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STABILITY OF INTELLIGENT AUTOMATIC CONTROL SYSTEMS

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Abstract. *In the present article, the authors develop a convenient engineering technique for the analysis and synthesis of automatic control systems with fuzzy controllers, developed on the basis of Yakubovich's method for the study of absolute stability in nonlinear multichannel systems. Modification of this technique for a simplified model of fuzzy PID controller (fuzzy P controller) is proposed. Here is a detailed mathematical foundation of criteria for fuzzy systems in general, and for the fuzzy P controller offered graphical-analytical method. The application of the developed methods is illustrated by various examples.*

Keywords: Fuzzy controllers, Nonlinear transformations in fuzzy controller, Stability of fuzzy control systems, Absolute stability of the equilibrium position, Absolute stability of processes, Intelligent automatic control systems

1. **Introduction.** In recent years, advances in engineering have allowed for relatively active introduction of fuzzy controllers into a variety of industrial systems. By comparison to these practical improvements, however, the development of the supporting theoretical basis is lagging behind. Development of such theory is vital if efficient progress is to be made with the analysis and synthesis of fuzzy controllers. Specialists are well aware that, since there are certain problems with the tuning of classic PID controllers, which have only three variable parameters, there are many more issues with fuzzy PID controllers, due to the theoretically far greater number of variable settings. However, the advantage of using this class of systems, as shown by the authors in [13,14,18,19], is obvious.

Over the last two decades, plenty of attention has been devoted to questions surrounding the investigation of fuzzy control systems [1-4]. Amongst the resulting work in the stability investigation, that of Tanaka [5-10] is the most noteworthy for its expansion upon both V. M. Popov's method and Lyapunov's second method.

In the present article, the authors outline a convenient engineering technique for the analysis and synthesis of automatic control systems (ACSs) with fuzzy controllers, developed on the basis of Yakubovich's method for the study of absolute stability in nonlinear multichannel systems [11], which is well-known in the literature. The main advantage of this technique is its simplicity and applicability to the fuzzy system with any number of control channels.

The proposed approach is based on the following basic provisions [12,13]:

– the transformations carried out in fuzzy controllers (FCs) are nonlinear in essence;

– by definition, the nonlinear transformations realized by FCs depend on individual FCs' settings, including the number of input and output terms, as well as the form and relative placement of membership functions;

– the creation of the FC, including the form of logical and linguistic models, and the organization of follow-up processing, is performed according to one of the models proposed by Mamdani, Larsen, Tsukamoto, and Sugeno [14];

– preliminary analysis shows that varying the model used has no significant impact on the character of the nonlinear transformations.

The relevance of the study is also due to the fact that the fuzzy inference technology makes it possible to provide a parallel interpretation of knowledge through specialized hardware with high performance, which makes fuzzy logic extremely perspective for development of intelligent fuzzy controllers for high-speed automatic control systems (ACS), operating under the impact of various uncertainties.

2. **Notation.** ACS – automatic control system; NE – nonlinear element; FO – fuzzy output; FC – fuzzy controller.

$W_{\mathrm{co}}(S)$ – transfer function of the control object; $W_{LP}(s)$ – transfer matrix of $h \times h$ linear parts in the ACS circuits; σ, ξ – nonlinear elements' input and output signals; h – the number of NE; $\varphi(\sigma)$ – transformation characteristics of the NE, nonlinear characteristics; $\beta_1, \ldots, \beta_h$ – sector boundaries, containing the characteristics of the nonlinear transformations realized by blocks FO_i; $\mu_d^{-1} = diag\left(\mu_1^{-1}, \ldots, \mu_h^{-1}\right)$ – diagonal matrix with diagonal elements $\mu_1^{-1}, \ldots, \mu_h^{-1}$; $\tau_d = diag(\tau_1, \ldots, \tau_h)$, $\vartheta_d = diag(\vartheta_1, \ldots, \vartheta_h)$ – diagonal matrices; ω – frequency; γ – boundary of the sector containing steep part of the characteristic of the nonlinear transformations implemented by blocks FO_i; μ_l – large angle of linear sector of approximated nonlinear transformation; μ_s – small angle of linear sector of approximated nonlinear transformation.

3. **Investigating the Absolute Stability of the Equilibrium Position of an ACS with Type-1 Fuzzy Controller.** Figure 1(a) shows the initial structure of an ACS with fuzzy PID controller, and Figure 1(b) shows the post-transform structure. Here, we see that the transfer function of the control object $W_{\mathrm{co}}(S)$ is introduced into all the separate parallel circuits. This structure for an ACS matches that of multi-channel nonlinear systems, for which absolute stability criteria can be modified as considered in [11].

The equations describing this sort of nonlinear ACS have the form

$$\sigma(s) = -W_{LP}(s)\xi(s), \tag{1}$$

$$\xi = \varphi(\sigma), \tag{2}$$

where the scalar vectors of the nonlinear elements' input and output signals are $\sigma = (\sigma_j)_{j=1}^{h}$, $\xi = (\xi_j)_{j=1}^{h}$, h is the number of NE_j, and $W_{LP}(s)$ is the transfer matrix of $h \times h$ linear parts in the ACS circuits. Equation (1) describes the transformation in the linear stationary part of the system, and Equation (2) defines the transformation characteristics of the nonlinear elements NE_j.

It should be noted that the nonlinear characteristics $\varphi_j(\sigma_j)$, realized by fuzzy calculators, are limited in amplitude. As such, if $\sigma_j \to \infty$, the lower boundary of the sector can be equated to zero. It follows that

$$0 \le \frac{\varphi_j(\sigma_j)}{\sigma_j} = \mu_j \le \beta_j, \quad j = 1, \ldots, h \tag{3}$$

if $\sigma \neq 0$ and $\varphi(0) = 0$, or alternatively

$$(\beta\sigma(t) - \varphi(\sigma, t))\varphi(\sigma, t) \ge 0 \tag{3a}$$

Based on the results of [12], condition (3) can be represented as seen in Figure 1(c), which shows that the nonlinear characteristic $\varphi_j(\sigma_j)$ lies within the sector $[0, \beta_j]$.

