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CHAOS CONTROL IN A 3D AUTONOMOUS SYSTEM

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Abstract. *This paper is devoted to investigating the problem of controlling chaos in a 3D autonomous chaotic dynamical system. Time-delayed feedback control method is applied to suppressing chaos to unstable equilibria or unstable periodic orbits. By adding the different feedback control term to the first, second equation of the 3D autonomous chaotic dynamical system, the stability and the occurrences of Hopf bifurcation of the controlled dynamical model are discussed and some sufficient conditions which ensure the stability and the occurrences of Hopf bifurcation are established. Some numerical simulations are presented to support theoretical predictions. Finally, main conclusions are given.*

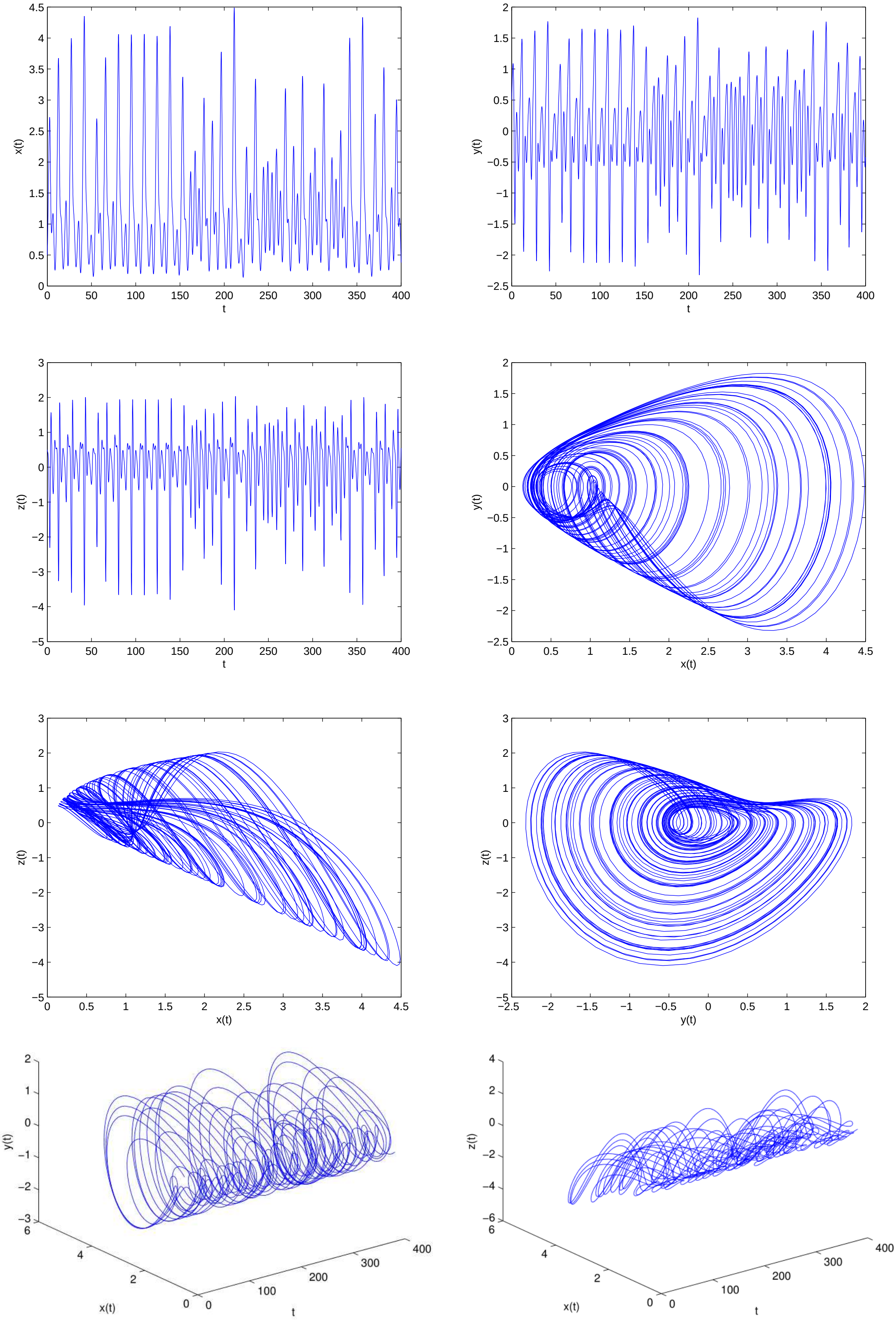
Keywords: 3D autonomous system, Chaos, Stability, Hopf bifurcation, Time-delayed feedback

1. **Introduction.** Chaos originates from the nonlinear interaction of system and is very sensitive to the configuration and initial condition of system. Chaotic systems play an important role in many fields such as secure communications, information processing, and high-performance circuit design for telecommunications [1]. During the last decade, many methods have been proposed to control chaos, i.e., to stabilize the chaotic dynamical systems to period motion, when chaos is not unwanted or undesirable. Recently, many excellent books were given by Moon [2], Chen and Dong [3], and Kapitaniak [4]. Moreover, numberous outstanding reports were presented by EI Naschie [5] and Kapitaniak et al. [6-21,28-31]. In 2013, Wang and Chen [22] have found a new chaotic attractor from the following 3D autonomous system

$$\begin{cases} \dot{x} = y, \\ \dot{y} = z, \\ \dot{z} = -y + 3y^2 - x^2 - xz - a, \end{cases} \tag{1}$$

where a is real constant. System (1) is chaotic. Figure 1 shows the time history plots of $t-x$, $t-y$ and $t-z$, phase plots $x-y$, $x-z$ and $y-z$ and space plots of $x-y-z$.

The aim of this paper is to investigate the dynamics of the 3D autonomous system (1) by considering the effect of delayed feedbacks. By making a detailed analysis on the characteristic equation of linearized system of the model, we theoretically prove that the Hopf bifurcation occurs in this model. Numerical results support the theoretical findings. The main contributions of this paper lie in four aspects. (i) The chaotic behavior of the 3D autonomous chaotic dynamical system has been controlled by applying three different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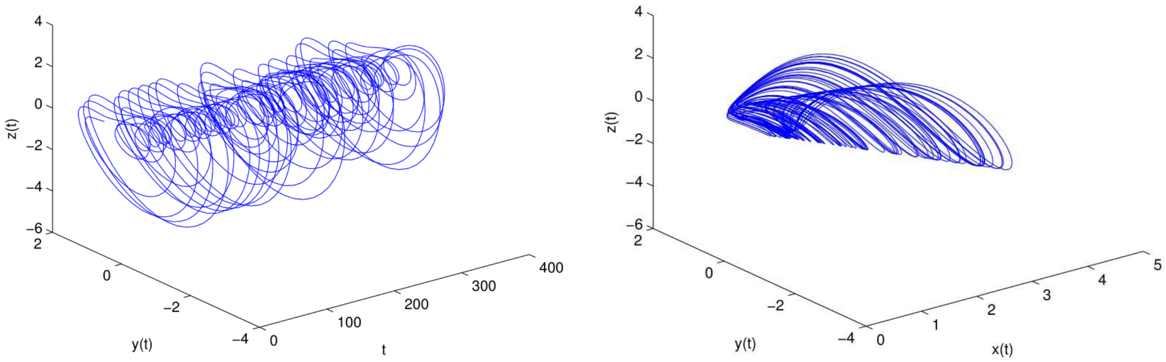


FIGURE 1. The new chaotic attractor of system (1) with $a = 0.05$ and the initial value is $(0.5, 0.5, 0.5)$.

feedback controllers. (ii) We establish some new sufficient conditions which guarantee the stability and the occurrences of Hopf bifurcation of the 3D system. (iii) The obtained results of this article complement some previous results of [6,18,19,21,23,28]. (iv) The control method of this paper can be applied to controlling some other similar chaotic systems.

2. **Controlling Chaos via Feedback Control Methods.** In this section, we shall apply the conventional feedback method to the dynamical system (1). Our aim is to drag the chaotic trajectories to the equilibrium or periodic orbits. Following the idea of Pyragas [23], we add two time-delayed forces $k_1[x(t)-x(t-\tau_1)]$ and $k_2[y(t)-y(t-\tau_2)]$ to the first and the second equations of system (1), respectively, and then system (1) reads as

$$\begin{cases} \dot{x} = y + k_1(x(t) - x(t-\tau_1)), \\ \dot{y} = z + k_2(y(t) - y(t-\tau_2)), \\ \dot{z} = -y + 3y^2 - x^2 - xz - a. \end{cases} \tag{2}$$

If

$$\text{(H1)}\ a > 0,$$

then system (1) has two symmetrical equilibria: $(\sqrt{a}, 0, 0)$ and $(-\sqrt{a}, 0, 0)$. If $a = 0$, then system (1) has one equilibrium: $(0, 0, 0)$. $a < 0$, then system (1) has no equilibrium, but system (1) still generates a chaotic attractor.

In the following, we only consider the dynamical behavior of the equilibrium $(x^*, 0, 0) = (\sqrt{a}, 0, 0)$ of system (2). As for another equilibrium, similar analysis can be carried out. Here we omit it. Now we consider three cases.

Case 1. Delayed feedback on the first equation

In this case, we will investigate the system (2) in which the variable x is influenced by the delayed feedback with $k_2 = 0$, i.e., system (2) takes the form

$$\begin{cases} \dot{x} = y + k_1(x(t) - x(t-\tau_1)), \\ \dot{y} = z, \\ \dot{z} = -y + 3y^2 - x^2 - xz - a. \end{cases} \tag{3}$$

The linearized system of Equation (3) around $(x^*, 0, 0)$ is given by

$$\begin{cases} \dot{x} = k_1 x - k_1 x(t-\tau_1) + y, \\ \dot{y} = z, \\ \dot{z} = -2x^* x - y - x^* z. \end{cases} \tag{4}$$

The characteristic equation of (4) takes the form

$$\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 + (b_2\lambda^2 + b_1\lambda + b_0)e^{-\lambda\tau_1} = 0, \tag{5}$$

where $a_0 = -2x^* - k_1$, $a_1 = 1 - k_1x^*$, $a_2 = x^* - k_1$, $b_0 = -k_1$, $b_1 = -k_1x^*$, $b_2 = -k_1$. In the sequel, we will deal with the distribution of roots of the transcendental equation (5).

Lemma 2.1. [24] *For the transcendental equation*

$$\begin{aligned} P(\lambda, e^{-\lambda\tau_1}, \ldots, e^{-\lambda\tau_m}) \;=\; & \lambda^n + p_1^{(0)}\lambda^{n-1} + \cdots + p_{n-1}^{(0)}\lambda + p_n^{(0)} \\ & + \left[p_1^{(1)}\lambda^{n-1} + \cdots + p_{n-1}^{(1)}\lambda + p_n^{(1)}\right] e^{-\lambda\tau_1} + \cdots \\ & + \left[p_1^{(m)}\lambda^{n-1} + \cdots + p_{n-1}^{(m)}\lambda + p_n^{(m)}\right] e^{-\lambda\tau_m} = 0, \end{aligned}$$

as $(\tau_1, \tau_2, \tau_3, \ldots, \tau_m)$ vary, the sum of orders of the zeros of $P\left(\lambda, e^{-\lambda\tau_1}, \ldots, e^{-\lambda\tau_m}\right)$ in the open right half plane can change, and only a zero appears on or crosses the imaginary axis.

