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ADAPTIVE DISTRIBUTED DYNAMIC SURFACE CONTROL FOR COOPERATIVE PATH FOLLOWING OF MULTI-ROBOT SYSTEMS WITH UNKNOWN UNCERTAINTIES UNDER DIRECTED GRAPHS

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Abstract. *This paper is concerned with the distributed cooperative path following control of multi-robot systems with unknown uncertainties under general directed graphs. The control design is categorized into two envelopes. Firstly, adaptive dynamic surface control allows for handling the unknown dynamical uncertainties and at the same time simplifies the cooperative path following controllers by introducing the first-order filters. Secondly, vehicles coordination is achieved by exchanging the path variables. The amount of communications is reduced effectively due to the distributed speed estimator, which means the condition that the global knowledge of the reference speed is relaxed. Moreover, the distributed protocol is developed to synchronize the path variable under general interconnected directed graphs. Under the proposed controllers, it can be shown that all signals in the closed-loop system are uniformly ultimately bounded. Finally simulations results are provided to demonstrate the effectiveness of the approach presented.*

Keywords: Cooperative control, Multi-robot systems, Dynamic surface control, Directed graph, Adaptive control

1. **Introduction.** In recent years, control of multiple networked vehicles is an emerging technology which has received great attention. The underlying idea of coordination control is that group of vehicles can perform many tasks more effectively than a single vehicle. Revelent applications contain the coordination of multiple robots, unmanned ocean vehicles, satellites, aircraft [1, 2, 3, 4, 5, 6, 7]. In this paper, we consider the problem of steering a group of multi-robot systems along pre-defined paths while keeping a desired formation pattern in time.

In the literature, a variety of methods has been developed about the cooperation of multi-robot systems. In [8], a combination of the virtual structure and path following approaches is applied to deriving the formation architecture without considering the communication constraints, which is limited in practice. The authors in [9] have presented a general framework for cooperative control of multi-robot systems in chained form without considering the orientation information. In [10], a Lyapunov-based technique and graph theory are combined to address the problem of coordinated path following. However, a key point of the existing results on [8, 10, 11, 12] is that the formation controllers are based on the traditional backstepping method. The controllers are complicated due to the needs to calculate numerical derivatives of the virtual control signals. It is well known that the traditional backstepping approach suffers from the explosion of complexity problem. While a dynamic surface control (DSC) method in [13] is proposed to avoid this problem by introducing the first-order filters. In [14], DSC for tracking problem of non-Lipschitz system is proposed. The DSC idea is extended to uncertain single-input single-output [15]. In [16], DSC for trajectory tracking of the non-holonomic mobile robots is solved. DSC method also has been applied to the single path following problem, but these papers do not consider the dynamic model with unknown parameters. This paper combines the adaptive control method for solving the unknown parameters with DSC method for coping with the cooperative path following.

In practice, in order to reduce the cost, it is necessary to consider the communication constraints among the vehicles. In [17, 18], the problem of coordinated path following under undirected graphs is presented. The authors in [19] strengthen the communication constraints under the directed graphs. However, a main assumption in [10, 17, 20] is that the global reference information needs to be known by all vehicles. In order to eliminate the problem, in [21], the distributed control strategy is applied for multi-agent linear systems. The authors in [22] present an algorithm for distributed estimation of the active leaders unmeasurable state variables. It is shown that we have not considered the cooperative problem under the directed graphs among the multi-robot systems, at the same time without the need to know the reference velocity.

Motivated by the above observations, this paper considers the cooperative path following problem of multi-robot systems with unknown uncertainties under directed graphs, partial knowledge to the reference velocity. Compared with [8, 10, 11, 12], the DSC method applied to solving the cooperative path following problem makes the control law simpler. Moreover, in contrast to [10, 17, 20], the designed distributed estimator reduces the communication cost. The control process can be divided into two aspects. Firstly, the adaptive path following controller is designed based on the DSC technique so that each mobile robot converges to the desired path. Secondly, with the analytic tool of graph theory, the speed and path variable are synchronized under directed graphs owing to the proposed synchronized control law, where the distributed speed estimate strategy is incorporated. Based on the Lyapunov analysis, it is proved that all signals in the closed-loop systems are uniformly ultimately bounded. The contributions of the paper are summarized as follows. (i) The adaptive DSC method is firstly applied to solving the cooperative path following problem of multi-robot systems with unknown uncertainties, which leads to a much more simpler controller than the traditional baskstepping-based design. (ii) A distributed speed estimator is proposed under directed graphs, which releases the condition that the global reference velocity is available to all vehicles. (iii) Owing to the aforementioned advantages, it can be applied to more extensive situations about the cooperative path following of mobile robots, in which case the amount of communications is reduced effectively and the dynamic model has unknown certainties.

The rest of the paper is organized as follows. Preliminaries and problem statement are presented in Section 2. Section 3 presents the cooperative path following controller design and stability analysis. Simulation results are showed in Section 4. Finally conclusions are given in Section 5.

2. Preliminaries and Problem Statement.

2.1. Notation.

Throughout this paper, let **1** and **0** denote column vector with all entries equal to one and equal to zero, respectively. Let $\mathbb{R}^{n\times m}$ represent a set of $n \times m$ real matrices. Given a real vector $x \in \mathbb{R}^n$, $\|x\|$ is the Euclidean norm of x. For a matrix P, $\lambda_{\min}(P)$, $\lambda_{\max}(P)$ represent its minimum and maximum eigenvalue, respectively.

2.2. Graph theory.

In this paper, the communication topology among the agents is denoted by the directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, \dots, n\}$ is the node set,

$\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges, in which an edge is represented by an ordered pair of distinct nodes and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the adjacent matrix associated with the directed graph $\mathcal{G}$. In the directed graph $\mathcal{G}$, $(i, j) \in \mathcal{E}$ means that the jth agent has access to the ith agent's information, but not vice visa. The communication graph graph $\mathcal{G}$ is strongly connected if there exists at least one node having a directed path to all of the other nodes. For the adjacent matrix $\mathcal{A} = [a_{ij}]$, $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$; otherwise it is zero. It is assumed that there is no self-edge, that is, $a_{ii} = 0$ for all nodes. The Laplacian matrix $L = [L_{ij}] \in \mathbb{R}^{n \times n}$ associated with the graph $\mathcal{G}$ is defined as $L_{ii} = \sum_{j=0, j \neq i}^{N} a_{ij}$ and $L_{ij} = -a_{ij}$, $i \neq j$.

Lemma 2.1. [23]. *The Laplacian matrix L associated with graph $\mathcal{G}$ has the property that it has a simple zero eigenvalue with vector **1** as a corresponding right eigenvector and all other eigenvalues have positive real parts if and only if $\mathcal{G}$ contains a directed spanning tree.*

