test run2

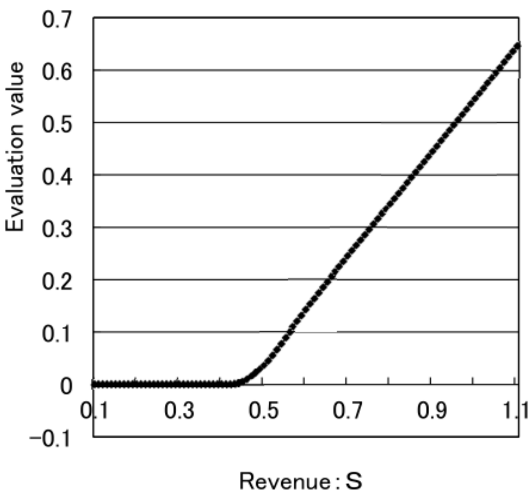

FIGURE 16. Calculation of revenue evaluation in Test run2

18 show that the same results are obtained for the evaluation of the revenue as for the guarantee premium.

Table 3 shows the parameter settings of Figures 19 through 24. Figures 19, 21, and 23 show the numerical simulation of the B-S equation (Equation (9)). In Figure 19, the value of S shows large changes due to the high volatility. In other words, the magnitude of volatility is inversely proportional to the throughput. However, for the small volatility in Figures 21 and 23, the magnitudes of the solution data show only a small variation. The

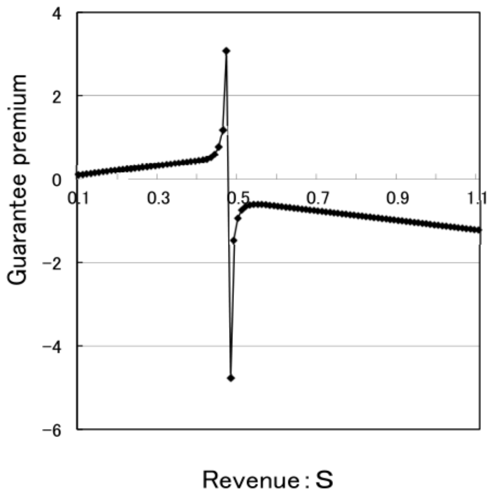

FIGURE 17. Calculation of break-even point guarantee premium in Test run3

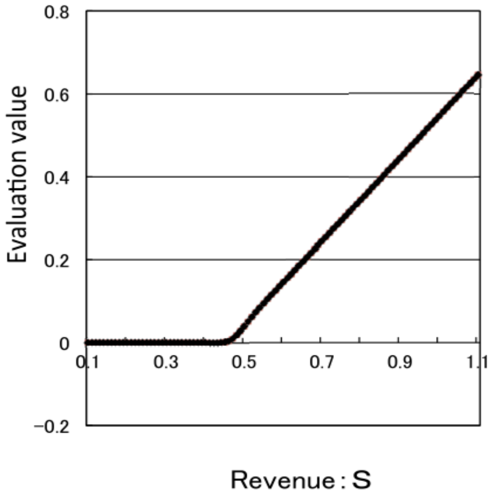

FIGURE 18. Calculation of revenue evaluation in Test run3

TABLE 3. Parameter settings in Test runs 1-3

Test run	Figure	K	average μ	Volatility σ
Test run1	Figure 19	0.6	0.73	0.29
Test run1	Figure 20	0.6	0.73	0.29
Test run2	Figure 21	0.6	0.92	0.06
Test run2	Figure 22	0.6	0.92	0.06
Test run3	Figure 23	0.6	0.92	0.03
Test run3	Figure 24	0.6	0.92	0.03

