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OPTIMAL PRICING DECISION FOR TIME-SENSITIVE PRODUCTS WITH ECOMMERCE SALES

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Abstract. *An ecommerce sale is a form of business strategy which utilizes the Internet to promote business by providing discounted prices. Customer demand is often dependent on price; therefore, it becomes the priority for the retailers to determine an optimal pricing strategy. In this study, two profit models of deterministic and stochastic demand on time-sensitive are developed and the optimal solutions of the models are derived. An algorithm for each model is developed to obtain a pricing strategy in which profit is maximized. Numerical examples and sensitivity analysis are presented to illustrate the model.*
Keywords: Ecommerce promotional sales, Time-sensitive product, Pricing, Internet

1. **Introduction.** An ecommerce sale is a form of business strategy which utilizes the Internet to promote business. Businesses can deliver their products, services and advertising through their commercial websites, where consumers can acquire information and make direct purchases with these websites. Information security is a major concern consumers are exposed to with the use of ecommerce. Discounted sales refer to retailers who use promotions to stimulate consumer demand [1-4]. The accessibility and convenience of ecommerce has made it increase necessary for business enterprises to incorporate ecommerce transactions into their business model. It is especially important for businesses that deal with time-sensitive products; examples include movie tickets, train, hotels and airplane tickets. Time-sensitive products are different from other traditional commodities, as it bears potentially higher loss after the end of the sales period. For example, as a result of globalization in recent decades, the demand for air transportation has increased, and with increasing accessibility to the Internet online travel companies servicing hotels and airfares were created. These websites purchase flight tickets from airline companies, and in turn resell these tickets on their own business websites. Online travel websites usually offer cheaper fares in order to promote online bookings and increase demand. Customers can search and compare airfare information among websites before placing their orders.

Kimes et al. [5] classified the time-sensitive products by price and demand, implemented diverse pricing options for different products and predicted market demand. Ward and Lee [6] suggested that branding can facilitate consumers' acceptance of electronic commerce. Chu [7] identified Internet users' needs and expectations towards airline/travel websites in Hong Kong. The results showed that online shopping behavior centers on more traditional products such as compact discs, cinema tickets, souvenirs, gifts, software programs, and books. Regarding their needs and expectations towards an ideal airline/travel website, Internet users believe that the content of it should be informative, interactive and attractive. Prasad et al. [8] offered a conceptual model linking market orientation, marketing competencies, and export performance and investigated the role of the Internet technology in these relationships. Wilson and Laskey [9] examined how online market research is utilized within UK market research agencies and what opportunities or problems of his new research tool are giving the market research industry. The main findings showed that the dramatic growth predicted by industry commentators was not evident among practicing market researchers. Aziz and Yasin [10] explored the relationship between market orientation and marketing competency and investigated the role of the Internet marketing integration in the market orientation-marketing competency linkage. Ansari et al. [11] developed a model of customer channel migration and apply it to a retailer that markets over the Web and through catalogs. Varadarajan and Yadav [12] presented a critical assessment of extant research on marketing strategy in an Internet-enabled environment viewed through the lens of research and speculated on the future of interactive marketing in the contexts of marketing practice, research in marketing and marketing education. Schlee and Harich [13] examined the skills and conceptual knowledge that employers require for marketing positions at different levels ranging from entry- or lower-level jobs to middle- and senior-level positions. Shih et al. [14] developed a search engine optimization mechanism for Internet marketing strategy that can be used by an enterprise to improve the ranking of its website in the search engine results. Mackey et al. [15] identified unique e-cigarette Internet vendor characteristics, including geographic location, promotional strategies, use of social networking, presence/absence of age verification, and consumer warning representation. Crespo-Almendros and Del Barrio-García [16] investigated the effect of online price discounts and free gifts on consumers' evaluation of the brand, in the context of an airline. The summary of the related literature to the time-sensitive products with ecommerce sales is presented in Table 1.

Most research in the past discussed the concept and technology of ecommerce sales, but little on the inventory problem. This study aims to determine what pricing strategy is most effective and to determine the optimal ordering quantities to reach a win-win scenario for both the business and customer. Two cases considering the systems with

Table 1. Summary of the related literature to the time-sensitive products with ecommerce sales

Authors	time-sensitive	ecommerce sales	inventory
Kimes et al. [5]	Yes	No	No
Ward and Lee [6]	No	Yes	No
Chu [7]	No	Yes	No
Prasad et al. [8]	No	Yes	No
Wilson and Laskey [9]	No	Yes	No
Aziz and Yasin [10]	No	Yes	No
Ansari et al. [11]	No	Yes	No
Varadarajan and Yadav [12]	No	Yes	No
Shih et al. [14]	No	Yes	No
Mackey et al. [15]	No	Yes	No
Crespo-Almendros and Del Barrio-García [16]	No	Yes	No

the demand rate are deterministic and stochastic. The following is the organization of this study. Section 1 introduces the background and the purpose of the study. Section 2 describes the assumptions, notations and shows the model development. Conclusion and further research are given in the last section.

2. **Model Development.**

Case 1: When the demand rate is deterministic.

In this case, the item of time-sensitive and price discount are considered.

2.1. **Assumptions and notations of case 1.** The mathematical models presented in case 1 have the following assumptions.

(1) There are no interdependencies between ordered items, and therefore a single item model is assumed.

(2) The demand rate is deterministic and stationary through time.

(3) The demand depends on the selling price of items.

(4) The replenishment is instantaneous.

(5) We assume that there are no shortages, due to the availability of ecommerce sales.

(6) The capacity of the warehouse is unlimited.

Meanwhile, the mathematical models have the following notations:

T	the selling period
t_1	the critical time of the largest demand during selling period
c_p	the unit wholesale purchase cost, \$/unit
K	the maximal unit selling price, \$/unit
Δ	increment rate for market price, $\Delta > 0$; market price: $c_p + \Delta(K - c_p)$
$p(t)$	the unit discounted selling price function of time t, \$/unit, $c_p < p(t) < K$
$D(t)$	The real demand function of time t
F	constant, used in Equation (5) and Equation (6)
G	constant, used in Equation (5) and Equation (6)
Q	the ordering quantity
c_o	the ordering cost, \$/order
δ_1	lower increment rate, that is $100(1 - \delta_1)\%$ off, decision variable
δ_2	higher increment rate, that is $100(1 - \delta_2)\%$ off, $0 < \delta_1 < \delta_2 < 1$; decision variable
h	inventory holding cost per item, \$/unit/unit time
TR	the total revenue per cycle
TC	the total cost per cycle
TPU	the net profit per unit time

2.2. **Analysis of the model in case 1.** In this section, a model is formulated to obtain the net profit. Throughout this case, a single product is assumed. The retailer orders a batch from the supplier of the products, Q, with the unit purchase cost c_p, and sells to customers on the Internet with discounted selling price $p(t)$. Since the customers' demand depends on the selling price $p(t)$, it is important for the retailer to know how to price the item, $p(t)$, for the optimal profit. Assume that the items (e.g., airplane tickets) are time-sensitive, due to the limited quantities, the retailers gradually increase the prices of the items as the critical time (one week before the flight date), t_1, is approaching, and decrease its prices a few days before the end (flight date) of selling period T. (Please refer to the unit market selling price in Figure 1; note that the unit market selling price denotes the unit price without Internet sales.) In this case, if the retailer improves management by Internet transaction, then the discounted selling price $p(t)$ will be used for promotion sake. (Please refer to the item's price in Figure 1.)

