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A LOW-COMPLEXITY BERNOULLI FILTER FOR SINGLE TARGET TRACKING

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Abstract. *To deal with the simultaneous process for performing large and intensive computational problem of the existing Bernoulli filter, we propose a low-complexity Bernoulli filter in this paper. Based on the random finite set (RFS) theory, this paper mainly focuses on the analysis and reduction on relative computational complexity. By using conventional measurement set, the operations of the transition and likelihood functions are optimized, and then the sequential Monte Carlo (SMC) implementation is further improved in the Bayes filtering framework. The numerical study results indicate that the proposed filter can track maneuvering single target in the cluttered environment with low relative computational complexity and high tracking accuracy.*
Keywords: Single target tracking, Bernoulli filter, Computational complexity, Clutter rate

1. **Introduction.** Single target tracking (STT) is to complete state estimates of moving target in cluttered environment. With the development of random finite set (RFS) theory, the STT has been widely used in surveillance fields without data association [1].

In recent years, many scholars have researched the STT with a great deal of success, and some articles have been published in international journals [2-7]. The related work is the Bernoulli filter under the assumption that the target state is considered as a Bernoulli RFS. Due to no closed-form solution, the filter is usually implemented using the sequential Monte Carlo (SMC) [2]. In [3], a mathematical formulation of the Bernoulli filter was proposed. Its update equations were derived for various measurement models encountered in practice, where the models were illustrated by some applications for location detection and estimation of dynamic systems. In [4], Ristic and Arulampalam discussed a Bernoulli filter for observer control for bearings-only tracking in cluttered environment and then developed the related SMC implementation based on information theoretic criterion. In [5], a Bernoulli filter was proposed for tracking maritime radiation source in the presence of measurement uncertainty. The tracking performance under different conditions, particularly those involving different duration of source opening and switching-off, indicated that the filter was robust. In [6], a joint detecting and tracking Bernoulli filter for single extended target in the presence of clutter measurements and missed detections was presented, in which simplification method was used to make it easy to be realized in dense clutter backgrounds. Lately, a novel Bernoulli filter and its SMC implementation were proposed to choose the particle proposal distribution for minimizing variance in [7]. Employing the auxiliary particle scheme, the robust particles from a discrete distribution were intelligently extracted to propagate with their indices. Due to the inherent defects, the SMC recursions of the existing Bernoulli filters have higher computational complexity that restricts tracking performance [8]. What is worse, we have to execute complicated operations in the prediction and update steps. How to improve tracking performance of the classical Bernoulli filter has become critically important in practice.

This work aims to perform large and intensive computational problem of the Bernoulli filter, provides an introduction to derive a low-complexity Bernoulli filter based on the Bayes recursion, as well as describing its SMC implementation that accommodates the target-generated measurements and state-dependent clutters. In addition, it emphasizes how to improve tracking accuracy and computational efficiency in actual detection environment. Thus, the mainly contribution of this work is threefold: i) we improve the Bernoulli recursion to achieve optimal measurement likelihood; ii) the updated weight of sampling particle reduces to simplification form based on the optimal measurement likelihood; iii) the computational complexity is synthetically analyzed to reveal the relationship between the Bernoulli recursions and the related detection parameters.

The remainder of this paper is organized as follows. In Section 2, the problems of the STT using the classical Bernoulli filter are briefly formulated. Section 3 discusses the low-complexity Bernoulli recursion and its SMC implementation. In Section 4, the numerical study is demonstrated to evaluate tracking performance of the proposed filter. In Section 5, we draw the conclusions with the next working plan for further research.

2. **Problem Statements.**

At time k, suppose that the target state set $X_k = \{x_{1,k}, \ldots, x_{n_k,k}\}$ is in the space $\mathcal{X} \subseteq \mathbb{R}^{n_k}$ and the measurement set $Z_k = \{z_{1,k}, \ldots, z_{m_k,k}\}$ is in the space $\mathcal{Z} \subseteq \mathbb{R}^{m_k}$, and then the stochastic dynamic system is described as [1,7,9]:

$$x_k = F_{k|k-1}(x_{k-1}) + v_{k-1} \tag{1}$$

$$z_k = h_k(x_k) + u_k \tag{2}$$

where $F_{k|k-1}(\cdot)$ denotes the transition function regarding evolution of the current state x_k, $h_k(\cdot)$ defines the relationship between x_k and the current measurement z_k, and v_{k-1} and u_k are the process and measurement noises. We have in hand the transitional probability density $\pi_{k|k-1}(x_k|x_{k-1})$ for x_{k-1} to x_k, and the single target likelihood $g_k(z_k|x_k)$ based on the conventional measurement z_k.