2.3. **Formation setup.** We consider a group of n mobile robots, which have n individual parameterized paths. A formation is achieved if and only if all the path parameters are synchronized. A modification of the conventional virtual structure method is applied here to generating the reference paths. We define the center of the virtual structure that moves along a given reference path $\Gamma_0(s_0) = [x_{d0}(s_0), y_{d0}(s_0)]^T$ with s_0 being the path parameter. The shape of the virtual structure can be varied by specifying the distance $l_i(x_{d0}(s_i), y_{d0}(s_i))$ from each mobile robot to the entry of the structure in Figure 1. When the center of the virtual structure moves along the path $\xi_0(s_0)$, the mobile robot i will follow their individual path $\Gamma_i(s_i) = [x_{di}(s_i), y_{di}(s_i)]^T$, $1 \leq i \leq n$ with s_i being the ith path parameter, given by

$$\Gamma_i(s_i) = \Gamma_0(s_i) + R(\theta_{d0}(s_i)) l_i(x_{d0}(s_i), y_{d0}(s_i)) \tag{1}$$

where $l_i(x_{d0}(s_i), y_{d0}(s_i)) = [l_{xi}(x_{d0}(s_i), y_{d0}(s_i)), l_{yi}(x_{d0}(s_i), y_{d0}(s_i))]^T$, $x'_{d0}(s_i) = \frac{\partial x_{d0}(s_0)}{\partial s_0}\big|_{s_0 = s_i}$, $\theta_{d0}(s_i) = \arctan\left(\frac{y'_{d0}(s_i)}{x'_{d0}(s_i)}\right)$ and $y'_{d0}(s_i) = \frac{\partial y_{d0}(s_0)}{\partial s_0}\big|_{s_0 = s_i}$, and

$$R(\theta_{d0}(s_i)) = \begin{bmatrix} \cos(\theta_{d0}(s_i)) & -\sin(\theta_{d0}(s_i)) \\ \sin(\theta_{d0}(s_i)) & \cos(\theta_{d0}(s_i)) \end{bmatrix}.$$

As we all know, the distance from the mobile robot to the center of the structure in the traditional virtual structure method is constant, which means that the shape of the virtual

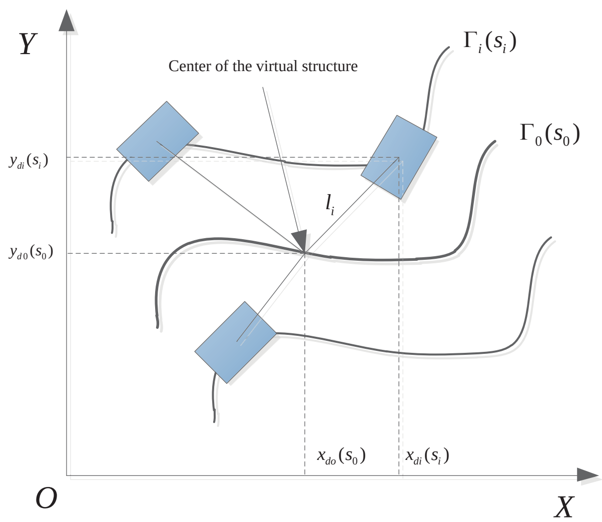

FIGURE 1. Formation coordinates

structure cannot be changed. However, we modify the traditional method to ensure the shape to be variable. Having generated the individual path for each mobile robot, the following tasks are to design a controller so that each vehicle tracks its own desired path, and all the path parameters of the reference paths are synchronized.

2.4. **Mobile robot dynamics.** In this section, we consider the ith two-wheeled driven mobile robot as shown in Figure 2, which has the following equations of motion [24]:

$$\begin{aligned} \dot{\eta}_i =& J_i(\eta_i)\nu_i \\ M_i\dot{\nu}_i + C_i(\dot{\eta}_i)\nu_i + D_i\nu_i + \Delta_i =& \tau_i \end{aligned} \tag{2}$$

where $\eta_i = [x_i, y_i, \theta_i]^T$ represents the position (x_i, y_i) and the heading θ_i of the robot i in the inertial reference frame XOY, $\nu_i = [\nu_{1i}, \nu_{2i}]^T$, with ν_{1i} and ν_{2i} being the angular velocities of the right and left wheels. $\tau_i = [\tau_{1i}, \tau_{2i}]^T$ with τ_{1i} and τ_{2i} denoting the control torques applied to the wheels of the robot i. $J_i(\eta_i)$ is the rotation matrix, M_i is a symmetric, positive definite inertia matrix, $C_i(\eta_i)$ denotes the centripetal and coriolis matrix, and D_i is the damping matrix. Δ_i is a bounded uncertainty. Matrices $J_i(\eta_i)$, M_i, $C_i(\eta_i)$, D_i in (2) are given by

$$J_i(\eta_i) = \frac{r_i}{2}\begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & b_i^{-1} \\ \cos(\theta_i) & -\sin(\theta_i) & -b_i^{-1} \end{bmatrix}, \quad M_i = \begin{bmatrix} m_{11i} & m_{12i} \\ m_{12i} & m_{11i} \end{bmatrix}$$

$$C_i(\eta_i) = \begin{bmatrix} 0 & c_i\dot{\theta}_i \\ -c_i\dot{\theta}_i & 0 \end{bmatrix}, \quad D_i = \begin{bmatrix} d_{11i} & 0 \\ 0 & d_{22i} \end{bmatrix}$$

with $m_{11i} = \frac{1}{4b_i^2}r_i^2(m_ib_i^2 + I_i) + I_{wi}$, $m_{12i} = \frac{1}{4b_i^2}r_i^2(m_ib_i^2 - I_i)$, $m_i = m_{ci} + m_{wi}$, $I_i = m_{ci}a_i^2 + 2m_{wi}b_i^2 + I_{ci} + 2I_{mi}$, $c_i = \frac{1}{2b_i}r_i^2m_{ci}a_i$, where m_{ci} and m_{wi} are the masses of the body and wheel with a motor. I_{ci}, I_{wi}, and I_{mi} are the moments of inertia of the body about the vertical axis through P_{ci} (center of mass), the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel diameter, respectively. a_i, b_i and r_i are defined in Figure 2. And the nonnegative constants d_{11i} and d_{22i} are damping coefficients.

Then we need to transform the model (2) to a simpler one. Let v_i and ω_i denote the linear and angular velocities of the mobile robot respectively. Then the relationship

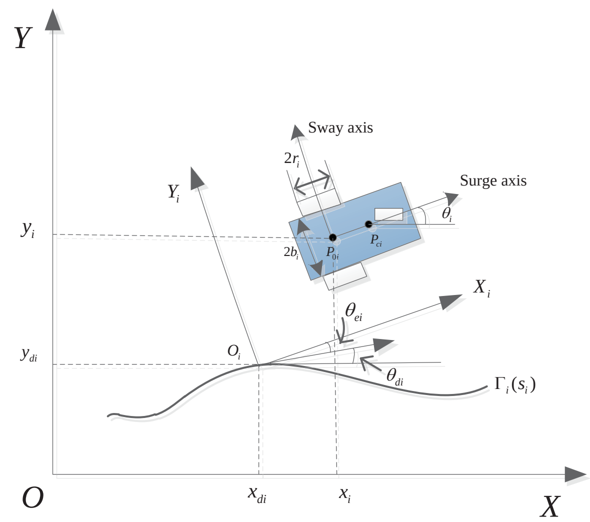

FIGURE 2. The ith mobile robot

between ν_{1i}, ν_{2i} and v_i, ω_i is described as follows:

$$\xi_i = B_i^{-1}\nu_i, \qquad B_i = \frac{1}{r_i}\begin{bmatrix} 1 & b_i \\ 1 & -b_i \end{bmatrix} \tag{3}$$

where $\xi_i = [v_i, \omega_i]^T$. With (3), we write (1) as

$$\begin{aligned} \dot{\eta}_i &= \bar{J}_i(\eta_i)\xi_i \\ \bar{M}_i\dot{\xi}_i + \bar{C}_i(\dot{\eta}_i)\xi_i + \bar{D}_i\xi_i + \bar{\Delta}_i &= \bar{B}_i\tau_i \end{aligned} \tag{4}$$

where $\bar{M}_i = B_i^{-1}M_iB_i$, $\bar{C}_i(\dot{\eta}_i) = B_i^{-1}C_i(\dot{\eta}_i)B_i$, $\bar{D}_i = B_i^{-1}D_iB_i$, $\bar{B}_i = B_i^{-1}$, $\bar{\Delta}_i = B_i^{-1}\Delta_i$, and

$$\bar{J}_i(\eta_i) = \begin{bmatrix} \cos(\theta_i) & \sin(\theta_i) & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

2.5. **Control objective.** In this paper we will solve the formation path following problem under the following assumptions.

