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EFFICIENCY EVALUATION WITH CASCADING DYNAMIC IN WATER DISTRIBUTION NETWORK

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Abstract. *The tasks of ensuring the safety of water supply and identifying the critical nodes in water distribution networks (WDNs) are significant to improve the ability of resistance to sudden disasters. It is also meaningful in guaranteeing the stable operation of urban infrastructure. The aim of the paper is to identify the critical nodes in WDN by calculating the critical ranking. The input indexes and output indexes are defined based on cascading dynamics. The cascading failure processes are simulated with the cascading dynamic model. Data envelopment analysis (DEA) is used to evaluate the nodal efficiency. The method is illustrated by a real WDN. The results show that the proposed method is able to identify the critical nodes and obtain the critical ranking. If the nodes have the same efficiency value, the relative efficiency is analyzed with tolerance parameter consequently. The step can distinguish the critical node further. Protective measures can be added to the critical nodes to improve the node reliability, which can prevent disaster effectively, and improve the security of the whole WDN.*
Keywords: Water distribution network, Cascading dynamics, DEA, Critical ranking

1. Introduction.

Cascading failures are common in most physical network. The physical networks are basic compositions of human life, including power grid, Internet, communication network and information system network. In such networks, emergencies like natural disasters and system failures can cause massive avalanche effects and reduce network services [1]. Research on cascading failure dynamics becomes a hot issue [2].

Water distribution network (WDN) is a complex engineering system that meets requirements of urban production and drinking, which is an indispensable basis to ensure people's life, production and development. Water supply safety is involved in the public safety. Due to the complex structures and various types of materials used in WDN, accidents like pipe burst and leakage occur frequently. Such accidents result in economic losses to water supply enterprises, disrupts water supply, and damages to water supply services. They are inconvenient to our life and industrial production, and produce threats to public safety and other underground pipelines. Furthermore, WDN is a complex and open system, and it is highly sensitive to natural disasters and system failures. Therefore, it is necessary to introduce the cascading failure model of complex network to study the changing process in WDN, and identify critical nodes. Sitzenfrei et al. [3] used GIS to establish the map of cascade risks, which combines hazard and cascade vulnerability. The applied research indicates that ignoring cascade incidents seriously underestimated risks of WDN. Shuang et al. [4] pointed out that in cascade effects, WDN reliability estimation should consider related uncertain factors, and simulate the peak of reliability and evolution periods of cascade failures. Shuang et al. [5] considered water supply emergency allocation strategies, and analyzed resistance effects of different emergency strategies in cascading failures of WDN from three aspects: uniform distribution, node betweenness and node pressure. Zhang et al. [6] built the model of water-electricity network cascading failures. Simulation effects indicate that when tolerance parameter is less than the critical value, compared with single network, interactive network is more susceptible to cascading failures.

Researches with graph theory in WDN can be classified as the topology-based approach. Nowadays, graph theory has received increased attention in water security and reliability. Zhuang et al. [7] used graph theory to identify the topological changes due to valves closing. Zheng et al. [8] built an optimization model for WDN. Using graph theory, the full WDN was decomposed into subnetworks, and differential evolution was employed to optimize each subnetwork. Herrera et al. [9] developed a graph-theoretic approach for assessing the resilience of large scale WDN. This approach worked well with large network but it did not have hydraulic simulations. Torres et al. [10] pointed out that the graph-based structures in WDNs were ideal for exploring engineered performance. Performance predictions could be carried out with statistical models.

The topology-based approaches used in WDNs have shown great benefits. However, strict topology-based approaches pay more attention to topological structures [11,12] and network weights [13] than flow attributes [14], i.e., supply and demand balance. On the contrary, the flow-based models [15] need to consider the entity attributes of network such as supply nodes, requirement nodes and transmission nodes. Therefore, topology-based WDN analysis should be combined with hydraulic simulation. As for the model of WDN cascading dynamic, it needs to concern dynamic iterative analysis of factors such as node pressure, flow quantity and flow direction, which assess the supply capacity and services of water network.