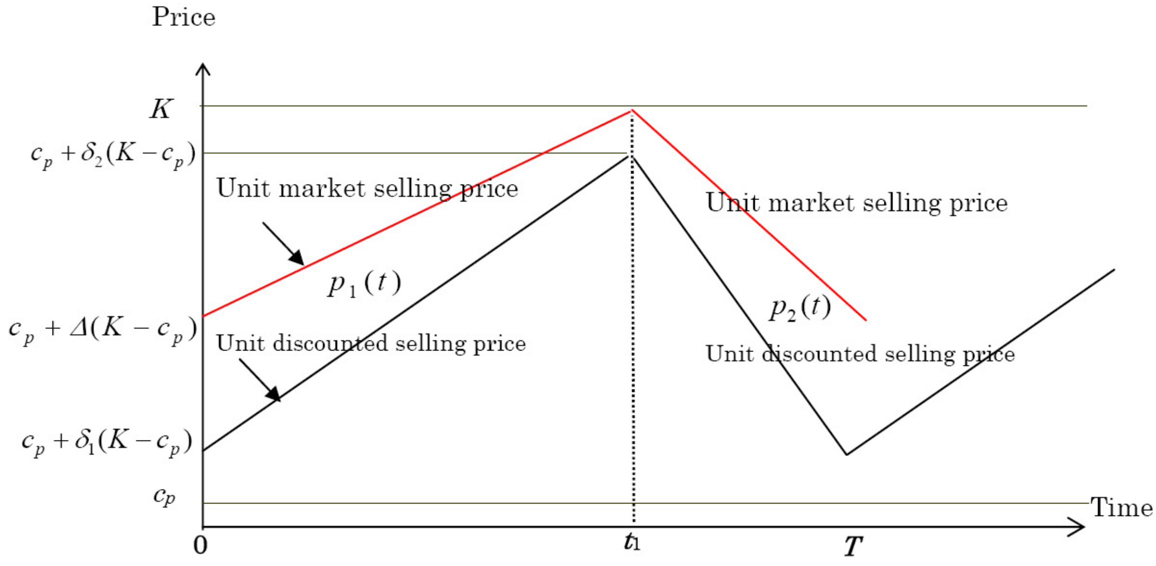

Figure 1. The figure of item's price (with v.s. without ecommerce sale)

From Figure 1, the item's price is piecewise linear; the unit market selling price at time 0 and T, is $c_p + \Delta(K - c_p)$, while the unit market selling price at time t_1 is K, where Δ is constant increment rate and K is the maximal unit selling price ($K - c_p$ is the maximal unit price difference). Thus, the unit market selling price function $p(t)$ is as follows (Please refer to Figure 1.),

$$p_o(t) = \begin{cases} p_{o1}(t), & 0 \leq t < t_1, \\ p_{o2}(t), & t_1 \leq t < T. \end{cases}$$

where

$$p_{o1}(t) = K + [K - c_p - \Delta(K - c_p)]\frac{t - t_1}{t_1},$$

$$p_{o2}(t) = K + [c_p + \Delta(K - c_p) - K]\frac{t - t_1}{T - t_1},$$

It is assumed that the customer's demand without Internet sale, $D_o(t)$, is as follows,

$$D_o(t) = \begin{cases} D_{o1}(t), & 0 \leq t < t_1, \\ D_{o2}(t), & t_1 \leq t < T. \end{cases}$$

where

$$D_{o1}(t) = \frac{F - p_{o1}(t)}{G}, \quad 0 \leq t < t_1,$$

$$D_{o2}(t) = \frac{F - p_{o2}(t)}{G}, \quad t_1 \leq t < T.$$

With F, G being constants, $F > K$. (It means the higher price, the lower demand, and F, K are adjusted parameters.)

Thus, the unit discounted selling price function $p(t)$ is as follows (Please refer to Figure 1.),

$$p(t) = \begin{cases} p_1(t), & 0 \leq t < t_1, \\ p_2(t), & t_1 \leq t < T. \end{cases} \tag{1}$$

where

$$p_1(t) = c_p + \delta_1(K - c_p) + \frac{(\delta_2 - \delta_1)(K - c_p)}{t_1}t, \quad 0 < \delta_1 < \delta_2 < 1, \quad 0 < t < t_1. \tag{2}$$

$$p_2(t) = c_p + \delta_1(K - c_p) + \frac{(\delta_2 - \delta_1)(K - c_p)}{t_1 - T}(t - T), \quad 0 < \delta_1 < \delta_2 < 1, \quad t_1 < t < T. \quad (3)$$

And δ_1 is lower increment rate of the selling price, δ_2 is higher increment rate of the selling price.

Responding to the unit discounted selling price, it is assumed that the customer's demand using Internet sale $D(t)$ is as follows,

$$D(t) = \begin{cases} D_1(t), & 0 \leq t < t_1, \\ D_2(t), & t_1 \leq t < T. \end{cases} \quad (4)$$

where

$$D_1(t) = \frac{F - p_1(t)}{G}, \quad 0 \leq t < t_1, \quad (5)$$

$$D_2(t) = \frac{F - p_2(t)}{G}, \quad t_1 \leq t < T. \quad (6)$$

From the above assumptions and notations, we know that the inventory level $I(t)$ at time t satisfies the following two differential equations (Please refer to Figure 2.):

$$dI(t)/dt = -D_2(t), \quad t_1 \leq t \leq T, \quad (7)$$

with initial condition $I(t) = 0$, one has

$$I(t) = \frac{(T-t)(2FT - 2c_pT + \delta_2 c_pT - \delta_2 KT + \delta_1 c_pT - \delta_1 KT + \delta_2 Kt - \delta_2 c_pt + 2c_pt_1 - 2Ft_1 + 2\delta_1 Kt_1 - 2\delta_1 c_pt_1 + \delta_1 c_pT - \delta_1 KT)}{2(T-t_1)G}, \quad (8)$$
$$t_1 \leq t \leq T.$$

Then

$$I(t_1) = \frac{(2FT - 2c_pT + \delta_2 c_pT - \delta_2 KT + \delta_1 c_pT - \delta_1 KT + \delta_2 Kt - \delta_2 c_pt + 2c_pt_1 - 2Ft_1 + 2\delta_1 Kt_1 - 2\delta_1 c_pt_1 + \delta_1 c_pT - \delta_1 KT)}{2G}. \quad (9)$$

And

$$dI(t)/dt = -D_1(t), \quad 0 \leq t < t_1, \quad (10)$$

with initial condition $\lim_{t \to t_1^+} I(t) = I(t_1)$, one has

$$I(t) = \frac{(\delta_1 - \delta_2)(c_p - K)t^2 - 2t_1(F - c_p - \delta_1 K + \delta_1 c_p)t + t_1 T(2F - 2c_p + \delta_2 c_p - \delta_2 K + \delta_1 c_p - \delta_1 K)}{2t_1 G}, \quad (11)$$
$$0 \leq t < t_1.$$

With the ordering quantity Q,

$$Q = I(0) = \frac{T(2F - 2c_p + \delta_2 c_p - \delta_2 K + \delta_1 c_p - \delta_1 K)}{2G}. \quad (12)$$

The inventory system is shown in Figure 2.