As we know, the classical Bernoulli filter computes the posterior spatial probability density function (p.d.f.) of target state $s_k(x_k) = \Pr(x_k|Z_{1:k})$ and the posterior probability of target existence $p_k = \Pr\{|X_k| = 1|Z_{1:k}\}$ in the RFS frame.

Suppose that the set X_k is given by a Bernoulli pair $(p_k, s_k(x_k))$, and then the posterior probability generating function (p.g.f.) can be written as:

$$s(X_k|Z_{1:k}) = \begin{cases} p_k s_k(x_k), & X_k = \{x_k\} \\ 1 - p_k, & X_k = \emptyset \end{cases} \tag{3}$$

where the first condition represents only one target with x_k, and the second condition denotes no target in the current scene.

Suppose that $p_{s,k|k-1}(x_{k-1})$ and $p_{b,k|k-1}$ are the probabilities of target survival and target birth, $b_{k|k-1}(x_k)$ is the target birth density, and then the state transitional p.g.f. is:

$$\Pi_{k|k-1}(X_k|X_{k-1}) = \begin{cases} p_{s,k|k-1}(x_{k-1})\,\pi_{k|k-1}(x_k|x_{k-1}), & X_k = \{x_k\},\ X_{k-1} = \{x_{k-1}\} \\ p_{b,k|k-1} b_{k|k-1}(x_k), & X_k = \{x_k\},\quad X_{k-1} = \emptyset \\ 1 - p_{s,k|k-1}(x_{k-1}), & X_k = \emptyset,\ X_{k-1} = \{x_{k-1}\} \\ 1 - p_{b,k|k-1}, & X_k = \emptyset,\ X_{k-1} = \emptyset \end{cases} \tag{4}$$

Similar to (3), we can get the restrictions of four conditions above.

Then, the predicted equations of the existence probability and spatial p.d.f. are:

$$p_{k|k-1} = p_{b,k|k-1}\left(1 - p_{k-1}\right) + p_{s,k|k-1}\left(x_{k-1}\right) p_{k-1} \tag{5}$$

$$s_{k|k-1}(x_k) = \frac{p_{k-1}\left\langle p_{s,k|k-1}(x_{k-1})\pi_{k|k-1}\left(x_k|x_{k-1}\right), s_{k-1}\left(x_{k-1}\right)\right\rangle}{p_{k|k-1}} \tag{6}$$

In (6), $\langle \varsigma, \zeta \rangle$ denotes the inner product of ς and ζ. Also, we find that $s_{k|k-1}(x_k)$ is directly dependent on the target existence.

Considering the cardinality distribution of independent identically distributed (i.i.d.) cluster RFS Z_k, we define the related p.g.f. as:

$$f(Z_k) = |Z_k|!\rho(|Z_k|) \prod_{z_k \in Z_k} p(z_k) \tag{7}$$

When Z_k follows the Poisson distribution, the cardinality distribution in (7) is given by:

$$\rho(|Z_k|) = \frac{e^{-\lambda}\lambda^{|Z_k|}}{|Z_k|!} \tag{8}$$

where λ is the mean clutter number.

On the other hand, we have in hand the clutter distribution $c_k(z_k|x_k)$, then the standard p.d.f. $p(z_k)$ in (7) can be written as:

$$p(z_k) = c_k(z_k|x_k) \tag{9}$$

After defining the clutter process $\lambda c_k\left(z_k|x_k\right)$, we can rewrite (7) as:

$$f(Z_k) = e^{-\lambda} \prod_{z_k \in Z_k} \lambda c_k\left(z_k|x_k\right) \tag{10}$$

Let $p_{D,k}(x_k)$ be the detection probability of passive sensor, and according to (10), the p.g.f. of likelihood function is:

$$G\left(Z_k|X_k\right) = \begin{cases} e^{-\lambda} \prod\limits_{z_k \in Z_k} \lambda c_k\left(z_k|x_k\right) \left(1 - p_{D,k}\left(x_k\right) + \sum\limits_{z_k \in Z_k} \frac{p_{D,k}(x_k) g_k(z_k|x_k)}{\lambda c_k(z_k|x_k)}\right), & X_k = \{x_k\} \\ e^{-\lambda} \prod\limits_{z_k \in Z_k} \lambda c_k\left(z_k|x_k\right), & X_k = \emptyset \end{cases} \tag{11}$$

Note that in the top equation in (11), the left term means the clutter process and the right term denotes the target component when $X_k = \{x_k\}$, whereas there is only the clutter process under the condition of $X_k = \emptyset$.