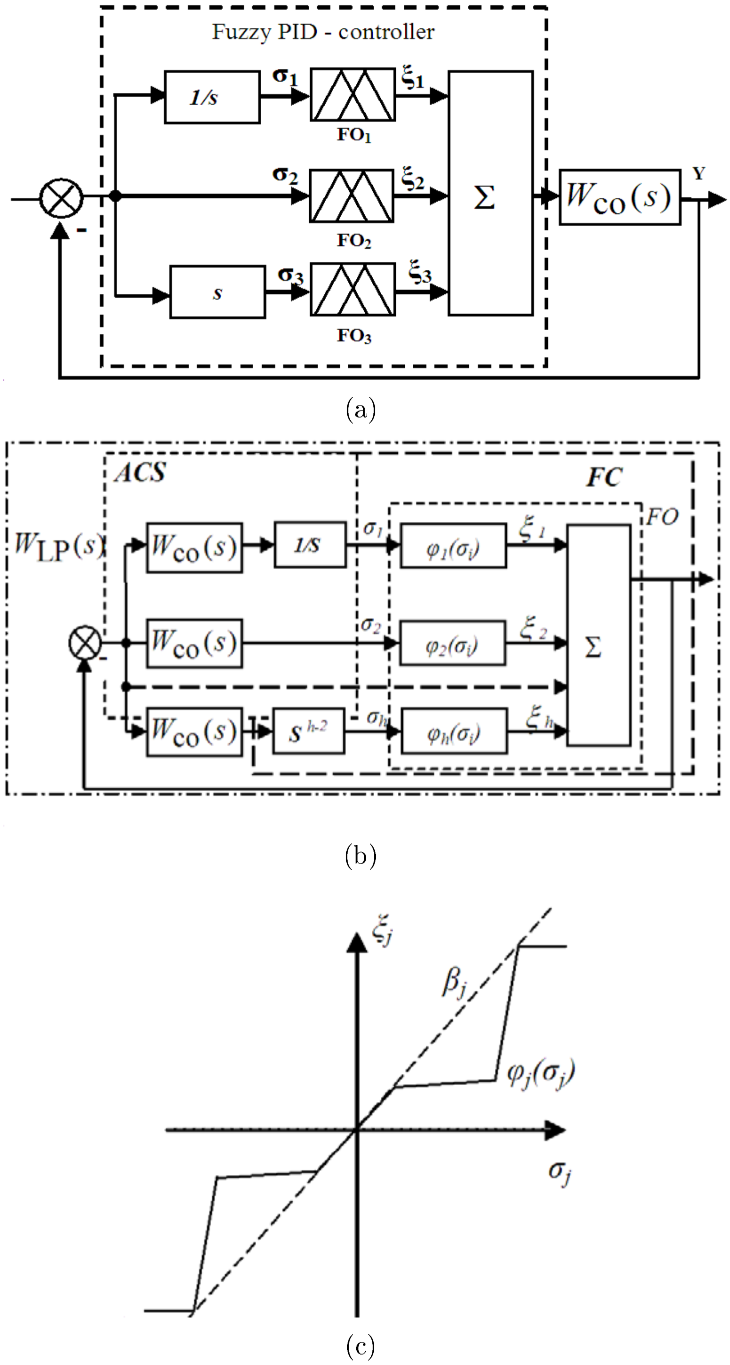


FIGURE 1. (a) Block diagram of an ACS with fuzzy PID-regulator, (b) block diagram of post-transformation ACS with fuzzy controller, and (c) the nonlinear characteristic $\varphi_j(\sigma_j)$ as represented by a fuzzy calculator

System (1), or the position of equilibrium of system (1) is absolutely stable in the angle (sector) $[0, \beta_j]$ if the zero solution of system (1) is asymptotically stable in the whole for any nonlinear function $\varphi_j(\sigma_j)$ satisfying the condition (3) [15-17].

If, during tuning of a fuzzy controller, a fuzzy calculator realizes the nonlinear transformation $\varphi_j(\sigma_j)$ but fails to satisfy condition (3), additional structural changes become necessary. Naturally, to maintain the condition of equivalence of both initial and transformed structures, the linear part of the system requires corresponding amendments.

For an ACS with FC, the absolute stability criteria of the equilibrium position can be expressed as follows.

Let the equation of the linear part of the ACS take the form (1), and the nonlinear characteristics $\varphi_j(\sigma_j)$ of the fuzzy controller corresponding to (2), satisfy condition (3). Let all the poles of the matrix elements $W_{LP}(s)$ be situated on the left-hand half-space, or have one pole on the imaginary axis (with stable or neutral linear parts in every circuit). We introduce the diagonal matrix $\mu_d^{-1} = diag\left(\mu_1^{-1}, \dots, \mu_h^{-1}\right)$ with diagonal elements $\mu_1^{-1}, \dots, \mu_h^{-1}$, (where $\mu_j^{-1} = 0$, if $\mu_j = \infty$), and also the diagonal matrices $\tau_d = diag(\tau_1, \dots, \tau_h), \vartheta_d = diag(\vartheta_1, \dots, \vartheta_h)$, where all $\tau_d > 0$. Let us suppose that for some $\tau_j > 0$, ϑ_j and that for all $-\infty < \omega < +\infty$, except where $\omega = 0$, the following relations are performed

$$\begin{aligned} &\det\left\{\tau_d\mu_d^{-1} + \mathrm{Re}\left[(\tau_d + j\omega\vartheta_d)\, W_{LP}(j\omega)\right]\right\} \neq 0, \\ &\text{where } -\infty \le \omega \le +\infty,\ \omega \neq 0, \\ &\tau_d\mu_d^{-1} + \mathrm{Re}\left\{\vartheta_d \lim_{\omega\to\infty} j\omega[W_{LP}(j\omega)]\right\} > 0. \end{aligned} \tag{4}$$

The $h\times h$-matrix on the left side of inequality (4) is positively defined, i.e., all its principal minors are positive. The ACS with FC combination in question is thus asymptotically stable on the whole.