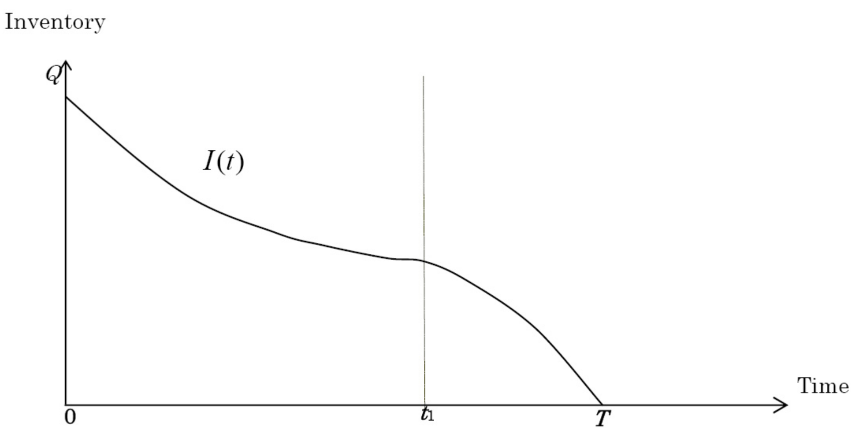

FIGURE 2. The figure of inventory system

The total revenue per cycle $TR(\delta_1,\delta_2)$ is as follows,

$$\begin{aligned} TR(\delta_1,\delta_2) &= \int_0^T D(t)p(t)dt \\ &= \Big[2Tc_p^2(3\delta_1+3\delta_2-\delta_1^2-\delta_2^2-\delta_1\delta_2-3)-2TK^2(\delta_1^2+\delta_2^2+\delta_1\delta_2) \\ &\quad - T\Big(6\delta_1Kc_p+6\delta_2Kc_p+3\delta_1Fc_p+3\delta_2Fc_p-4\delta_1^2Kc_p-4\delta_2^2Kc_p \\ &\quad - 6Fc_p-3\delta_1KF-3\delta_2KF-4\delta_1\delta_2Kc_p\Big)\Big]\Big/(6G). \end{aligned} \tag{13}$$

The total cost per cycle $TC(\delta_1,\delta_2)$ is as follows,

$$\begin{aligned} TC(\delta_1,\delta_2) &= \text{Purchase cost} + \text{Inventory cost} + \text{Ordering cost} \\ &= QC_p + \int_0^T I(t)dth + C_o, \end{aligned} \tag{14}$$

where

$$\begin{aligned} \int_0^T I(t)dt = \Big[&T^2(\delta_2c_p-\delta_2K+2\delta_1c_p-2\delta_1K+3F-3c_p) \\ &- t_1T(\delta_2-\delta_1)(K-c_p)\Big]\Big/(6G). \end{aligned} \tag{15}$$

The net profit per unit time $TPU(\delta_1,\delta_2)$ is as follows,

$$TPU(\delta_1,\delta_2) = \frac{1}{T}\left[TR(\delta_1,\delta_2)-TC(\delta_1,\delta_2)\right]. \tag{16}$$

Since the unit discounted selling price $p(t)$ of the item is lower than that of market price $c_p+\Delta(K-c_p)$ for marketing, then the problem can be formulated as follows:

$$\begin{aligned} \text{Maximize:}\quad & TPU(\delta_1,\delta_2) \\ \text{Subject to:}\quad & 0 \le \delta_1 \le \delta_2 \le 1,\ 0 \le \delta_1 \le \Delta. \end{aligned} \tag{17}$$

From Equation (17), the domain of the problem is closed and bounded, which means the optimum of the problem occurs at either relative maximum of $TPU(\delta_1,\delta_2)$ in the interior of the domain or at the boundary of the domain, $\delta_1=0$, $\delta_1=\Delta$, $\delta_1=\delta_2$, $\delta_2=1$. The following solution procedure is used.

Solution procedure

Step 1. Check the concavity of $TPU(\delta_1,\delta_2)$. (Hessian matrix function of $TPU(\delta_1,\delta_2)$ is positive.)

Step 2. Find both the relative maximum of $TPU(\delta_1,\delta_2)$ in the interior of the domain and at the boundary of the domain.

Step 3. Find the maximal value of Step 2, and the optimum is obtained. Stop.

Consider

$$\frac{\partial^2 TPU}{\partial\delta_1^2} = \frac{-2(c_p-K)^2}{3G} < 0, \tag{18}$$

$$\frac{\partial^2 TPU}{\partial\delta_2^2} = \frac{-2(c_p-K)^2}{3G} < 0, \tag{19}$$

$$\frac{\partial^2 TPU}{\partial\delta_1\partial\delta_2} = \frac{(c_p-K)^2}{3G}, \tag{20}$$

and

$$\text{Hessian}(\delta_1, \delta_2) = \frac{(c_p - K)^4}{3G^2} > 0, \tag{21}$$

which leads to the function $TPU(\delta_1, \delta_2)$ being strictly convex with respect to (δ_1, δ_2). The positive-definite Hessian matrix results in optimal (δ_1^*, δ_2^*) values without restriction. Hence, setting $\frac{\partial TPU}{\partial \delta_1} = 0$, and $\frac{\partial TPU}{\partial \delta_2} = 0$, the optimal discount rate δ_1^*, δ_2^* without restriction, can be derived by MAPLE 13 software as follows:

$$\delta_1 = \frac{F - c_p + Th - t_1 h}{2(K - c_p)}, \text{ and the optimal higher discount rate } \delta_1^* = \min\{\delta_1, 1\}. \tag{22}$$

$$\delta_2 = \frac{F - c_p + t_1 h}{2(K - c_p)}, \text{ and the optimal lower discount rate } \delta_2^* = \max\{\delta_2, 0\}. \tag{23}$$

2.3. **Numerical results and sensitivity analysis of case 1.**

2.3.1. *Example.*

Example 2.1. *To validate the theory, the numerical parameters are as follows:*
$T = 6$ *months,* $t_1 = 5.5$ *months,* $c_p = 500/unit$*,* $K = 2000/unit$*,* $\Delta = 0.3$*,* $h = \$2/unit$*,* $c_o = \$300/cycle$*,* $F = 2100$*, and* $G = 2$*.*
The problem can be formulated as follows:

$$\textit{Maximize:} \quad TPU(\delta_1, \delta_2) = -375000\delta_1^2 - 375000\delta_2^2 - 375000\delta_1\delta_2 + 601625\delta_1 + 602875\delta_2 - 4850. \tag{24}$$

$$\textit{Subject to:} \quad 0 \le \delta_1 \le \delta_2 \le 1, \ \ 0 \le \delta_1 \le 0.3. \tag{25}$$