When $\tau_1 = 0$, (5) has the form

$$\lambda^3 + (a_2 + b_2)\lambda^2 + (a_1 + b_1)\lambda + a_0 + b_0 = 0. \tag{6}$$

It is easy to see that all roots of (6) have a negative real part if the following condition

$$\text{(H2)}\ a_2 + b_2 > 0, \quad a_0 + b_0 > 0, \quad (a_2 + b_2)(a_1 + b_1) > a_0 + b_0$$

holds. Then the equilibrium point $(x^*, 0, 0)$ is locally asymptotically stable when the conditions (H1) and (H2) hold.

For $\omega > 0$, $i\omega$ is a root of (5) if and only if

$$-\omega^3 i - a_2\omega^2 + a_1\omega i + a_0 + \left(-b_2\omega^2 + b_1\omega i + b_0\right) e^{-\omega\tau i} = 0.$$

Separating the real and imaginary parts gives

$$\begin{cases} (b_0 - b_2\omega^2)\cos\omega\tau_1 + b_1\omega\sin\omega\tau_1 = a_2\omega^2 - a_0, \\ b_1\omega\cos\omega\tau_1 + \left(b_0 - b_2\omega^2\right)\sin\omega\tau_1 = \omega^3 - a_1\omega. \end{cases} \tag{7}$$

It follows from (7) that

$$\left(b_0 - b_2\omega^2\right)^2 + (b_1\omega)^2 = \left(a_2\omega^2 - a_0\right)^2 + \left(\omega^3 - a_1\omega\right)^2,$$

which is equivalent to

$$\omega^6 + p_1\omega^4 + q_1\omega^2 + r_1 = 0, \tag{8}$$

where $p_1 = a_2^2 - b_2^2 - 2a_1$, $q_1 = a_1^2 - 2a_0a_2 - b_1^2 + 2b_0b_2$, $r_1 = a_0^2 - b_0^2$.

Denote $z = \omega^2$, and then (8) takes the following form

$$z^3 + p_1z^2 + q_1z + r_1 = 0. \tag{9}$$

Let

$$h(z) = z^3 + p_1z^2 + q_1z + r_1. \tag{10}$$

Song et al. [25] obtained the following results on the distribution of roots of Equations (5) and (9).

Lemma 2.2. *For the polynomial equation (9),*

(i) If $r_1 < 0$, then Equation (9) has at least one positive root;

(ii) If $r_1 \geq 0$ and $\Delta_1 = p_1^2 - 3q_1 \leq 0$, then Equation (9) has no positive roots;

(iii) If $r_1 \geq 0$ and $\Delta_1 = p_1^2 - 3q_1 > 0$, then Equation (9) has positive roots if and only if $z_1^* = \frac{-p_1+\sqrt{\Delta_1}}{3}$ and $h(z_1^*) \leq 0$.

Suppose that Equation (10) has positive roots. Without loss of generality, we assume that it has three positive roots, denoted by z_1, z_2 and z_3, respectively. Then Equation (8) has three positive roots

$$\omega_1 = \sqrt{z_1}, \quad \omega_2 = \sqrt{z_2}, \quad \omega_3 = \sqrt{z_3}.$$

By (7), we derive

$$\cos\omega_k\tau_1 = \frac{(a_2\omega_k^2 - a_0)(b_0 - b_2\omega_k^2) - (\omega_k^3 - a_1\omega_k)\, b_1\omega_k}{(b_0 - b_2\omega_k^2)^2 - (b_1\omega_k)^2}.$$

Thus, if we denote

$$\tau_{1k}^{(j)} = \frac{1}{\omega_k}\left\{ \arccos\left(\frac{(a_2\omega_k^2 - a_0)(b_0 - b_2\omega_k^2) - (\omega_k^3 - a_1\omega_k) b_1\omega_k}{(b_0 - b_2\omega_k^2)^2 - (b_1\omega_k)^2} \right) + 2j\pi \right\}, \tag{11}$$

where $k = 1, 2, 3$; $j = 0, 1, 2, \ldots$, then $\pm i\omega_k$ are a pair of imaginary roots of Equation (5) with $\tau_{1k}^{(j)}$. Define

$$\tau_{1_0} = \tau_{1k0}^{(0)} = \min_{k\in\{1,2,3\}} \left\{\tau_{1k}^{(0)}\right\}, \quad \omega_0 = \omega_{k0}. \tag{12}$$

The following Lemma 2.3 is taken from Song et al. [25].

Lemma 2.3. *For the third degree exponential polynomial equation (5), we have*

(i) if $r_1 \geq 0$ and $\Delta_1 = p_1^2 - 3q_1 \leq 0$, then all roots with positive real parts of Equation (5) have the same sum as those of the polynomial Equation (6) for all $\tau_1 \geq 0$;

(ii) if either $r_1 < 0$ or $r \geq 0$, $\Delta_1 = p_1^2 - 3q_1 > 0$, $z_1^* = \frac{-p_1+\sqrt{\Delta_1}}{3} > 0$ and $h(z_1^*) \leq 0$, then all roots with positive real parts of Equation (5) have the same sum as those of the polynomial equation (6) for all $\tau_1 \in [0, \tau_{1_0})$.

Let $\lambda(\tau_1) = \alpha(\tau_1) + i\omega(\tau_1)$ be a root of (5) around $\tau_1 = \tau_{1_0}^{(j)}$, and $\alpha\left(\tau_{1_0}^{(j)}\right) = 0$ and $\omega\left(\tau_{1_0}^{(j)}\right) = \omega_k$. Differentiating both sides of (5) with respect to τ_1 yields

$$\begin{aligned}&\left[3\lambda^2 + 2a_2\lambda + a_1 + \left(2b_2\lambda + b_1 - \tau_1\left(b_2\lambda^\lambda 2 + b_1\lambda + b_0\right)\right)e^{-\lambda\tau_1}\right]\frac{d\lambda}{d\tau_1}\\ &= \lambda e^{-\lambda\tau_1}\left(b_2\lambda^2 + b_1\lambda + b_0\right),\end{aligned}$$

which gives

$$\left[\frac{d\lambda}{d\tau_1}\right]^{-1} = -\frac{2\lambda^3 + a_2\lambda^2 - a_0}{\lambda^2\left(\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0\right)} + \frac{b_2\lambda - b_0}{\lambda^2(b_2\lambda^2 + b_1\lambda + b_0)} - \frac{\tau_1}{\lambda}.$$

Let $\lambda = i\omega_k, \tau_1 = \tau_{1k}^{(j)}$, and then we have

$$\left[\frac{d\lambda}{d\tau_1}\right]^{-1}\Bigg|_{\lambda=i\omega_k,\tau_1=\tau_{1k}^{(j)}} = \frac{-2\omega_k^3 i - a_2\omega_k^2 - a_0}{\omega_k^2\left(a_0 - a_2\omega_k^2 - i\left(\omega_k^3 - a_1\omega_k\right)\right)} + \frac{b_2\omega_k^2 + b_0}{\omega_k^2\left(b_0 - b_2\omega_k^2 + b_1\omega_k i\right)} - \frac{\tau_{1k}^{(j)}}{i\omega_k}.$$

Then

$$\begin{aligned}&\mathrm{Re}\left\{\left[\frac{d\lambda}{d\tau_1}\right]^{-1}\Bigg|_{\lambda=i\omega_k,\tau_1=\tau_{1k}^{(j)}}\right\}\\ &= -\frac{1}{\omega_k^2}\left[\frac{a_0^2 - (a_2^2 - 2a_1)\omega_k^4 - 2\omega_k^6}{(a_0 - a_2\omega_k^2)^2 + (\omega_k^3 - a_1\omega_k)^2} - \frac{b_0^2 - b_2^2\omega_k^4}{b_1^2\omega_k^2 + (b_0 - b_2\omega_k^2)^2}\right]\\ &= \frac{2\omega_k^6 + p\omega_k^4 - r}{\omega_k^2(b_0 - b_2\omega_k^2)^2 + b_1^2\omega_k^2} = \frac{3\omega_k^4 + 2p\omega_k^2 + q}{(b_0 - b_2\omega_k^2)^2 + b_1^2\omega_k^2},\end{aligned}$$

where Re{.} is the real part of $\cdot$. We assume that the following condition holds.

$$\text{(H3) } 3\omega_k^4 + 2p_1\omega_k^2 + q_1 \neq 0.$$

According to above analysis and the results of Yang [26] and Kuang [27], we have

Theorem 2.1. *If* (*H*1) *and* (*H*2) *hold, then the equilibrium* $(x^*, 0, 0)$ *of system (3) is asymptotically stable for* $\tau \in [0, \tau_0)$. *Under the conditions* (*H*1) *and* (*H*2), *if the condition* (*H*3) *holds, then system (3) undergoes a Hopf bifurcation at the equilibrium* $(x^*, 0, 0)$ *when* $\tau_1 = \tau_{1_0}^{(j)}$, $j = 0, 1, 2, \ldots$.

Case 2. Delayed feedback on the second equation

In this case, we will investigate system (2) in which the variable y is influenced by the delayed feedback with $k_1 = 0$, i.e., system (2) takes the form

$$\begin{cases} \dot{x} = y, \\ \dot{y} = z + k_2(y(t) - y(t - \tau_2)), \\ \dot{z} = -y + 3y^2 - x^2 - xz - a. \end{cases} \tag{13}$$

The linearized system of Equation (13) around $(x^*, 0, 0)$ is given by

$$\begin{cases} \dot{x} = y, \\ \dot{y} = k_2 y + z - k_2 y(t - \tau_2), \\ \dot{z} = -2x^* x - y - x^* z. \end{cases} \tag{14}$$

The characteristic equation of (14) takes the form

$$\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0 + \left(d_2\lambda^2 + d_1\lambda\right) e^{-\lambda\tau_2} = 0, \tag{15}$$

where $c_0 = -2x^*$, $c_1 = 1 - k_2x^*$, $c_2 = x^* - k_2$, $d_1 = -k_2x^*$, $d_2 = -k_2$. Next, we will analyze the distribution of roots of the transcendental equation (15).

When $\tau_2 = 0$, (15) reads as

$$\lambda^3 + (c_2 + d_2)\lambda^2 + (c_1 + d_1)\lambda + c_0 = 0. \tag{16}$$

All roots of (16) have a negative real part if the following condition

$$\text{(H4) } c_2 + d_2 > 0, \quad c_0 > 0, \quad (c_2 + d_2)(c_1 + d_1) > c_0$$

holds. Then the equilibrium point $(x^*, 0, 0)$ is locally asymptotically stable when the conditions (H1) and (H2) are satisfied.