The safety and effective supply in WDN is significant to improve the ability of resisting disasters. At the same time, the stable operation of WDN can play a supporting role in the process of urbanization. The paper evaluates the critical nodes which can easily lead to large-scale failures in WDN from these two aspects. In addition, most researches of the physical network just study network topology with the graph theory. In this paper, the flow-based cascading dynamic model is built with graph theory and WDN hydraulic calculation. The identified critical nodes are more accordant with the real WDN operating rules, which means that the model has a practical character.

The paper considers the flow features in cascading failure model. The input indexes and output indexes are defined under cascading dynamics in WDN. DEA (Data Envelopment Analysis) model is used to assess node efficiency. The method in this paper studies WDN topology structure and analyzes the loss of network services. To the nodes with the same efficiency values, the relative efficiency is further evaluated with the tolerance parameter. A more specific ranking is given according to the comprehensive results of node efficiencies and relative efficiencies. A case study indicates that the method can effectively simulate dynamic behaviors of cascading failures in WDN, and assess node efficiency by sorting. It provides decision support for protecting critical nodes in WDN.

2. Dynamical Model of Cascading Failures in WDN.

Cascading failure is a step-to-step failure process. It is a hot issue in infrastructure networks safety. The focusing problem in this paper is to obtain the critical ranking in WDN experiencing cascading failures. The simulation can be divided into two phases. The first phase is cascading dynamic simulation. The failure starts from a certain node. It triggers WDN topological and hydraulic redistribution. Secondary failure nodes are produced if their pressure beyond capacity. The stable condition means that no new failure is generated. The second phase is efficiency evaluation. With the defined input and output indexes, DEA is used to evaluate node efficiency. If more than one node gets the same efficiency value, the relative efficiency is then conducted to find which one is more critical. Finally, the critical ranking of WDN can be obtained, and the critical nodes can be identified.

2.1. **The hydraulic feature of WDN.** WDN is an underground network system with complex topological structures and operational control process [14]. If the network has stable water quantity and reliable level of water pressure, it can ensure normal development of different social industries and people's living standards. The design objective of water supply refers to transfering specified water flow to users in specified water pressure. Therefore, the node pressure P is used to measure hydraulic structure in WDN. When node pressure is too high, sudden pipe burst causes water resource waste and other disasters such as road damages [16,17]. When node pressure is too low, water supply shortage or inadequate node pressure may impact normal corporate production and cause indirect losses.

2.2. **The cascading dynamic model.** Cascading failures can be measured based on extra load of nodes. In the network, due to economic and technical reasons, nodes' bearing capacity is limited [18]. Once the load exceeds the bearing capacity, invalidation phenomenon occurs in the network. Considering the flow attributes of WDN, the node pressure P is selected as the load. According to Motter and Lai [19] model, the node capacity, i.e., the maximum node pressure $P_{k,\max}$ can be defined as:

$$P_{k,\max} = (1+\alpha)P_{k,ser}, \tag{1}$$

where α is a tolerance parameter, which controls the bearing capacity of nodes. $P_{k,\max}$ is the maximum node pressure of the kth node.

WDN is a type of physical network. Different users have different requirements on node pressure. Meanwhile, each node should meet the minimum demands of node pressure by fire control. Therefore, it is necessary to define node pressure constraint for each node. The expression is as follows:

$$P_{k,\max} > P_k > P_{k,\min}, \quad k = 1, 2, \ldots, N, \tag{2}$$

where $P_{k,\min}$ is the minimum node pressure of the kth node. The minimum water pressure should be designed according to local water supply standards or design drawings of WDN.

Nodes should meet node pressure constraint conditions. Excessively high water pressure may cause pipeline leakage or burst. Excessively low water pressure may cause shortage of water supply. Therefore, what defines the failure of water supply nodes are node pressure higher than the maximum pressure ($P_k \geq P_{k,\max}$) or lower than the minimum pressure ($P_k \leq P_{k,\min}$).