Firstly, we consider the interior of the domain, $0 < \delta_1 < \delta_2 < 1$*,* $0 < \delta_1 < 0.3$*. Using Equation (22) and Equation (23), the solution is* $\delta_1 = 0.534$*,* $\delta_2 = 0.537$*. However, this solution does not satisfy the constraint (25). Secondly, we consider the boundary of the domain: (a)* $\{\delta_1 = 0, 0 \le \delta_2 \le 1\}$*, (b)* $\{\delta_1 = \delta_2, 0 \le \delta_2 \le 0.3\}$*, (c)* $\{\delta_1 = 0.3, 0.3 \le \delta_2 \le 1\}$*, (d)* $\{\delta_2 = 1, 0 \le \delta_1 \le 0.3\}$*. In (a), the maximum is* $TPU(0, 0.804) = \$237456$*; in (b), the maximum is* $TPU(0.3, 0.3) = \$255250$*; in (c), the maximum is* $TPU(0.3, 0.654) = \$302199$*, and in (d), the maximum is* $TPU(0.3, 1) = \$257262$*. Hence, by comparison, the optimal profit per year is* $TPU(0.3, 0.654) = \$302199$*, that is, the lower discount rate,* $\delta_1 = 0.3$*, the higher discount rate,* $\delta_2 = 0.654$ *and the optimal ordering quantity is* $Q^* = 2654$ *units. Therefore, the results show that when not incorporating ecommerce sales the net profit per year is* $TPU(0.3, 1) = \$191393$*. Utilizing and incorporating ecommerce sales results in an increase in profit by* $(302199/191393) - 1 = 57.9\%$*.*

Example 2.2. *The numerical parameters are the same as Example 2.1 except* $\Delta = 0.6$*. The problem can be formulated as follows:*

$$\textit{Maximize:} \quad TPU(\delta_1, \delta_2) = -375000\delta_1^2 - 375000\delta_2^2 - 375000\delta_1\delta_2 + 601625\delta_1 + 602875\delta_2 - 4850. \tag{26}$$

$$\textit{Subject to:} \quad 0 \le \delta_1 \le \delta_2 \le 1, \quad 0 \le \delta_1 \le 0.6. \tag{27}$$

Using Equation (22) and Equation (23), the interior solution of the domain is $\delta_1 = 0.534$*,* $\delta_2 = 0.537$*,* $TPU(0.534, 0.537) = \$317556$*. In the boundary of the domain: (a)* $\{\delta_1 = 0, 0 \le \delta_2 \le 1\}$*, the maximum is* $TPU(0, 0.804) = \$237456$*; (b)* $\{\delta_1 = \delta_2, 0 \le \delta_2 \le 0.6\}$*, the maximum is* $TPU(0.535, 0.535) = \$317555$*; (c)* $\{\delta_1 = 0.6, 0.6 \le \delta_2 \le 1\}$*, the maximum is* $TPU(0.6, 0.6) = \$312850$*; (d)* $\{\delta_2 = 1, 0 \le \delta_1 \le 0.6\}$*, the maximum is* $TPU(0.302, 1) = \$257264$*. Hence, by comparison, the optimal profit per year is* $TPU(0.534, 0.537) = \$317556$*, the lower discount rate,* $\delta_1 = 0.534$*, the higher discount rate,* $\delta_2 = 0.537$ *and the optimal ordering quantity is* $Q^* = 2391$ *units.*

2.3.2. *Sensitivity analysis of case 1.* In order to utilize the effect of the Internet sale for items with timing and expiration date, different parameters values in Example 2.1 are assumed. Tables 2 to 10 show the changes in δ_1^*, δ_2^*, Q^*, $TPU(\delta_1^*, \delta_2^*)$, and % profit change for variables T, t_1, c_p, K, Δ, h, c_o, F and G, respectively. Table 2 shows the selling period (T) at 5.6, 5.7, ..., 6.4, and other variables unchanged. It is shown that as T increases, the lower increment rate, δ_1^* and higher increment rate, δ_2^* remain constant, $TPU(\delta_1^*, \delta_2^*)$ and % profit change decrease, but Q^* increases. Table 3 shows the critical time (t_1) at 5.1, 5.2, ..., 5.9, and other variables unchanged. It is shown that as t_1 increases, δ_1^*, δ_2^*, Q^*, and % profit change all remain constant, but $TPU(\delta_1^*, \delta_2^*)$ increases.

Table 4 shows the unit purchase cost, (c_p) at 100, 200, ..., 900, and other variables unchanged. It is shown that as c_p increases, δ_1^* remains constant, δ_2^* increases, but the Q^*, $TPU(\delta_1^*, \delta_2^*)$, and % profit change all decrease. Table 5 shows the maximal unit selling price, (K) at 1600, 1700, ..., 2400, and other variables unchanged. It is shown that as K increases, δ_1^* remains constant, δ_2^* and Q^*, decrease, but $TPU(\delta_1^*, \delta_2^*)$, and % profit change increase. Table 6 shows the discount rate of market price, (Δ) at 0.1, 0.15, ..., 0.5, and other variables unchanged. It is shown that as Δ increases, δ_1^*, $TPU(\delta_1^*, \delta_2^*)$, and % profit change all increase, but δ_2 and Q^* decrease.

Table 7 shows the unit inventory holding cost, (h) at 0.4, 0.8, ..., 3.6, and other variables unchanged. It is shown that as h increases, δ_1^* remains constant, δ_2^* increases,

Table 2. Sensitivity analysis for the selling period, T

$T=6$, $t_1=5.5$, $c_p=500$, $K=2000$, $\Delta=0.3$, $h=2$, $c_o=300$, $F=2100$, $G=2$					
T	δ_1^*	δ_2^*	Q^*	$TPU(\delta_1^*, \delta_2^*)$	% profit change
6.4	0.3	0.654	2830	302008	−0.1%
6.3	0.3	0.654	2786	302056	0%
6.2	0.3	0.654	2742	302104	0%
6.1	0.3	0.654	2698	302151	0%
{6}	0.3	0.654	2654	302199	–
5.9	0.3	0.654	2610	302247	0%
5.8	0.3	0.654	2566	302295	0%
5.7	0.3	0.654	2521	302343	0%
5.6	0.3	0.654	2477	302390	0.1%

Note: 1. % profit increase denotes percent profit change. 2. The value in { } is the parameter of Example 2.1. 3. * denotes the optimum.