For $\tilde{\omega} > 0$, $i\tilde{\omega}$ is a root of (15) if and only if

$$-\tilde{\omega}^3 i - c_2\tilde{\omega}^2 + c_1\tilde{\omega} i + c_0 + \left(-d_2\tilde{\omega}^2 + d_1\tilde{\omega} i\right) e^{-\tilde{\omega}\tau_2 i} = 0.$$

Separating the real and imaginary parts gives

$$\begin{cases} d_2\tilde{\omega}^2 \cos\tilde{\omega}\tau_2 + d_1\tilde{\omega} \sin\tilde{\omega}\tau_2 = c_2\tilde{\omega}^2 - c_0, \\ d_1\tilde{\omega} \cos\tilde{\omega}\tau_2 + d_2\tilde{\omega}^2 \sin\tilde{\omega}\tau_2 = \tilde{\omega}^3 - c_1\tilde{\omega}. \end{cases} \tag{17}$$

It follows from (17) that

$$\left(d_2\tilde{\omega}^2\right)^2 + (d_1\tilde{\omega})^2 = \left(c_2\tilde{\omega}^2 - c_0\right)^2 + \left(\tilde{\omega}^3 - c_1\tilde{\omega}\right)^2,$$

which is equivalent to

$$\tilde{\omega}^6 + p_2\tilde{\omega}^4 + q_2\tilde{\omega}^2 + r_2 = 0, \tag{18}$$

where

$$p_2 = c_2^2 - d_2^2 - 2c_1, \quad q_2 = c_1^2 - 2c_0c_2 - d_1^2, \quad r_2 = c_0^2.$$

Denote $\tilde{z} = \tilde{\omega}^2$, and then (18) takes the following form

$$\tilde{z}^3 + p_2\tilde{z}^2 + q_2\tilde{z} + r_2 = 0. \tag{19}$$

Let

$$\tilde{h}(\tilde{z}) = \tilde{z}^3 + p_2\tilde{z}^2 + q_2\tilde{z} + r_2. \tag{20}$$

Obviously, $r_2 \geq 0$. Thus it follows from Song et al. [25] that we have the results on the distribution of roots of Equations (15) and (19) as follows.

Lemma 2.4. *For the polynomial equation (19),*

(ii) If $\Delta_2 = p_2^2 - 3q_2 \leq 0$, then Equation (19) has no positive roots;

(iii) If $\Delta_2 = p_2^2 - 3q_2 > 0$, then Equation (19) has positive roots if and only if $\tilde{z}_1^* = \frac{-p_2+\sqrt{\Delta_2}}{3}$ and $\tilde{h}(\tilde{z}_1^*) \leq 0$.

Suppose that Equation (20) has positive roots. Without loss of generality, we assume that it has three positive roots, denoted by $\tilde{z}_1$, $\tilde{z}_2$ and $\tilde{z}_3$, respectively. Then Equation (18) has three positive roots

$$\tilde{\omega}_1 = \sqrt{\tilde{z}_1}, \quad \tilde{\omega}_2 = \sqrt{\tilde{z}_2}, \quad \tilde{\omega}_3 = \sqrt{\tilde{z}_3}.$$

By (17), we derive

$$\cos\tilde{\omega}_k\tau_2 = \frac{d_2\tilde{\omega}_k^2\left(c_0 - c_2\tilde{\omega}_k^2\right) - \left(\omega_k^3 - c_1\tilde{\omega}_k\right)d_1\tilde{\omega}_k}{\left(-d_2\tilde{\omega}_k^2\right)^2 - \left(d_1\tilde{\omega}_k\right)^2}.$$

Thus, if we denote

$$\tau_{2k}^{(j)} = \frac{1}{\tilde{\omega}_k}\left\{\arccos\left(\frac{d_2\tilde{\omega}_k^2\left(c_0 - c_2\tilde{\omega}_k^2 - c_0\right) - \left(\tilde{\omega}_k^3 - c_1\tilde{\omega}_k\right)d_1\tilde{\omega}_k}{\left(-d_2\tilde{\omega}_k^2\right)^2 - \left(b_1\tilde{\omega}_k\right)^2}\right) + 2j\pi\right\}, \tag{21}$$

where $k = 1, 2, 3$; $j = 0, 1, 2, \ldots$, then $\pm i\tilde{\omega}_k$ are a pair of imaginary roots of Equation (15) with $\tau_{2k}^{(j)}$. Define

$$\tau_{2_0} = \tau_{2k0}^{(0)} = \min_{k\in\{1,2,3\}}\left\{\tau_{2k}^{(0)}\right\}, \quad \tilde{\omega}_0 = \tilde{\omega}_{k0}. \tag{22}$$

The following Lemma 2.5 is taken from Song et al. [25].

Lemma 2.5. *For the third degree exponential polynomial equation (15), we have (i) if $\Delta_2 = p^2 - 3q \leq 0$, then all roots with positive real parts of Equation (15) have the same sum as those of the polynomial equation (16) for all $\tau_2 \geq 0$;*

(ii) if $\Delta_2 = p_2^2 - 3q_2 > 0$, $\tilde{z}_1^* = \frac{-p_2+\sqrt{\Delta_2}}{3} > 0$ and $\tilde{h}(\tilde{z}_1^*) \leq 0$, then all roots with positive real parts of Equation (15) have the same sum as those of the polynomial equation (16) for all $\tau_2 \in [0, \tau_{2_0})$.

Let $\lambda(\tau_2) = \tilde{\alpha}(\tau_2) + i\tilde{\omega}(\tau_2)$ be a root of (15) around $\tau_2 = \tau_{2_0}^{(j)}$, and $\tilde{\alpha}\left(\tau_{2_0}^{(j)}\right) = 0$ and $\tilde{\omega}\left(\tau_{2_0}^{(j)}\right) = \tilde{\omega}_k$. Differentiating both sides of (15) with respect to τ_2 yields

$$\left[3\lambda^2 + 2c_2\lambda + c_1 + \left(2d_2\lambda + d_1 - \tau_2(d_2\lambda^2 + d_1\lambda)\right)e^{-\lambda\tau_2}\right]\frac{d\lambda}{d\tau_2} = \lambda e^{-\lambda\tau_2}\left(d_2\lambda^2 + d_1\lambda\right),$$

which gives

$$\left[\frac{d\lambda}{d\tau_2}\right]^{-1} = -\frac{2\lambda^3 + c_2\lambda^2 - c_0}{\lambda^2(\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0)} + \frac{d_2\lambda}{\lambda^2(d_2\lambda^2 + d_1\lambda)} - \frac{\tau_2}{\lambda}.$$

Let $\lambda = i\tilde{\omega}_k, \tau_2 = \tau_{2k}^{(j)}$, and then we have

$$\left[\frac{d\lambda}{d\tau_2}\right]^{-1}\Bigg|_{\lambda=i\tilde{\omega}_k,\tau_2=\tau_{2k}^{(j)}} = \frac{-2\tilde{\omega}_k^3 i - c_2\tilde{\omega}_k^2 - c_0}{\tilde{\omega}_k^2\left(c_0 - c_2\tilde{\omega}_k^2 - i\left(\tilde{\omega}_k^3 - c_1\tilde{\omega}_k\right)\right)} + \frac{d_2\tilde{\omega}_k^2}{\tilde{\omega}_k^2\left(-d_2\tilde{\omega}_k^2 + d_1\tilde{\omega}_k i\right)} - \frac{\tau_{2k}^{(j)}}{i\tilde{\omega}_k}.$$

Then

$$\begin{aligned}
&\mathrm{Re}\left\{ \left[\frac{d\lambda}{d\tau_2}\right]^{-1} \Bigg|_{\lambda=i\tilde{\omega}_k,\tau_2=\tau_{2k}^{(j)}} \right\} \\
&= -\frac{1}{\tilde{\omega}_k^2}\left[\frac{c_0^2-(c_2^2-2c_1)\tilde{\omega}_k^4-2\tilde{\omega}_k^6}{(c_0-c_2\tilde{\omega}_k^2)^2+(\tilde{\omega}_k^3-c_1\tilde{\omega}_k)^2} - \frac{-d_2^2\tilde{\omega}_k^4}{d_1^2\tilde{\omega}_k^2+(-d_2\tilde{\omega}_k^2)^2}\right] \\
&= \frac{2\tilde{\omega}_k^6+p_2\tilde{\omega}_k^4-r_2}{\tilde{\omega}_k^2\left(-d_2\tilde{\omega}_k^2\right)^2+d_1^2\tilde{\omega}_k^2} = \frac{3\tilde{\omega}_k^4+2p_2\tilde{\omega}_k^2+q_2}{(-d_2\tilde{\omega}_k^2)^2+d_1^2\tilde{\omega}_k^2},
\end{aligned}$$

where Re$\{.\}$ is the real part of $\cdot$. We assume that the following condition holds.

$$\text{(H5) } 3\tilde{\omega}_k^4+2p_2\tilde{\omega}_k^2+q_2\neq 0.$$

In view of above analysis and the results of Yang [26] and Kuang [27], we get

Theorem 2.2. *If* $(H1)$ *and* $(H4)$ *hold, then the equilibrium* $(x^*,0,0)$ *of system (13) is asymptotically stable for* $\tau_2\in[0,\tau_{2_0})$. *Under the conditions* $(H1)$ *and* $(H4)$, *if the condition* $(H5)$ *holds, then system (13) undergoes a Hopf bifurcation at the equilibrium* $(x^*,0,0)$ *when* $\tau_2=\tau_{2_0}^{(j)}$, $j=0,1,2,\ldots$.

Case 3. Delayed feedback on the first and second equations

In this case, we will investigate the system (2) in which the variable x and y are influenced by the delayed feedback with $k_1\neq 0$, $k_2\neq 0$. For simplicity, we assume that $k_i=k$ $(i=1,2)$ and $\tau=\tau_i$ $(i=1,2)$, then system (2) takes the form

$$\begin{cases} \dot{x}=y+k(x(t)-x(t-\tau)), \\ \dot{y}=z+k(y(t)-y(t-\tau)), \\ \dot{z}=-y+3y^2-x^2-xz-a. \end{cases} \tag{23}$$

The linearized system of Equation (23) around $(x^*,0,0)$ is given by

$$\begin{cases} \dot{x}=k_1x-kx(t-\tau)+y, \\ \dot{y}=k_1x+z-kx(t-\tau), \\ \dot{z}=-2x^*x-y-x^*z, \end{cases} \tag{24}$$

The characteristic equation of (24) takes the form

$$\lambda^3+m_2\lambda^2+m_1\lambda+m_0+(n_2\lambda^2+n_1\lambda+n_0)e^{-\lambda\tau}+(s_1\lambda+s_0)e^{-2\lambda\tau}=0, \tag{25}$$

where $m_0=-2x^*-k+k^2x^*$, $m_1=k^2+1-2kx^*$, $m_2=x^*-k$, $n_0=k$, $n_1=2kx^*$, $n_2=2k$, $s_0=-k^2x^*$, $s_1=-k^2$.

Multiplying $e^{\lambda\tau}$ on both sides of (25), it is obvious to obtain

$$(\lambda^3+m_2\lambda^2+m_1\lambda+m_0)e^{\lambda\tau}+(n_2\lambda^2+n_1\lambda+n_0)+(s_1\lambda+s_0)e^{-\lambda\tau}=0. \tag{26}$$

Now we will analyze the distribution of roots of the transcendental equation (26).