Here is inequality (4) in explicit form:

$$\det\left\{\begin{bmatrix} \tau_1\mu_1^{-1} & 0 & \cdots & 0 \\ 0 & \tau_2\mu_2^{-1} & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & \tau_h\mu_h^{-1} \end{bmatrix} + \mathrm{Re}\left(\begin{vmatrix} \tau_2 + j\omega\vartheta_2 & 0 & \cdots & 0 \\ 0 & \tau_2 + j\omega\vartheta_2 & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & \tau_h + j\omega\vartheta_h \end{vmatrix} W_{LP}(j\omega)\right)\right\} \neq 0 \tag{5}$$

and

$$\begin{bmatrix} \tau_1\mu_1^{-1} & 0 & \cdots & 0 \\ 0 & \tau_2\mu_2^{-1} & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & \tau_h\mu_h^{-1} \end{bmatrix} + \mathrm{Re}\left(\begin{bmatrix} \vartheta_1 & 0 & \cdots & 0 \\ 0 & \vartheta_2 & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & \vartheta_h \end{bmatrix} \lim_{\omega\to\infty} j\omega W_{LP}(j\omega)\right) > 0 \tag{6}$$

where

$$W_{LP}(j\omega) = \begin{bmatrix} \dfrac{W_{\mathrm{co}}(j\omega)}{j\omega} & \dfrac{W_{\mathrm{co}}(j\omega)}{j\omega} & \cdots & \dfrac{W_{\mathrm{co}}(j\omega)}{j\omega} \\ W_{\mathrm{co}}(j\omega) & W_{\mathrm{co}}(j\omega) & \cdots & W_{\mathrm{co}}(j\omega) \\ \cdots & \cdots & \cdots & \cdots \\ j\omega^{h-2}W_{\mathrm{co}}(j\omega) & j\omega^{h-2}W_{\mathrm{co}}(j\omega) & \cdots & j\omega^{h-2}W_{\mathrm{co}}(j\omega) \end{bmatrix}. \tag{7}$$

Calculating the determinant (5) where $\tau_j > 0$, we obtain

$$\begin{aligned} &1 + \beta_1\mathrm{Re}\left(\frac{1}{j\omega}W_{\mathrm{co}}(j\omega)\right) + \vartheta_1\beta_1 j\omega\mathrm{Re}\frac{1}{j\omega}W_{\mathrm{co}}(j\omega) + \beta_2\mathrm{Re}W_{\mathrm{co}}(j\omega) \\ &+ \vartheta_2\beta_2\mathrm{Re}(j\omega W_{\mathrm{co}}(j\omega)) + \ldots + \beta_h\mathrm{Re}\left((j\omega)^{h-2}W_{\mathrm{co}}(j\omega)\right) \\ &+ \vartheta_h\beta_h j\omega\mathrm{Re}\left((j\omega)^{h-2}W_{\mathrm{co}}(j\omega)\right) \neq 0, \quad -\infty < \omega < +\infty, \quad \omega \neq 0, \end{aligned} \tag{8}$$

where $\beta_1, \ldots, \beta_h$ are the sector boundaries, containing the characteristics of the nonlinear transformations realized by blocks FO_i $(i = 1, \ldots, h)$ in the structure of the fuzzy controller. If the principal minors of the matrix are calculated on the left-hand side of inequality (6) and checked for positivity, provided that $\beta_i > 0$, and $i = 1, \ldots, h$, we obtain the following system of inequalities:

$$\begin{cases} 1 + \vartheta_1\beta_1 \text{Re}\left(\lim\limits_{\omega\to\infty} W_{\text{co}}(j\omega)\right) > 0, \\ 1 + \vartheta_1\beta_1 \text{Re}\left(\lim\limits_{\omega\to\infty} W_{\text{co}}(j\omega)\right) + \vartheta_2\beta_2 \text{Re}\left(\lim\limits_{\omega\to\infty} j\omega W_{\text{co}}(j\omega)\right) > 0, \\ 1 + \vartheta_1\beta_1 \text{Re}\left(\lim\limits_{\omega\to\infty} W_{\text{co}}(j\omega)\right) + \vartheta_2\beta_2 \text{Re}\left(\lim\limits_{\omega\to\infty} j\omega W_{\text{co}}(j\omega)\right) \\ \quad + \vartheta_3\beta_3 \text{Re}\left(\lim\limits_{\omega\to\infty} (j\omega)^2 W_{\text{co}}(j\omega)\right) > 0, \\ \ldots \\ 1 + \vartheta_1\beta_1 \text{Re}\left(\lim\limits_{\omega\to\infty} W_{\text{co}}(j\omega)\right) + \vartheta_2\beta_2 \text{Re}\left(\lim\limits_{\omega\to\infty} j\omega W_{\text{co}}(j\omega)\right) \\ \quad + \ldots + \vartheta_h\beta_h \text{Re}\left(\lim\limits_{\omega\to\infty} (j\omega)^{h-1} W_{\text{co}}(j\omega)\right) > 0. \end{cases} \tag{9}$$

As an example, let us investigate the absolute stability of the equilibrium position of an ACS (Figure 1(a)) with a fuzzy PID controller and a third-order industrial control object: $W_{\text{co}}(s) = 15\left[(s+1)(0.5s+1)(0.1s+1)\right]^{-1}$. The fuzzy controller's settings are presented in Figure 2 for FO_1, in Figure 3 for FO_2, and in Figure 4 for FO_3.

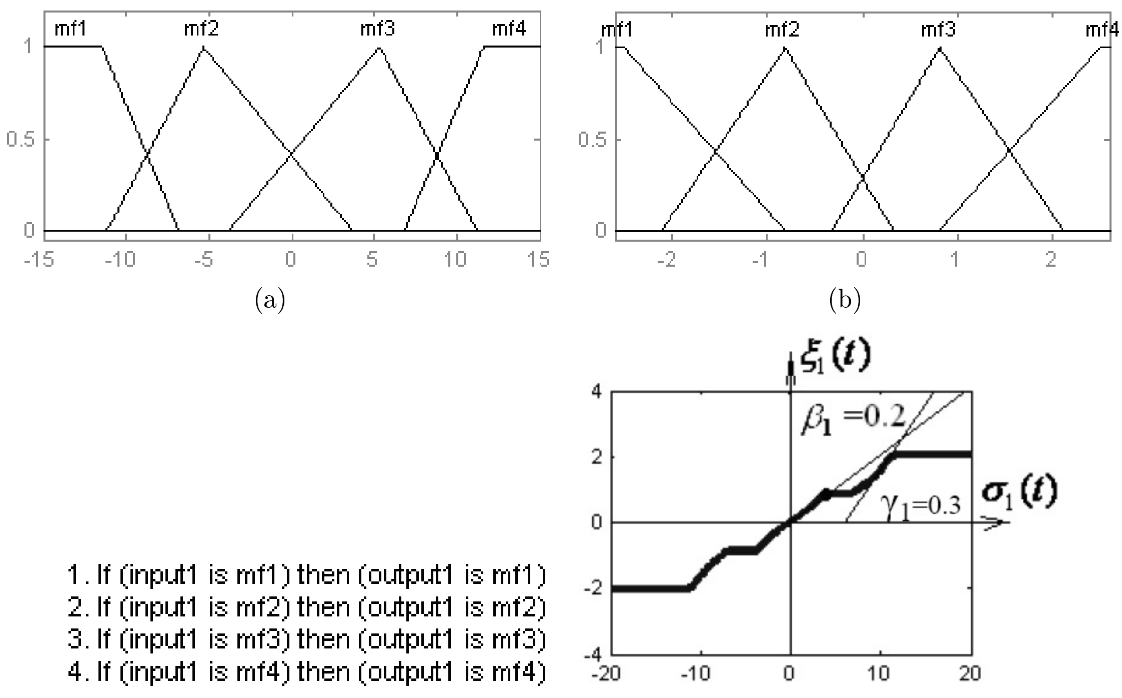


Figure 2. Input linguistic variable (a), output linguistic variable (b), production rules (c), and nonlinear transformation (d) in the integral channel of fuzzy PID controller