Table 3. Sensitivity analysis for the critical time, t_1

$T=6$, $c_p=500$, $K=2000$, $\Delta=0.3$, $h=2$, $c_o=300$, $F=2100$, $G=2$					
t_1	δ_1	δ_2	Q^*	$TPU(\delta_1^*, \delta_2^*)$	% profit change
5.9	0.3	0.654	2654	302235	0%
5.8	0.3	0.654	2654	302226	0%
5.7	0.3	0.654	2654	302217	0%
5.6	0.3	0.654	2654	302208	0%
{5.5}	0.3	0.654	2654	302199	–
5.4	0.3	0.654	2654	302190	0%
5.3	0.3	0.654	2654	302182	0%
5.2	0.3	0.654	2654	302173	0%
5.1	0.3	0.654	2654	302164	0%

TABLE 4. Sensitivity analysis for the unit purchase cost, c_p

$T = 6,\ t_1 = 5.5,\ c_p =,\ K = 2000,\ \Delta = 0.3,\ h = 2,\ c_o = 300,\ F = 2100,\ G = 2$					
c_p	δ_1	δ_2	Q^*	$TPU(\delta_1^*, \delta_2^*)$	% profit change
900	0.3	0.673	1994	169009	−44.1%
800	0.3	0.667	2159	198707	−34.2%
700	0.3	0.662	2324	230804	−23.6%
600	0.3	0.658	2489	265302	−12.2%
{500}	0.3	0.654	2654	302199	–
400	0.3	0.65	2819	341497	13%
300	0.3	0.647	2984	383194	26.8%
200	0.3	0.645	3149	427292	41.4%
100	0.3	0.643	3314	473789	56.8%

TABLE 5. Sensitivity analysis for the maximal unit selling price, K

$T = 6,\ t_1 = 5.5,\ c_p = 500,\ \Delta = 0.3,\ h = 2,\ c_o = 300,\ F = 2100,\ G = 2$					
K	δ_1	δ_2	Q^*	$TPU(\delta_1^*, \delta_2^*)$	% profit change
2400	0.3	0.485	2564	310914	2.9%
2300	0.3	0.52	2586	309073	2.3%
2200	0.3	0.559	2609	307007	1.6%
2100	0.3	0.604	2631	304716	0.8%
{2000}	0.3	0.654	2654	302199	–
1900	0.3	0.711	2676	299458	−0.9%
1800	0.3	0.778	2699	296492	−1.9%
1700	0.3	0.855	2721	293301	−2.9%
1600	0.3	0.946	2744	289884	−4.1%

TABLE 6. Sensitivity analysis for the discount rate of market price, Δ

$T = 6,\ t_1 = 5.5,\ c_p = 500,\ K = 2000,\ h = 2,\ c_o = 300,\ F = 2100,\ G = 2$					
Δ	δ_1	δ_2	Q^*	$TPU(\delta_1^*, \delta_2^*)$	% profit change
0.5	0.5	0.554	2429	317237	5%
0.45	0.45	0.579	2485	315587	4.4%
0.4	0.4	0.604	2541	312531	3.4%
0.35	0.35	0.629	2598	308068	1.9%
{0.3}	0.3	0.654	2654	302199	–
0.25	0.25	0.679	2710	294924	−2.4%
0.2	0.2	0.704	2766	286243	−5.3%
0.15	0.15	0.729	2823	276156	−8.6%
0.1	0.1	0.754	2879	264662	−12.4%

but Q^*, $TPU(\delta_1^*, \delta_2^*)$, and % profit change all decrease. Table 8 shows the ordering cost, c_o at 100, 150, ..., 500, and other variables unchanged. It is shown that as c_o increases, δ_1^*, δ_2^*, Q^*, and % profit change all remain constant, only $TPU(\delta_1^*, \delta_2^*)$ decreases. Table 9 shows the constant, (F) at 2000, 2025, ..., 2200, and other variables unchanged. It is shown that as F increases, δ_1^* remains constant, while δ_2^*, Q^*, $TPU(\delta_1^*, \delta_2^*)$, and % profit change all increase.

TABLE 7. Sensitivity analysis for the unit inventory holding cost, h

$T = 6$, $t_1 = 5.5$, $c_p = 500$, $K = 2000$, $\Delta = 0.3$, $c_o = 300$, $F = 2100$, $G = 2$					
h	δ_1	δ_2	Q^*	$TPU(\delta_1^*, \delta_2^*)$	% profit change
3.6	0.3	0.657	2647	300257	−0.6%
3.2	0.3	0.656	2649	300742	−0.5%
2.8	0.3	0.655	2630	301227	−0.3%
2.4	0.3	0.655	2652	301713	−0.2%
{2}	0.3	0.654	2654	302199	–
1.6	0.3	0.653	2656	302686	0.2%
1.2	0.3	0.652	2657	303173	0.3%
0.8	0.3	0.652	2659	303661	0.5%
0.4	0.3	0.651	2661	304149	0.6%

TABLE 8. Sensitivity analysis for the ordering cost, c_o

$T = 6$, $t_1 = 5.5$, $c_p = 500$, $K = 2000$, $\Delta = 0.3$, $h = 2$, $F = 2100$, $G = 2$					
c_o	δ_1	δ_2	Q^*	$TPU\ (\delta_1^*, \delta_2^*)$	% profit change
500	0.3	0.654	2654	302166	0%
450	0.3	0.654	2654	302174	0%
400	0.3	0.654	2654	302183	0%
350	0.3	0.654	2654	302191	0%
{300}	0.3	0.654	2654	302199	–
250	0.3	0.654	2654	302208	0%
200	0.3	0.654	2654	302216	0%
150	0.3	0.654	2654	302224	0%
100	0.3	0.654	2654	302233	0%

TABLE 9. Sensitivity analysis for the constant, F

$T = 6$, $t_1 = 5.5$, $c_p = 500$, $K = 2000$, $\Delta = 0.3$, $h = 2$, $c_o = 300$, $G = 2$					
F	δ_1	δ_2	Q^*	$TPU(\delta_1^*, \delta_2^*)$	% profit change
2200	0.3	0.704	2841	338606	12%
2175	0.3	0.691	2795	329328	9%
2150	0.3	0.679	2748	320168	5.9%
2125	0.3	0.666	2701	311125	3%
{2100}	0.3	0.654	2654	302199	–
2075	0.3	0.641	2607	293391	−2.9%
2050	0.3	0.629	2560	284699	−5.8%
2025	0.3	0.616	2513	276125	−8.6%
2000	0.3	0.604	2466	267668	−11.4%

Table 10 shows the constant, (G) at 0.4, 0.8, ..., 3.6, and other variables unchanged. It is shown that as G increases, δ_1^* and δ_2^* remain constant, while Q^*, $TPU(\delta_1^*, \delta_2^*)$, and % profit change all decrease. The graphic presentation of sensitivity analysis is in Figure 3.

Case 2: When the demand rate is stochastic.

In this case, the item of price discount is considered.

