International Journal of Innovative Computing, Information and Control
Volume 13, Number 2, April 2017
ICIC International ©2017 ISSN 1349-4198
pp. 679–686

NOVEL OUTPUT TRACKING APPROACH FOR A CLASS OF LINEAR NEUTRAL SYSTEMS

Yujie Xu and Jing Zhang

Department of Basic Course Teaching
Beijing Union University
No. 97, Beisihuan East Road, Chaoyang District, Beijing 100101, P. R. China
bjxyjie@126.com

Received September 2016; revised January 2017

Abstract. *In this paper, a novel output tracking approach is proposed for a class of linear neutral systems. Dynamic equations of the error signal and the desired output are constructed on the basis of the considered system. An augmented error system is deduced to make use of the desired output in control design. And a delay-independent controller is derived for the augmented error system. Control design of the original system contains feedback of the state, integrals of the desired output and error signals in this paper. Finally, an illustrative example is presented to show the effectiveness of the proposed control design.*

Keywords: Neutral system, Asymptotic stability, Augmented error system, Output tracking

1. Introduction.

Time-delay is frequently encountered in many areas such as automatic control systems, neural networks, and population dynamic models. It is well known that time-delay often leads to control systems' poor performance, and makes the operation situation not be reflected in time. Neutral systems contain time-delay both in the state, and in the derivatives of the state. Such systems often appear in many places such as heat exchangers, population ecology, partial element equivalent circuit (PEEC) and distributed networks containing lossless transformation lines [1,2]. On the other hand, a lot of ordinary delay systems can be modeled mathematically as neutral systems, e.g., lossless transmission model, standard delay systems, and standard distributed delay systems [3]. In view of the extensive background and wide representatively, increasing attention has been paid on neutral systems. And many results have been reported in [4,6]. Lyapunov functional is the most common approach to analyze the stability of neutral systems [7-9].

However, research on tracking control for neutral systems has not been reported as widely as that on stability analysis. To the best of our knowledge, only a few results on output tracking for neutral systems have been reported in the last two decades, which mainly focus on H_∞ output tracking control. It makes a certain norm of the transfer matrix from disturbance input to controlled output less than a constant, which is usually called output tracking performance. A reference model without disturbance input is needed in H_∞ tracking control to guarantee that the output of the systems tracks the output of the given reference model well in the H_∞ sense. Based on this thought, [10,11] derive tracking control respectively for switched neutral systems and uncertain delay system.

Instead of a reference model, this paper combines the desired output with the considered neutral system. First, the desired output is assumed to be known, which occurs in robot path control, motor control system and so on [12]. Then an augmented error system is constructed. Control design for the original neutral system contains not only state feedback, but also integrals of the desired output, and integrators on error signals which may help decrease static error [13].

The rest of this paper is organized as follows. Section 2 gives the problem description. In Section 3, stability results for the augmented error system are investigated, and control design for the neutral system is derived. In Section 4, a numerical example is given to show the effectiveness of the results. Section 5 concludes the paper.

2. **Problem Statement and Preliminaries.** Consider the neutral system as follows:

$$\begin{cases} \dot{x}(t) - G\dot{x}(t-h) = A_0 x(t) + A_1 x(t-d) + Bu(t), \\ y(t) = Cx(t), \quad (t > 0) \\ x(\theta) = \phi(\theta), \quad \theta \in [-\max\{h, d\}, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $y(t) \in \mathbf{R}^p$ is the output of the system, and $u(t) \in \mathbf{R}^m$ is the control input. $\phi(\theta)$ is the compatible vector valued a continuous function representing the initial condition, G, A_0, A_1, B and C are known real matrices with appropriate dimensions, and $d > 0$ ($h > 0$) is the time-delay in the state (the derivative of the state).

Assume that the desired output of system (1) is $r(t) \in \mathbf{R}^p$. Then the error signal is defined as $e(t) = y(t) - r(t)$, that is

$$e(t) = Cx(t) - r(t). \tag{2}$$

To establish our main results, it is necessary to present the following assumptions and lemma.

Assumption 2.1. $\rho(G) < 1$, *where* $\rho(G)$ *denotes the spectral radius of* G.