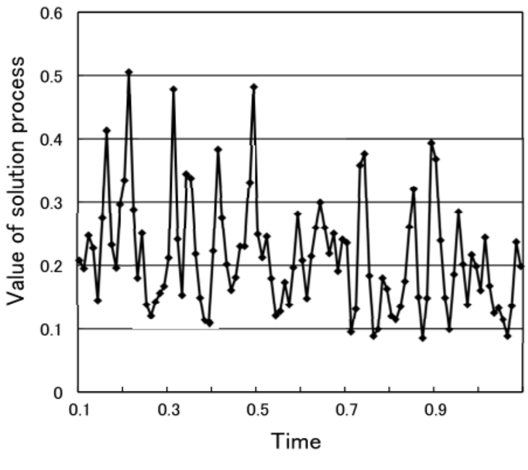

FIGURE 19. Calculation of solution processes in cash flow model Equation (7) in Test run1

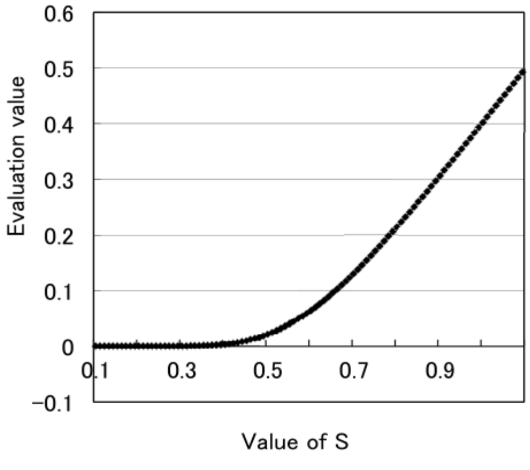

FIGURE 20. Calculation of B-S equation of Equation (15) in Test run1

volatility is highest for Figure 20 and is lowest for Figure 24; the magnitude of volatility is therefore inversely proportional to the magnitude of the cash flow data. In other word, a small volatility indicates that the production business is stable.

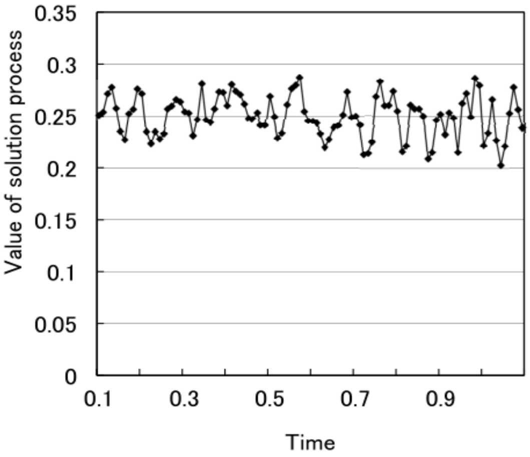

FIGURE 21. Calculation of solution processes in cash flow model Equation (7) in Test run2

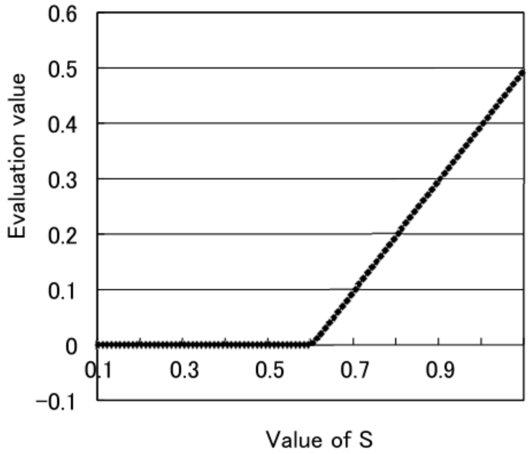

FIGURE 22. Calculation of B-S equation of Equation (15) in Test run2

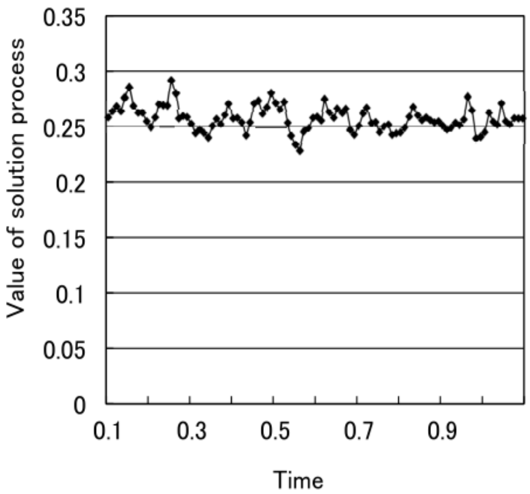

FIGURE 23. Calculation of solution processes in cash flow model Equation (7) in Test run3