Consequently, the updated equations of the existence probability and spatial p.d.f. are:

$$p_k = \frac{1 - \Delta_k}{p_{k|k-1}^{-1} - \Delta_k} \tag{12}$$

$$s_k(x_k) = \frac{1 - p_{D,k}(x_k) + \sum\limits_{z_k \in Z_k} \frac{p_{D,k}(x_k) g_k(z_k|x_k)}{\lambda c_k(z_k|x_k)}}{1 - \Delta_k} s_{k|k-1}(x_k) \tag{13}$$

where Δ_k is given by:

$$\Delta_k = p_{D,k}(x_k) \left(1 - \sum_{z_k \in Z_k} \frac{\left\langle g_k\left(z_k|x_k\right), s_{k|k-1}(x_k)\right\rangle}{\lambda c_k\left(z_k|x_k\right)}\right) \tag{14}$$

Remark 2.1. *Note that the propagations of p_k and $s_k(x_k)$ depend on Δ_k, which are coupled in the classical Bernoulli filter. Then, $s_k(x_k)$ has high computational complexity because of computing Δ_k. With the increase of the number of z_k, the integration of $g_k(z_k|x_k)$ and $s_{k|k-1}(x_k)$ becomes more complicated, and the time-consuming on the sum operation in (13) and (14) is inevitable. How to reduce running load by distinguishing effective measurement from state-independent clutters has important significance. Therefore, we will optimize measurement likelihood to simplify the representation of $s_k(x_k)$.*

3. **Proposed Bernoulli Filter.**

3.1. **Filtering principle.**

We utilize three RFSs to define measurement equation at time k, i.e., the primary target-generated measurement $T_k(x_k)$, the suspicious target-generated measurement $S_k(x_k)$, and the state-independent clutter C_k [10,11]:

$$Z_k = T_k(x_k) \cup S_k(x_k) \cup C_k \tag{15}$$

With respect to $T_k(x_k)$, we have the following definition:

$$T_k(x_k) = \begin{cases} \{z_k^*\}, & p_{D,k}(x_k)\, g_k(z_k^*|x_k) \\ \emptyset, & 1 - p_{D,k}(x_k) \end{cases} \tag{16}$$

where the probability of not obtaining primary measurement z_k^* from x_k is $1 - p_{D,k}(x_k)$.

For simplification, we unite $S_k(x_k)$ and C_k in (15) because they are defined by the non-primary measurement z_k:

$$K_k(x_k) = S_k(x_k) \cup C_k \tag{17}$$

In (17), $K_k(x_k)$ is a union of two statistically independent RFSs, and its intensity is:

$$v_{K,k}(z_k|x_k) = v_{S,k}(z_k) + v_{C,k}(z_k|x_k) \tag{18}$$

where the intensities $v_{S,k}(\cdot|x_k)$ and $v_{C,k}(\cdot)$ are corresponding to $S_k(x_k)$ and C_k.

Independent on x_k, each z_k follows the i.i.d. based on the probability density:

$$c_k(z_k) = c_k(z_k|x_k) = \frac{v_{K,k}(z_k|x_k)}{\langle v_{K,k}(z_k|x_k), 1\rangle} \tag{19}$$

Remark 3.1. *Assuming no suspicious target-generated measurement, (17) and (18) reduce to the single-target Bayes recursion because of $S_k(x_k) = 0$ or $v_{S,k}(z_k) = 0$. Due to no new target appearing or old target disappearing, except exactly one target present ensures consistency with the dynamic model. Then (18) becomes $v_{K,k}(z_k|x_k) = v_{C,k}(z_k|x_k)$, and the computational complexity of (19) can be simplified into the standard normalized form.*

According to the Bayes recursion, in the prediction step, we can predict the p.d.f.:

$$s_{k|k-1}(x_k|Z_{1:k-1}) = \left\langle \pi_{k|k-1}(x_k|x_{k-1}), s_{k-1}(x_{k-1}|Z_{1:k-1})\right\rangle \tag{20}$$

where $s_{k-1}(x_{k-1}|Z_{1:k-1})$ denotes the posterior p.d.f. at time $k-1$.

Then, the corresponding p.d.f. is updated when the measurement set Z_k is available:

$$s_k(x_k|Z_{1:k}) = \frac{\phi_k(Z_k|x_k)\, s_{k-1}(x_{k-1}|Z_{1:k-1})}{\langle \phi_k(Z_k|x_k), s_{k-1}(x_{k-1}|Z_{1:k-1})\rangle} \tag{21}$$

In (21), we should find the optimized measurement likelihood.

Consequently, suppose that the Bernoulli recursion accommodates at most one target-generated measurement, i.e., $p_{b,k|k-1} = 0$, then (5) reduces to:

$$p_{k|k-1} = p_{s,k|k-1}(x_k)p_{k-1} \tag{22}$$

In (22), there is only the target survival term and no target birth term.