When $\tau=0$, (26) has the form

$$\lambda^3+(m_2+n_2)\lambda^2+(m_1+m_1+s_1)\lambda+m_0+n_0+s_0=0. \tag{27}$$

It is easy to see that all roots of (26) have a negative real part if the following condition

$$\text{(H6) } m_2+n_2>0, \quad m_0+n_0+s_0>0, \quad (m_2+n_2)(m_1+n_1)>m_0+n_0+s_0$$

is satisfied. Then the equilibrium point $(x^*,0,0)$ is locally asymptotically stable when the conditions (H1) and (H6) hold.

For $\bar{\omega}>0$, $i\bar{\omega}$ is a root of (26) if and only if

$$\left(-\bar{\omega}^3i-m_2\bar{\omega}^2+m_1\bar{\omega}i+m_0\right)e^{\bar{\omega}\tau i}+\left(-n_2\bar{\omega}^2+n_1\bar{\omega}i+n_0\right)+\left(s_1\bar{\omega}i+s_0\right)e^{-\bar{\omega}\tau i}=0.$$

Separating the real and imaginary parts gives

$$\begin{cases} (m_0 - m_2\bar{\omega}^2 + s_0)\cos\bar{\omega}\tau + (s_1\bar{\omega} - m_1\bar{\omega} + \bar{\omega}^3)\sin\bar{\omega}\tau = n_2\bar{\omega}^2 - n_0, \\ (s_1\bar{\omega} + m_1\bar{\omega} - \bar{\omega}^3)\cos\bar{\omega}\tau + (m_0 - m_2\bar{\omega}^2 - s_0)\sin\bar{\omega}\tau = -n_1\bar{\omega}. \end{cases} \tag{28}$$

It follows from (28) that

$$\cos\bar{\omega}\tau = \frac{(n_2\bar{\omega} - n_0)(m_0 - m_2\bar{\omega}^2 - s_0) + n_1\bar{\omega}(s_1\bar{\omega} - m_1\bar{\omega} + \bar{\omega}^3)}{(m_0 - m_2\bar{\omega}^2)^2 - s_0^2 - (s_1\bar{\omega})^2 + (m_1\bar{\omega} - \bar{\omega}^3)^2} \tag{29}$$

and

$$\sin\bar{\omega}\tau = \frac{(n_2\bar{\omega} - n_0)(s_0\bar{\omega} + m_1\bar{\omega} - \bar{\omega}^3) + n_1\bar{\omega}(m_0 - m_2\bar{\omega}^2 + s_0)}{(m_0 - m_2\bar{\omega}^2)^2 - s_0^2 - (s_1\bar{\omega})^2 + (m_1\bar{\omega} - \bar{\omega}^3)^2}, \tag{30}$$

which is equivalent to

$$\bar{\omega}^{12} + \theta_5\bar{\omega}^{10} + \theta_4\bar{\omega}^8 + \theta_3\bar{\omega}^6 + \theta_2\bar{\omega}^4 + \theta_1\bar{\omega}^2 + \theta_0 = 0, \tag{31}$$

where

$$\begin{aligned}
\theta_0 &= -n_0^2(m_0 - s_0)^2, \\
\theta_1 &= 2(m_1^2 - 2m_0m_2 - s_1^2)(m_0^2 - s_0^2) \\
&\quad -2n_0(2m_0n_0 - n_0s_0 + n_1s_1 - m_1n_1)(m_0 - s_0), \\
\theta_2 &= m_1^2 - 2m_0m_2 - s_1^2 + 2(m_2^2 - 2m_1)(m_0^2 - s_0^2) \\
&\quad -(2m_0n_0 - n_0s_0 + n_1s_1 - m_1n_1)^2 - 2n_0(n_1 - m_0n_0)(m_0 - s_0), \\
\theta_3 &= 2(m_0^2 - s_0^2) + 2(m_2^2 - 2m_1)(m_1^2 - 2m_0m_2 - s_1^2) \\
&\quad -(n_2s_1 + m_1n_2 - m_2n_1 - n_0) \\
&\quad -2(n_1 - m_0n_0)(2m_0n_0 - n_0s_0 + n_1s_1 - m_1n_1), \\
\theta_4 &= [m_2^2 - 2m_1 + 2(m_1^2 - 2m_0m_2 - s_1^2) - (n_1 - m_0n_0)^2 \\
&\quad +2n_2(n_2s_1 + m_1n_2 - m_2n_1 - n_0)]^2, \\
\theta_5 &= 2(m_2^2 - 2m_1) - n_2^2.
\end{aligned}$$

Denote $\bar{z} = \bar{\omega}^2$, and then (28) takes the following form

$$\bar{z}^6 + \theta_5\bar{z}^5 + \theta_4\bar{z}^4 + \theta_3\bar{z}^3 + \theta_2\bar{z}^2 + \theta_1\bar{z} + \theta_0 = 0. \tag{32}$$

Let

$$\bar{h}(\bar{z}) = \bar{z}^6 + \theta_5\bar{z}^5 + \theta_4\bar{z}^4 + \theta_3\bar{z}^3 + \theta_2\bar{z}^2 + \theta_1\bar{z} + \theta_0. \tag{33}$$

Since $\theta_0 < 0$ and $\lim_{\bar{z}\to+\infty}\bar{h}(\bar{z}) = +\infty$, and then (32) has at least one positive root. Without loss of generality, we assume that (32) has six positive roots, denoted by $\bar{z}_1$, $\bar{z}_2$, $\bar{z}_3$, $\bar{z}_4$, $\bar{z}_5$, $\bar{z}_6$, respectively. Then Equation (31) has six positive roots

$$\bar{\omega}_1 = \sqrt{\bar{z}_1}, \quad \bar{\omega}_2 = \sqrt{\bar{z}_2}, \quad \bar{\omega}_3 = \sqrt{\bar{z}_3}, \quad \bar{\omega}_4 = \sqrt{\bar{z}_4}, \quad \bar{\omega}_5 = \sqrt{\bar{z}_5}, \quad \bar{\omega}_6 = \sqrt{\bar{z}_6}.$$

If we denote

$$\tau_k^{(j)} = \frac{1}{\bar{\omega}_k}\left\{\arccos\left(\frac{(n_2\bar{\omega} - n_0)(m_0 - m_2\bar{\omega}^2 - s_0) + n_1\bar{\omega}(s_1\bar{\omega} - m_1\bar{\omega} + \bar{\omega}^3)}{(m_0 - m_2\bar{\omega}^2)^2 - s_0^2 - (s_1\bar{\omega})^2 + (m_1\bar{\omega} - \bar{\omega}^3)^2}\right) + 2j\pi\right\}, \tag{34}$$

where $k = 1, 2, 3$; $j = 0, 1, 2, \ldots$, then $\pm i\bar{\omega}_k$ are a pair of imaginary roots of Equation (26) with $\tau_k^{(j)}$. Define

$$\tau_0 = \tau_{k0}^{(0)} = \min_{k\in\{1,2,3,4,5,6\}}\left\{\tau_k^{(0)}\right\}, \quad \bar{\omega}_0 = \bar{\omega}_{k0}. \tag{35}$$

Let $\lambda(\tau) = \bar{\alpha}(\tau) + i\bar{\omega}(\tau)$ be a root of (26) around $\tau = \tau_0^{(j)}$, and $\bar{\alpha}\left(\tau_0^{(j)}\right) = 0$ and $\bar{\omega}\left(\tau_0^{(j)}\right) = \bar{\omega}_k$. Differentiating both sides of (25) with respect to τ yields

$$\left[\frac{d\lambda}{d\tau}\right]^{-1} = \frac{(3\lambda^2 + 2m_2\lambda + m_1)e^{\lambda\tau} + 2n_2\lambda + n_1 + s_1 e^{-\lambda\tau}}{-\lambda(\lambda^3 + m_2\lambda^2 + m_1\lambda + m_0)e^{\lambda\tau} + \lambda(s_1\lambda + s_0)e^{-\lambda\tau}} - \frac{\tau}{\lambda}.$$

Let $\lambda = i\bar{\omega}_k$, $\tau = \tau_k^{(j)}$, and then we have

$$\left[\frac{d\lambda}{d\tau}\right]^{-1}\Bigg|_{\lambda = i\bar{\omega}_k, \tau = \tau_k^{(j)}} = \frac{A_1 + iA_2}{B_1 + iB_2} - \frac{\tau_k^{(j)}}{i\bar{\omega}_k}.$$

where

$$\begin{aligned}
A_1 &= (m_1 - 3\bar{\omega}_k^2 + s_1)\cos\bar{\omega}_k\tau_k^{(j)} - 2m_2\bar{\omega}_k\cos\bar{\omega}_k\tau_k^{(j)},\\
A_2 &= 2m_2\bar{\omega}_k^2\cos\bar{\omega}_k\tau_k^{(j)} + (m_1 - 3\bar{\omega}_k^2 - s_1)\sin\bar{\omega}_k\tau_k^{(j)} + 2n_2\bar{\omega}_k,\\
B_1 &= \omega\left[\left(n_1\bar{\omega}_k - \bar{\omega}_k^3 - s_1\bar{\omega}_k\right)\cos\bar{\omega}_k\tau_k^{(j)} + \left(m_0 - m_1\bar{\omega}_k^2 + s_0\right)\sin\bar{\omega}_k\tau_k^{(j)}\right],\\
B_2 &= \omega\left[\left(s_0 - m_0 + m_2\bar{\omega}_k^2\right)\cos\bar{\omega}_k\tau_k^{(j)} + \left(s_1\bar{\omega}_k + n_1\bar{\omega}_k - \bar{\omega}_k^3\right)\sin\bar{\omega}_k\tau_k^{(j)}\right].
\end{aligned}$$

Then

$$\mathrm{Re}\left\{\left[\frac{d\lambda}{d\tau}\right]^{-1}\Bigg|_{\lambda = i\omega_k, \tau = \tau_k^{(j)}}\right\} = \mathrm{Re}\left\{\frac{A_1 + iA_2}{B_1 + iB_2}\right\} = \frac{A_1B_1 + A_2B_2}{B_1^2 + B_2^2},$$

where Re$\{.\}$ is the real part of $\cdot$. We assume that the following condition holds.

(H7) $A_1B_1 + A_2B_2 \neq 0$.

Based on above analysis and the results of Yang [26] and Kuang [27], we have

Theorem 2.3. *If* (*H*1) *and* (*H*6) *hold, then the equilibrium* $(x^*, 0, 0)$ *of system (23) is asymptotically stable for* $\tau \in [0, \tau_0)$. *Under the conditions* (*H*1) *and* (*H*6), *if the condition* (*H*7) *holds, then system (23) undergoes a Hopf bifurcation at the equilibrium* $(x^*, 0, 0)$ *when* $\tau = \tau_0^{(j)}$, $j = 0, 1, 2, \ldots$.