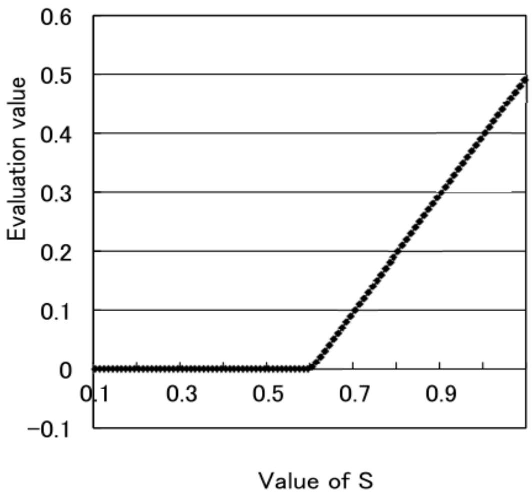

FIGURE 24. Calculation of B-S equation of Equation (15) in Test run3

4.3. **Dynamic simulation of production processes.** Regarding rate-of-return deviation, it was found that it conforms to log-normal distribution. From the analysis of mathematical models about rate-of-return deviation, we obtained the following conclusion.

First of all, analysis was made focusing attention on business rate-of-return deviation (hereinafter, referred to as rate-of-return deviation). As a result, it is reported that rate-of-return deviation has power-law characteristics [5]. Generally, disnormality of rate-of-return deviation in business is well known about a stock price fluctuation model, although with conditions. For example, there exists widely-known Levy process [16].

However, almost all of the reported actual data was entirely limited to stock price data. As another example, also in applying Real Option, many of the return fluctuation models are of a lognormal stochastic differential equation, and there is also one that handles a jump process [17]. Moreover, we think that, as far as the present writers and the like know, there has been no report that handles power-law distribution focusing attention on rate-of-return deviation of a privately-owned company of equipment manufacturing business. Further, regarding a maketo-order production department (production-number based manufacturing system), in relation between rate-of-return deviation and a sales amount in the case of recent production departments, a model of rate-of-return deviation becomes Langevin type. However, in reality, when a "fluctuation" becomes large, force to adjust expected values of them will be added. For example, force to adjust expected values by suppressing an order entry volume, or by making a production amount increase (or decrease) transiently will be added.

Therefore, if an amount of money of order entries and an amount of money of production are stochastic, accumulated excessive order entries becomes of Brownian motion, and thus a random "fluctuation" occurs in hour to hour order entries and production even though it might be of a small degree. In addition, a rate of return is distributed log-normally, a cash flow of a target company proportional to a rate of return will be also distributed log-normally, naturally [5]. Therefore, we attempted to perform a dynamic simulation of the production process by utilizing the simulation system that NTT DATA Mathematical Systems Inc. (www.msi.co.jp) has developed. With respect to the meaning of the individual parts in Figure 25, we conducted a simulation of the following procedure.

- Individual components are commercialized by flowing from the "process1" to "process6".
- In each process, including the six workers in parallel, the slowest worker waited till the work was completed.
- When the work of each process was completed, it moved to the next process.
- Simultaneously as each process was completed, it recorded the working time of each process.