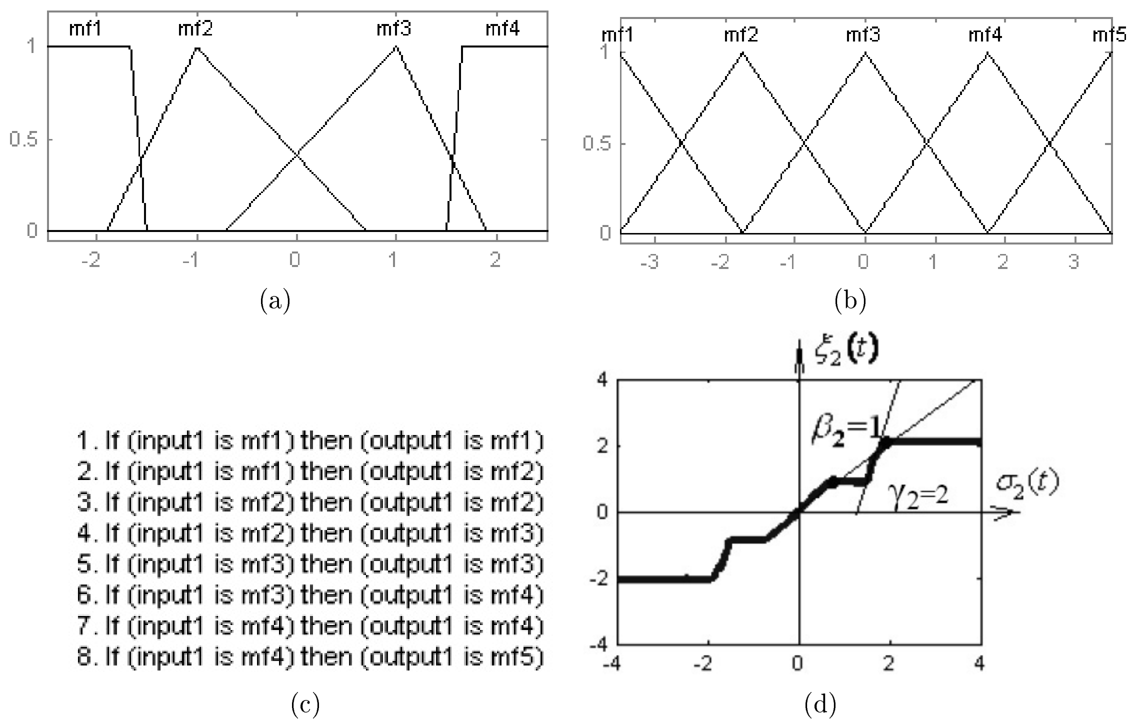


Figure 3. Input linguistic variable (a), output linguistic variable (b), production rules (c), and non-linear transformation (d) in the proportional channel of fuzzy PID controller

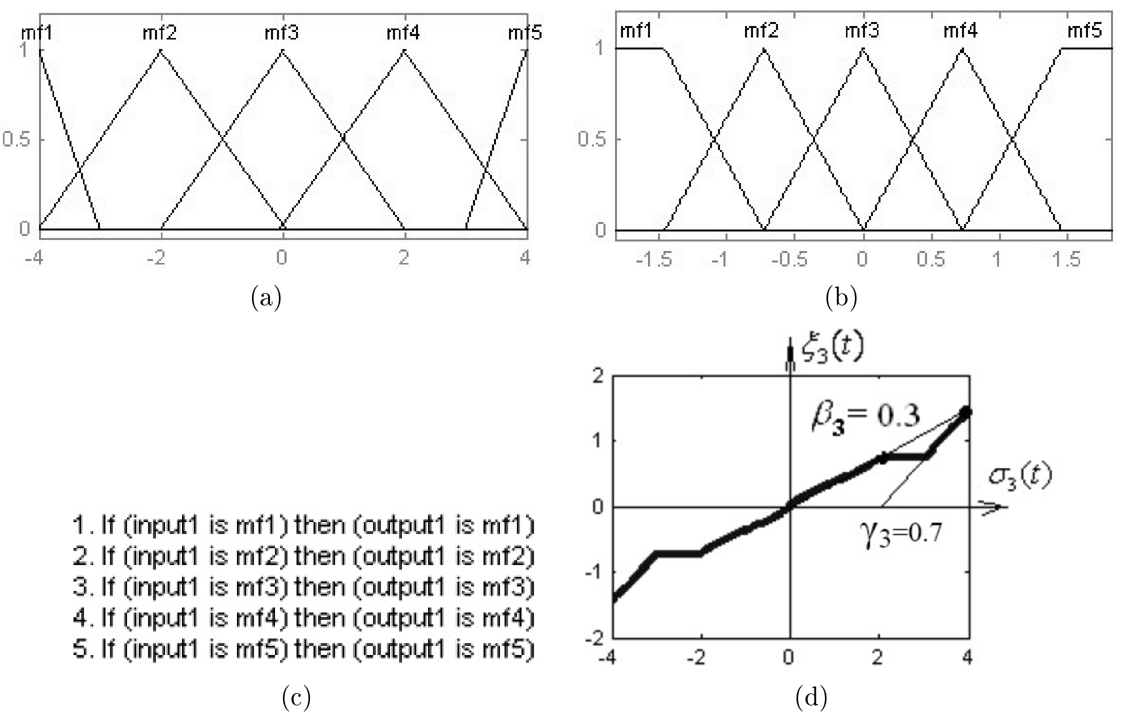


Figure 4. Input linguistic variable (a), output linguistic variable (b), production rules (c), nonlinear transformation (d) in the differential channel of fuzzy PID controller