Assumption 2.2. *The desired output* $r(t)$ *is a piecewise-continuously differentiable function satisfying* $r^{(s)}(t) \equiv 0$. *If it is differential up to infinite order, we can expand it into polynomial with finite terms in error range, which is allowable in engineering.*

Remark 2.1. *Assumption 2.1 guarantees that the operator* $\dot{x}(t) - G\dot{x}(t-h)$ *is stable independent of delay* h *[14,15].*

Remark 2.2. *For Assumption 2.2,* $r(t)$ *is in low requirement; hence the method of the paper is suitable for more extensive target signals, such as stair-step signal. The left or the right derivatives can be taken as the derivative of* $r(t)$ *on the underivable points.*

Lemma 2.1. *(Schur complement [16]) Given the constant symmetric matrices* Ω_1, Ω_2, Ω_3, *where* $\Omega_1 = \Omega_1^{\mathrm{T}}$ *and* $\Omega_2 = \Omega_2^{\mathrm{T}} > 0$, *then* $\Omega_1 + \Omega_3^{\mathrm{T}} \Omega_2^{-1} \Omega_3 < 0$, *if and only if*

$$\begin{bmatrix} \Omega_1 & \Omega_3^{\mathrm{T}} \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0, \quad \text{or} \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^{\mathrm{T}} & \Omega_1 \end{bmatrix} < 0.$$

3. **Main Results.** In this section, we will present our main results on control design for system (1).

3.1. **Construction of an augmented error system.** In order to make use of the desired output $r(t)$, the dynamic equation of the tracking error $e(t)$ is necessary. From (2), we get

$$\dot{e}(t) - \alpha\dot{e}(t-h) = C\dot{x}(t) - \alpha C\dot{x}(t-h) - \dot{r}(t) + \alpha\dot{r}(t-h), \tag{3}$$

where α is a real constant and satisfies $0 < \alpha < 1$. By Remark 2.1, the operator $\dot{e}(t) - \alpha\dot{e}(t-h)$ is stable. Equation (3) is the error system of (1).

Now, $r(t)$ is contained in Equation (3). It is better to combine Equation (3) with system (1) for utilization of $r(t)$. So a new vector $X(t) = \begin{bmatrix} \dot{x}^{\mathrm{T}}(t) & x^{\mathrm{T}}(t) & e^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$ is defined. From (1) and (3), we deduce that

$$\dot{X}(t) - \bar{G}\dot{X}(t-h) = \bar{A}_0 X(t) + \bar{A}_1 X(t-h) + \bar{A}_2 X(t-d) + \bar{B}\dot{u}(t) + G_r(\dot{r}(t) - \alpha\dot{r}(t-h)), \quad (4)$$

where

$$\bar{G} = \begin{bmatrix} G & 0 & 0 \\ * & G & 0 \\ * & * & \alpha I_p \end{bmatrix}, \ \bar{A}_0 = \begin{bmatrix} A & 0 & 0 \\ I_n & 0 & 0 \\ C & 0 & 0 \end{bmatrix}, \ \bar{A}_1 = \begin{bmatrix} 0 & 0 & 0 \\ -G & 0 & 0 \\ -\alpha C & 0 & 0 \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \ G_r = \begin{bmatrix} 0 \\ 0 \\ -I \end{bmatrix}.$$

Notice $0 < \alpha < 1$, so $\dot{X}(t) - \bar{G}\dot{X}(t-h)$ is stable by Assumption 2.1 and Remark 2.1.

Formula (4) is an augmented error system. It is obvious that if $\dot{u}(t)$ can be deduced from system (4), the control input $u(t)$ for system (1) can be derived correspondingly. It is our goal that the known desired output $r(t)$ can be utilized. So next we combine $r(t)$ with the state vector $X(t)$ of system (4) through some transformation method.