2.4. **Assumptions and notations of case 2.** The mathematical models presented in case 2 have the following assumptions:

TABLE 10. Sensitivity analysis for the constant, G

$T=6,\ t_1=5.5,\ c_p=500,\ K=2000,\ \Delta=0.3,\ h=2,\ c_o=300,\ F=2100$					
G	δ_1	δ_2	Q^*	$TPU(\delta_1^*,\delta_2^*)$	% profit change
3.6	0.3	0.654	1474	167866	−44.5%
3.2	0.3	0.654	1659	188856	−37.6%
2.8	0.3	0.654	1896	215842	−28.6%
2.4	0.3	0.654	2212	251824	−16.7%
{2}	0.3	0.654	2654	302199	–
1.6	0.3	0.654	3317	377762	25%
1.2	0.3	0.654	4423	503699	66.7%
0.8	0.3	0.654	6635	755573	150%
0.4	0.3	0.654	13269	1511000	400.1%

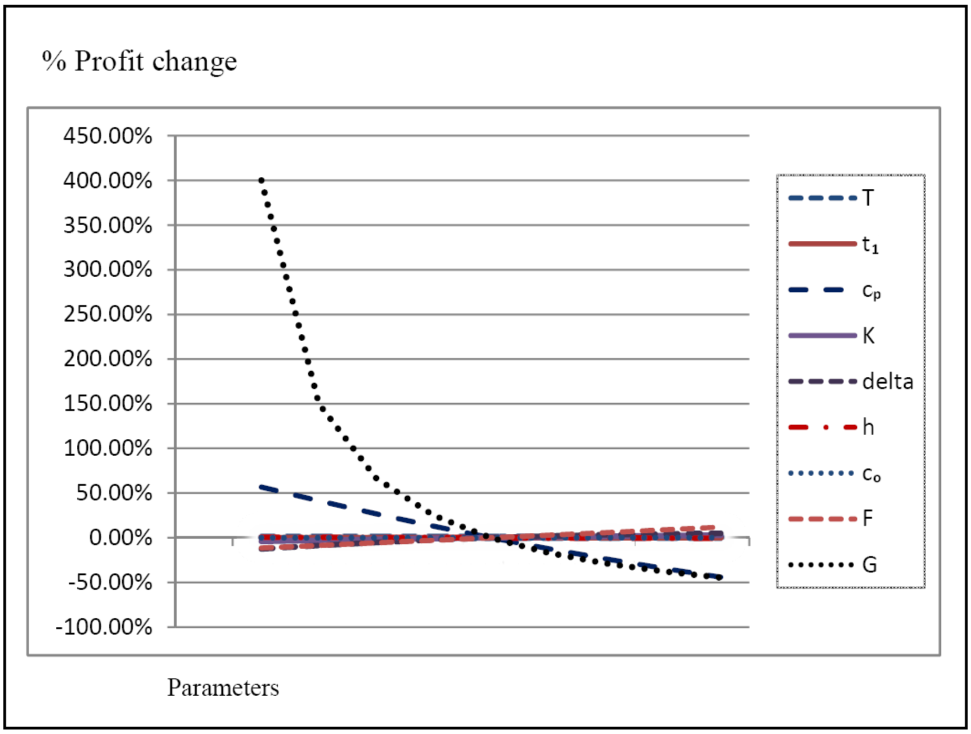

FIGURE 3. The graphic presentation of sensitivity analysis in Example 2.1

(1) There are no interdependencies between ordered items, and therefore, a single item model is assumed.
(2) The demand rate is stochastic through time.
(3) When the sale quantity is less than the ordering batch, the leftover is sold with lower salvage value.
(4) When the demand is more than the ordering batch, shortage backordered is not allowed and the shortage cost occurs.
The following notations are used in case 2:

$E\pi$ the expected profit for the retailor
Q the ordering quantity for the retailor; decision variable
Q^* the optimal ordering quantity for the retailor
p_1 the wholesale price per unit; constant

p_2 the upper bound of selling price per unit; constant; it is commonly assumed as the market price
δ the increased price; $0 < \delta < p_2 - p_1$
b the upper bound of selling quantity
$p(\delta)$ the selling price per unit
s the salvage value per unit $s < p_1$
r the shortage cost per unit; represent costs of lost goodwill
x the random demand with the PDF (Probability Density Function), $f(x)$, and CDF (Cumulative Distribution Function), $F(x)$

2.5. **Analysis of the model in case 2.** The classic single-period inventory problem with random demand always referred to the newsboy problem model such as ordering and selling out newspaper, milk, flight ticket. The model is proposed by Silver and Peterson [17]. There are three conditions with the model: (a) single item; (b) single-period; (c) the leftover is directly sold out with low price. Rogers and Tsubakitani [18] considered a general, non-linear mathematical formulation with the objective of minimizing total penalty costs for expected backorders and a budget constraint upon holding costs to determine amounts to stock at each point. Khouja [19] (2000) extended the single-period problem to the case in which demand is price-dependent and multiple discounts with prices under the control of the newsvendor are used to sell excess inventory. Zheng and Liu [20] investigated a single-period supply chain problem with one retailer and one manufacturer under the demand of fuzzy random variable.

In this section, a model is formulated to obtain the expected profit. The retailor orders a batch of the products, Q, and sells to customers. The unit wholesale price of the product is p_1. The unit selling price is a function of increased price, $p(\delta)$. When the sale quantity is less than the batch Q, the leftover is sold with the unit salvage value s. When the demand is more than the batch, Q, the shortage occurs. Here, shortage backordered is not allowed and the shortage unit cost is r. If the customers' demand is x, the retailor will order an optimal batch of the products according to its optimal expected profit.

If the retailor manages the unit selling price of the products for marketing and business purposes, then the consumers' perceived value and purchase decisions are usually influenced by the low price and convenience. However, the customer demand will decrease due to the higher selling price simultaneously. Thus, in this study the random demand depends on the unit selling price, $p(\delta)$. That means the PDF, $f(x)$, of the random demand x depends on δ. The retailor's expected profit function $E\pi$ is given as follows:

$$\begin{aligned} E\pi(Q,\delta) &= \int_0^Q \{[p(\delta)-p_1]x - (p_1-s)(Q-x)\} f(x,\delta)dx \\ &\quad + \int_Q^{B(\delta)} \{[p(\delta)-p_1]Q - (x-Q)r\} f(x,\delta)dx. \end{aligned} \tag{28}$$

Our problem can be formulated as:

$$\begin{aligned} \text{Max:}\quad & E\pi(Q,\delta) \\ \text{Subject to:}\quad & 0 < \delta < p_2 - p_1. \end{aligned} \tag{29}$$

The concavity of the expected profit function is an optimality condition. The partial derivatives of $E\pi(Q,\delta)$ are as follows:

$$\frac{\partial}{\partial Q} E\pi(Q,\delta) = \int_0^Q (s-p_1) f(x,\delta)dx + \int_Q^{B(\delta)} (p(\delta)-p_1+r) f(x,\delta)dx \tag{30}$$

$$\frac{\partial}{\partial\delta}E\pi(Q,\delta) = \int_0^Q \left\{\left[\frac{\partial}{\partial\delta}p(\delta)\right]xf(x,\delta) + (p(\delta)-p_1)x - (p_1-s)(Q-x)\left[\frac{\partial}{\partial\delta}f(x,\delta)\right]\right\}dx$$
$$+ \int_Q^{B(\delta)} \left\{\left[\frac{\partial}{\partial\delta}p(\delta)\right]Qf(x,\delta) + (p(\delta)-p_1)Q + r(Q-x)\left[\frac{\partial}{\partial\delta}f(x,\delta)\right]\right\}dx$$
$$+ \left[\frac{\partial}{\partial\delta}B(\delta)\right][(p(\delta)-p_1)Q - (B(\delta)-Q)r]f(B(\delta,\delta)). \tag{31}$$

For the concavity of the expected profit function, the positive Hessian matrix function $\left(\text{i.e.,} \frac{\partial^2}{\partial Q^2}E\pi(Q,\delta) \times \frac{\partial^2}{\partial\delta^2}E\pi(Q,\delta) - \left(\frac{\partial^2}{\partial Q\partial\delta}E\pi(Q,\delta)\right)^2\right)$ is a necessary condition. Due to the complexity of $E\pi(Q,\delta)$, it is hard to prove the optimality. We then investigate the model by an illustrative case study.