Usually, the lower probability of detection is unrealistic for actual target tracking. When the detection probability is $p_D(x_k) \doteq 100\%$, the undetected probability $1 - p_D(x_k) \to 0$. According to z_k^*, (13) reduces to:

$$s_k(x_k) \doteq \frac{\sum\limits_{z_k^* \in Z_k} \frac{p_{D,k}(x_k) g_k\left(z_k^*|x_k\right)}{\lambda c\left(z_k^*\right)} s_{k|k-1}\left(x_k\right)}{\sum\limits_{z_k^* \in Z_k} \left\langle \frac{p_{D,k}(x_k) g_k\left(z_k^*|x_k\right)}{\lambda c\left(z_k^*\right)}, s_{k|k-1}(x_k) \right\rangle} \tag{23}$$

At this time, (14) can be rewritten as:

$$\Delta_k \doteq p_{D,k}\left(x_k\right) \sum_{z_k^* \in Z_k} \frac{\left\langle g_k\left(z_k^*|x_k\right), s_{k|k-1}\left(x_k\right)\right\rangle}{\lambda c_k\left(z_k^*\right)} \tag{24}$$

Subsequently, we use $\sum\limits_{z_k^*, z_k \in Z_k} e^{-\lambda} \lambda \left(c(z_k) + c\left(z_k^*\right)\right)$ to multiply both numerator and denominator in (23). According to (21), we have the following recursion:

$$\begin{aligned} s_k(x_k) &\doteq \frac{p_{D,k}(x_k) \sum\limits_{z_k^* \in Z_k} g_k\left(z_k^*|x_k\right) \prod\limits_{z_k \in Z_k, z_k \neq z_k^*} e^{-\lambda} \lambda c_k(z_k) s_{k|k-1}\left(x_k\right)}{\left\langle p_{D,k}(x_k) \sum\limits_{z_k^* \in Z_k} g_k\left(z_k^*|x_k\right) \prod\limits_{z_k \in Z_k, z_k \neq z_k^*} e^{-\lambda} \lambda c_k\left(z_k\right), s_{k|k-1}\left(x_k\right) \right\rangle} \\ &= \frac{\phi_k\left(Z_k|x_k\right) s_{k|k-1}\left(x_k\right)}{\left\langle \phi_k\left(Z_k|x_k\right), s_{k|k-1}(x_k)\right\rangle} \end{aligned} \tag{25}$$

Note that (25) is the generalized form of $s_k(x_k)$, where $\phi_k\left(Z_k|x_k\right)$ can be obtained:

$$\phi_k\left(Z_k|x_k\right) = p_{D,k}(x_k) \sum_{z_k^* \in Z_k} g_k\left(z_k^*|x_k\right) \prod_{z_k \in Z_k, z_k \neq z_k^*} e^{-\lambda} \lambda c_k(z_k) \tag{26}$$

where $\phi_k\left(Z_k|x_k\right)$ is the Radon-Nikodym derivative of probability distribution of Z_k given x_k. It is easy to derive the completed $\phi_k\left(Z_k|x_k\right)$ when the undetected component exists:

$$\begin{aligned} \phi_k\left(Z_k|x_k\right) = &\left(1 - p_{D,k}(x_k)\right) e^{-\lambda} \prod_{z_k \in Z_k} \lambda c_k(z_k) \\ &+ p_{D,k}(x_k) \sum_{z_k^* \in Z_k} g_k\left(z_k^*|x_k\right) e^{-\lambda} \prod_{z_k \in Z_k, z_k \neq z_k^*} \lambda c_k(z_k) \end{aligned} \tag{27}$$

3.2. **SMC implementation.** According to the Bayes recursion above, the related SMC implementation is derived in this subsection.

Prediction step. Suppose that the posterior density at time $k-1$ is approximated by a set of weighted particles $\left\{x_{k-1}^{(i)}, w_{k-1}^{(i)}\right\}_{i=1}^{L_{k-1}}$:

$$p_{k-1}\left(x_{k-1}|Z_{1:k-1}\right) \doteq \sum_{i=1}^{L_{k-1}} w_{k-1}^{(i)} \delta\left(x_{k-1} - x_{k-1}^{(i)}\right) \tag{28}$$

where $\delta(\cdot)$ denotes the Dirac delta function, L_{k-1} is the number of particles, $x_{k-1}^{(i)}$ is the state of the ith particle, and $w_{k-1}^{(i)}$ is its normalized weight.