It is necessary to obtain the dynamic equation of the desired output $r(t)$. Considering Assumption 2.2, we define $R(t) = \begin{bmatrix} r^{\mathrm{T}}(t) & \dot{r}^{\mathrm{T}}(t) & \cdots & r^{(s-1)\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$. Then the dynamic equation of $R(t)$ can be described as

$$\dot{R}(t) - \alpha\dot{R}(t-h) = G_R R(t) - \alpha G_R R(t-h), \quad (5)$$

where

$$G_R = \begin{bmatrix} 0 & I_p & 0 & \cdots & 0 \\ 0 & 0 & I_p & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & I_p \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Note $z(t) = \begin{bmatrix} X(t) \\ R(t) \end{bmatrix}$, and Formulas (4) and (5) are equivalent to the following system

$$\dot{z}(t) - G_z\dot{z}(t-h) = A_z z(t) + A_h z(t-h) + A_d z(t-d) + B_1\dot{u}(t), \quad (6)$$

where

$$G_z = \begin{bmatrix} \bar{G} & 0 \\ 0 & \alpha I \end{bmatrix}, \ A_z = \begin{bmatrix} \bar{A}_0 & E_R \\ 0 & G_R \end{bmatrix}, \ A_h = \begin{bmatrix} \bar{A}_1 & -\alpha E_R \\ 0 & -\alpha G_R \end{bmatrix},$$

$$A_d = \begin{bmatrix} \bar{A}_2 & 0 \\ 0 & 0 \end{bmatrix}, \ B_1 = \begin{bmatrix} \bar{B} \\ 0 \end{bmatrix}, \ E_R = \begin{bmatrix} 0 & G_r & \cdots & 0 \end{bmatrix}.$$

System (6) is the augmented error system that we need, and the operator $\dot{z}(t) - G_z\dot{z}(t-h)$ is stable. In system (6), time-delay h appears not only in neutral term but also in the augmented state vector $z(t)$, which is different from the original neutral system. The corresponding unforced system without $\dot{u}(t)$ of system (6) is denoted as

$$\dot{z}(t) - G_z\dot{z}(t-h) = A_z z(t) + A_h z(t-h) + A_d z(t-d). \quad (7)$$

Next, the asymptotic stability of system (7) is investigated. And a feedback controller of (6) is derived. Thus, control design of system (1) can be derived.

3.2. **Asymptotic stability analysis of the unforced system.** The stability of the unforced system is presented in the following theorem.

Theorem 3.1. *System (7) is asymptotically stable if there exist* $P > 0$, $Q_j > 0$ $(j = 1, 2, 3)$ *and nonnegative scalars* ε_i $(i = 1, 2, 3)$ *such that the following inequalities hold:*

$$\begin{bmatrix} PA_z + A_z^{\mathrm{T}}P + 4I + m\varepsilon_3 I + m\varepsilon_1 I & PA_d & PA_h & PG_z & A_z^{\mathrm{T}} \\ * & -2I + m\varepsilon_2 I & 0 & 0 & A_d^{\mathrm{T}} \\ * & * & -2I + m\varepsilon_4 I & 0 & A_h^{\mathrm{T}} \\ * & * & * & -Q_3 & G^{\mathrm{T}} \\ * & * & * & * & -R^{-1} \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} -\varepsilon_1 I & 0 & -I \\ * & -\varepsilon_2 I & -I \\ * & * & -Q_1 \end{bmatrix} < 0, \quad \begin{bmatrix} -\varepsilon_3 I & 0 & -I \\ * & -\varepsilon_4 I & -I \\ * & * & -Q_2 \end{bmatrix} < 0, \tag{9}$$

where $m = \max\{h, d\}$, *and* $R = dQ_1 + hQ_2 + Q_3$.

Proof: Construct a Lyapunov-Krasovskii functional for system (7) as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t),$$

where $V_1(t)=z^{\mathrm{T}}(t)Pz(t)$, $V_2(t)=\int_{-d}^{0}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(s)Q_1\dot{z}(s)\mathrm{d}s\mathrm{d}\theta$, $V_3(t)=\int_{-h}^{0}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(s)Q_2\dot{z}(s)\mathrm{d}s\mathrm{d}\theta$, $V_4(t) = \int_{t-h}^{t} \dot{z}^{\mathrm{T}}(\theta)Q_3\dot{z}(\theta)\mathrm{d}\theta$. Then, the time derivative of $V(t)$ along the trajectory of system (7) is given by