2.3. **The renewal function of node pressure and flow.** The incidence relation between node pressure and flow can be calculated based on Wagner's function [20]. This function can calculate nodes' demand and has been widely applied [21,22]. Considering the limitation condition on the maximum capacity in complex network, when nodes pressure is greater than the maximum node pressure, they will be in the status of overload and collapse. Based on the constraint condition of maximum node pressure and Wagner's function, the renewal function between users' actual demand of WDN and node pressure is:

$$Q'_{k,act,t} = \begin{cases} 0 & P'_{k,t} \leq P_{k,\min} \\ Q_{k,req}\sqrt{\frac{P'_{k,t}-P_{k,\min}}{P_{k,ser}-P_{k,\min}}} & P_{k,\min} < P'_{k,t} < P_{k,ser} \\ Q_{k,req} & P_{k,ser} \leq P_k < P_{k,\max} \\ 0 & P'_{k,t} \geq P_{k,\max} \end{cases}, \tag{3}$$

where $Q_{k,act}$ is the actual demand of the kth node. $Q_{k,req}$ is the require demand of the kth node. P_k is the kth node pressure. $P_{k,\min}$ is the minimum node pressure of the kth node. $P_{k,ser}$ is the service node pressure of the kth node. $P_{k,\max}$ is the maximum node pressure of the kth node.

Use the pressure-driven method to solve node equations with steady flow [23]. The advantage of this method is that when pipe failure occurs, negative water pressure in failure condition can be avoided. The disadvantage is that so far, there has not been developed and universal network hydraulic analysis software package that provide solutions for the pressure-driven method. Therefore, the paper uses MATLAB to call EPANET 2 Toolkit. It also analyzes the WDN pressure and actual demand in cascade failures based on Equation (3).

2.4. **DEA model and the cascading dynamic indexes.** As a systemic analysis method, DEA can be used to evaluate the relative efficiency of decision making unit (DMU) generated by multiple inputs and outputs [24]. Through maximizing the input-output ratio, this method does not require to specify functional relationship and weight hypothesis for each input and output. It can work out the efficiency value simply by observation data, which assess the effective level of different DMUs.

Assume that there are n DMUs. Each DMU represents a node in WDN. Each DMU has t input indexes X_{ok} $(o = 1, \ldots, t)$ and s output indexes Y_{rk} $(r = 1, \ldots, s)$. To study their effectiveness, C^{2}R model [25] can be used in assessment:

$$\begin{aligned} E_k = \max & \sum_{r=1}^{s} u_r Y_{rk} \\ \text{s.t.} \quad & \sum_{r=1}^{s} u_r Y_{rj} - \sum_{o=1}^{t} v_o X_{oj} \leq 0 \quad j = 1, \ldots, n \\ & \sum_{i=1}^{m} v_o X_{oj} = 1 \\ & u_r \geq \varepsilon, \quad v_o \geq \varepsilon \quad \forall r, o \end{aligned}, \tag{4}$$

where u_r and v_o are output and input weights. ε is the small non-Archimedean quantity. Take $\varepsilon = 10^{-7}$ in calculation. The linear programming requires n optimization for identifying optimum outputs and inputs weight of each DMU to calculate efficiency. DEA model can be used in complex decision-making system as it does not require the relationship between inputs and outputs.

2.4.1. *Input indexes.* In the DEA model, as input factors, input indexes should be smaller in values. Therefore, the total number of failure nodes and the cascade propagation velocity are used as input indexes. Cascading failures in WDN can cause large-scale secondary failures. Less secondary failure nodes produce larger robustness of WDN. There are two input indexes.

(1) Define the total number of failure nodes as the sum of all failure nodes in WDN when it returns to the stable status, i.e., when no new failure node emerges.

(2) The cascade propagation velocity measures the number of failure nodes occurring in each time step. The definition is:

$$V = \frac{N - N'}{T}, \tag{5}$$

where T is the total number of iterations in cascading failures, which measures the propagation time of cascade propagation in WDN.

2.4.2. *Output indexes.* As output factors, output indexes should be larger in values. The total length of valid pipes and the total actual demand are used as output indexes.

(1) Define that the total length of valid pipes is the sum of pipes' length related to the failure nodes after cascading failures. When cascading failures occur in WDN, in order to avoid more losses, the upstream and downstream pipes of failure nodes are closed. The closed pipes are regarded as failure pipes. The total length of valid pipes measures the scale of coverage after cascading failures are over. Larger values are better.

(2) Define that the total actual flow is the sum of actual flow of valid nodes. The actual flow of failure node is set as 0 according to the loss of service function. The total actual flow measures the residue service functions of WDN. Larger values are better.