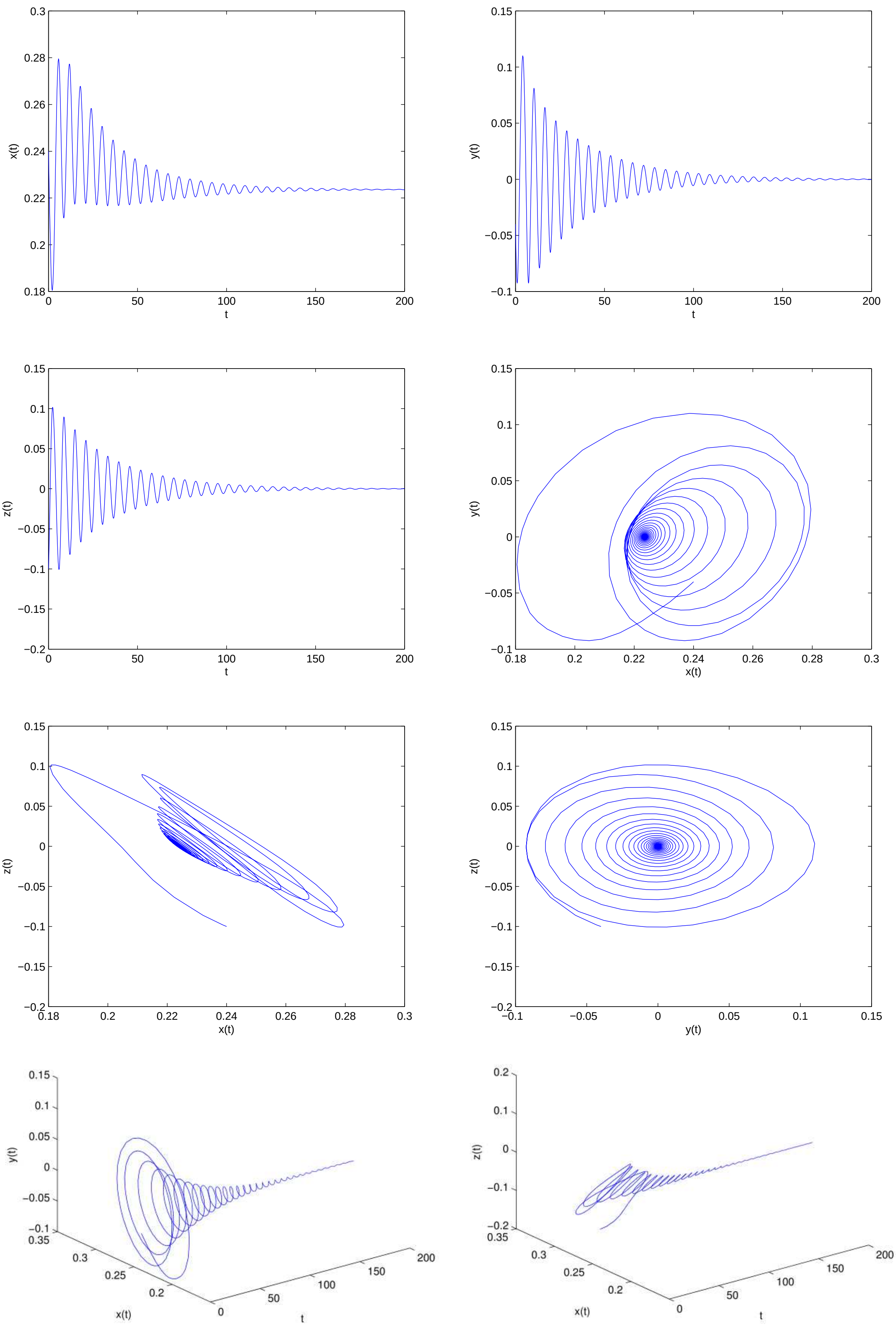
Remark 2.1. *Kapitaniak [6] studied the chaos control without feedback, Yassen [18] and Liao and Lin [19] studied the chaos control with adaptive control method, Song and Wei [21] considered the control of chaos for Chen's system by adding a delayed feedback term to a certain equation, Pyragas [23] analyzed the continuous control of chaos by self-controlling feedback, Zhou and Yang [28] investigated the chaos control of a 3D autonomous system with the stability transformation method. In this paper, we investigate the chaos control with adding different feedback term to different equations to suppress the chaos of the chaotic system. From this viewpoint, our results are new and complement some previous results of [6,18,19,21,23,28].*

3. **Computer Simulations.**

In this section, we present some numerical results of system (2) to verify the analytical predictions obtained in the previous section. Let us consider the following system:

$$\begin{cases} \dot{x} = y + k_1(x(t) - x(t - \tau_1)),\\ \dot{y} = z + k_2(y(t) - y(t - \tau_2)),\\ \dot{z} = -y + 3y^2 - x^2 - xz - 0.05. \end{cases} \tag{36}$$

It is easy to see that system (36) has an equilibrium $E(0.2236, 0, 0)$. For $k_1 = -2$, $k_2 = 0$, we can obtain that (H1)-(H3) are fulfilled. Let $j = 0$ and by means of Matlab 7.0 software,


x(t)
t
y(t)
t
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x(t)
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x(t)
z(t)
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z(t)
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t

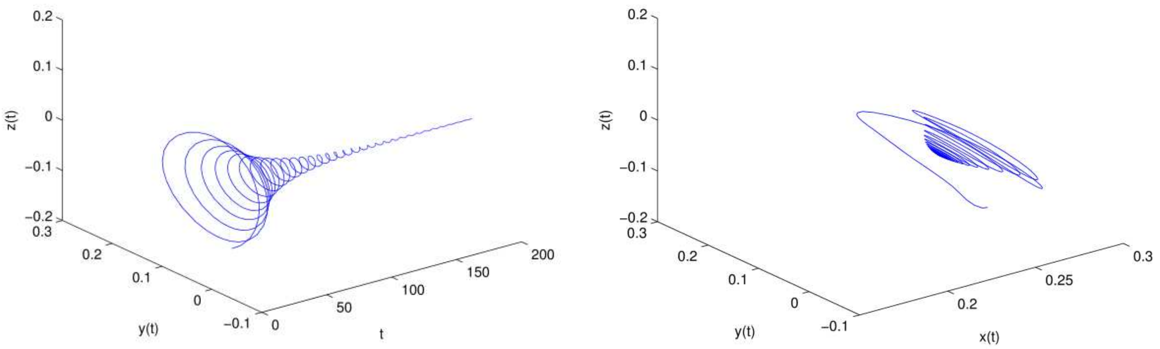
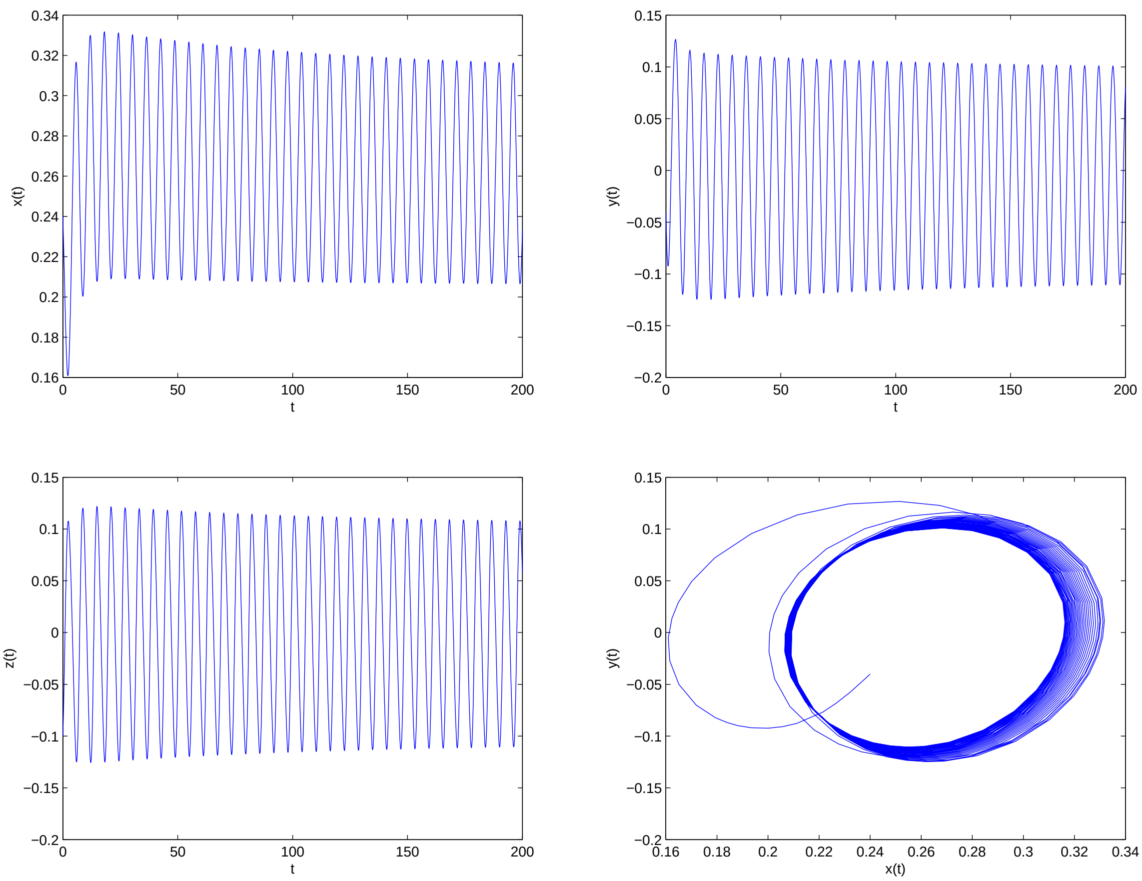


FIGURE 2. Chaos vanishes when $\tau_1 = 0.45 < \tau_{1_0} \approx 0.5$. The equilibrium $E(0.2236, 0, 0)$ is asymptotically stable. The initial value is $(0.24, -0.04, -0.1)$.

we derive $\omega_{1_0} \approx 0.5541, \tau_{1_0} \approx 0.5$. Thus the equilibrium $E(0.2236, 0, 0)$ is asymptotically stable for $\tau_1 < \tau_{1_0} \approx 0.5$ which is illustrated in Figure 2 (Figure 2 shows the time history plots of $t - x$, $t - y$ and $t - z$, phase plots $x - y$, $x - z$ and $y - z$, space plots of space plots of $t - x - y$, $t - x - z$, $t - y - z$ and $x - y - z$). When $\tau_1 = \tau_{1_0} \approx 0.5$, Equation (36) undergoes a Hopf bifurcation at the equilibrium $E(0.2236, 0, 0)$, i.e., a small amplitude periodic solution occurs near $E(0.2236, 0, 0)$. When τ_1 is close to $\tau_{1_0} \approx 0.5$ it can be shown in Figure 3 (Figure 3 shows the time history plots of $t - x$, $t - y$ and $t - z$, phase plots