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t).$$

Differentiating $V_1(t)$ along system (7) yields

$$\begin{aligned} \dot{V}_1 &= 2z^{\mathrm{T}}(t)P\dot{z}(t) \\ &= 2z^{\mathrm{T}}(t)P\left[A_z z(t) + A_h z(t-h) + A_d z(t-d) + G_z \dot{z}(t-h)\right] \\ &= z^{\mathrm{T}}(t)\left(PA_z + A_z^{\mathrm{T}}P\right)z(t) + z^{\mathrm{T}}(t)\left(PA_h + A_h^{\mathrm{T}}P\right)z(t-h) \\ &\quad + z^{\mathrm{T}}(t)\left(PA_d + A_d^{\mathrm{T}}P\right)z(t-d) + z^{\mathrm{T}}(t)\left(PG_z + G_z^{\mathrm{T}}P\right)\dot{z}(t-h). \end{aligned}$$

And the derivatives of $V_2(t)$, $V_3(t)$, and $V_4(t)$ are respectively given by

$$\begin{aligned} \dot{V}_2(t) &= \dot{z}^{\mathrm{T}}(t)(dQ_1)\dot{z}(t) - \int_{t-d}^{t} \dot{z}^{\mathrm{T}}(s)Q_1\dot{z}(s)\mathrm{d}s, \\ \dot{V}_3(t) &= \dot{z}^{\mathrm{T}}(t)(hQ_2)\dot{z}(t) - \int_{t-h}^{t} \dot{z}^{\mathrm{T}}(s)Q_2\dot{z}(s)\mathrm{d}s, \\ \dot{V}_4(t) &= \dot{z}^{\mathrm{T}}(t)Q_3\dot{z}(t) - \dot{z}^{\mathrm{T}}(t-h)Q_3\dot{z}(t-h). \end{aligned}$$

Utilizing Newton-Leibniz formula that $\int_{t-d}^{t} \dot{z}(s)\mathrm{d}s = z(t) - z(t-d)$ and $\int_{t-h}^{t} \dot{z}(s)\mathrm{d}s = z(t) - z(t-h)$, the following equalities are established:

$$W_1 = 2\begin{bmatrix} z^{\mathrm{T}}(t) & z^{\mathrm{T}}(t-d) \end{bmatrix}\begin{bmatrix} I \\ I \end{bmatrix}\left[z(t) - z(t-d) - \int_{t-d}^{t} \dot{z}(s)\mathrm{d}s\right] = 0,$$

$$W_2 = 2\begin{bmatrix} z^{\mathrm{T}}(t) & z^{\mathrm{T}}(t-h) \end{bmatrix}\begin{bmatrix} I \\ I \end{bmatrix}\left[z(t) - z(t-h) - \int_{t-h}^{t} \dot{z}(s)\mathrm{d}s\right] = 0.$$

Additionally, let $m = \max\{h, d\}$. For $\begin{bmatrix} \varepsilon_1 I & 0 \\ * & \varepsilon_2 I \end{bmatrix} > 0$ and $\begin{bmatrix} \varepsilon_3 I & 0 \\ * & \varepsilon_4 I \end{bmatrix} > 0$, we have

$$\begin{aligned} W_3 &= m \begin{bmatrix} z^{\mathrm{T}}(t) & z^{\mathrm{T}}(t-d) \end{bmatrix} \begin{bmatrix} \varepsilon_1 I & 0 \\ * & \varepsilon_2 I \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-d) \end{bmatrix} \\ &\quad - \int_{t-d}^{t} \begin{bmatrix} z^{\mathrm{T}}(t) & z^{\mathrm{T}}(t-d) \end{bmatrix} \begin{bmatrix} \varepsilon_1 I & 0 \\ * & \varepsilon_2 I \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-d) \end{bmatrix} \mathrm{d}s \geq 0, \end{aligned}$$

$$\begin{aligned} W_4 &= m \begin{bmatrix} z^{\mathrm{T}}(t) & z^{\mathrm{T}}(t-h) \end{bmatrix} \begin{bmatrix} \varepsilon_3 I & 0 \\ * & \varepsilon_4 I \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-h) \end{bmatrix} \\ &\quad - \int_{t-d}^{t} \begin{bmatrix} z^{\mathrm{T}}(t) & z^{\mathrm{T}}(t-d) \end{bmatrix} \begin{bmatrix} \varepsilon_3 I & 0 \\ * & \varepsilon_4 I \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-d) \end{bmatrix} \mathrm{d}s \geq 0. \end{aligned}$$

Then $\dot{V}(t) \leq \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + W_1 + W_2 + W_3 + W_4$.