Update step. Suppose that $q_k\left(x_k|x_{k-1}, Z_k\right)$ is the proposal density distribution that represents the posterior p.d.f. at time k:

$$x_k^{(i)} \sim q_k\left(x_k|x_{k-1}, Z_k\right) \tag{29}$$

Then, the posterior $p_k\left(x_k|Z_{1:k}\right)$ is approximated by the set $\left\{x_k^{(i)}, \tilde{w}_k^{(i)}\right\}_{i=1}^{L_{k|k-1}}$:

$$p_k\left(x_k|Z_{1:k}\right) \doteq \sum_{i=1}^{L_{k|k-1}} \tilde{w}_k^{(i)} \delta\left(x_k - x_k^{(i)}\right) \tag{30}$$

where $L_{k|k-1}$ is the updated number of particles, and the updated weight $\tilde{w}_k^{(i)}$ is given by:

$$\tilde{w}_k^{(i)} = \frac{\frac{\phi_k\left(Z_k\middle|x_k^{(i)}\right)\pi_{k|k-1}\left(x_k^{(i)}\middle|x_{k-1}^{(i)}\right)}{q_k\left(x_k^{(i)}\middle|x_{k-1}^{(i)}, Z_k\right)} w_{k-1}^{(i)}}{\sum_{i=1}^{L_{k|k-1}} \frac{\phi_k\left(Z_k\middle|x_k^{(i)}\right)\pi_{k|k-1}\left(x_k^{(i)}\middle|x_{k-1}^{(i)}\right)}{q_k\left(x_k^{(i)}\middle|x_{k-1}^{(i)}, Z_k\right)} w_{k-1}^{(i)}} \tag{31}$$

For simplification, we select $q_k\left(x_k\middle|x_{k-1}, Z_k\right)$ as $\pi_{k|k-1}\left(x_k^{(i)}\middle|x_{k-1}^{(i)}\right)$, and then (31) reduces to:

$$\tilde{w}_k^{(i)} = \frac{\phi_k\left(Z_k\middle|x_k^{(i)}\right) w_{k-1}^{(i)}}{\sum_{i=1}^{L_{k|k-1}} \phi_k\left(Z_k\middle|x_k^{(i)}\right) w_{k-1}^{(i)}} \tag{32}$$

Resampling and state estimation step. Considering a few particles having a significant and non-zero weight after iterations, we resample L_k particles from the set $\left\{x_k^{(i)}, \tilde{w}_k^{(i)}\right\}_{i=1}^{L_{k|k-1}}$ to relieve particle degeneracy where the weight of the ith particle equals $w_k^{(i)} = 1/L_k$. Then, a new particle set $\left\{x_k^{(i)}, w_k^{(i)}\right\}_{i=1}^{L_k}$ is achieved.

Finally, the target state estimates can be obtained as:

$$\hat{x}_k = \sum_{i=1}^{L_k} w_k^{(i)} x_k^{(i)} \tag{33}$$

Remark 3.2. *Note that (32) represents the kernel estimate of updated weight, which follows the Bayes theory. When $p_D\left(x_k^{(i)}\right)$ is a fixed constant and its upper bound approximates 1, it can be deleted in (26) to reduce complexity of computing $\tilde{w}_k^{(i)}$. At this time, we have:*

$$\phi_k\left(Z_k\middle|x_k^{(i)}\right) = \sum_{z_k^* \in Z_k} g_k\left(z_k^*\middle|x_k^{(i)}\right) \prod_{z_k \in Z_k, z_k \neq z_k^*} e^{-\lambda}\lambda c_k(z_k) \tag{34}$$

3.3. **Analysis of computational complexity.** Given that z_k^*, x_k, and x_{k-1} are known, the computational complexities of $\pi_{k|k-1}\left(x_k^{(i)}\middle|x_{k-1}^{(i)}\right)$ and $g_k\left(z_k\middle|x_k^{(i)}\right)$ are $O(\alpha)$ and $O(\beta)$. Usually, a single sensor interrogates a certain scene containing n targets and L_k particles per target. Then the computational complexity of the classical Bernoulli filter is [10]:

$$O\left(n(\alpha + n!\beta)L_k\right) \tag{35}$$

In the proposed Bernoulli filter, the complexity of $\pi_{k|k-1}\left(x_k^{(i)}\middle|x_{k-1}^{(i)}\right)$ is reduced by half because of the assumption of no target birth except for target survival. Regarding $g_k\left(z_k^*\middle|p_{D,k}\left(x_k^{(i)}\right)\right)$, its relative computational complexity is $O\left(n\beta/|Z_k|\right)$ owing to the number of z_k^* is 1. Under the situation of different values of $|Z_k|$, we can find that the larger the value of $|Z_k|$ is, the smaller the relative computational complexity is. Therefore, the total relative complexity is:

$$O\left(\left(\frac{n\alpha}{2}+\frac{(n-1)!n^2\beta}{|Z_k|}\right)L_k\right)\xrightarrow[n=1]{|Z_k|=p_{D,k}n+\lambda}O\left(\left(\frac{\alpha}{2}+\frac{\beta}{p_{D,k}\left(x_k^{(i)}\right)+\lambda}\right)L_k\right) \tag{36}$$

Note that the total relative complexity approximates to $O\left(\alpha L_k/2\right)$ with the increase of λ, whereas the original value of the classical filter in (35) is $O\left((\alpha+\beta)L_k\right)$ in the same situation. Obviously, we can find that the running load in the whole recursion is shortened.