2.6. **An illustrative case study of case 2.** In this section, the practical selling price and probability distribution are used to explain the results of the previous section. The selling price per unit $P(\delta)$ is assumed as

$$P(\delta) = p_1 + \delta, \quad 0 < \delta < p_2 - p_1. \tag{32}$$

which means $p_1 < P(\delta) < p_2$. The random demand is uniformly distributed over the range 0 and $B(\delta)$, where

$$B(\delta) = \frac{bp_1}{(p(\delta))^a}, \tag{33}$$

is a function of δ with positive constant b (b is the upper bound of the selling quantity). This means that a higher selling price would decrease the demand. Thus, the PDF of the supplier's demand is

$$f(x,\delta) = \frac{1}{B(\delta)}. \tag{34}$$

The numerical examples are provided to illustrate the model.

2.6.1. *Example.*

Example 2.3. *Given $p_2 = 200$, $p_1 = 120$, $a = 1.2$, $b = 2500$, $s = 15$, and $r = 5$, then (Calculated by mathematical software Maple 13)*

$$E\pi(Q,\delta) = +8.3*10^8Q^2(120+\delta)^{2/5}\delta^3 + 1.3*10^{15}Q^2(120+\delta)^{2/5} + 3.75*10^{20}$$
$$- 6.25*10^{16}\delta Q(120+\delta)^{1/5} - 5*10^{14}\delta^2Q(120+\delta)^{1/5}$$
$$- 3*10^{17}Q(120+\delta)^{1/5}\big]\big\}$$

Hessian matrix of $E\pi(Q,\delta)$

$$= \frac{\partial^2}{\partial Q^2}E\pi(Q,\delta) \times \frac{\partial^2}{\partial\delta^2}E\pi(Q,\delta) - \left(\frac{\partial^2}{\partial\delta\partial Q}E\pi(Q,\delta)\right)^2$$
$$= -\frac{1}{(120+\delta)^4}\Big\{1*10^{-20}\Big[6.6*10^{26}\delta + 8.4*10^{24}\delta^2 + 4.7*10^{22}\delta^3 + 1*10^{20}\delta^4$$
$$+1.9*10^{28} + 5.4*10^{20}\delta Q^2(120+\delta)^{2/5} - 1.5*10^{24}\delta Q(120+\delta)^{1/5}$$
$$-2.1*10^{20}\delta^3Q(120+\delta)^{1/5} - 2.5*10^{22}\delta^2Q(120+\delta)^{1/5}$$
$$+1.3*10^{17}\delta^3Q^2(120+\delta)^{2/5} + 1.1*10^{19}\delta^2Q^2(120+\delta)^{2/5}$$
$$-8.7*10^{17}\delta^4Q(120+\delta)^{1/5} - 1.5*10^{15}\delta^5Q(120+\delta)^{1/5}$$

$$+8.2 * 10^{14}\delta^4 Q^2(120+\delta)^{2/5} + 2.8 * 10^{12}\delta^5 Q^2(120+\delta)^{2/5}$$
$$+3.9 * 10^{9}\delta^6 Q^2(120+\delta)^{2/5} + 1 * 10^{22} Q^2(120+\delta)^{2/5}$$
$$-3.5 * 10^{25} Q(120+\delta)^{1/5}\Big]\Big\}.$$

The concavity of $E\pi(Q,\delta)$ is illustrated in Figures 4 and 5. Figure 4 presents the shape of $E\pi(Q,\delta)$ on $[0,500]\times[0,80]$. Figure 5 presents the shape of Hessian matrix function of $E\pi(Q,\delta)$ on $[0,500]\times[0,80]$. Set $\frac{\partial}{\partial Q}E\pi(Q,\delta)$ and $\frac{\partial}{\partial \delta}E\pi(Q,\delta)$ equal to zero, using Software Maple 13, $Q^* = 233$ and $\delta^* = 80$ are derived, the selling price per unit is $p(\delta^*) = \$200$, and the optimal expected profit for the supplier is $E\pi(Q^*,\delta^*) = \$8585$.

2.6.2. *Sensitivity analysis of case 2.* In order to utilize the effect of the limited production quantity, different parameters values in Example 2.3 are assumed. Tables 11 to 16 show the changes in Q^*, $p(\delta^*)$ and $E\pi(Q^*,\delta^*)$ for variables p_2, p_1, a, b, s, and r, respectively. Table 11 shows the upper bound of selling price (p_2) at 160, 170, ..., 240, and other variables unchanged. It is shown that as p_2 increases, the optimal ordering quantity Q^*,

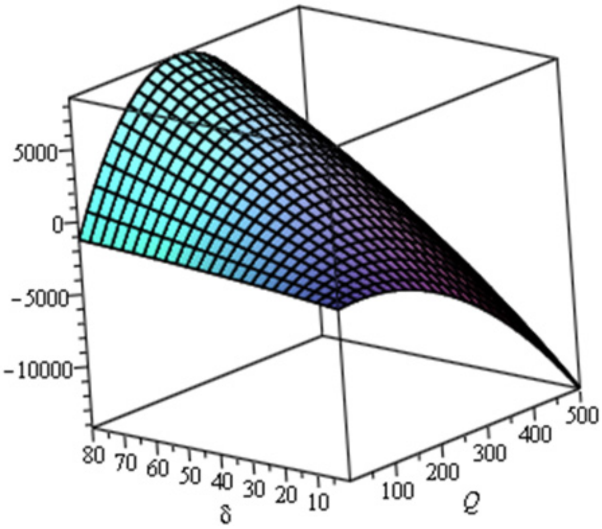

FIGURE 4. Shape of $E\pi(Q,\delta)$ on $[0,500]\times[0,80]$

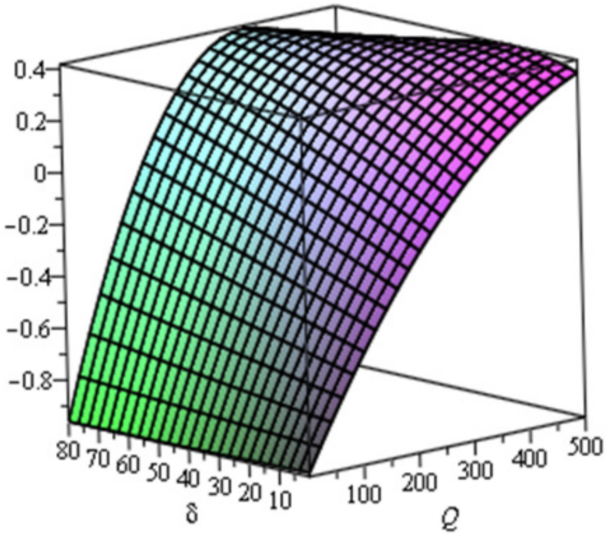

FIGURE 5. Shape of Hessian matrix of $E\pi(Q,\delta)$ on $[0,500]\times[0,80]$

the unit selling price, $p(\delta^*)$, the expected profit, $E\pi(Q^*,\ \delta^*)$ all increase. Table 12 shows the upper bound of selling price (p_1) at 80, 90, ..., 160, and other variables unchanged. It is shown that as p_1 increases, Q^* increases firstly and then decreases, $p(\delta^*)$ and $E\pi(Q^*, \delta^*)$ all decrease.