Assumption 2.1. *For each value of s_i, there exists a unique value of $x_{di}(s_i)$ and $y_{di}(s_i)$.*

Assumption 2.2. *There exists a positive constant κ such that $\|\Delta_i\| \le \kappa$.*

Assumption 2.3. *The dimensional terms r_i, a_i and b_i of the mobile robot i are known. The parameters related to mass, inertia and damping m_{11i}, m_{12i}, d_{11i}, d_{12i}, c_i are unknown but constant.*

Assumption 2.4. *The reference path to be tracked by mobile robot i is $\Gamma_i(s_i) = [x_{di}(s_i), y_{di}(s_i)]^T$ satisfying the following conditions: there exist strictly positive constants ε_{1i}, ε_{2i} such that:*

$$\varepsilon_{1i} \le x_{di}'^2(s_i) + y_{di}'^2(s_i) \le \varepsilon_{2i},$$

where $x'_{di}(s_i)$ and $y'_{di}(s_i)$ are defined as

$$x'_{di}(s_i) = \frac{\partial x_{di}(s_i)}{\partial s_i}, \qquad y'_{di}(s_i) = \frac{\partial y_{di}(s_i)}{\partial s_i}$$

Assumption 2.5. *In this paper, for the sake of simplicity, we assume that the robot does not slip and slide.*

Remark 2.1. *Assumption 2.4 means that each path is regular with respect to the path parameter. If the path is not regular, we can break it into different regular paths and consider each path separately. In addition, we only consider that the virtual structure moves forward.*

Let $\eta_{di}(s_i) = [x_{di}(s_i), y_{di}(s_i), \theta_{di}(s_i)]^T$, $i = 1, \ldots, n$ be a series of desired paths parameterized by continuous variable s_i. Our purpose is to design a controller τ_i under Assumption 2.1, such that all signals in the closed loop system are uniformly ultimately bounded.

$$\lim_{t\to\infty}\|\eta_i - \eta_{di}\| \le \epsilon_{1i} \tag{5}$$

$$\lim_{t\to\infty}\|\dot{s}_i - v_r\| \le \epsilon_{2i} \tag{6}$$

$$\lim_{t\to\infty}\|s_i - s_j\| \le \epsilon_{3i} \tag{7}$$

where $\theta_{di} = \arctan\left(y'_{di}(s_i)/x'_{di}(s_i)\right)$, ϵ_{1i}, ϵ_{2i} and ϵ_{3i} are some small constants, v_r is a constant reference speed which is assigned by the virtual leader. The objective (5) means that each mobile robot moves on the desired path and its linear velocity is tangential to its own path. The objective (6) means that the speed is synchronized. Moreover, the objective (7) guarantees synchronization of all path parameters so that the formation of the mobile robots is achieved.

3. **Formation Path Following Controller Design.** In this section, we will consider the dynamics (4) and use the dynamic surface control method to design controller τ_i for the robot i to achieve the formation control aim (5).

3.1. **Path following error dynamic.** In order to prepare for the control design, we first interpret path following errors [25] in the frame $X_iO_iY_i$ (see Figure 2), where the origin O_i is a point on the path $\Gamma_i(s_i)$, and O_iX_i and $O_{bi}X_{bi}$ axes are parallel to the surge axis and sway axis of the mobile robot, respectively. So, we have:

$$\begin{bmatrix} x_{ei} \\ y_{ei} \\ \theta_{ei} \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & \sin(\theta_i) & 0 \\ -\sin(\theta_i) & \cos(\theta_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i - x_{di} \\ y_i - y_{di} \\ \theta_i - \theta_{di} \end{bmatrix} \tag{8}$$

where θ_{di} is the angel between the path and the OX axis defined by

$$\theta_{di} = \arctan\left(y'_{di}(s_i)/x'_{di}(s_i)\right).$$

Differentiating both sides of (8) along the solution of the first equation of (4) results in the following kinematic error dynamics:

$$\begin{aligned} \dot{x}_{ei} &= v_i - v_{di}\cos(\theta_{ei}) + y_{ei}\omega_i \\ \dot{y}_{ei} &= v_{di}\sin(\theta_{ei}) - x_{ei}\omega_i \\ \dot{\theta}_{ei} &= \omega_i - \omega_{di} \\ \bar{M}_i\dot{\xi}_i &= -\bar{C}_i(\dot{\eta}_i)\xi_i - \bar{D}_i\xi_i - \bar{\Delta}_i + \bar{B}_i\tau_i \end{aligned} \tag{9}$$

where we have defined

$$v_{di} = \sqrt{x'^2_{di}(s_i) + y'^2_{di}(s_i)}\,\dot{s}_i = \bar{v}_{di}\dot{s}_i \tag{10}$$

$$\omega_{di} = \frac{x'_{di}(s_i)y''_{di}(s_i) - x''_{di}(s_i)y'_{di}(s_i)}{x'^2_{di}(s_i) + y'^2_{di}(s_i)}\dot{s}_i = \bar{\omega}_{di}\dot{s}_i \tag{11}$$

Remark 3.1. *From the equation of v_{di} and ω_{di}, we can see that the speed of the robot on the path is specified by the derivative of the path parameter $\dot{s}_i$. In addition, $\dot{s}_i$ for the ith robot will be used as an additional control input for formation feedback and synchronization of path parameters in the following part.*

3.2. **Control design.** In this part, the adaptive dynamic surface control method is applied to stabilizing the error dynamics (9). Meanwhile, the derivative of the path parameter is utilized as an extra control input to synchronize the speed and path parameters. The controller design follows three steps.

3.2.1. *Kinematic control design.* In this step, the aim is to stabilize the $(x_{ei}, y_{ei}, \theta_{ei})$ dynamics. At first, we define the following variables:

$$v_{ei} = v_i - \bar{v}_i \tag{12}$$

$$\bar{\theta}_{ei} = \theta_{ei} - \bar{\theta}_i \tag{13}$$

where $\bar{v}_i$ and $\bar{\theta}_i$ are virtual controls to be specified later.

Step 1. Stabilizing the x_{ei} dynamics. Replacing the expression (12) into the first equation of (9), we obtain

$$\dot{x}_{ei} = v_{ei} + \bar{v}_i - \bar{v}_{di}\dot{s}_i\cos(\theta_{ei}) + y_{ei}\omega_i \tag{14}$$

then choosing the virtual control law $\bar{v}_i$ as

$$\bar{v}_i = -k_{1i}x_{ei} + \bar{v}_{di}\dot{s}_i\cos(\theta_{ei}) \tag{15}$$

where k_{1i} is a small positive constant. Finally we get

$$\dot{x}_{ei} = -k_{1i}x_{ei} + v_{ei} + y_{ei}\omega_i \tag{16}$$

where we have defined $\kappa_{1i} = -\bar{v}_{di}\cos(\theta_{ei})$.

Step 2. Stabilizing the y_{ei} dynamics. Define the virtual control $\bar{\theta}_i$ as

$$\bar{\theta}_i = -\arctan(k_{2i}y_{ei}) \tag{17}$$

where k_{2i} is a small positive constant.

Substituting (17) into the second equation of (8) gives

$$\dot{y}_{ei} = -\frac{k_{2i}\bar{v}_{di}\dot{s}_i y_{ei}}{\sqrt{1+k_{2i}^2y_{ei}^2}} + \Phi_i\bar{v}_{di}\dot{s}_i - x_{ei}\omega_i \tag{18}$$

where $\Phi_i = \big(\sin\big(\bar{\theta}_{ei}\big)\cos\big(\bar{\theta}_i\big) + \big(\cos\big(\bar{\theta}_{ei}\big) - 1\big)\sin\big(\bar{\theta}_i\big)\big)$.