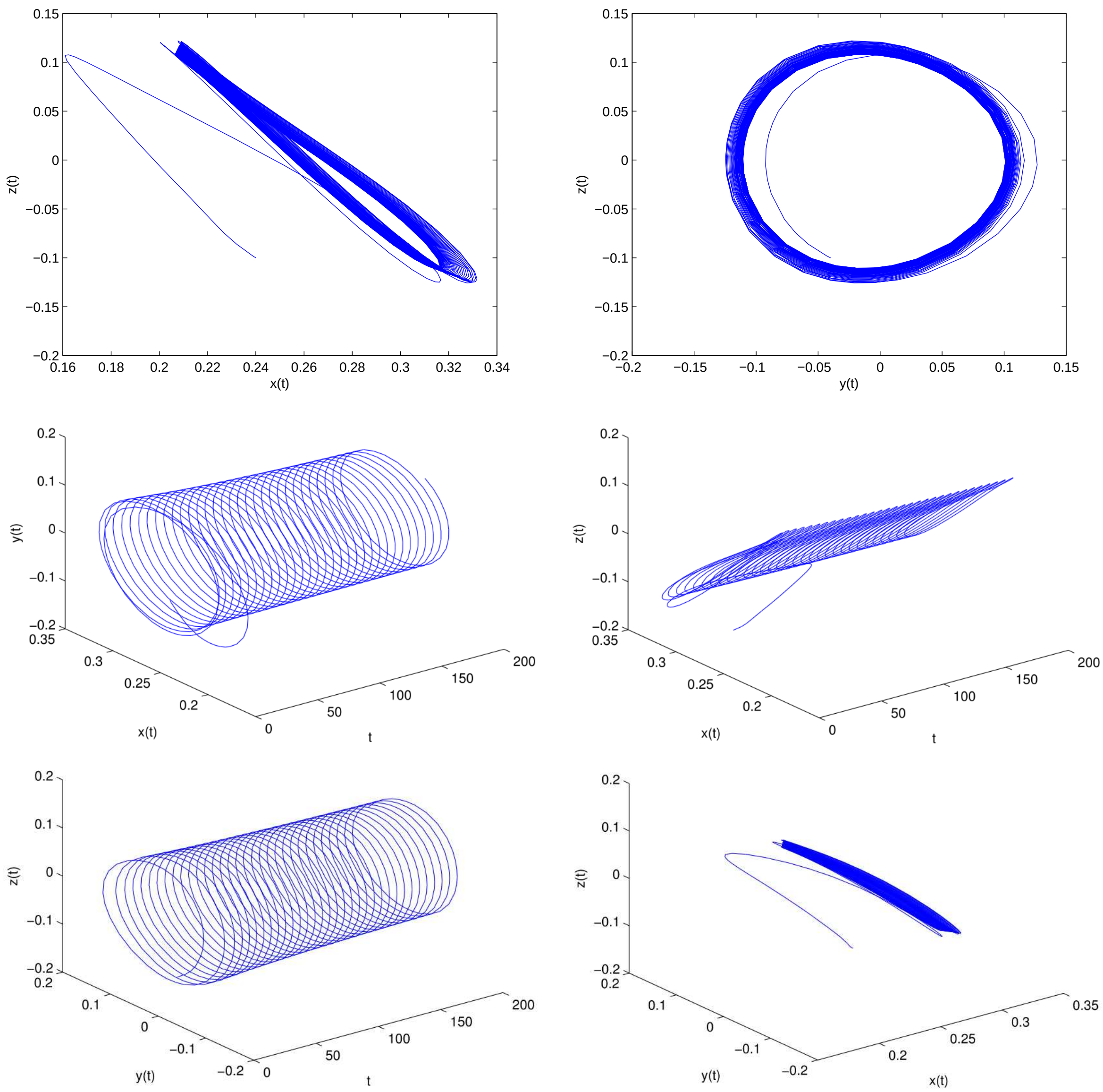
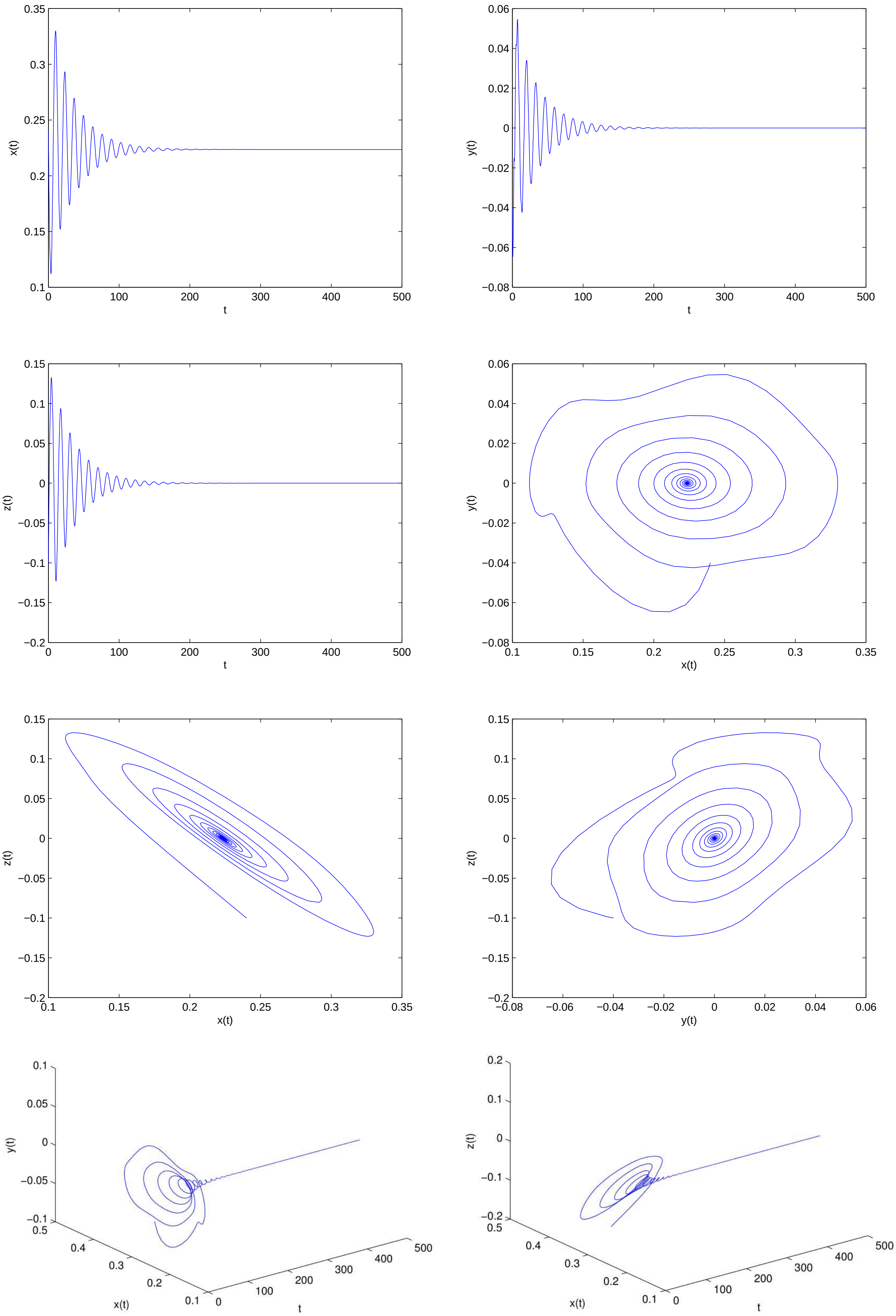


FIGURE 3. Chaos vanishes when $\tau_1 = 0.8 > \tau_{1_0} \approx 0.5$. Hopf bifurcation occurs from the equilibrium $E(0.2236, 0, 0)$. The initial value is $(0.24, -0.04, -0.1)$.

$x-y$, $x-z$ and $y-z$, space plots of space plots of $t-x-y$, $t-x-z$, $t-y-z$ and $x-y-z$).

For $k_1 = 0$, $k_2 = -2$, we can obtain that (H1), (H4) and (H5) are fulfilled. Let $j = 0$ and we have $\tilde{\omega}_2 \approx 0.3976, \tau_{2_0} \approx 1.8$. Thus, the equilibrium $E(0.2236, 0, 0)$ is asymptotically stable for $\tau_2 < \tau_{2_0} \approx 1.8$ which is illustrated in Figure 4 (Figure 4 shows the time history plots of $t-x$, $t-y$ and $t-z$, phase plots $x-y$, $x-z$ and $y-z$, space plots of space plots of $t-x-y$, $t-x-z$, $t-y-z$ and $x-y-z$). When $\tau_2 = \tau_{2_0} \approx 1.8$, Equation (36) undergoes a Hopf bifurcation around the equilibrium $E(0.2236, 0, 0)$. When τ_2 is close to $\tau_{2_0} \approx 1.8$ it can be shown in Figure 5 (Figure 5 shows the time history plots of $t-x$, $t-y$ and $t-z$, phase plots $x-y$, $x-z$ and $y-z$, space plots of space plots of $t-x-y$, $t-x-z$, $t-y-z$ and $x-y-z$).


x(t)
t
y(t)
t
z(t)
t
y(t)
x(t)
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x(t)
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t
z(t)
x(t)
t

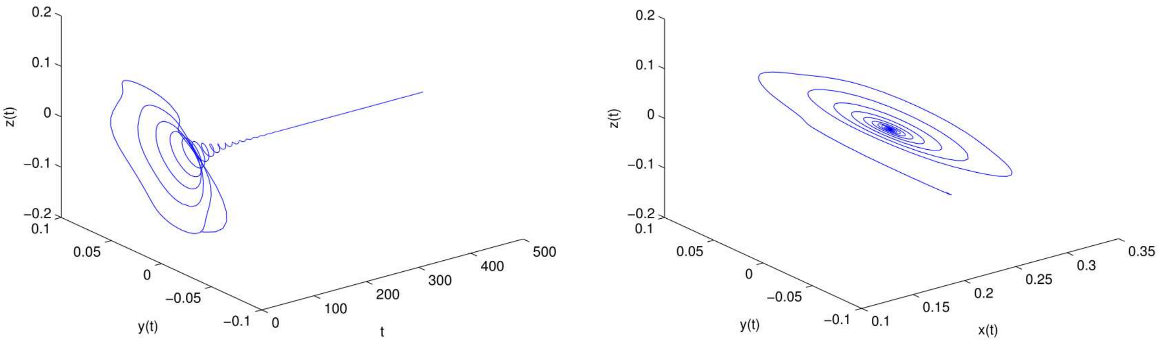
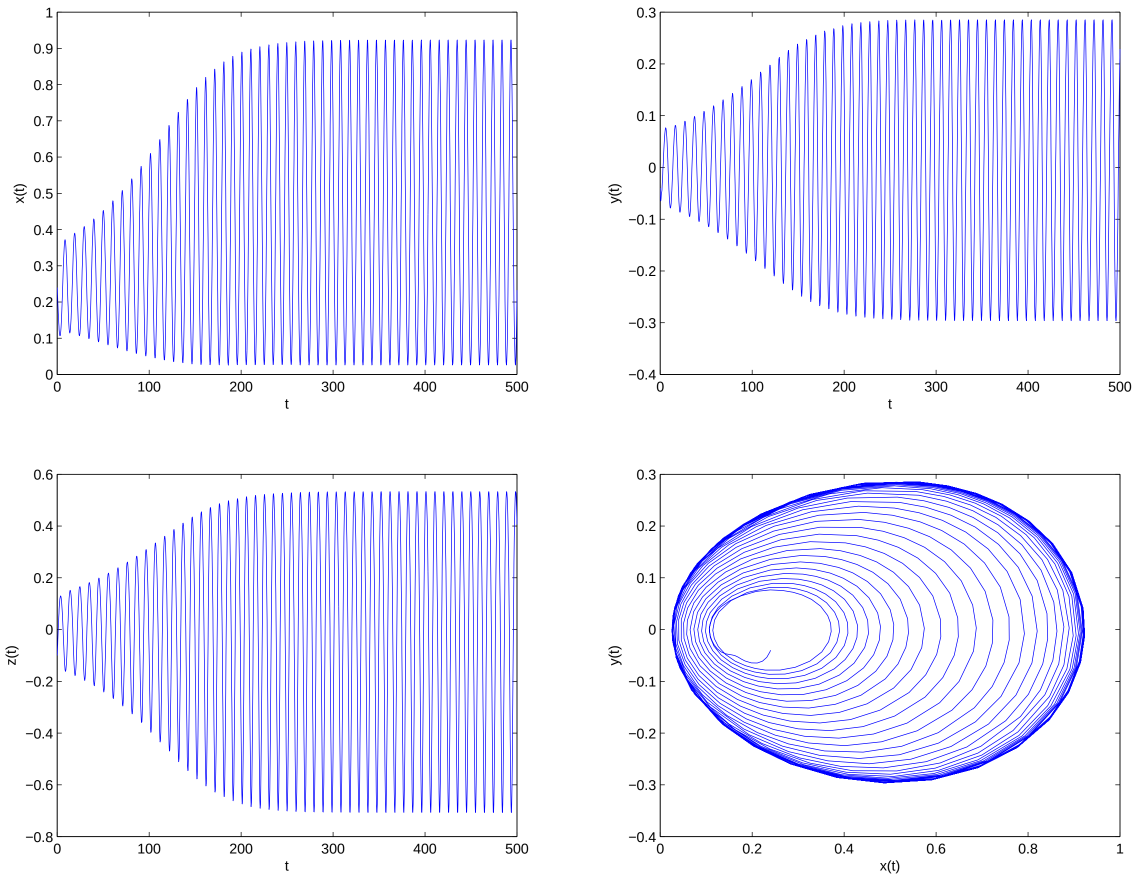