Denote

$$\begin{aligned} &\eta(t) = \begin{bmatrix} z^{\mathrm{T}}(t) & z^{\mathrm{T}}(t-d) & z^{\mathrm{T}}(t-h) & \dot{z}^{\mathrm{T}}(t-h) \end{bmatrix}^{\mathrm{T}}, \\ &\xi_d(t) = \begin{bmatrix} z^{\mathrm{T}}(t) & z^{\mathrm{T}}(t-d) & \dot{z}^{\mathrm{T}}(s) \end{bmatrix}^{\mathrm{T}}, \quad \xi_h(t) = \begin{bmatrix} z^{\mathrm{T}}(t) & z^{\mathrm{T}}(t-h) & \dot{z}^{\mathrm{T}}(s) \end{bmatrix}^{\mathrm{T}}. \end{aligned}$$

We have

$$\dot{V}(t) \leq \eta^{\mathrm{T}}(t)\left(\Omega_1 + \Upsilon^{\mathrm{T}} R \Upsilon\right)\eta(t) + \textstyle\int_{t-d}^{t} \xi_d^{\mathrm{T}}(t)\Omega_2\xi_d(t)\mathrm{d}s + \int_{t-h}^{t} \xi_h^{\mathrm{T}}(t)\Omega_3\xi_h(t)\mathrm{d}s$$

where $\Omega_1 = \begin{bmatrix} PA_z + A_z^{\mathrm{T}}P + 4I + m\varepsilon_3 I + m\varepsilon_1 I & PA_d & PA_h & PG_z \\ * & -2I + m\varepsilon_2 I & 0 & 0 \\ * & * & -2I + m\beta I & 0 \\ * & * & * & -Q_3 \end{bmatrix}$,

$\Omega_2 = \begin{bmatrix} -\varepsilon_1 I & 0 & -I \\ * & -\varepsilon_2 I & -I \\ * & * & -Q_1 \end{bmatrix}$, $\Omega_3 = \begin{bmatrix} -\varepsilon_3 I & 0 & -I \\ * & -\varepsilon_4 I & -I \\ * & * & -Q_2 \end{bmatrix}$, $\Upsilon = \begin{bmatrix} A_z & A_d & A_h & G \end{bmatrix}$, $R = dQ_1 + hQ_2 + Q_3$. When $\Omega_1 + \Upsilon^{\mathrm{T}} R \Upsilon < 0$, $\Omega_2 < 0$ and $\Omega_3 < 0$, we get $\dot{V}(t) < 0$ which can assure the asymptotic stability of system (7). By Lemma 2.1, $\Omega_1 + \Upsilon^{\mathrm{T}} R \Upsilon < 0$ equals that

$$\begin{bmatrix} PA_z + A_z^{\mathrm{T}}P + 4I + m\varepsilon_3 I + m\varepsilon_1 I & PA_d & PA_h & PG_z & A_z^{\mathrm{T}} \\ * & -2I + m\varepsilon_2 I & 0 & 0 & A_d^{\mathrm{T}} \\ * & * & -2I + m\varepsilon_4 I & 0 & A_h^{\mathrm{T}} \\ * & * & * & -Q_3 & G^{\mathrm{T}} \\ * & * & * & * & -R^{-1} \end{bmatrix} < 0.$$

Therefore, (8) and (9) equal $\dot{V}(t) < 0$. Hence, system (7) is asymptotically stable. In Theorem 3.1, stability of system (7) is relative not only to matrices A_z, A_h, A_d and G_z, but also to time-delay h and d. Control design of system (6) will be obtained on the basis of Theorem 3.1.

3.3. **Control design for the augmented error system and original neutral system.** Theorem 3.1 provides a sufficient condition for stability of system (7), where there is no input. In this section, we first suppose a feedback controller for system (6). According to Theorem 3.1, we obtain the feedback gain matrix as described in the following theorem.