Then we introduce two new smoothed states α_{θ_i} and α_{v_i}, which can be obtained by passing $\bar{\theta}_i$ and $\bar{v}_i$ through low-pass filters with positive time constants γ_{1i} and γ_{2i}, respectively.

$$\gamma_{1i}\dot{\alpha}_{\theta_i} + \alpha_{\theta_i} = \bar{\theta}_i, \qquad \alpha_{\theta_i}(0) = \bar{\theta}_i(0) \tag{19}$$

$$\gamma_{2i}\dot{\alpha}_{v_i} + \alpha_{v_i} = \bar{v}_i, \qquad \alpha_{v_i}(0) = \bar{v}_i(0) \tag{20}$$

In addition, we define two new error variables $\tilde{\theta}_i$ and $\tilde{v}_i$ as

$$\begin{aligned} \tilde{\theta}_i &= \alpha_{\theta_i} - \bar{\theta}_i \\ \tilde{v}_i &= \alpha_{v_i} - \bar{v}_i \end{aligned} \tag{21}$$

Furthermore, we consider a candidate of Lyapunov function as $V_{1i} = \frac{1}{2}x_{ei}^2 + \frac{1}{2}y_{ei}^2$, whose time derivative along (14) and (18) is given by

$$\dot{V}_{1i} = -k_{1i}x_{ei}^2 - \frac{k_{2i}\bar{v}_{di}\dot{s}_i y_{ei}^2}{\sqrt{1+k_{2i}^2y_{ei}^2}} + x_{ei}v_{ei} + \Phi_i\bar{v}_{di}\dot{s}_i y_{ei} \tag{22}$$

Step 3. Stabilizing the $\bar{\theta}_{ei}$ dynamic. We define the new variable:

$$\omega_{ei} = \omega_i - \bar{\omega}_i \tag{23}$$

where $\bar{\omega}_i$ is the virtual control to be specified later. Differentiating both sides (13) along the third equation of (9) and (19) gives

$$\dot{\bar{\theta}}_{ei} = \omega_i - \bar{\omega}_{di}\dot{s}_i - \dot{\alpha}_{\theta_i} + \dot{\tilde{\theta}}_i \tag{24}$$

So we can choose the virtual control $\bar{\omega}_i$ as

$$\bar{\omega}_i = -k_{3i}\bar{\theta}_{ei} + \bar{\omega}_{di}\dot{s}_i + \dot{\alpha}_{\theta_i} \tag{25}$$

where k_{3i} is also a small positive constant.

Finally we get the $\bar{\theta}_{ei}$ dynamic like

$$\dot{\bar{\theta}}_{ei} = -k_{3i}\bar{\theta}_{ei} + \omega_{ei} + \dot{\tilde{\theta}}_i \tag{26}$$

Similarly, let $\bar{\omega}_i$ pass through a first-order filter with a positive time constant γ_{3i} to obtain a new state α_{ω_i} as

$$\begin{aligned} \gamma_{3i}\dot{\alpha}_{\omega_i} + \alpha_{\omega_i} &= \bar{\omega}_i \\ \alpha_{\omega_i}(0) &= \bar{\omega}_i(0) \end{aligned} \tag{27}$$

In addition, we propose the condidate of Lyapunov function as $V_{2i} = V_{1i} + \frac{1}{2}\bar{\theta}_{ei}^2$. Differentiating V_{2i} with respect to time, we get

$$\dot{V}_{2i} = \dot{V}_{1i} - k_{3i}\bar{\theta}_{ei}^2 + \omega_{ei}\bar{\theta}_{ei} + \bar{\theta}_{ei}\dot{\tilde{\theta}}_i \tag{28}$$

Remark 3.2. *In this part, we apply the dynamic surface method to constructing a kinematic controller that stabilizes the position and heading errors of each mobile robot with respect to its own path. Then we will continue to design the dynamic controller τ_i for the dynamic model of the mobile robot with unknown parameters.*

3.2.2. *Dynamic control design.* At first, we introduce the dynamic model of the mobile robot which has been presented in Section 2.

$$\bar{M}_i\dot{\xi}_i = -\bar{C}_i(\dot{\eta}_i)\xi_i - \bar{D}_i\xi_i - \bar{\Delta}_i + \bar{B}_i\tau_i \tag{29}$$

Then we define the new variable:

$$\xi_{ei} = \xi_i - \xi_{di} \tag{30}$$

where $\xi_i = [v_i, \omega_i]^T$ and $\xi_{di} = [\alpha_{v_i}, \alpha_{w_i}]^T$, $\xi_{ei} = [\xi_{e1i}, \xi_{e2i}]^T$. Differentiating (30) along the the dynamic Equation (29), we get

$$\bar{M}_i\dot{\xi}_{ei} = -\bar{C}_i(\dot{\eta}_i)\xi_i - \bar{D}_i\xi_i - \bar{M}_i\dot{\xi}_{di} - \bar{\Delta}_i + \bar{B}_i\tau_i \tag{31}$$

And we can rewrite the linear parametrization of the expression $-\bar{C}_i(\dot{\eta}_i)\xi_i - \bar{D}_i\xi_i - \bar{M}_i\dot{\xi}_{di}$ into the new form $\Omega_i\Theta_i$, where Ω_i refers to the regressor matrix and Θ_i is the vector which contains all the unknown parameters of the mobile robot. They are given by

$$\begin{aligned} \Omega_i &= \begin{bmatrix} \omega_i^2 & -v_i & -\omega_i & -\dot{v}_{di} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -v_i\omega_i & -v_i & -\omega_i & -\dot{\omega}_{di} \end{bmatrix} \\ \Theta_i &= \begin{bmatrix} c_ib_i & \bar{d}_{11i} & \bar{d}_{12i} & \bar{m}_{1i} & c_ib_i^{-1} & \bar{d}_{21i} & \bar{d}_{22i} & \bar{m}_{2i} \end{bmatrix}^T \end{aligned} \tag{32}$$

where $\bar{d}_{11i} = \frac{d_{11i}+d_{22i}}{2}$, $\bar{d}_{12i} = \frac{b_i(d_{11i}-d_{22i})}{2}$, $\bar{d}_{21i} = \frac{d_{11i}-d_{22i}}{2b_i}$, $\bar{d}_{22i} = \frac{d_{11i}+d_{22i}}{2}$, $\bar{m}_{1i} = m_{11i}+m_{12i}$, and $\bar{m}_{2i} = m_{11i} - m_{12i}$.

So Equation (31) becomes

$$\bar{M}_i\dot{\xi}_{ei} = \Omega_i\Theta_i + \bar{B}_i\tau_i - \bar{\Delta}_i \tag{33}$$