FIGURE 4. Chaos vanishes when $\tau_2 = 1.2 < \tau_{2_0} \approx 1.8$. The equilibrium $E(0.2236, 0, 0)$ is asymptotically stable. The initial value is $(0.24, -0.04, -0.1)$.

For $k_1 = -2$, $k_2 = -2$, we can obtain that (H1), (H6) and (H7) are fulfilled. Let $j = 0$ and we obtain $\bar{\omega}_0 \approx 0.3782$, $\tau_0 \approx 0.4$. Thus, the equilibrium $E(0.2236, 0, 0)$ is asymptotically stable for $\tau < \tau_0 \approx 0.4$ which is illustrated in Figure 6 (Figure 6 shows the time history plots of $t-x$, $t-y$ and $t-z$, phase plots $x-y$, $x-z$ and $y-z$, space plots of space plots of $t-x-y$, $t-x-z$, $t-y-z$ and $x-y-z$). When $\tau = \tau_0 \approx 0.4$, Equation (36) undergoes a Hopf bifurcation at the equilibrium $E(0.2236, 0, 0)$. When τ is close to $\tau_0 \approx 0.4$ it can be shown in Figure 7 (Figure 7 shows the time history plots of



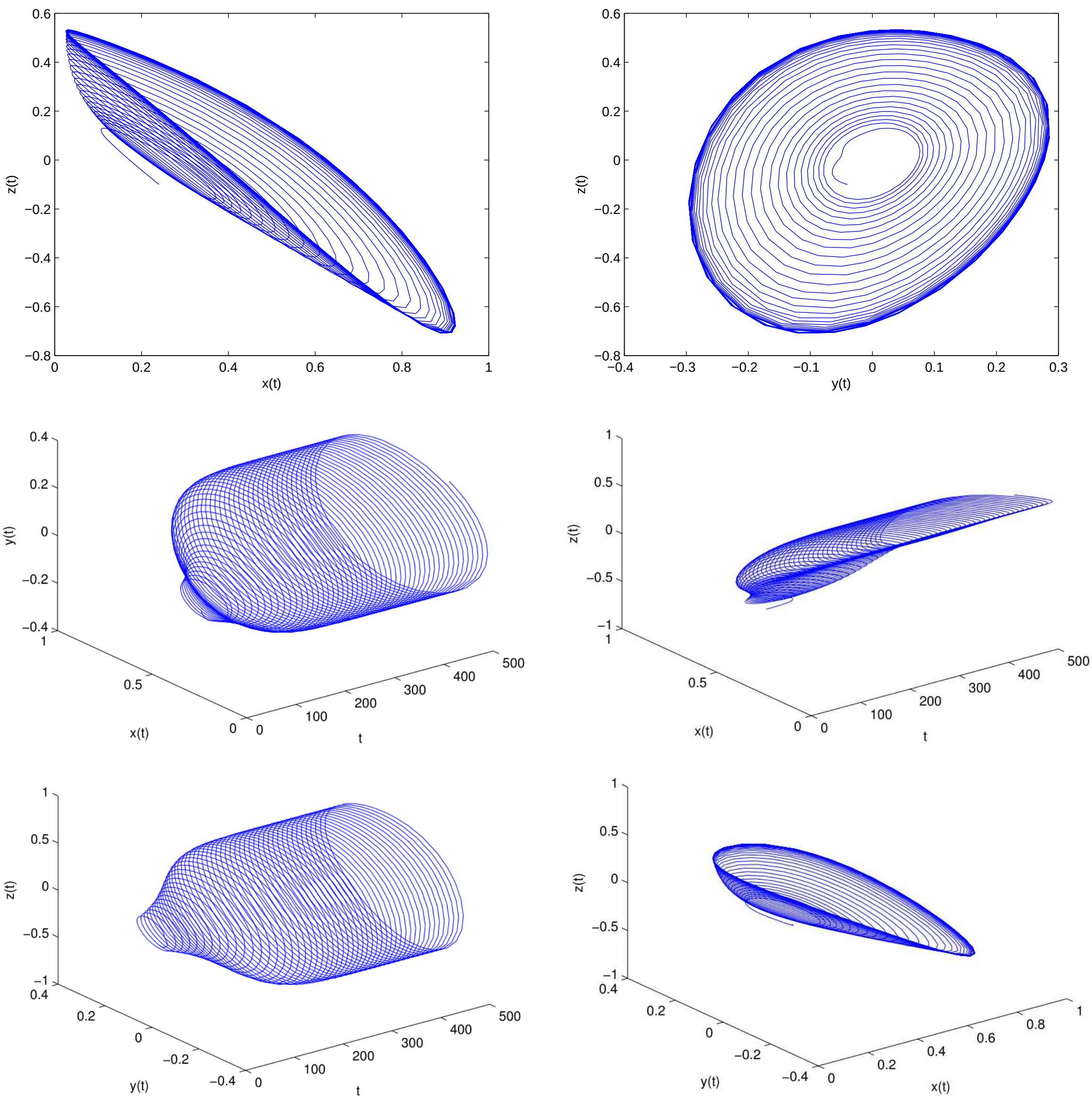
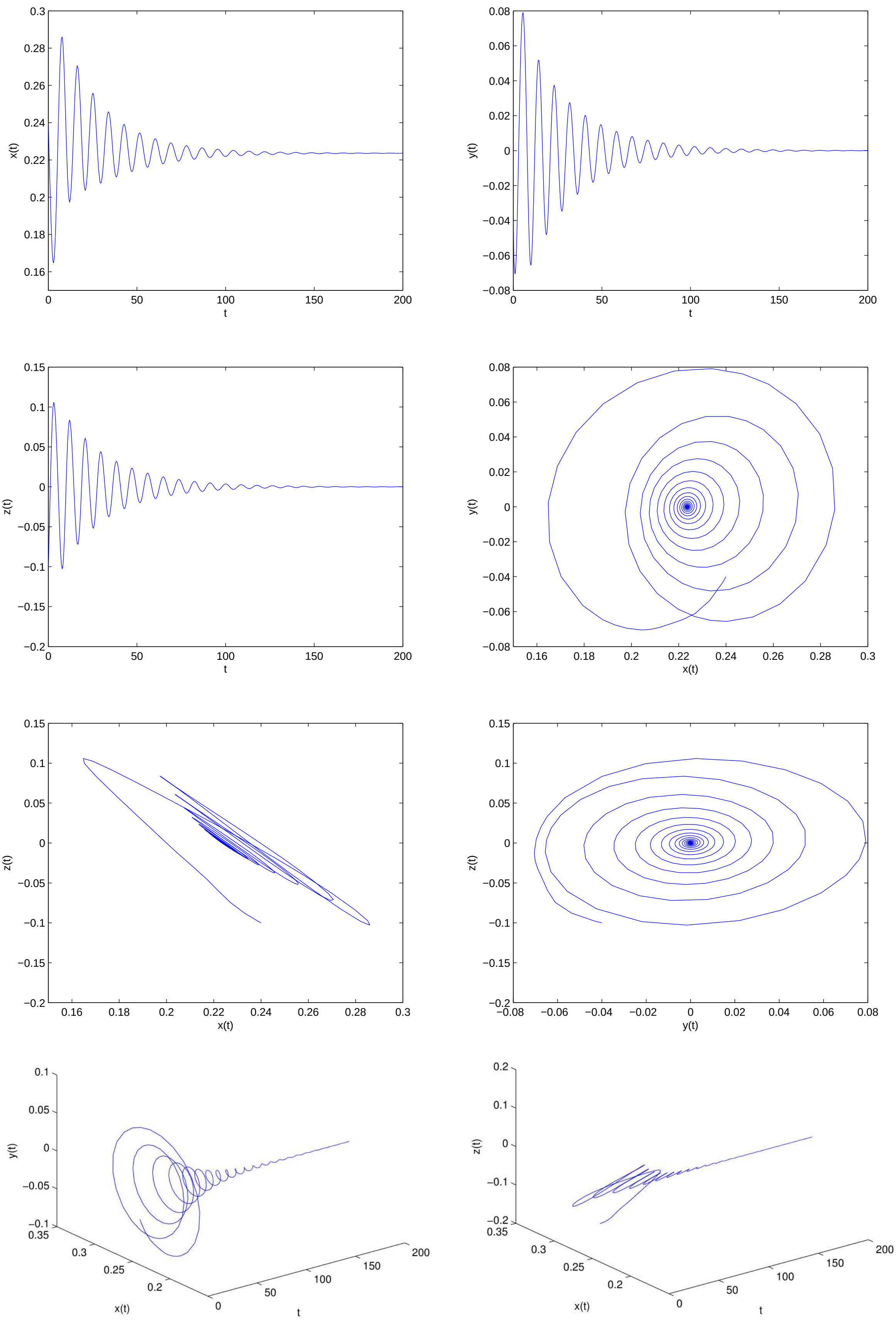


FIGURE 5. Chaos vanishes when $\tau_2 = 2.3 < \tau_{2_0} \approx 1.8$. Hopf bifurcation occurs from the equilibrium $E(0.2236, 0, 0)$. The initial value is $(0.24, -0.04, -0.1)$.

$t-x$, $t-y$ and $t-z$, phase plots $x-y$, $x-z$ and $y-z$, space plots of space plots of $t-x-y$, $t-x-z$, $t-y-z$ and $x-y-z$).