Theorem 3.2. *There exists a state-feedback controller $\dot{u}(t) = Kz(t)$ for system (6), if there are matrices $W > 0$, $Q_j > 0$ $(j = 1, 2, 3)$, Y and nonnegative scalars ε_i $(i = 1, 2, 3, 4)$ satisfying inequalities (9) and the following inequality.*

$$\begin{bmatrix} \Pi & A_d & A_h & G_z & WA_z^{\mathrm{T}} + Y^{\mathrm{T}}B_1^{\mathrm{T}} \\ * & -2I + m\varepsilon_2 I & 0 & 0 & A_d^{\mathrm{T}} \\ * & * & -2I + m\varepsilon_4 I & 0 & A_h^{\mathrm{T}} \\ * & * & * & -Q_3 & G^{\mathrm{T}} \\ * & * & * & * & -R^{-1} \end{bmatrix} < 0, \tag{10}$$

where $\Pi = A_zW + B_1Y + WA_z^{\mathrm{T}} + Y^{\mathrm{T}}B_1^{\mathrm{T}} + 4I + m\varepsilon_3 I + m\varepsilon_1 I$.

Moreover, if (9) and (10) are feasible, the gain matrix is given by $K = YW^{-1}$.

Proof: Take $\dot{u}(t) = Kz(t)$ into system (6). The closed-loop system of (6) is

$$\dot{z}(t) - G_z\dot{z}(t-h) = (A_z + B_1K)z(t) + A_hz(t-h) + A_dz(t-d).$$

Substituting A_z with $A_z + B_1K$ in (8) obtains

$$\begin{bmatrix} PA_z + \Gamma^{\mathrm{T}}P + 4I + m\varepsilon_3 I + m\varepsilon_1 I & PA_d & PA_h & PG_z & \Gamma^{\mathrm{T}} \\ * & -2I + m\varepsilon_2 I & 0 & 0 & A_d^{\mathrm{T}} \\ * & * & -2I + m\varepsilon_4 I & 0 & A_h^{\mathrm{T}} \\ * & * & * & -Q_3 & G^{\mathrm{T}} \\ * & * & * & * & -R^{-1} \end{bmatrix} < 0, \tag{11}$$

where $\Gamma = A_z + B_1K$.

Perform a congruence transformation to (11) by *diag* $\{P^{-1}, I, I, I, I\}$ and define $P^{-1} = W$, $Y = KW$. Thus, (10) is gained and it is obvious that $K = YW^{-1}$.

For $z(t) = \begin{bmatrix} \dot{x}^{\mathrm{T}}(t) & x^{\mathrm{T}}(t) & e^{\mathrm{T}}(t) & R^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$, let $K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}$, and we get that

$$\dot{u}(t) = k_1\dot{x}(t) + k_2x(t) + k_3e(t) + k_4R(t).$$

According to the initial condition of system (1), it is not difficult to deduce from the above equality that the control design of system (1) is

$$u(t) = k_1x(t) + k_2\int_0^t x(t)\mathrm{d}s + k_3\int_0^t e(t)\mathrm{d}s + k_4\int_0^t R(t)\mathrm{d}s - k_1\phi(0).$$

It is realized that control $u(t)$ contains not only the state feedback, but also the integrals of state, error signal and the desired output.

4. **Numerical Example.** In this section, we present a numerical example to illustrate the effectiveness of the proposed method. This example comes from a modified NDDE model which is motivated by a small PEEC in [2], where delays in the state and the derivate of the state are equal. Consider system (1) with

$$\begin{aligned} &G = \begin{bmatrix} -0.4 & 0.2 \\ 0.01 & -0.1 \end{bmatrix}, \ A_0 = \begin{bmatrix} -1 & -0.5 \\ -2 & 0.6 \end{bmatrix}, \ A_1 = \begin{bmatrix} -1.5 & -0.5 \\ -0.5 & -0.9 \end{bmatrix}, \\ &B = \begin{bmatrix} -0.5 \\ 1.2 \end{bmatrix}, \ C = \begin{bmatrix} 1.2 & 1.25 \end{bmatrix}, \ \phi(t) = \begin{bmatrix} -t/10 \\ \sin(t) \end{bmatrix}, \ \alpha = 0.8. \end{aligned} \tag{12}$$

The desired output is chosen as a stair-step function, which is often used and belongs to typical signals in control engineer.

$$r(t) = \begin{cases} 0, & t < 10 \\ (t-10)/20, & t \in [10, 30] \\ 1, & t > 30 \end{cases}$$