At this step, we can design the actual control input τ_i and updated laws for the unknown system parameter vector Θ_i for each mobile robot i. So we choose

$$\tau_i = \bar{B}_i^{-1}\left[-k_{4i}\xi_{ei} - \Omega_i\hat{\Theta}_i - \hat{\kappa}_i\mathrm{sgn}(\xi_{ei})\right] \tag{34}$$

$$\dot{\hat{\Theta}}_i = K_{5i}\mathbf{proj}\left(\Omega_i^T\xi_{ei}, \hat{\Theta}_i\right) \tag{35}$$

$$\dot{\hat{\kappa}}_i = \eta_i(\|\xi_{ei}\| - \rho_i\hat{\kappa}_i) \tag{36}$$

where $\hat{\Theta}_i$ is an estimate of Θ_i, $\hat{\kappa}_i$ is an estimate of κ_i, and we define $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$, $\tilde{\kappa}_i = \kappa_i - \hat{\kappa}_i$, The gain matrix K_{5i} is symmetry, positive definite, and k_{4i} is a positive constant. And the operator **proj** is the Lipchitz continuous projection algorithm [26] as follows:

$$\begin{aligned} &\mathbf{proj}(\pi, \hat{\sigma}) = \pi, && \text{if, } \Xi(\hat{\sigma}) \le 0 \\ &\mathbf{proj}(\pi, \hat{\sigma}) = \pi, && \text{if, } \Xi(\hat{\sigma}) \ge 0 \text{ and } \Xi_{\hat{\sigma}}(\hat{\sigma}) \le 0 \\ &\mathbf{proj}(\pi, \hat{\sigma}) = (1 - \Xi(\hat{\sigma}))\pi && \text{if, } \Xi(\hat{\sigma}) > 0 \text{ and } \Xi_{\hat{\sigma}}(\hat{\sigma}) > 0 \end{aligned}$$

where $\Xi(\hat{\sigma}) = (\hat{\sigma}^2 - \sigma_M^2)\,/\,(k^2 + 2k\sigma_M)$, $\Xi_{\hat{\sigma}}(\hat{\sigma}) = \frac{\partial\Xi(\hat{\sigma})}{\partial\hat{\sigma}}$. k is an arbitrarily small positive constant. $\hat{\sigma}$ is an estimate of σ and $|\sigma| \le \sigma_M$. The projection algorithm is such that if $\dot{\hat{\sigma}} = \mathbf{proj}(\pi, \hat{\sigma})$ and $\hat{\sigma}(t_0) \le \sigma_M$ then

(a) $\hat{\sigma}(t) \leq \sigma_M + k$, $\forall 0 \ \leq t_0 \leq t \leq \infty$
(b) $\mathbf{proj}(\pi, \hat{\sigma})$ is Lipschitz continuous
(c) $|\mathbf{proj}(\pi, \hat{\sigma})| \leq \pi$
(d) $\tilde{\sigma}\mathbf{proj}(\pi, \hat{\sigma}) \geq \tilde{\sigma}\pi$ with $\tilde{\sigma} = \sigma - \hat{\sigma}$

Finally, we propose a Lyapunov function $V_{3i} = V_{2i} + \frac{1}{2}\xi_{ei}^T \bar{M}_i \xi_{ei} + \frac{1}{2}\tilde{\Theta}_i^T K_{5i}^{-1} \tilde{\Theta}_i + \frac{1}{2\eta_i}\tilde{\kappa}_i^2$. Differentiating V_{3i} along (33), (34), (35) and (36), using the property (d) of the projection algorithm results in

$$\dot{V}_{3i} \leq \dot{V}_{2i} - k_{4i}\xi_{ei}^T \xi_{ei} + \rho_i \tilde{\kappa}_i \hat{\kappa}_i \tag{37}$$

Remark 3.3. *By applying the DSC technique, our design leads to a much simpler control law than the traditional backstepping-based design. In traditional backstepping approach, the high order derivative of $\bar{\theta}_i$ and $\bar{v}_i$ have to appear in the dynamic control τ. As a result, the expression of τ will be more complicated. While the DSC method can overcome the problem of explosion of complexity.*

3.2.3. *Coordination controller design.* In this section, to achieve the control objective (6) and (7), cooperative control for multiple vehicles needs to be developed by the directed communication network over which the vehicles exchange information about path parameter s_i. To satisfy the constraints under the directed communication network, the synchronization control law for the vehicle i can only depend on its own states and the information exchanged with its neighbors.

Since not all the vehicles can obtain the value of common reference speed, some of them have to estimate it throughout the process. Let $\mathcal{N}_i$ be the set of those vehicles that are neighbors of vehicle i, and $\mathcal{N}_0$ be the set of labels of those vehicles that are neighbors of the virtual leader. In such a way, the common reference speed assigned by a virtual leader is available to only one or a subset of vehicles. So we achieve the speed distributed control. It is given by

$$\hat{v}_{ri} = -\sum_{j \in \mathcal{N}_i} a_{ij}(\hat{v}_{ri} - \hat{v}_{rj}) - \sum_{j \in \mathcal{N}_0} h_i(\hat{v}_{ri} - v_r) \tag{38}$$

where h_i is the connection weight between vehicle i and the virtual leader. $h_i = 1$ if the virtual leader is available to the ith vehicle and $h_i = 0$ otherwise.

Suppose that the communication topology between the n followers and the virtual leader can be represented by an extended directed graph $\bar{\mathcal{G}} = \left(\bar{\mathcal{V}}, \bar{\mathcal{E}}\right)$, where $= 0, 1, \ldots, n$. A directed graph among the n robots is defined as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ related to $\mathcal{G}$ is defined as $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$; otherwise, $a_{ij} = 0$. The new Laplacian matrix $\bar{L}$ is given as

$$\bar{L} = \begin{bmatrix} 0 & 0_{1\times N} \\ -h & L + H \end{bmatrix}$$

where $h = [h_1, \ldots, h_n]^T$, and $H = \mathrm{diag}[h_1, \ldots, h_n]^T$.

Lemma 3.1. [27]. *For the nonsingular M-matrix $L + H$, there exists a diagonal matrix $G = \mathrm{diag}(g_1, \ldots, g_n)$ with $g_i > 0$, $i = 1, \ldots, n$, such that $G(L + H) + (L + H)^T G > 0$. And the positive definite matrix G can be given with $[g_1, \ldots, g_n]^T = L^{-1}\mathbf{1}$.*

And the $\hat{v}_r = [\hat{v}_{r1}, \ldots, \hat{v}_{rn}]^T \in \mathbb{R}^n$, then we define $\tilde{v}_r = \hat{v}_r - v_r\mathbf{1}_n$, then we have the following structure

$$\dot{\tilde{v}}_r = -(L + H)\tilde{v}_r \tag{39}$$

Since then, all the vehicles can directly or indirectly get the information of the reference speed. Then for the convenience of the cooperative controller design, we define the variable

$\mu_i = \dot{s}_i - \hat{v}_{ri}$, and then the dynamics of the coordinated variable can be written as

$$\begin{aligned} \dot{\mu} &= u \\ \dot{s} &= \mu + \hat{v}_r \end{aligned} \tag{40}$$

where $\mu = [\mu_1, \ldots, \mu_n]^T$ and $s = [s_1, \ldots, s_n]^T$.

Now the coordination control objective can be stated as finding a control law u to make $s_i = s_j$. In this paper, we suppose that the communication topology between the vehicles is represented by a directed graph that has at least one globally reachable vertex. With this assumption, then there exists a nonsingular matrix F, defined as [28]

$$F = [1, F_1], \qquad F^{-1} = \left[\beta^T, F_2\right]^T \tag{41}$$

such that

$$F^{-1}LF = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0}^T & L_{11} \end{bmatrix} \tag{42}$$

where $\mathbf{0} = [0, \ldots, 0]^T$ is a vector of appropriated dimensions. β is a left eigenvector associated with the 0 eigenvalue, and L_{11} is nonsingular and has the property that all its eigenvalues have positive real parts. It can be easily concluded that

$$F_2\mathbf{1} = \mathbf{0}, \qquad L = F_1 L_{11} F_2 \tag{43}$$

Define the cooperation error $\tilde{s} = F_2 s \in \mathbb{R}^{n-1}$. Given the properties of $F_2\mathbf{1} = \mathbf{0}$, then $\tilde{s} = 0$ is equivalent to $s_i = s_j$, $\forall i, j$. The dynamic (40) can be rewritten as

$$\begin{aligned} \dot{\mu} &= u \\ \dot{\tilde{s}} &= F_2\mu \end{aligned} \tag{44}$$

Considering the coordination dynamics (40), we propose the following coordination control law as

$$u_i = -p\mu_i - q\sum_{j\in\mathcal{N}_i}(s_i - s_j) \tag{45}$$

where p, q are positive control gains to be designed.