The control object's frequency response takes the form:

$$W_{co}(j\omega)=15\left(\frac{-0.65\omega^2+1}{(\omega^2+1)\,(0.25\omega^2+1)\,(0.01\omega^2+1)}-j\frac{-0.05\omega^3+1.6\omega}{(\omega^2+1)\,(0.25\omega^2+1)\,(0.01\omega^2+1)}\right)$$

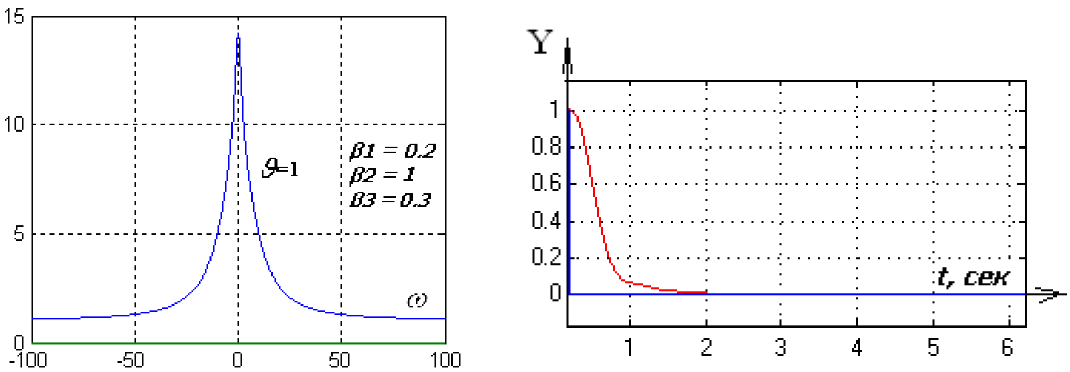
The absolute stability conditions of equilibrium positions (8) and (9) for the investigated ACS take the following form:

$$\begin{aligned}1+15\Bigg(&\beta_1\frac{0.05\omega^2-1.6}{(\omega^2+1)\,(0.25\omega^2+1)\,(0.01\omega^2+1)}+\vartheta_1\beta_1\frac{-0.65\omega^2+1}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}\\ &+\beta_2\frac{-0.65\omega^2+1}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}+\vartheta_2\beta_2\frac{-0.05\omega^4+1.6\omega^2}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}+\dots\\ &+\beta_3\frac{-0.05\omega^4+1.6\omega^2}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}+\vartheta_3\beta_3\frac{0.65\omega^4-\omega^2}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}\Bigg)\neq 0,\end{aligned} \tag{10}$$

$$\begin{cases}1+15\vartheta_1\beta_1\dfrac{-0.65\omega^2+1}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}>0,\\ 1+15\Bigg(\vartheta_2\beta_2\dfrac{-0.05\omega^4+1.6\omega^2}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}\\ \quad+\vartheta_1\beta_1\dfrac{-0.65\omega^2+1}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}\Bigg)>0,\\ 1+15\Bigg(\vartheta_3\beta_3\dfrac{0.65\omega^4+\omega^2}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}\\ \quad+\vartheta_2\beta_2\dfrac{-0.05\omega^4+1.6\omega^2}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}\\ \quad+\vartheta_1\beta_1\dfrac{-0.65\omega^2+1}{(\omega^2+1)(0.25\omega^2+1)(0.01\omega^2+1)}\Bigg)>0.\end{cases} \tag{11}$$

Figure 5(a) shows a graphical solution of inequality (10) for $\beta_1=0.2$, $\beta_2=1$, $\beta_3=0.3$, and Figure 5(b) demonstrates an example of its movement towards the position of equilibrium. The position of equilibrium is absolutely stable.

We now re-configure the FC's proportional channel as detailed in Figure 6. In this case, $\beta_2=6$. For $\beta_1=0.2$, $\beta_2=6$, $\beta_3=0.3$, the graphical solution of inequality (10) is



(a) (b)

FIGURE 5. A graphical solution of inequality (10) for $\beta_1=0.2$, $\beta_2=1$, $\beta_3=0.3$ and $W_{co}(s)=15\left[(s+1)(0.5s+1)(0.1s+1)\right]^{-1}$ (a), and its free movement towards the equilibrium position (b)

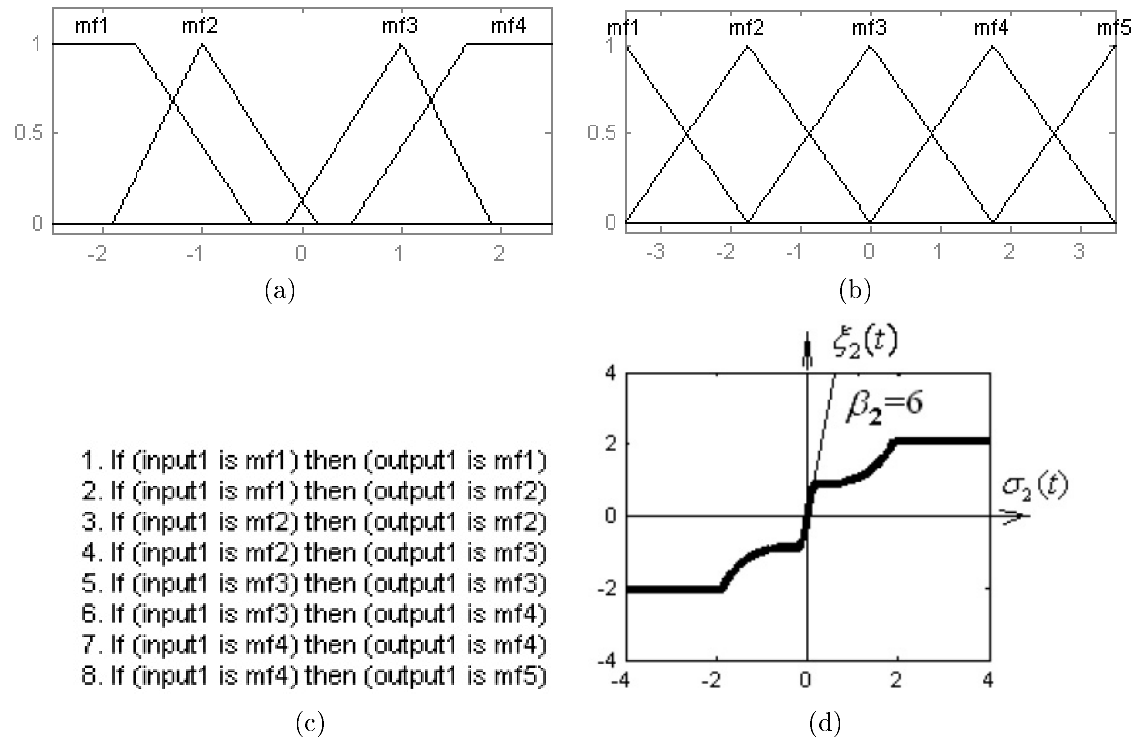


Figure 6. Input linguistic variable (a), output linguistic variable (b), production rules (c), and non-linear transformation (d) in the proportional channel of fuzzy PID controller