Table 13 shows the constant (a) at 1, 1.05, ..., 1.4, and other variables unchanged. It is shown that as a increases, Q^*, and $E\pi(Q^*, \delta^*)$ all increase, but $p(\delta^*)$ remains constant. Table 14 shows the upper bound of selling quantity (b) at 2100, 2200, ..., 2900, and other

Table 11. Sensitivity analysis for the upper bound of unit selling price p_2

$p_1 = 120,\ a = 1.2,\ b = 2500,\ s = 15,\ r = 5$			
p_2	Q^*	δ^*	$E\pi\,(Q^*, \delta^*)$
160	204	40	2888
170	217	50	4393
180	226	60	5856
190	230	70	7256
200	233	80	8585
210	233	90	9837
220	232	100	11012
230	230	110	12114
240	227	120	13144

Table 12. Sensitivity analysis for the unit wholesale price p_1

$p_2 = 200,\ a = 1.2,\ b = 2500,\ s = 15,\ r = 5$			
p_1	Q^*	δ^*	$E\pi\,(Q^*, \delta^*)$
80	228	120	13384
90	236	110	12595
100	239	100	11486
110	238	90	10126
120	233	80	8585
130	222	70	6929
140	207	60	5227
150	188	50	3548
160	164	40	1961

Table 13. Sensitivity analysis for the constant a

$p_2 = 200,\ p_1 = 120,\ a = 1.2,\ b = 2500,\ s = 15,\ r = 5$			
a	Q^*	δ^*	$E\pi\,(Q^*, \delta^*)$
1	671	80	24770
1.05	515	80	19005
1.1	395	80	14582
1.15	303	80	11188
1.2	233	80	8585
1.25	178	80	6587
1.3	137	80	5054
1.35	105	80	3878
1.4	81	80	2975

TABLE 14. Sensitivity analysis for the upper bound of selling quantity b

$p_2 = 200$, $p_1 = 120$, $a = 1.2$, $s = 15$, $r = 5$			
b	Q^*	δ^*	$E\pi(Q^*, \delta^*)$
2100	195	80	7211
2200	205	80	7554
2300	214	80	7898
2400	223	80	8241
2500	233	80	8585
2600	242	80	8928
2700	251	80	9271
2800	260	80	9615
2900	270	80	9958

TABLE 15. Sensitivity analysis for the unit salvage value s

$p_2 = 200$, $p_1 = 120$, $a = 1.2$, $b = 2500$, $r = 5$			
s	Q^*	δ^*	$E\pi(Q^*, \delta^*)$
11	228	80	8381
12	229	80	8431
13	230	80	8482
14	231	80	8533
15	233	80	8585
16	234	80	8637
17	235	80	8690
18	236	80	8743
19	238	80	8797

TABLE 16. Sensitivity analysis for the unit shortage cost r

$p_2 = 200$, $p_1 = 120$, $a = 1.2$, $b = 2500$, $s = 15a$			
r	Q^*	δ^*	$E\pi(Q^*, \delta^*)$
1	226	80	8909
2	228	80	8827
3	229	80	8745
4	231	80	8663
5	233	80	8585
6	234	80	8506
7	236	80	8427
8	237	80	8350
9	238	80	8274

variables unchanged. It is shown that as b increases, Q^*, and $E\pi(Q^*, \delta^*)$ all increase, but $p(\delta^*)$ remains constant. Table 15 shows the unit salvage value (s) at 11, 12, ..., 19, and other variables unchanged. It is shown that as s increases, Q^*, and $E\pi(Q^*, \delta^*)$ all increase, but $p(\delta^*)$ remains constant. Table 16 shows the unit shortage cost (r) at 1, 2, ..., 9, and other variables unchanged. It is shown that as r increases, Q^* increases, $E\pi(Q^*, \delta^*)$ decreases, and $p(\delta^*)$ remains constant. The graphic presentation of sensitivity analysis is in Figure 6.

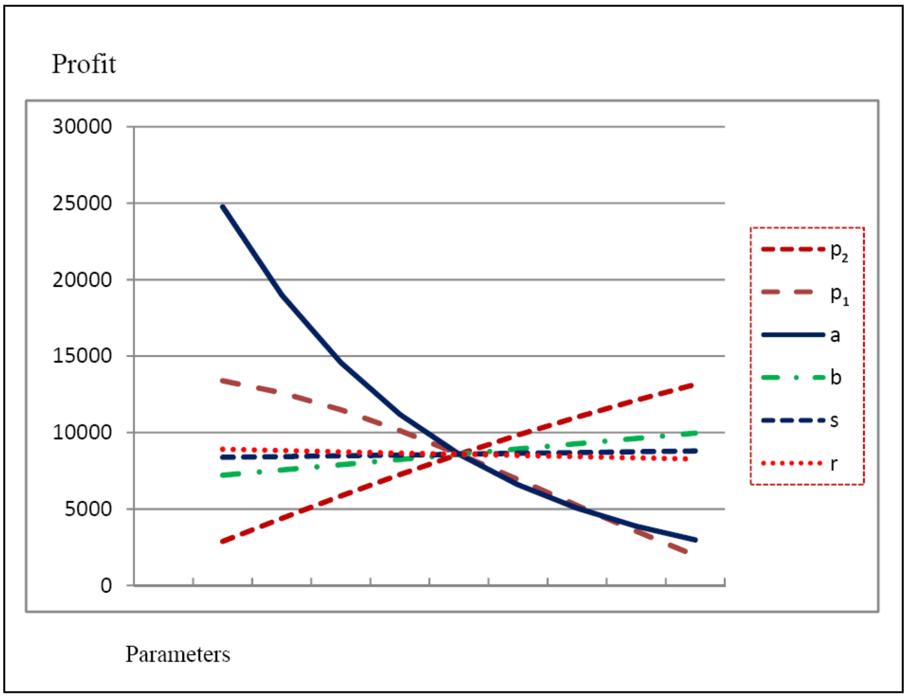

FIGURE 6. The graphic presentation of sensitivity analysis

3. **Conclusion.** Ecommerce sale is a form of business strategy which utilizes the Internet to promote business. The retailer orders a batch from the supplier and sells to customers via the Internet with discount promotional selling prices. Since customers' demand is often dependent on the selling price, it is very important for business to determine a pricing strategy in order to increase sales. In this study, two profit models with the demand rate of deterministic and stochastic are developed and the optimal solution of the models is derived. This study will also help the business managers understand the nature of Internet market pricing dynamic.

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