4. **Numerical Study and Discussion.**

In the numerical study, we will verify the low-complexity Bernoulli filter. The experimental environment was: Intel$^{\text{TM}}$ Core$^{\text{TM}}$ i5, RAM 4 GB, Winows$^{\text{TM}}$ 7 and MATLAB$^{\text{TM}}$ V8.0.

4.1. **Scenario.**

Given that the maneuvering target tracking in cluttered environment is very representative, in this scenario, we suppose that a constant turn (CT) motion target travels in a top half-disc region of $[-2000, 2000] \times [0, 2000]$ m^2. The sensor is located on the origin $(0, 0)$ m with the detection probability of 98.5%. The surveillance time is 60 s and the sampling period is 1 s. Suppose that the target executes the anti-clock CT motion of the turn rate 0.01 rad/s and the velocity $(-10, 30)$ m/s from initial position coordinate $(10, 10)$ m during the surveillance time. The equations of the dynamic system in (1) and (2) are:

$$x_k=\begin{bmatrix}1 & 0.99998 & 0 & -0.00500 & 0\\ 0 & 0.99995 & 0 & -0.01000 & 0\\ 0 & 0.00500 & 1 & 0.99998 & 0\\ 0 & 0.01000 & 0 & 0.99995 & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}x_{k-1}+\begin{bmatrix}0.5 & 0 & 0\\ 1 & 0 & 0\\ 0 & 0.5 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}v_{k-1} \tag{37}$$

$$z_k=\begin{bmatrix}\arctan\left(y_k/x_k\right)\\ \sqrt{x_k^2+y_k^2}\end{bmatrix}+u_k \tag{38}$$

where the standard deviations of v_{k-1} and u_k are diag$(100, 10, 100, 10, 1)$ and diag$(2\pi/180, 10)$, and diag$(\cdot)$ denotes the diagonal matrix. Besides, the probability of target survival is 99%, and the mean clutter rate in surveillance region is 10.

In order to evaluate tracking performance, we use the 1-order optimal sub-pattern assignment (OSPA) distance $d^{(c)}\left(X,\hat{X}\right)$ in (39).

$$\begin{cases} d^{(c)}\left(X,\hat{X}\right)=\dfrac{1}{\left|\hat{X}\right|}\left(\min\limits_{\theta\in\Theta_{|\hat{X}|}}\sum\limits_{i=1}^{|X|}d^{(c)}\left(x_i,\hat{x}_{\theta(i)}\right)+c\left(\left|\hat{X}\right|-\left|\hat{X}\right|\right)\right) & |X|\le\left|\hat{X}\right|\\ d^{(c)}\left(X,\hat{X}\right)=\min\left(c,\|x-\hat{x}\|\right) & |X|>\left|\hat{X}\right|\end{cases} \tag{39}$$

where $X=\{x_i\}_{i=1}^{o}$ and $\hat{X}=\{\hat{x}_i\}_{i=1}^{e}$ are the original and estimated state sets, $\Theta_{|\hat{X}|}$ is the permutation set in $\left|\hat{X}\right|$, and c ($c>0$) is the cut-off parameter that determines the weight of penalties assigned to cardinality and localization error. In this scenario, we set $c=100$ m to define the maximal value of the OSPA distance that determines cardinality error as well as position error. It also represents a threshold at which no distinction is made between whether X and $\hat{X}$ are paired together, or whether one of them is unassigned when the other is an exact match.

4.2. **Discussion.** Figure 1 shows the true track of the target and current measurements. Note that the target track among random clutters in the surveillance region is a continuous curve, which represents that the target is executing the CT motion. Subsequently, the true track, measurements and two filter estimates in the x and y coordinates are plotted in Figures 2 and 3. As seen, two filters can estimate the position of target, whereas the proposed Bernoulli filter gives stable position-estimates. For comparison, the position-estimates from the classical Bernoulli filter are biased during the surveillance time. We can find that there is a false alarm happened in the x and y coordinates on the 40th s. Figure 4 demonstrates the number-estimates versus time for two filters. It can be seen that the proposed filter produces cardinality results essentially in agreement with the ground truth. However, the classical filter over-estimates a target on the 40th s. The reason can be explained that the classical filter mistakes a surrounding clutter for actual target. Figure 5 compares the OSPA distance of two filters under consideration. Note that the classical filter settles to distance error, whilst the proposed filter achieves the lower error as a direct result of always approaching the true position. During the surveillance time, the OSPA distance is lower than that of the classical filter. Especially, the intensive peak on the 40th s represents cardinality error owing to the given value of c. In Figure 6, we plot the relative computational complexity under different clutter rates. Compared with the classical filter, the proposed filter gives decreasing the relative complexity with the increase of λ.