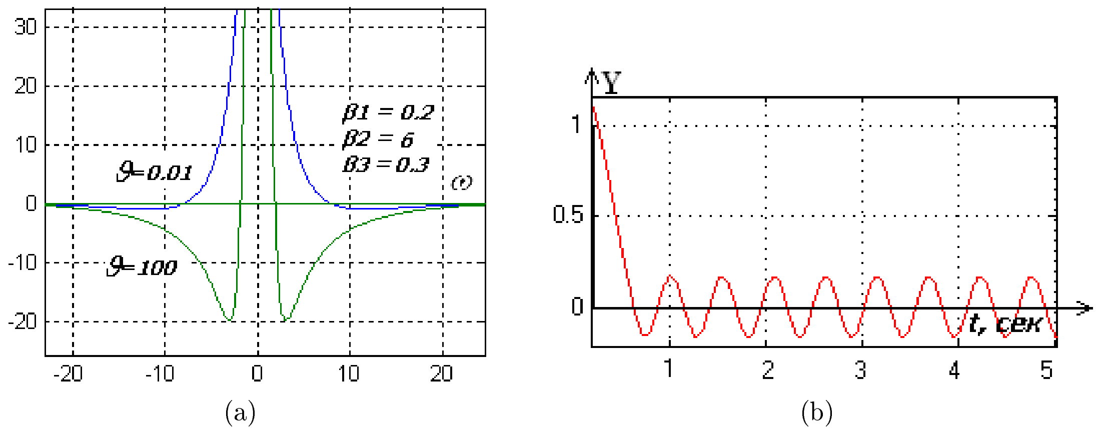


Figure 7. A graphical solution of inequality (10) for $\beta_1 = 0.2$, $\beta_2 = 6$, $\beta_3 = 0.3$ and $W_{\text{co}}(s) = 15\left[(s+1)(0.5s+1)(0.1s+1)\right]^{-1}$ (a) and free movement towards the equilibrium position (b)

presented in Figure 7(a), and Figure 7(b) shows its free movement towards the equilibrium position. Here, the condition of absolute stability is not fulfilled.

4. **Investigating the Absolute Stability of Processes in ACSs with Type-1 Fuzzy Controllers.** When investigating the absolute stability of processes in ACSs with FCs – just as with investigation of the equilibrium position's absolute stability – the system's structure is converted to the form presented in Figure 1(b). The nonlinear

characteristics $\varphi_j(\sigma_j)$ satisfy the sector restrictions.

$$0 \leq \frac{\varphi_j(\sigma_j)}{\sigma_j} \leq \beta_j, \quad j = 1, \ldots, h \tag{12}$$

and

$$0 < \frac{d\xi_j(t)}{d\sigma_j(t)} \leq \gamma_j, \quad (j = 1, \ldots, h). \tag{13}$$

Thus, the absolute stability criteria for processes in an ACS with FC can be expressed as follows.

Let the equations of the ACS's linear part have the form (1), and the nonlinear characteristics $\varphi_j(\sigma_j)$ of the fuzzy regulator correspond to (12) and (13). Let all the poles of the elements of the matrix $W_{LP}(s)$ be arranged on the left-hand half-space, or have one pole on the imaginary axis (with stable or neutral linear parts in all circuits), and $\gamma = diag(\gamma_1, \ldots, \gamma_h)$ be the diagonal matrix with the indicated diagonal elements. Suppose that condition (14), below, is performed for all $-\infty < \omega < +\infty$, except $\omega = 0$.

$$\det \mathrm{Re}\{[I + \gamma W_{LP}(j\omega)]\} \neq 0 \quad (-\infty < \omega < +\infty),\ \omega \neq 0, \tag{14}$$

where I is the identity matrix $h \times h$.

As such, an ACS with FC as described by Equations (1) and (2) is exponentially absolutely stable.

The absolute stability criteria for processes in an ACS with FC will take the following matrix form:

$$\det\ \mathrm{Re}\left\{\begin{bmatrix} 1 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 1 \end{bmatrix} + \begin{bmatrix} \gamma_1 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & \gamma_h \end{bmatrix} W_{LP}(j\omega)\right\} \neq 0 \tag{15}$$

$$-\infty < \omega < +\infty, \quad \omega \neq 0.$$

Calculating the determinant of matrix (15), we obtain an inequality which defines the boundary of the absolute-stability for processes in an ACS with an FC and either neutral or stable linear part:

$$1 + \mathrm{Re}\left[\gamma_1 \left(\frac{1}{(j\omega)} W_{\mathrm{co}}(j\omega)\right) + \gamma_2 W_{\mathrm{co}}(j\omega) + \cdots + \gamma_h \left((j\omega)^{h-2} W_{\mathrm{co}}(j\omega)\right)\right] \neq 0 \tag{16}$$

$$-\infty < \omega < +\infty, \quad \omega \neq 0.$$

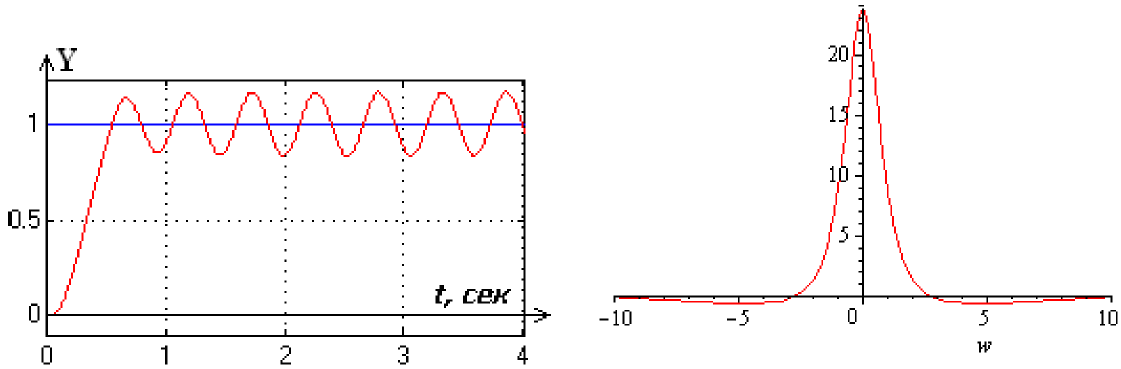
Consider the example of an absolute stability investigation of processes in an ACS with an industrial control object, which has the transfer function $W_{\mathrm{co}}(s) = 15[(s+1)(0.5s+1)(0.1s+1)]^{-1}$, and where the fuzzy PID controller's parameters correspond to $\gamma_1 = 0.3$, $\gamma_2 = 2$, $\gamma_3 = 0.7$.