According to the calculated input index matrix and output index matrix, DEA model can be used to calculate the efficiency of each node in WDN. Larger efficiency values indicate that when cascading failures are over, more service functions are retained in WDN, and the failure nodes will not cause large-scale service losses. On the contrary, smaller efficiency values indicate that the failure node may cause massive secondary failures and worse consequences. The node with the smallest efficiency value is the critical node in WDN.

2.5. **Simulation process of cascading dynamic in WDN.** The simulation process of cascading dynamic in WDN based on DEA is shown in Figure 1. The specific steps are listed below.

Step 1: According to WDN topological structures, set up the incidence matrix.

Firstly, load data information on nodes, including information on water resource and demand nodes. Basic information of demand nodes includes node number, node base demand and elevation. Load data information on pipes, including pipe number, starting and ending nodes, pipe length, pipe diameter, and coefficient factor (C-factor). These factors determine the computational formula for head loss.

Secondly, according to information on water hydraulic and topology, use MATLAB to call EPANET 2 Toolkit, calculate node pressure.

Thirdly, define the time step. The WDN is not attacked at this moment. Set the time step as $t = 0$.

Step 2: Set the tolerance parameter α. Identify the minimum value, the maximum value and the incremental iteration change for the tolerance parameter α.

Step 3: According to the tolerance parameter α in Step 2, use Equation (1) to calculate the maximum node pressure.

Step 4: Select the initial failure node. Identify the simulation scope of the failure nodes. Within this scope, successively simulate cascading behaviors caused by each node as initial failure node. Set up matrix FailureNodeProcess and FailureLinkProcess to record failure nodes and failure pipes in each time step.

Step 5: Close the upstream and downstream pipes of failure nodes. Update the topological structures of WDN. Use MATLAB to call EPANET 2 Toolkit, and calculate node pressure of each normal node after failure occurs.