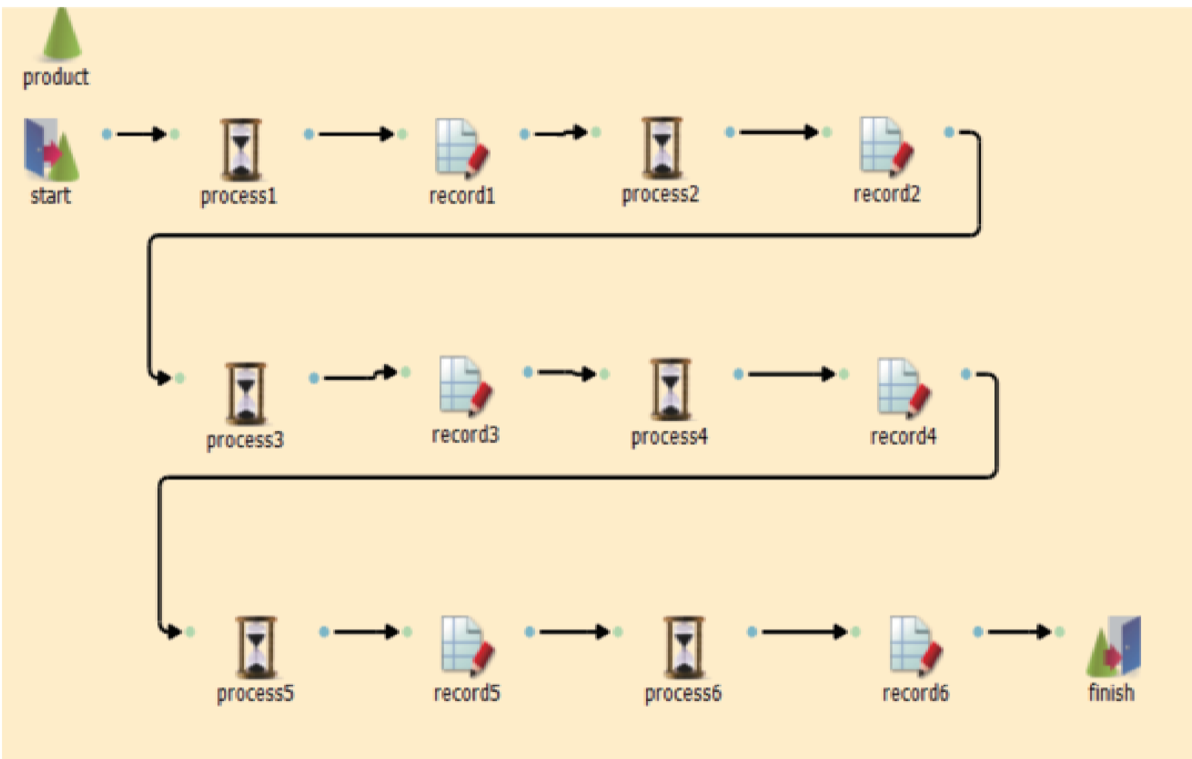

FIGURE 25. Simulation model of production flow system

The meaning of each item in Tables 4 and 5 is as follows.

- "Process No." indicates each "process1" to "process6".
- "Average" indicates the average working time.
- "STD" represents the standard deviation (Min) at each process time of "process1" to "process6".
- Worker's efficiency (W.E1-W.E6) represents that six workers are the ratio of the average working time (AVE) in each process.

"record" calculates the worker's operating time, which is obtained by multiplying the specified WE data for the log-normally distributed random numbers in Table 4.

Figure 26 shows the operating time of process 1-6 (record 1-record 6). As the working time of the synchronous process is less volatile, the work efficiency became higher than the asynchronous process. In Figure 26, the total working time of asynchronous and synchronous processes are 1241.7 (sec) and 586.4 (sec) respectively. The synchronous process shows more better production efficiency than the asynchronous process.

5. **Conclusion.** The dynamic mathematical model, which includes the effect of procurement of the specific production equipment from the supplier through a market, is derived and is given by Equation (7). The average and volatility are derived from the actual data

Table 4. Working data for six production asynchronous processes

Process No.	No.1	No.2	No.3	No.4	No.5	No.6
Average	20	22	25	22	25	21
STD	2.1	2.5	1.6	1.9	2.0	1.9
W.E 1	0.83	1.0	0.66	0.76	0.88	0.91
W.E 2	1.27	1.26	1.21	1.31	1.17	1.20
W.E 3	0.96	1.11	1.01	1.12	0.88	0.89
W.E 4	0.92	0.96	1.06	0.98	0.91	0.9
W.E 5	1.2	1.03	1.07	0.89	1.03	1.1
W.E 6	1.09	1.1	1.2	0.98	1.13	0.89