With such a system, inequality (16) is represented thus:

$$1 + 15\frac{\gamma_3\left(0.05\omega^4 + 1.6\omega^2\right) + \gamma_2\left(-0.65\omega^2 + 1\right) + \gamma_1\left(0.05\omega^2 - 1.6\right)}{\left(\omega^2 + 1\right)\left(0.25\omega^2 + 1\right)\left(0.01\omega^2 + 1\right)} \neq 0 \tag{17}$$

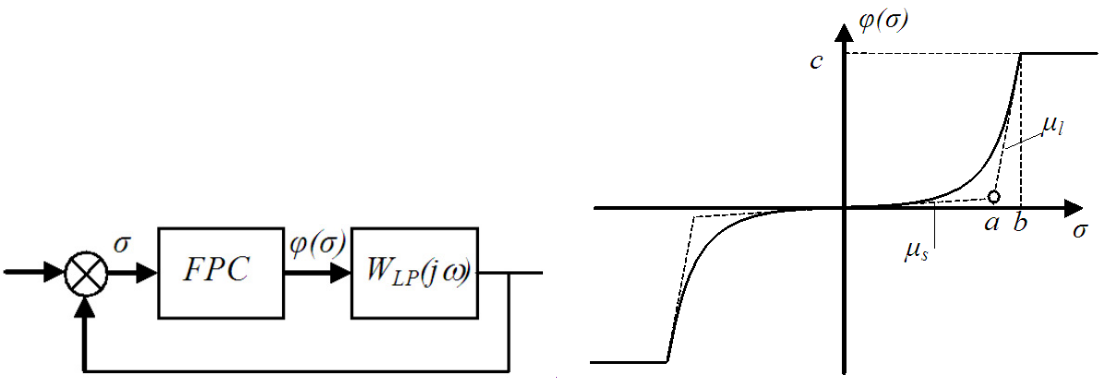
$$-\infty < \omega < +\infty, \quad \omega \neq 0.$$

Figure 8(b) demonstrates a graphical solution for inequality (17), where $\gamma_1 = 0.3$, $\gamma_2 = 2$, $\gamma_3 = 0.7$. It is visible here that, for the processes of the ACS with FC in question, the stability criterion is not fulfilled. Figure 8(a) demonstrates the transient at the control object output, with the constant setpoint ($h(t) = 1$).



(a) (b)

FIGURE 8. Graph of the transient (a) and a graphical solution of the condition of absolute stability of ACS processes $W_{\text{co}}(s) = 15\left[(s+1)(0.5s+1)(0.1s+1)\right]^{-1}$ (b), where the nonlinear PID-controller's parameters correspond to $\gamma_1 = 0.3$, $\gamma_2 = 2$, $\gamma_3 = 0.7$



(a) (b)

FIGURE 9. (a) Diagram of a control system with fuzzy P-controller, (b) nonlinear transformation in the fuzzy P-controller

5. **Investigating the Absolute Stability of an ACS with Fuzzy P-Controller.** An analysis of the practical accomplishments of both Russian and foreign specialists demonstrates that in many cases, improvements to qualitative variables can be achieved not only through use of fuzzy PID controllers (which are controllers of a general type), but also by simplifying its modifications – for example, through use of a fuzzy P-controller, as demonstrated by [18]. Such an improvement can be achieved by realizing the nonlinear transformation $\varphi(\sigma)$ in the fuzzy P-controller, similar to that shown in Figure 9. In this diagram, the transformation is approximated as two linear sectors, with one large angle (μ_l) and one small one (μ_s).

It is clear that the above process of investigating absolute stability becomes significantly simpler in the case of a fuzzy P-controller. To illustrate, the conditions of equilibrium position absolute stability are presented in graph form in Figure 10(a). Figure 10(b) presents the condition of process stability where $W_{LP}(j\omega)$ is the transfer function of the linear part of the system in question, and $W^*_{LP}(j\omega)$, the transfer function of the post-transformation linear part of the same system. Examples of the movement of this system towards the equilibrium position, and processes where $0 < \sigma < a$ and $a < \sigma < b$ obtained by this model, are presented in Figures 11(a) and 11(b).

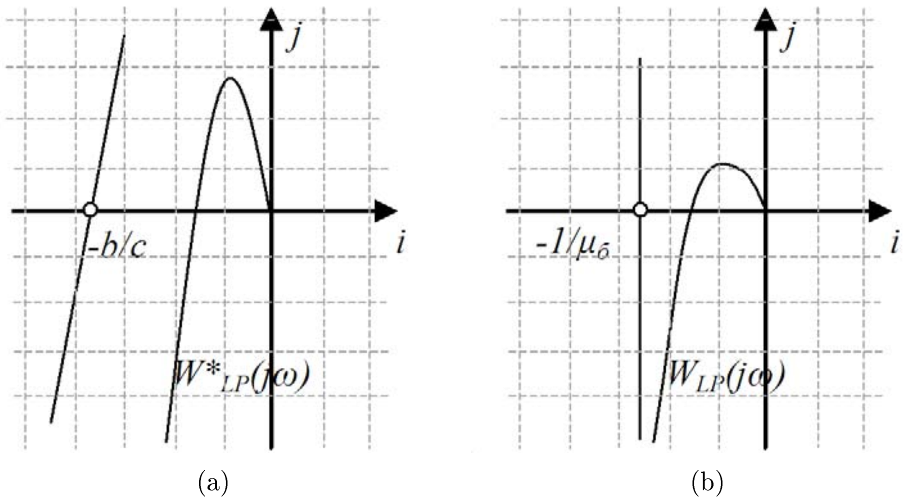


FIGURE 10. Graphical investigation of absolute stability of the equilibrium position (a) and processes (b) in ACS with fuzzy P-controller