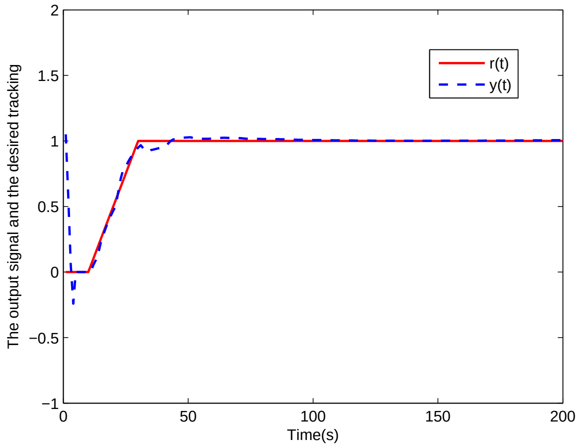

FIGURE 1. Output of system (1) with (12) and $h = 0.1$, $d = 0.02$

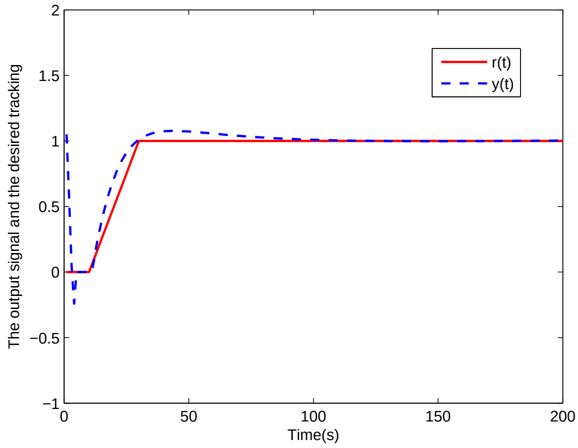

FIGURE 2. Output of system (1) with (12) and $h = 0.02$, $d = 0.1$

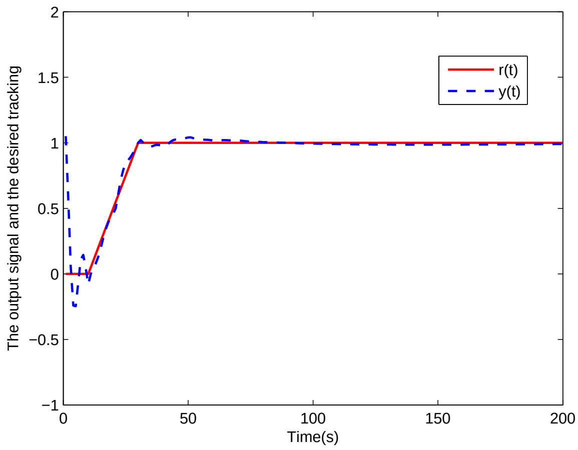

FIGURE 3. Output of system (1) with (12) and $h = d = 0.1$

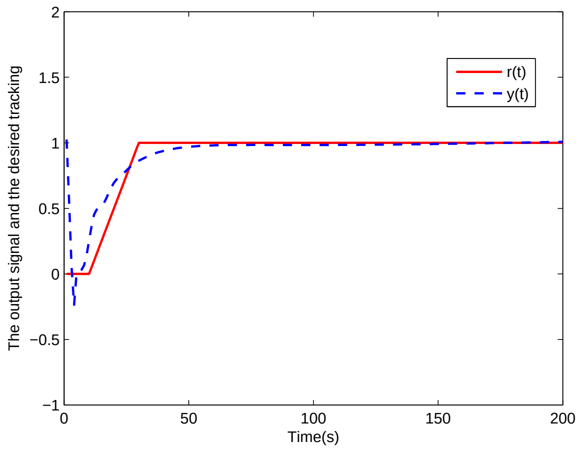

FIGURE 4. Output of system (1) with (12) and $h = d = 0.02$

We compare the output response on the condition that whether delay in the derivate of the state equals that in the state. We respectively choose $h = 0.1$, $d = 0.02$ and $h = 0.02$, $d = 0.1$, the corresponding output responses are shown in Figures 1 and 2 by Theorem 3.2. Then the delays are assumed to be $h = d = 0.1$, $h = d = 0.02$.