Table 5. Working data for six production synchronous processes

Process No.	No.1	No.2	No.3	No.4	No.5	No.6
Average	20	20	20	20	20	20
STD	1.1	1.5	1.2	1.4	1.0	1.4
W.E 1	1.0	1.0	1.0	1.0	1.0	1.0
W.E 2	1.0	1.0	1.2	1.3	1.1	1.2
W.E 3	1.7	1.1	1.0	1.1	1.0	1.0
W.E 4	1.0	1.0	1.0	1.0	1.0	1.0
W.E 5	1.0	1.0	1.0	1.0	1.0	1.0
W.E 6	1.0	1.3	1.2	1.0	1.1	1.0

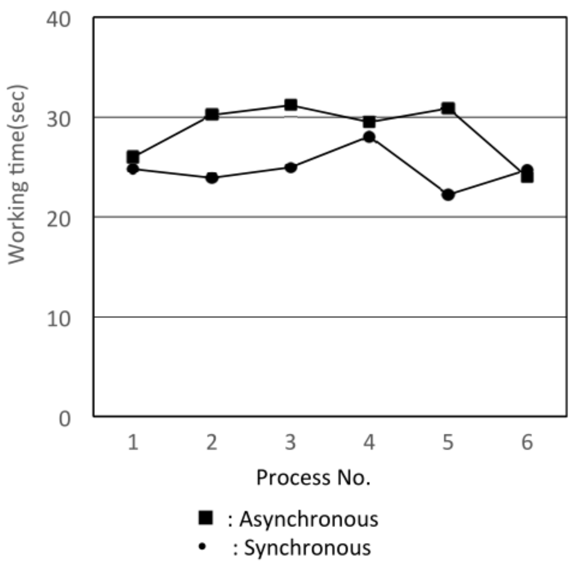

Figure 26. Working time for process number one through six

Table 1 of Test run1 through Test run3. Therefore, we can construct a model for the evaluation of the production business including the suppliers.

At present, an investment by a production business into new equipment incurs a great amount of risk. Here, we provide a corporate strategy for the production business including the suppliers that greatly reduce the risks due to the purchase of the new equipment. Furthermore, we have confirmed that by expressing the cash flow model by a log-normal stochastic differential equation, the results of the research conducted in mathematical finance can also be used for the evaluation of a manufacturing company. This provides useful tools for the evaluation of a business plan.

Companies function in an increasingly complex and globalized financial environment, requiring greater sophistication in their financial management. We believe that the present research will make a significant contribution to the cash flow management of small-to-midsize firms.

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REFERENCES

[1] K. Shirai and Y. Amano, Prduction density diffusion equation and production, *IEEJ Trans. Electronics, Information and Systems*, vol.132-C, no.6, pp.983-990, 2012.

[2] K. Shirai and Y. Amano, Model of production system with time delay using stochastic bilinear equation, *Asian Journal of Management Science and Applications*, vol.1, no.1, pp.83-103, 2015.

[3] K. Shirai and Y. Amano, A study on mathematical analysis of manufacturing lead time -application for deadline scheduling in manufacturing system-, *IEEJ Trans. Electronics, Information and Systems*, vol.132-C, no.12, pp.1973-1981, 2012.

[4] K. Shirai, Y. Amano and S. Omatu, Process throughput analysis for manufacturing process under incomplete information based on physical approach, *International Journal of Innovative Computing, Information and Control*, vol.9, no.11, pp.4431-4445, 2013.

[5] K. Shirai, Y. Amano, S. Omatu and E. Chikayama, Power-law distribution of rate-of-return deviation and evaluation of cash flow in a control equipment manufacturing company, *International Journal of Innovative Computing, Information and Control*, vol.9, no.3, pp.1095-1112, 2013.

[6] K. Shirai and Y. Amano, Self-similarity of fluctuations for throughput deviations within a production process, *International Journal of Innovative Computing, Information and Control*, vol.10, no.3, pp.1001-1016, 2014.