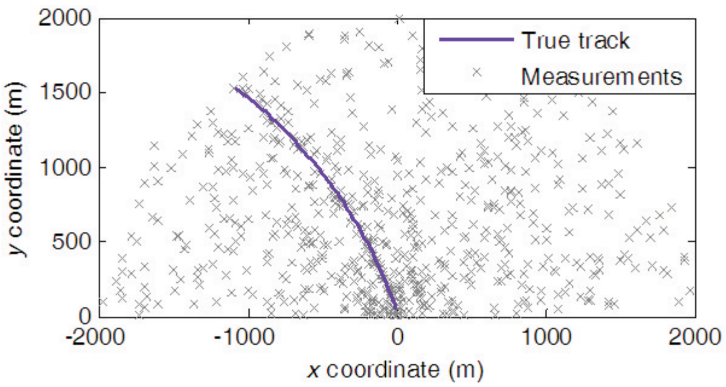

FIGURE 1. Target track and measurements

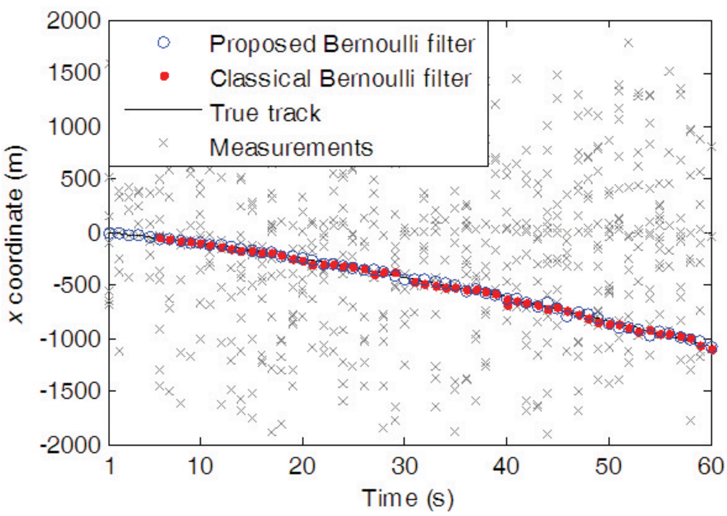

FIGURE 2. Position-estimates in x coordinate

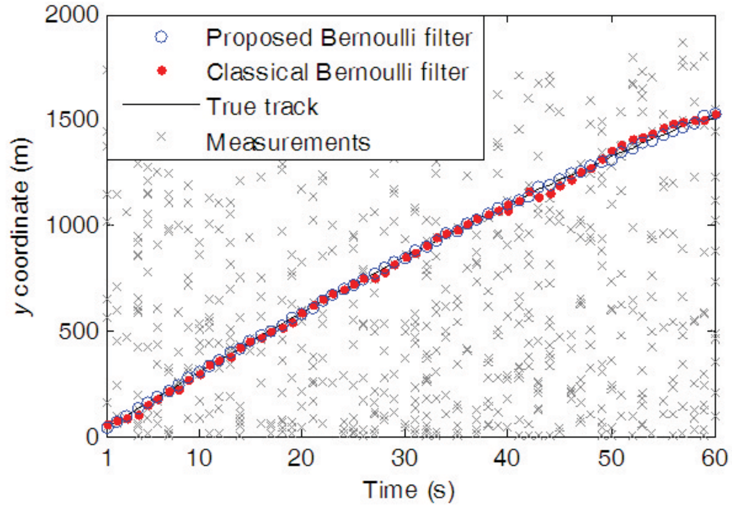

FIGURE 3. Position-estimates in y coordinate

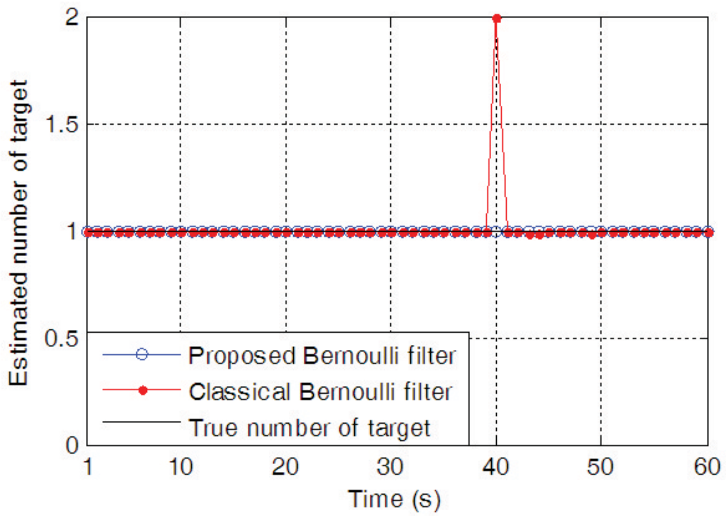

FIGURE 4. Target number-estimates

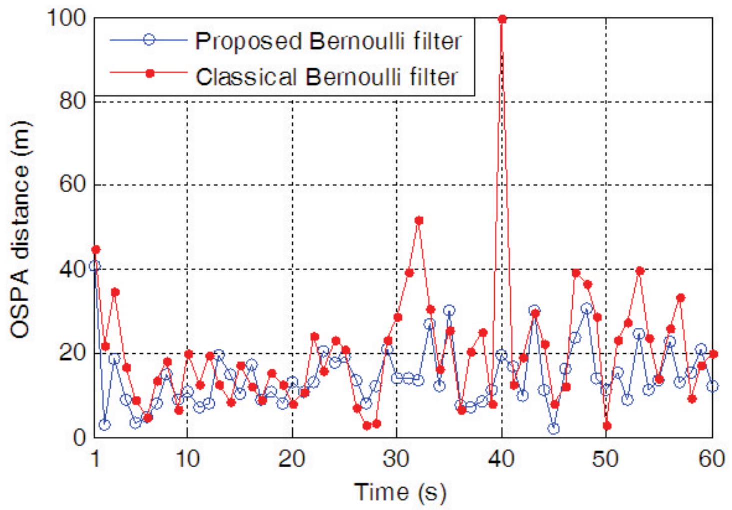

FIGURE 5. OSPA distance

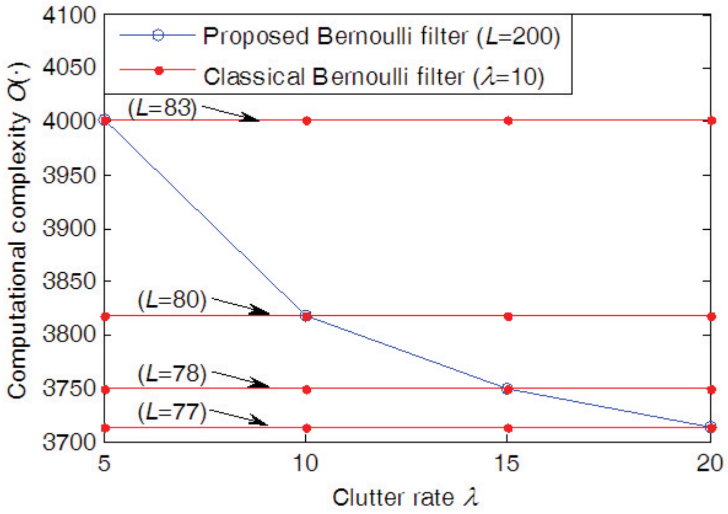

FIGURE 6. Relative computational complexity

TABLE 1. Tracking performance statistics

Kinds of filters	Target number	OSPA distance (m)	Max error of track (m)	Computational complexity
Proposed Bernoulli filter	Mean: 1.0003 Std: 0.0046	Mean: 14.42 Std: 7.37	x coordinate: 10.73 y coordinate: 24.08	$\lambda = 10$, $L = 200$ $O(3818)$
Classical Bernoulli filter	Mean: 1.0156 Std: 0.1273	Mean: 20.85 Std: 15.12	x coordinate: 13.84 y coordinate: 32.56	$\lambda = 10$, $L = 80$ $O(3818)$

In Table 1, the tracking performance statistics are shown. Combining Figures 2-6, we can find that the proposed filter has smaller number-estimates and OSPA distance. Regarding the track bias in the x and y coordinates, the proposed filter reports the expected position improvements. The classical filter requires 80 particles to get the value $O(3818.18)$ under the circumstance of $\lambda = 10$. In terms of the same computational complexity, the required particle number of the proposed filter can achieve about 200, which means the relative computational complexity reduces 150%. Therefore, the proposed Bernoulli filter has remarkable solution, no matter tracking accuracy or computational efficiency.

5. **Conclusions.** This paper discusses a novel Bernoulli filter for the STT. With respect to computational complexity of tracking process, we simplify the Bayes recursion using the optimized transition and likelihood functions. The proposed filter accommodates the target-generated measurement and state-dependent clutters. Besides, its SMC implementation distinguishes the actual target from random clutters. The numerical study suggests that the proposed filter provides promising results with low relative computational complexity and high tracking accuracy. Further works will continue the development of the proposed filter to track multi-target with different dynamic motions. Moreover, the extended target tracking should be considered.

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