Comparing with the above figures, we find that when delay in the derivate of state is smaller, the output can change in advance of the desired output, and its curve is smooth.

5. **Conclusions.**

By use of the desired output and the error signal, output tracking control for a class of linear neutral systems is studied in this paper. Control design is presented based on the augmented error system. A Lyapunov-Krasovskii functional and some integral identities are used to analyze the stability of the augmented error unforced system. Controllers in this paper can make use of the desired output, for integrals of the error signal and the desired output are both contained in control design. A numerical example has been given to demonstrate the validity of results in this paper.

Utilization of the desired output makes the tracking better and decreases the static error. It is particular that the parameter matrices and time-delays in this paper are constants. There exist many neutral systems where the parameters and delays are not constant. So tracking control of the neutral systems with uncertainty or time-varying delays will be studied. And we also need to investigate of how to promote the method of this paper to ordinary delay systems.

Acknowledgment. This work is partially supported by "New Start" Academic Research Projects of Beijing Union University (No. Zk10201503), and the National Natural Science Foundation of China (No. 11501030). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] V. B. Kolmanovskii and A. Myshkis, *Applied Theory of Functional Differential Equations*, Kluwer Academic Publishers, Netherlands, 1992.

[2] A. Bellen, N. Guglielmi and A. E. Ruehli, Methods for linear systems of circuit delay differential equations of neutral type, *IEEE Trans. Circuits and Systems I: Fundamental Theory and Application*, vol.46, no.1, pp.212-216, 1999.

[3] H. Li, *Robust Control for Neutral Delay Systems*, Northwestern Polytechnical University Press, Xi'an, 2006.

[4] P. Balasubramaniam, R. Krishnasamy and R. Rakkiyappan, Delay-dependent stability of neutral systems with time-varying delays using delay-decomposition approach, *Applied Mathematical Modelling*, vol.36, no.5, pp.2253-2261, 2012.

[5] G. Hu, Computable stability criterion of linear neutral systems with unstable difference operators, *Journal of Computational and Applied Mathematics*, vol.271, no.12, pp.223-232, 2014.

[6] S. Arik, An analysis of stability of neutral-type neural systems with constant time delays, *Journal of the Franklin Institute*, vol.351, no.11, pp.4949-4959, 2014.

[7] B. Li, A further note on stability criteria of uncertain neutral systems with mixed delays, *Chaos, Solitons and Fractals*, vol.77, pp.72-83, 2015.

[8] J. Duda, A Lyapunov functionl for a neutral system with distributed time delay, *Mathematics and Computers in Simulation*, vol.119, pp.171-181, 2016.

[9] V. L. Kharitonov, Predictor based stabilization of neutral type systems with input delay, *Automatica*, vol.52, no.1, pp.125-134, 2015.

[10] S. Liu and Z. Xiang, Exponential H_∞ output tracking control for switched neutral system with time-varying delay and nonlinear perturbations, *Circuits, Systems, and Signal Processing*, vol.32, pp.103-121, 2013.

[11] J. Xia, J. H. Park, T. H. Lee and B. Zhang, H_∞ tracking of uncertain stochastic time-delay systems: Memory state-feedback controller design, *Applied Mathematics and Computation*, vol.249, no.9, pp.356-370, 2014.

[12] T. Tsuchiya and T. Egami, *Digital Preview and Predictive Control*, Beijing Science and Technology Press, Beijing, 1994.

[13] T. Katayama and T. Hirono, Design of an optimal servomechanism with preview action and its dual problem, *International Journal of Control*, vol.45, no.2, pp.407-420, 1987.

[14] Q. Han, A discrete delay decomposition approach to stability of linear retarded and neutral systems, *Automatica*, vol.45, no.2, pp.517-524, 2009.

[15] T. Li, J. Zhao and G. M. Dimirovski, Hysteresis switching design for stabilization of a class of switched neutral systems, *Asian Journal of Control*, vol.15, no.4, pp.1149-1157, 2013.

[16] O. M. Kwon, J. H. Park and S. M. Lee, Augmented Lyapunov functional approach to stability of uncertain neutral systems with time-varying delays, *Applied Mathematics and Computation*, vol.207, no.1, pp.202-212, 2009.