[7] K. Shirai, Y. Amano and S. Omatu, Consideration of phase transition mechanisms during production in manufacturing processes, *International Journal of Innovative Computing, Information and Control*, vol.9, no.9, pp.3611-3626, 2013.

[8] K. Shirai and Y. Amano, Calculating phase transition widths in production flow processes using an average regression model, *International Journal of Innovative Computing, Information and Control*, vol.11, no.3, pp.1075-1091, 2015.

[9] K. Shirai and Y. Amano, On-off intermittency management for production process improvement, *International Journal of Innovative Computing, Information and Control*, vol.11, no.3, pp.815-831, 2015.

[10] S. J. Baderstone and V. J. Mabin, A review goldratt's theory of constraints (TOC) – Lessons from the international literature, *Operations Research Society of New Zealand the 33rd Annual Conference*, University of Auckland, New Zealand, 1998.

[11] K. Shirai, Y. Amano and S. Omatu, Improving throughput by considering the production process, *International Journal of Innovative Computing, Information and Control*, vol.9, no.12, pp.4917-4930, 2013.

[12] K. Shirai and Y. Amano, Production throughput evaluation using the vasicek model, *International Journal of Innovative Computing, Information and Control*, vol.11, no.1, pp.1-17, 2015.

[13] K. Shirai, Y. Amano and S. Omatu, Propagation of working-time delay in production, *International Journal of Innovative Computing, Information and Control*, vol.10, no.1, pp.169-182, 2014.

[14] K. Shirai and Y. Amano, Analysis of production processes using a lead-time function, *International Journal of Innovative Computing, Information and Control*, vol.12, no.1, pp.125-138, 2016.
[15] P. Wilmott, *Derivatives*, John Wiley & Sons, 1998.
[16] M. Jeanblanc, S. Koeppel and Y. Miyahara, Minimal f-q martingale measures for exponential Levy processes, *Annals of Applied Probability*, vol.17, pp.1615-1638, 2007.
[17] S. G. Kou, A jump-diffusion model for option pricing, *Management Science*, pp.1086-1101, 2002.

Appendix A. **Calculation Process of Obtaining Equation (6).** Therefore, the second term of Equation (5) is

$$\begin{aligned}\text{(The second term)} &= \int_L^U (py+q)f(y)dy + C_0\\ &= \int_L^\infty py \cdot f(y)dy - \int_U^\infty q \cdot f(y)dy + C_0 \qquad (18)\end{aligned}$$

From Equation (18), the first term of Equation (18) is

$$\text{(The first term)} = p\int_L^\infty f(y)\cdot ydy = \int_L^\infty py \cdot f(y)dy + \int_L^\infty q(U)\cdot f(y)dy \qquad (19)$$

$$\begin{aligned}\text{(The first term)} &= \int_L^\infty py \cdot f(y)dy\\ &= p\int_L^\infty \frac{1}{\sqrt{2\pi}\sigma(y/y_0)} \exp\left[-\frac{(\ln y - \ln y_0 - \mu)^2}{2\sigma^2}\right]dy \qquad (20)\end{aligned}$$

In Equation (20), let $\ln y = x$, $y = e^x$ and then $dy = e^x dx$.

$$\begin{aligned}\text{(The first term)} &= p\int_L^\infty y\cdot f(y)dy\\ &= p\int_L^\infty \frac{y_0}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(x-\ln y_0-\mu)^2}{2\sigma^2}\right]e^x dx \qquad (21)\end{aligned}$$

Further, let $z = (x - \ln y_0 - \mu)/\sigma$ and then $dx = \sigma dz$.