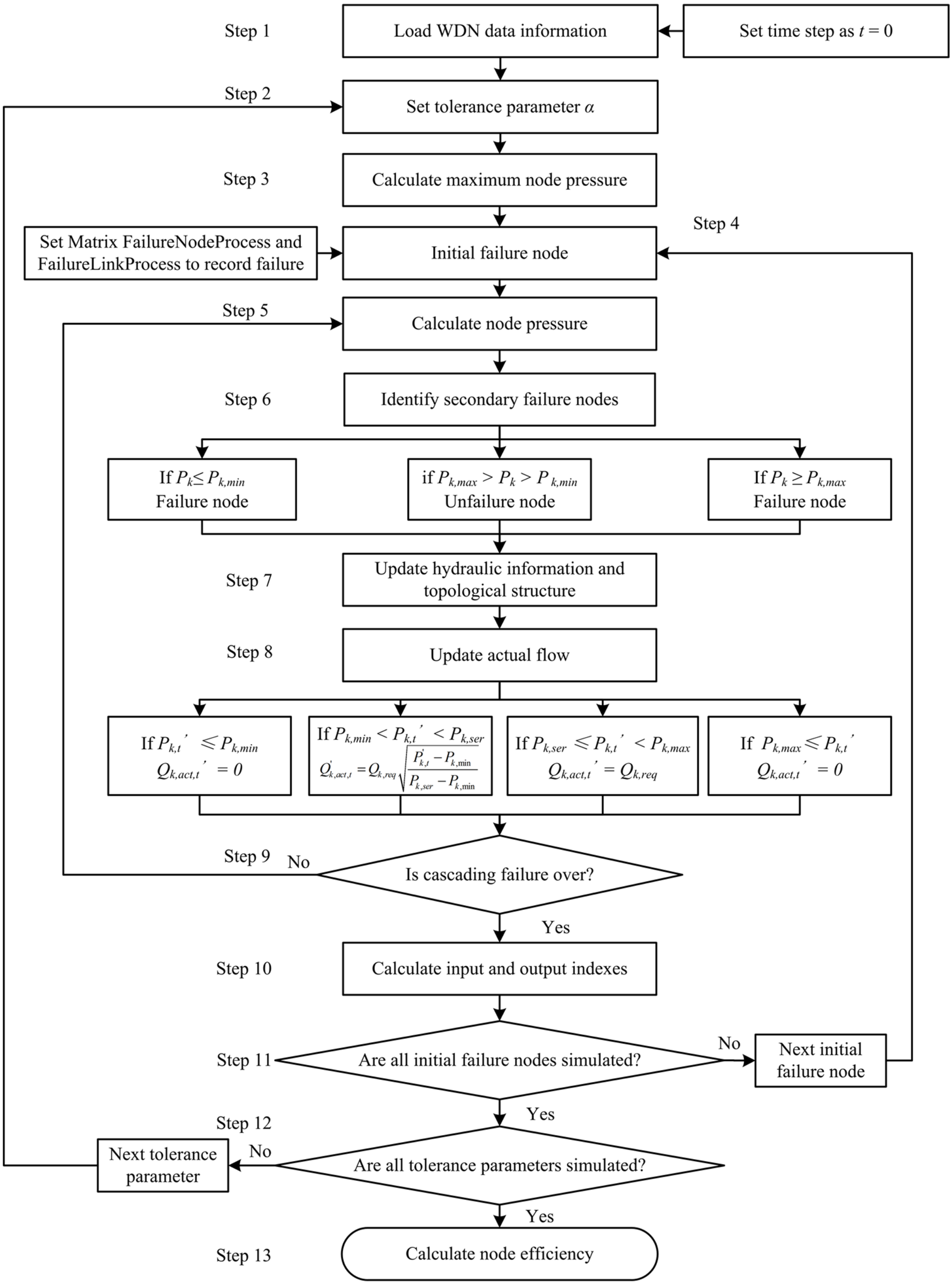

FIGURE 1. Simulation process flowchart of cascading failures in WDN based on DEA

Step 6: According to node pressure calculated in Step 5, identify whether new secondary failure nodes are activated. Based on Equation (2), if node pressure is greater than maximum node pressure or less than minimum node pressure, then it can be recognized as new secondary failure nodes.

Step 7: According to information on new failure nodes, update hydraulic information, i.e., update the flow direction of pipes. Update topological structure, i.e., update the incidence matrix.

Step 8: According to node pressure in Step 5, as well as hydraulic and topological information in Step 7, use Equation (1) to calculate the actual flow, which is used in the next iteration.

Step 9: Judge whether cascading failures are over in WDN, i.e., whether WDN returns to the stable status.

Read data in the t and $t-1$ lines of matrix FailureNodeProcess and FailureLinkProcess. If the t and $t-1$ lines of these two matrixes have the same data, it means that the cascading failures are over. If not, then continue the iteration simulation in Step 5. If cascading failures are over, turn to Step 10.

Assume that when cascading failures are over, the total time step is T in WDN. There is an extra simulation occurring in iteration of cascading failures due to the judgment comparison in t and $t-1$ lines. Therefore, the extra simulation should be deducted while calculating the total iteration step T in cascading failures, i.e., the total time step in cascading failures of WDN is $T = t - 1$.

Step 10: Calculate the total number of failure nodes and cascade propagation velocity of input indexes in WDN under the condition of current tolerance parameter and initial failure node. Calculate the total length of valid pipes and total actual flow of output indexes. Set up the input index matrix and output index matrix.

Step 11: Decide whether all initial failure nodes in WDN are simulated. If so, turn to Step 12. If not, switch to Step 4 and continue simulation.

Step 12: Decide whether all tolerance parameters are simulated. If so, turn to Step 13. If not, switch to Step 2 and continue iteration simulation.

Step 13: According to the input matrix and output matrix of each node, calculate efficiency values of nodes.

3. **Case Study.**

3.1. **Case.**

The case is a simplified WDN, whose characteristics were derived from a real Italian system [26]. It includes 23 water nodes (Node 1 – Node 23), 1 reservoir (Node 24), and 34 pipes (Pipe 1 – Pipe 34). The topological structures, node numbers, and pipe numbers are shown in Figure 2. Basic information of nodes is shown in Table 1. Basic information of pipes is shown in Table 2. Node 24 is an elevated reservoir with total heads of 36.4 m. The total pipe length is 6143 m, with pipe lengths ranging from 100 to 368 m. Pipe diameters vary from 158.2 to 1023.1 mm, and base demands vary from 7.575 to 17.034 L/s. The Hazen-Williams formula is used in hydraulic computation. The minimum value in design of node pressure is 10 m [26,27].