Proposition 3.1. *Consider the coordination dynamics described by (40). Suppose the communication graph has at least one globally reachable vertex. When applying the control law (45) to synchronizing the path parameters, then the control objective (7) is achieved if the parameters $p > 0$, $q > 0$ are chosen such that*

$$\frac{p^2}{q} > \max_{0\neq\varsigma\in\sigma(L)}\left(\frac{\mathrm{Im}(\varsigma)^2}{\mathrm{Re}(\varsigma)}\right) \tag{46}$$

where $\sigma(\cdot)$ stands for the spectrum of a matrix and $\mathrm{Im}(\cdot)$ and $\mathrm{Re}(\cdot)$ denote the imaginary and real part of a complex number, respectively.

Proof: The control law can be written in vector form as

$$u = -P\mu - QLs \tag{47}$$

where $P = pI$ and $Q = qI$. Then the closed-loop system is given by

$$\begin{aligned} \dot{\mu} &= -P\mu - QF_1L_{11}\tilde{s} \\ \dot{\tilde{s}} &= F_2\mu \end{aligned} \tag{48}$$

We show that the closed-loop matrix

$$A_{cl} = \begin{bmatrix} -P & -QF_1L_{11} \\ F_2 & 0 \end{bmatrix}$$

is a stability matrix. Using a similarly transformation $\text{diag}(I, F)$, it can be shown that $\lambda = -p$ is an eigenvalue of A_{cl} and the rest of the eigenvalues are given by the roots of $\det(\lambda(\lambda + p)I + qL_{11}) = 0$. Let $\varsigma \in \sigma(L_{11})$ with $Re(\varsigma) > 0$ be an arbitrary eignvaule of L_{11}. From the last equation it follows that $\varsigma = -\lambda(\lambda + p)/q$. Obviously, to every ς there correspond two possible eignvalues which have negative real part if p and q satisfy (45).

3.3. **Stability analysis.** In this section, we shall analyze the stability of the closed system. At first we define the new variable $\tilde{\omega}_i$ as

$$\tilde{\omega}_i = \alpha_{\omega_i} - \bar{\omega} \tag{49}$$

Together with Equation (21), we have three new variable $\tilde{\theta}_i$, $\tilde{v}_i$, and $\tilde{\omega}_i$ as

$$\tilde{\theta}_i = \alpha_{\theta_i} - \bar{\theta}_i, \quad \tilde{v}_i = \alpha_{v_i} - \bar{v}_i, \quad \tilde{\omega}_i = \alpha_{\omega_i} - \bar{\omega} \tag{50}$$

Differentiating the variables, we have the following relationship

$$\begin{aligned}
\dot{\tilde{\theta}}_i &= -\frac{\tilde{\theta}_i}{\gamma_{1i}} + \Pi_{1i}\left(x_{ei}, y_{ei}, \bar{\theta}_{ei}, \Psi_i, \xi_{ei}, \tilde{\theta}_i, \tilde{v}_i, \eta_{di}, \eta'_{di}, \eta''_{di}\right) \\
\dot{\tilde{v}}_i &= -\frac{\tilde{v}_i}{\gamma_{2i}} + \Pi_{2i}\left(x_{ei}, y_{ei}, \bar{\theta}_{ei}, \Psi_i, \xi_{ei}, \tilde{\theta}_i, \tilde{v}_i, \eta_{di}, \eta'_{di}, \eta''_{di}\right) \\
\dot{\tilde{\omega}}_i &= -\frac{\tilde{\omega}_i}{\gamma_{3i}} + \Pi_{2i}\left(x_{ei}, y_{ei}, \bar{\theta}_{ei}, \Psi_i, \xi_{ei}, \tilde{\theta}_i, \tilde{v}_i, \tilde{\omega}_i, \eta_{di}, \eta'_{di}, \eta''_{di}\right)
\end{aligned} \tag{51}$$

where $\eta'_{di} = \partial(\eta_{di}(s_i))/\partial s_i$ and $\eta''_{di} = \partial^2(\eta_{di}(s_i))/\partial s_i^2$.

We now state the main result for the formation tracking control problem in the following theorem.

Theorem 3.1. *Consider a network of non-holonomic mobile robots with the dynamic given by (2). Suppose the communication graph has at least one globally reachable vertex. And the proposed system satisfies Assumptions 2.1-2.4; then for any initial conditions satisfying $V(0) \le \delta$, where δ is any positive constant, under the control input (34), adaptive control law (35), (36), the coordinated control law (45) and distributed speed update law (38), all signals in the closed-loop systems are UUB. Finally, we solve the control objective (5), (6) and (7).*

Proof: Consider the following Lyapunov function candidate

$$V = \sum_{i=1}^{n}\left(V_{3i} + \frac{1}{2}\tilde{\theta}_i^2 + \frac{1}{2}\tilde{v}_i^2 + \frac{1}{2}\tilde{\omega}_i^2\right) + \frac{1}{2}\tilde{v}_r^T G v_r \tag{52}$$

Then differentiating (52) with respect to tome and substituting (37), (39) and (51) yields

$$\begin{aligned}
\dot{V} \le \sum_{i=1}^{n} &\left(-k_{1i}x_{ei}^2 - \frac{k_{2i}\bar{v}_{di}\dot{s}_i y_{ei}^2}{\sqrt{1 + k_{2i}^2 y_{ei}^2}} - k_{3i}\bar{\theta}_{ei}^2 - k_{4i}\xi_{ei}^T\xi_{ei} - \frac{\tilde{\theta}_i^2}{\gamma_{1i}} - \frac{\tilde{v}_i^2}{\gamma_{2i}} - \frac{\tilde{\omega}_i^2}{\gamma_{3i}} + x_{ei}v_{ei}\right. \\
&\left. + \Phi_i\bar{v}_{di}\dot{s}_i y_{ei} + \omega_{ei}\bar{\theta}_{ei} + \bar{\theta}_{ei}\dot{\tilde{\theta}}_i + \tilde{\theta}_i\Pi_{1i} + \tilde{v}_i\Pi_{2i} + \tilde{\omega}_i\Pi_{3i} + \rho_i\tilde{\kappa}_i\hat{\kappa}_i\right) \\
&- \tilde{v}_r(GL + L^T G)v_r
\end{aligned} \tag{53}$$

Then based on Assumption 2.3 and the condition $V_0 \le \delta$, we can consider the compact set as follows

$$\Lambda_{1i} = \left\{(\eta_{di}, \eta'_{di}, \eta''_{di}) : \|\eta_{di}\|^2 + \|\eta'_{di}\|^2 + \|\eta''_{di}\|^2 \le \varepsilon\right\}$$

$$\Lambda_{2i} = \left\{ \sum_{j=1}^{i} x_{ej}^2 + y_{ej}^2 + \bar{\theta}_{ej}^2 + \Psi_j^2 + \xi_{ej}^2 + \tilde{\theta}_j^2 + \tilde{v}_j^2 \le 2p \right\}$$
$$\Lambda_{3i} = \left\{ \sum_{j=1}^{i} x_{ej}^2 + y_{ej}^2 + \bar{\theta}_{ej}^2 + \Psi_j^2 + \xi_{ej}^2 + \tilde{\theta}_j^2 + \tilde{v}_j^2 + \tilde{\omega}_j^2 \le 2p \right\}$$

where ε and δ are positive constants. There exist positive constants c_{1i}, c_{2i} and c_{3i} such that $\|\Pi_{1i}\| \le c_{1i}$ on $\Lambda_{1i} \times \Lambda_{2i}$, $\|\Pi_{2i}\| \le c_{2i}$ on $\Lambda_{1i} \times \Lambda_{2i}$, and $\|\Pi_{3i}\| \le c_{3i}$ on $\Lambda_{1i} \times \Lambda_{3i}$. In addition, from the above details, we know $\bar{v}_{di}$ and $\dot{s}_i$ are all bounded functions, and then define $|\bar{v}_{di}\dot{s}_i| \le \zeta$, where ζ is a positive constant. Furthermore, $v_{ei} = \xi_{e1i} + \tilde{v}_i$ and $\omega_{ei} = \xi_{e2i} + \tilde{\omega}_i$. Use Young's inequality.