The first term of Equation (21) is

$$\begin{aligned}&\text{(The first term)}\\ &= py_0\int_{\ln L}^\infty \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}z^2\right)\exp(\sigma z + \ln y_0 + \mu)\cdot\sigma dz\\ &= py_0\int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}z^2\right)\exp(\sigma z + \ln y_0 + \mu)dz\\ &= py_0\int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}}\cdot e^{\sigma z}\cdot e^{\ln y_0 + \mu}dz = py_0^2\int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(z^2 - 2\sigma z + \sigma^2\right) + \frac{1}{2}\sigma^2}\cdot e^\mu dz\\ &= py_0^2\int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(z-\sigma)^2}\cdot e^{\mu + \frac{1}{2}\sigma^2}dz = py_0^2 e^{\left(\mu + \frac{1}{2}\sigma^2\right)}\Phi\left(\frac{\ln(L/y_0) - (\mu + \sigma^2)}{\sigma}\right) \qquad (22)\end{aligned}$$

Applying the same method to the first term of Equation (18), the second term of Equation (18) becomes

$$\begin{aligned}&\text{(The second term)}\\ &= q\int_U^\infty f(y)dy = q\int_U^\infty \frac{1}{\sqrt{2\pi}\sigma(y/y_0)}\exp\left\{-\frac{(\ln y - \ln y_0 - \mu)^2}{2\sigma^2}\right\}dy \qquad (23)\end{aligned}$$

In Equation (23), let $\ln y = x$, $y = e^x$ and then $dy = e^x dx$.

$$\begin{aligned}(\text{The second term}) &= qy_0 \int_{\ln U}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\ln y_0-\mu)^2}{2\sigma^2}\right\} dx \\ &= q\int_{\frac{\ln U - \ln y_0 - \mu}{\sigma}}^{\infty} e^{-\frac{1}{2}z^2} dz = q\Phi\left(\frac{\ln(U/y_0)-\mu}{\sigma}\right) \end{aligned} \tag{24}$$

$$F = +py_0^2 e^{\left(\mu+\frac{1}{2}\sigma^2\right)}\Phi(d_1) - q\Phi(d_2) + C_0 \tag{25}$$

Appendix B. **Test-run1 through Test-run3 Results Using Production Flow Process.** In Table 6, the circle mark represents the working delay by comparing with WS data (working standard).

TABLE 6. Total manufacturing time at each stage for each worker (Test run1)

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181

TABLE 7. Standard deviation of Table 6

K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

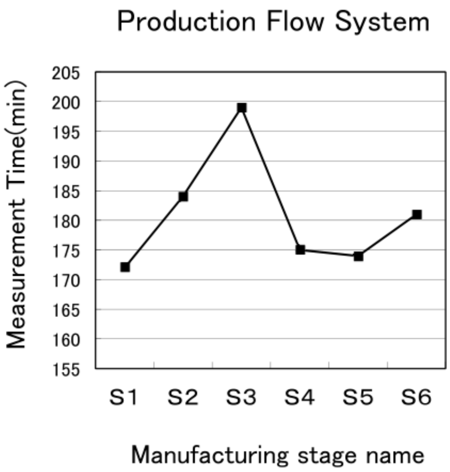

FIGURE 27. Total work time for each stage (S1-S6) in Table 6

FIGURE 28. STD data for each stage (S1-S6) in Table 6

TABLE 8. Total manufacturing time at each stage for each worker (Test run2)

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180

TABLE 9. Deviation of Table 8

K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

TABLE 10. Total manufacturing time at each stage for each worker, K5 (*): Previous process (Test run3)

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	18	18	18
K2	20	18	18	18	18	18	18
K3	20	21	21	21	21	21	21
K4	16	13	11	11	13	13	13
K5	16	*	*	*	*	*	*
K6	16	18	18	18	18	18	18
K7	16	14	14	13	14	14	13
K8	20	22	22	22	22	22	22
K9	20	20	20	20	20	20	20
Total	148	144	143	141	144	144	143

TABLE 11. Standard deviation of values stated in Table 10, K5: Previous process

K1	0.67	0.33	0.67	0.67	0.67	0.67
K2	0.67	0.67	0.67	0.67	0.67	0.67
K3	0.33	0.33	0.33	0.33	0.33	0.33
K4	1	1.67	1.67	1	1	1
K5	*	*	*	*	*	*
K6	0.67	0.67	0.67	0.67	0.67	0.67
K7	0.67	0.67	1	0.67	0.67	1
K8	0.67	0.67	0.67	0.67	0.67	0.67
K9	0	0	0	0	0	0