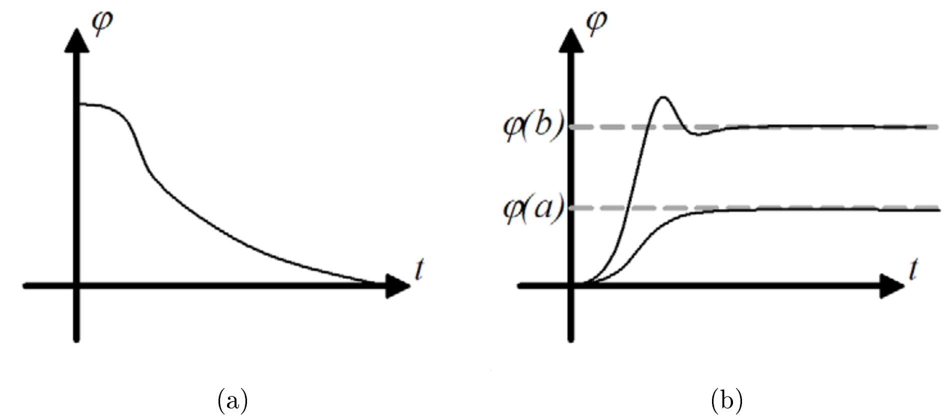


FIGURE 11. Examples of the system's movement towards the equilibrium position (a) and processes (b) where $0 < \sigma < a$ and $a < \sigma < b$

6. **Conclusions.** It will be clear that the condition of absolute stability is stronger for the processes. Therefore, it is best to select this condition as a basis for calculating the stability of a system with fuzzy P-controller. At the same time, the use of more rigid conditions naturally leads to a reduction of the stability zone on the plane of the gain factor, and a corresponding lack of steady state precision. However, it should be emphasized that, if the absolute stability condition is broken, it does not follow that the processes are unstable. It is therefore possible, in principle, to increase the system's gain factor such that the processes' absolute stability condition is broken, but simultaneously exercise control beyond the stability region using other methods. One example is the harmonic balance [19] method, which yields fairly precise results for third-order (or higher) control objects with transfer functions.

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REFERENCES

[1] G. Feng, A survey on analysis and design of model-based fuzzy control systems, *IEEE Trans. Fuzzy Systems*, vol.14, no.5, pp.676-697, 2006.

[2] Y. Y. Cao and P. M. Frank, Stability analysis and synthesis of nonlinear time-delay systems via linear Takagi-Sugeno fuzzy models, *Fuzzy Sets and Systems*, vol.124, no.2, pp.213-229, 2001.

[3] H. K. Lam and F. H. F. Leung, *Stability Analysis of Fuzzy-Model-Based Control Systems: Linear-Matrix-Inequality Approach*, Springer, 2011.

[4] H. K. Lam, F. H. F. Leung and P. K. S. Tam, Design and stability analysis of fuzzy model based nonlinear controller for nonlinear systems using genetic algorithm, *IEEE Trans. Syst., Man and Cybern., Part B: Cybernetics*, vol.33, no.2, pp.250-257, 2003.

[5] Y.-J. Chen, M. Tanaka, K. Tanaka and H. O. Wang, Stability analysis and region-of-attraction estimation using piecewise polynomial Lyapunov functions: Polynomial fuzzy model approach, *IEEE Trans. Fuzzy Systems*, vol.23, no.4, pp.1314-1322, 2015.

[6] K. Tanaka, T. Hori and H. O. Wang, A fuzzy Lyapunov approach to fuzzy control system design, *American Control Conference*, Arlington, pp.4790-4795, 2001.

[7] K. Tanaka and M. Sugeno, Stability analysis of fuzzy systems using Lyapunov's direct method, *North American Fuzzy Information Society Annual Meeting*, Toronto, vol.1, pp.133-136, 1990.

[8] K. Tanaka, T. Hori and H. O. Wang, A multiple Lyapunov function approach to stabilization of fuzzy control systems, *IEEE Trans. Fuzzy Systems*, vol.11, no.4, pp.582-589, 2003.

[9] K. Tanaka and T. Ikeda, Absolute stability conditions in a fuzzy phase-lead compensation and their extension to MIMO systems, *IEEE Trans. Industrial Electronics*, vol.45, no.2, pp.333-340, 1998.

[10] W. D. Esterhuizen, H. O. Wang and K. Tanaka, Stability and stabilization conditions for Takagi-Sugeno fuzzy model via polyhedral Lyapunov functions, *American Control Conference*, Washington DC, pp.5637-5642, 2013.

[11] V. A. Yakubovich, Frequency conditions for the absolute stability of control systems with multiple non-linear or linear non-stationary units, *Automation and Remote Control*, no.6, 1967.

[12] I. M. Makarov and V. M. Lokhin, *Artificial Intelligence and Complex Objects Control*, Edwin Mellen Press, 2000.

[13] I. M. Makarov, V. M. Lokhin, S. V. Man'ko and M. P. Romanov, *Artificial Intelligence and Intelligent Control Systems*, Branch Inform, Technology and Computer, RAS Systems, Nauka, Moscow, 2006.

[14] A. Piegat, *Fuzzy Modeling and Control*, Physica Verlag Heidelberg, 2001.

[15] D. P. Kim, The theory of automatic control, V.2, in *Multi-dimensional, Nonlinear, Optimal and Adaptive Systems 2 - Vol.*, 2007.

[16] A. I. Lurie and V. N. Postnikov, On the theory of stability of controlled systems, *Applied Mathematics and Mechanics*, vol.8, no.3, pp.246-248, 1944.

[17] V. M. Popov, Criterii suficiente de stabilitate simptotica in mare pentru sistemele automate on mai multe organe de executie, *Studei si Cercetari de Energetica*, vol.9, no.1, 1959.

[18] N. A. Kazachek, V. M. Lokhin, M. S. Melikhov and S. V. Yepishin, *Research of Features of Fuzzy Controllers for the Control of Industrial Objects and Software Tools Development of Automated Synthesis and Settings*, Ministry of Education and Science of the Russian Federation (Project No. 784 within the Base Part of the State Task in Scientific Research on the Assignment No. 2014/112 at 2014).

[19] N. Kazachek, V. Lokhin, S. Manko and M. Romanov, Research of periodic oscillations in control systems with fuzzy controllers, *International Journal of Innovative Computing, Information and Control*, vol.11, no.3, pp.985-997, 2015.