$$\begin{aligned}
&|\tilde{\theta}\Pi_{1i}| \le \frac{1}{2}\tilde{\theta}_i^2 + \frac{1}{2}c_{1i}^2 \\
&|\tilde{v}_i\Pi_{2i}| \le \frac{1}{2}\tilde{v}_i^2 + \frac{1}{2}c_{2i}^2 \\
&|\tilde{\omega}_i\Pi_{3i}| \le \frac{1}{2}\tilde{\omega}_i^2 + \frac{1}{2}c_{3i}^2 \\
&|x_{ei}v_{ei}| \le x_{ei}^2 + \frac{1}{2}\tilde{v}_i^2 + \frac{1}{2}\xi_{ei}^2 \\
&|\Phi_i\bar{v}_{di}\dot{s}_i y_{ei}| \le \frac{1}{2}\zeta^2 y_{ei}^2 + \frac{1}{2}\Phi_i^2 \\
&|\omega_{ei}\bar{\theta}_{ei}| \le \bar{\theta}_{ei}^2 + \frac{1}{2}\tilde{\omega}_i^2 + \frac{1}{2}\xi_{ei}^2 \\
&|\bar{\theta}_{ei}\dot{\tilde{\theta}}_i| \le \frac{\gamma_{1i}+1}{2\gamma_{1i}}\bar{\theta}_{ei}^2 + \frac{\tilde{\theta}_i^2}{2\gamma_{1i}} + \frac{1}{2}c_{1i}^2 \\
&|\tilde{\kappa}_i\hat{\kappa}_i| \le \frac{1}{2}\kappa_i^2 - \frac{1}{2}\tilde{\kappa}_i^2
\end{aligned} \tag{54}$$

Substituting Equation (54) into (53), we get

$$\begin{aligned}
\dot{V} \le \sum_{i=1}^{n} \Bigg[&-(k_{1i}-1)x_{ei}^2 - \left(\frac{k_{2i}\zeta}{\Delta_i} - \frac{\zeta^2}{2}\right) y_{ei}^2 - \left(k_{3i} - \frac{3\gamma_{1i}+1}{2\gamma_{1i}}\right)\bar{\theta}_{ei}^2 - (k_{4i}-1)\xi_{ei}^T\xi_{ei} \\
&- \frac{1}{2}\rho_i\tilde{\kappa}_i^2 - \left(\frac{1-\gamma_{1i}}{2\gamma_{1i}}\right)\tilde{\theta}_i^2 - \left(\frac{1-\gamma_{2i}}{\gamma_{2i}}\right)\tilde{v}_i^2 - \left(\frac{1-\gamma_{3i}}{\gamma_{3i}}\right)\tilde{\omega}_i^2 + \Upsilon_{1i} \Bigg]
\end{aligned} \tag{55}$$

where $\Upsilon_i = c_{1i}^2 + \frac{1}{2}c_{2i}^2 + \frac{1}{2}c_{3i}^2 + \frac{1}{2}\rho_i\kappa_i^2 + 1$. By assigning $k_{1i}^* = k_{1i} - 1$, $k_{2i}^* = \frac{k_{2i}\zeta}{\Delta_i} - \frac{\zeta^2}{2}$, $k_{3i}^* = k_{3i} - \frac{3\gamma_{1i}+1}{2\gamma_{1i}}$, $k_{4i}^* = \frac{k_{4i}-1}{\lambda_{\max}M_i}$, $\gamma_{1i}^* = \frac{1-\gamma_{1i}}{2\gamma_{1i}}$, $\gamma_{2i}^* = \frac{1-\gamma_{2i}}{\gamma_{2i}}$, $\gamma_{3i}^* = \frac{1-\gamma_{3i}}{\gamma_{3i}}$, and then substituting them into (55), it leads to

$$\dot{V} \le -2\beta(V - V_\delta) + \Upsilon \tag{56}$$

where $\beta = \min\left(k_{1i}^*, k_{2i}^*, k_{3i}^*, k_{4i}^*, \gamma_{1i}^*, \gamma_{2i}^*, \gamma_{3i}^*, \varepsilon_s\right) > 0$, $V_\delta = \frac{1}{2}\tilde{v}_r^T G v_r + \frac{1}{2}\tilde{\Theta}_i^T K_{5i}^{-1}\tilde{\Theta}_i$ and $\Upsilon = \sum_{i=1}^{n}\left(c_{1i}^2 + \frac{1}{2}c_{2i}^2 + \frac{1}{2}c_{3i}^2 + 1 + \frac{1}{2}\tilde{\Theta}_i^T K_{5i}^{-1}\tilde{\Theta}_i\right)$.

It is noted that (56) implies $\dot{V} \le 0$ when $\beta > \dfrac{\Upsilon}{2p}$. And solving (56) gives to

$$V \le \frac{\Upsilon}{2\beta} + \left[V(0) - \frac{\Upsilon}{2\beta}\right] e^{-2\beta t} \tag{57}$$

It shows that V is uniformly ultimately bounded. Accordingly, all error signals in the closed-loop system are semi-globally uniformly ultimately bounded. So the control objective (5), (6) and (7) are satisfied, where the error bound $\|\epsilon_{1i}\| \leq \sqrt{\Upsilon/\beta}$ and $\|\epsilon_{2i}\| \leq \sqrt{\Upsilon/\beta}$

Remark 3.4. *By adjusting the design parameters k_{1i}, k_{2i}, k_{3i}, k_{4i}, γ_{1i}, γ_{2i}, γ_{3i}, ε_s, the error set $\sqrt{\Upsilon/\beta}$ in the closed-loop system can be made arbitrarily small. Here are some suggestions: (i) increasing k_{1i}, k_{2i}, k_{3i}, k_{4i}, ε_s and decreasing γ_{1i}, γ_{2i}, γ_{3i} to increase β, subsequently reduces the bound Υ/β; (ii) decreasing K_{5i} helps to decrease Υ, and then reduce Υ/β.*

4. **Simulation Results.** To illustrate the effectiveness of our formation path following controller, we run some simulations in this section. These four mobile robots assemble and maintain an in-line formation, which means that these four robots are requested to follow the desired path with respect to the path parameter s_i by having them aligned along a common vertical line. We assume that these four robots are identical, the nominal parameters of the EtsRo mobile robots taken from [10]. The physical parameters are given by:

$$\begin{array}{llll} r_i = 0.04 & b_i = 0.1 & a_i = 0.02 & m_{ci} = 2.3 \\ m_{wi} = 0.28 & I_{ci} = 0.01 & I_{wi} = 0.0056 & I_{mi} = 0.002 \end{array}$$

And the control gains, the initial conditions and the parameters involved to update the path parameters are as follows: $k_{1i} = 2$, $k_{2i} = 3$, $k_{3i} = 1.5$, $k_{4i} = 3$, $K_{5i} = \mathrm{diag}[0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1]$, $v_r = 0.6$, $\gamma_{1i} = \gamma_{2i} = \gamma_{3i} = 0.01$, $\eta_i = 0.001$, $\rho_i = 1$, $s_i(0) = 0$, $p = 2$, $q = 1$, $H = \mathrm{diag}[1, 0, 0, 0]$, $\eta_1(0) = [-8, 5, 0]^T$, $\eta_2(0) = [-7, 1.5, \pi/2]^T$, $\eta_3(0) = [-5.5, -1, \pi/4]^T$ and $\eta_4(0) = [-6, -6, \pi/2]^T$.