4. **Conclusions.** In this paper, a feedback control method is applied to suppressing chaotic behavior of a 3D autonomous system within the chaotic attractor. By adding a time-delayed force to the first equation of 3D autonomous system, we have focused on the local stability of the equilibrium $(\sqrt{a}, 0, 0)$ and local Hopf bifurcation of the delayed 3D autonomous system. It is showed that if the conditions (H1) and (H2) are satisfied, then 3D autonomous system is asymptotically stable for $\tau_1 \in [0, \tau_{1_0})$. If (H1)-(H3) hold true, a sequence of Hopf bifurcations occurs around the equilibrium $(\sqrt{a}, 0, 0)$, that is, a family of periodic orbits bifurcates from the equilibrium $(\sqrt{a}, 0, 0)$. By adding a time-delayed force to the second equation of 3D autonomous system, we have analyzed the


x(t)
t
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0
50
100
150
200
y(t)
t
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
−0.08
z(t)
t
0.15
0.1
0.05
−0.05
−0.1
−0.15
−0.2
y(t)
x(t)
z(t)
x(t)
z(t)
y(t)
y(t)
x(t)
t
0.35
0.25
z(t)
x(t)
t

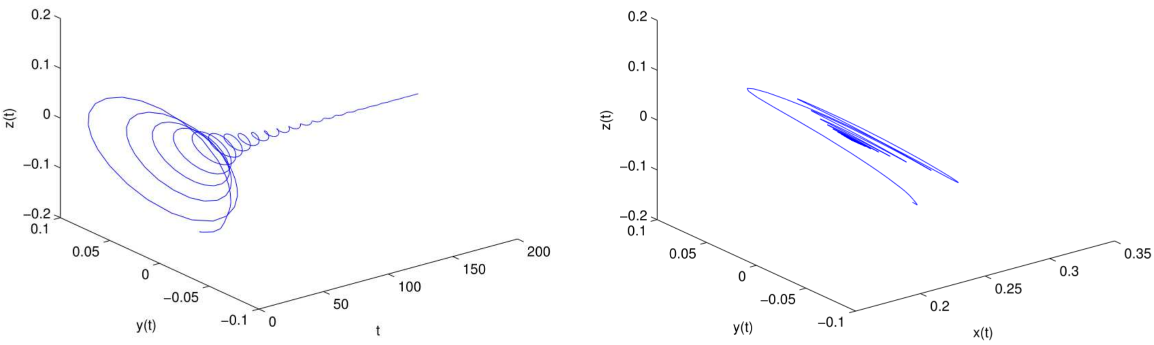
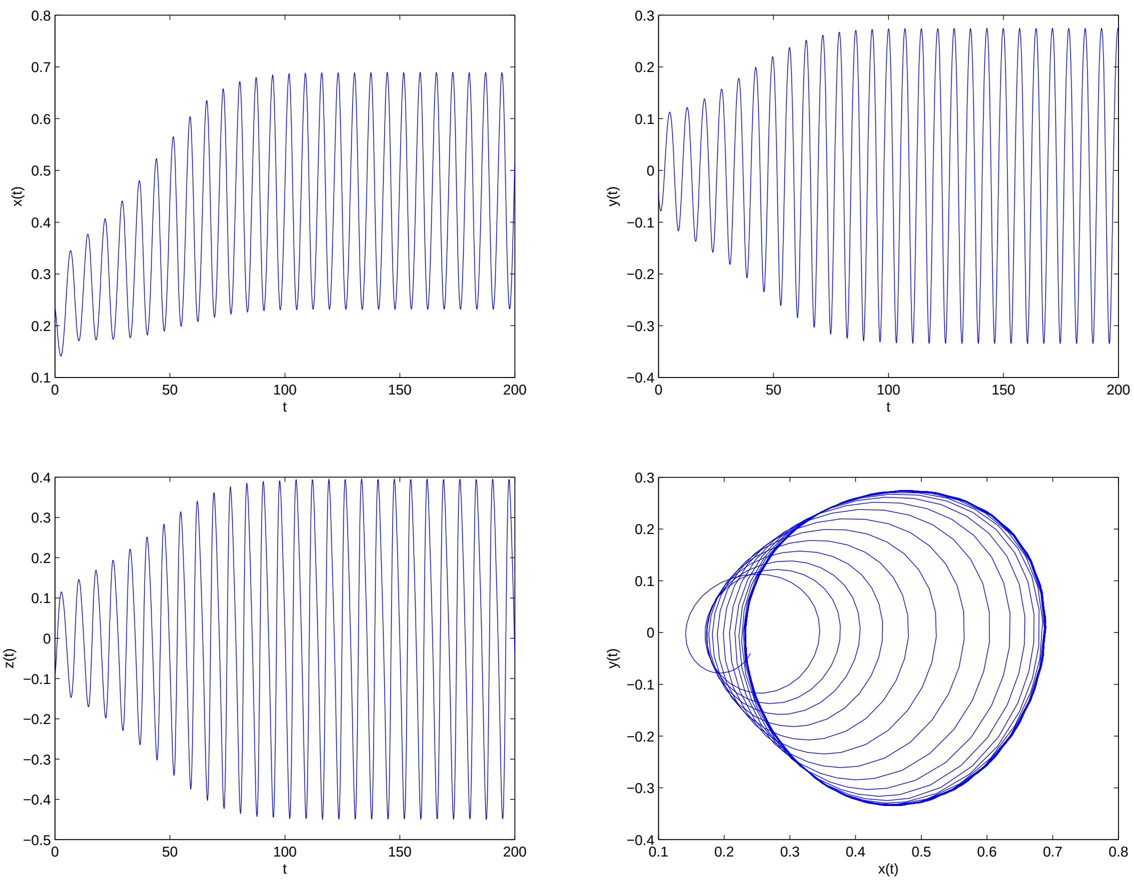


FIGURE 6. Chaos vanishes when $\tau = 0.25 < \tau_0 \approx 0.4$. The equilibrium $E(0.2236, 0, 0)$ is asymptotically stable. The initial value is $(0.24, -0.04, -0.1)$.

local stability of the equilibrium $(\sqrt{a}, 0, 0)$ and local Hopf bifurcation of the delayed 3D autonomous system. It is showed that if the conditions (H1) and (H4) are satisfied, then 3D autonomous system is asymptotically stable for $\tau_2 \in [0, \tau_{2_0})$. If (H1), (H4) and (H5) hold true, a sequence of Hopf bifurcations occurs around the equilibrium $(\sqrt{a}, 0, 0)$. By adding a time-delayed force to the first and the second equations of 3D autonomous system, we have discussed the local stability of the equilibrium $(\sqrt{a}, 0, 0)$ and local Hopf bifurcation of the delayed 3D autonomous system. We showed that if the conditions (H1) and (H6) are fulfilled, then 3D autonomous system is asymptotically stable for $\tau \in [0, \tau_0)$.



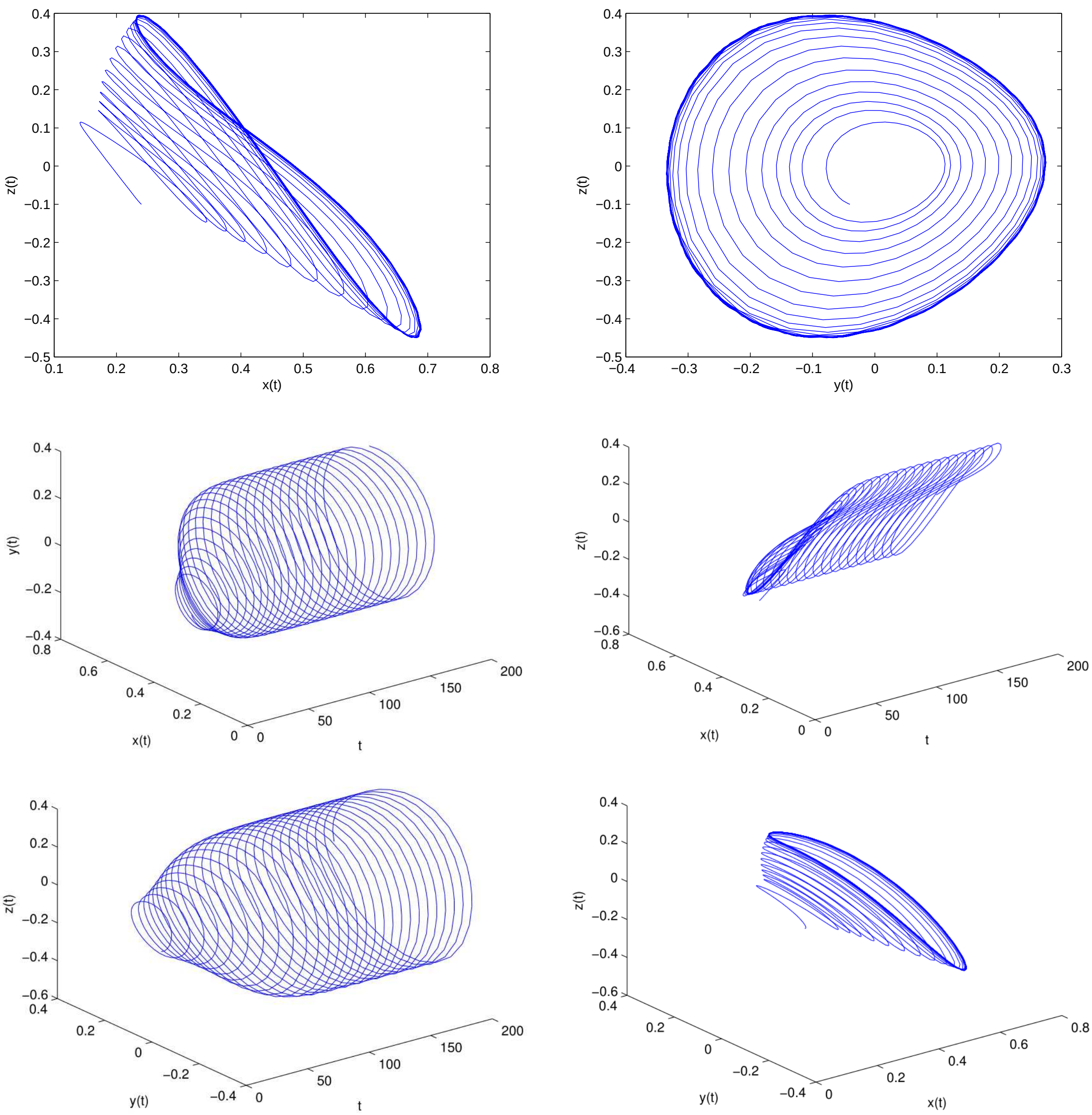


FIGURE 7. Chaos vanishes when $\tau = 0.5 > \tau_0 \approx 0.4$. Hopf bifurcation occurs from the equilibrium $E(0.2236, 0, 0)$. The initial value is $(0.24, -0.04, -0.1)$.

If (H1), (H6) and (H7) hold true, a sequence of Hopf bifurcations occurs around the equilibrium $(\sqrt{a}, 0, 0)$. All the cases show that chaos vanishes and can be suppressed. Some computer simulations are carried out to visualize the theoretical results. To the best of our knowledge, there are few results on the chaos control by feedback control with distributed time delay, which might be our future research topic.

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