The reference path of the center of the virtual structure is chosen as $\Gamma_0(s_0) = [s_0 - 3.5, \cos(s_0)]^T$. The distance from the vehicles to the center of the virtual structure are chosen as $l_1(x_{d0}(s_1), y_{d0}(s_1)) = \left(\frac{-2\sin(s_1)}{\sqrt{1+\sin^2(s_1)}}, \frac{2}{\sqrt{1+\sin^2(s_1)}}\right)$, $l_2(x_{d0}(s_2), y_{d0}(s_2)) = (0, 0)$, $l_3(x_{d0}(s_3), y_{d0}(s_3)) = \left(\frac{2\sin(s_3)}{\sqrt{1+\sin^2(s_3)}}, \frac{-2}{\sqrt{1+\sin^2(s_3)}}\right)$ and $l_4(x_{d0}(s_4), y_{d0}(s_4)) = \left(\frac{4\sin(s_4)}{\sqrt{1+\sin^2(s_4)}}, \frac{-4}{\sqrt{1+\sin^2(s_4)}}\right)$.

And we assume the system parameters are unknown. The communication directed graph is given as in Figure 3. In Figure 4, it shows that the mobile robots are successfully towards their desired paths and the vehicles ultimately attain their desired formation configuration. And the path tracking and path parameter errors are shown in Figure 5, Figure 6, Figure 7, and Figure 8. It can be shown that all the errors converge to a small neighborhood around the origin. It is clear that all the signals are ultimately bounded.

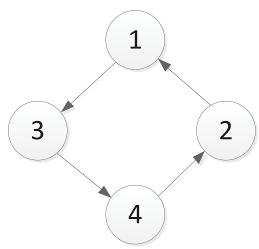

FIGURE 3. The communication graph

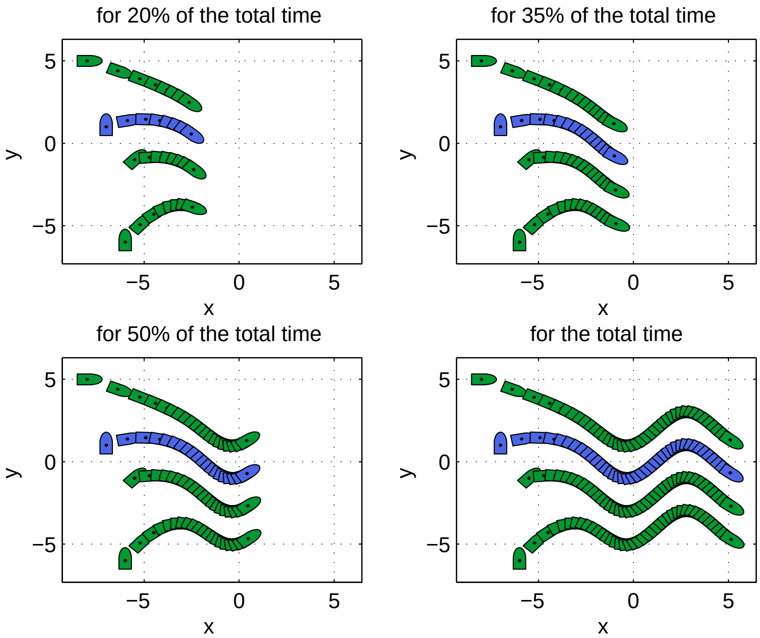

FIGURE 4. Robot position and orientation

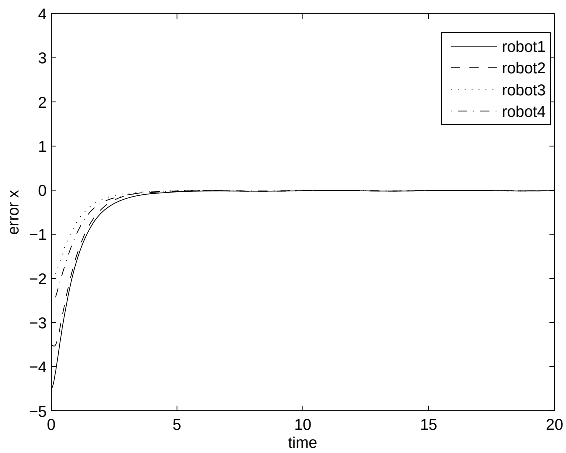

FIGURE 5. Path following errors x_{ei}

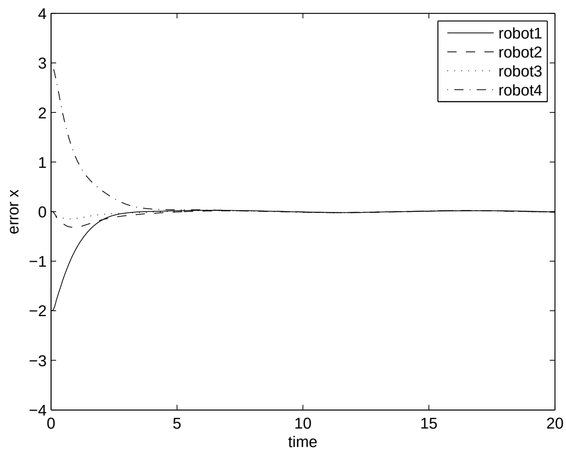

FIGURE 6. Path following errors y_{ei}

5. **Conclusion.** This paper has addressed the distributed cooperative path following control of multi-robot systems with unknown uncertainties under general directed graphs. The cooperative path following controllers are devised on the adaptive DSC technique, the coordinated protocol, and the distributed velocity estimator. It has been shown that the closed-loop signals are UUB and the compact set of the error can be made arbitrarily small by choosing the control parameters appropriately. In addition, simulation results have demonstrated the effectiveness of the method. From a multiple vehicle point of view, problems revolving around communication among vehicles, such as switching topologies (time-varying network topologies) and time delays, pose many challenging problems that deserve further research.

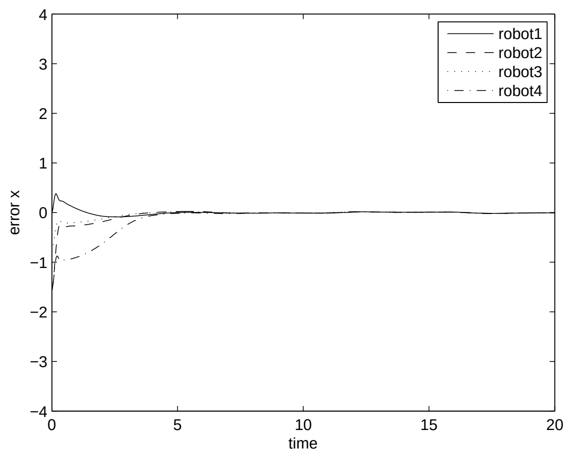

FIGURE 7. Path following errors θ_{ei}

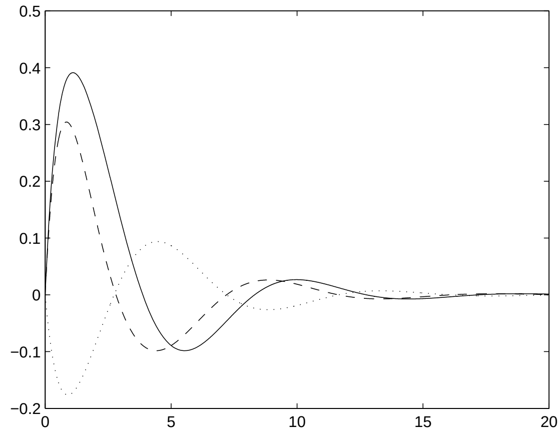

FIGURE 8. Path parameter error

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