3.2. **Cascading failure process.**

When $\alpha = 0.4$, the cascading failure process of each node is shown in Table 3. It can be seen that cascading failures exist in WDN. Different nodes activate cascading dynamic in different failure paths. If cascading failures are not promptly controlled, the more significant nodes may cause large-scale secondary failures, and reduce WDN's service functions.

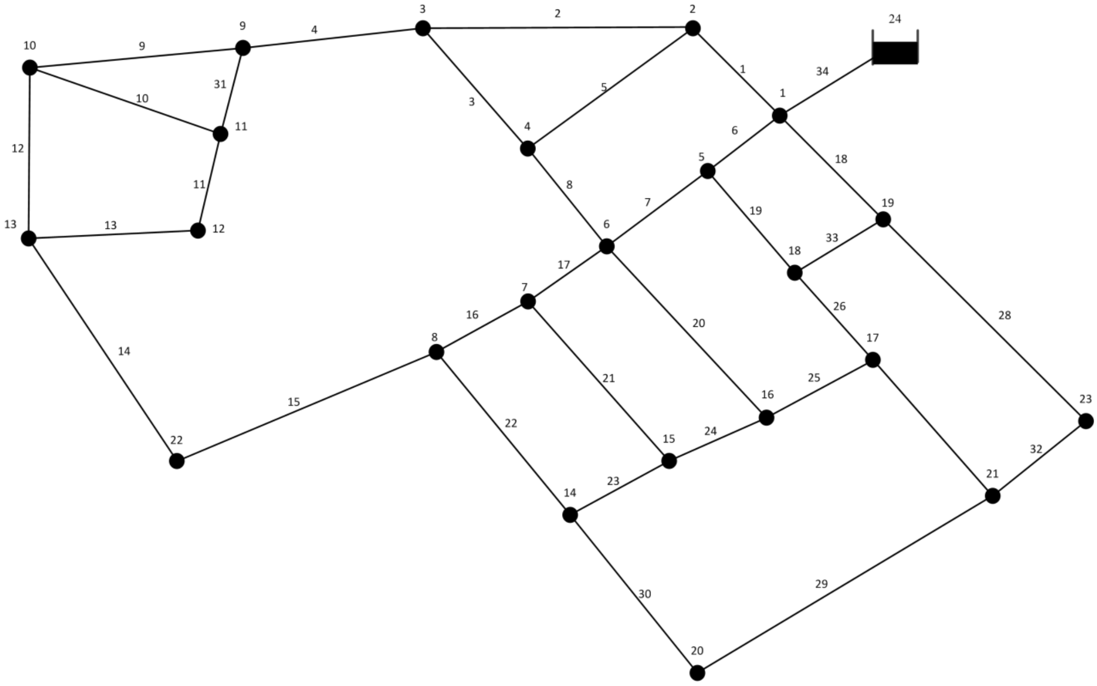

Figure 2. The layout of WDN

Table 1. Data of WDN nodes

Node number	Base demand (L/s)	Elevation (m)
1	10.863	6.4
2	17.034	7
3	14.947	6
4	14.28	8.4
5	10.133	7.4
6	15.35	9
7	9.114	9.1
8	10.51	9.5
9	12.182	8.4
10	14.579	10.5
11	9.007	9.6
12	7.575	11.7
13	15.2	12.3
14	13.55	10.6
15	9.226	10.1
16	11.2	9.5
17	11.469	10.2
18	10.818	9.6
19	14.675	9.1
20	13.318	13.9
21	14.631	11.1
22	12.012	11.4
23	10.326	10
24		36.4

TABLE 2. Data of WDN pipes

Pipe number	Starting node	Ending node	Pipe diameter (mm)	Pipe length (m)	C-factor
1	1	2	348.5	327	124
2	2	3	955.7	290	123
3	3	4	483	100	118
4	3	9	400.7	290	126
5	2	4	791.9	100	114
6	1	5	404.4	368	123
7	5	6	390.6	327	124
8	6	4	482.3	100	115
9	9	10	934.4	100	118
10	11	10	431.3	184	120
11	11	12	513.1	100	114
12	10	13	428.4	184	126
13	13	12	419	100	123
14	22	13	1,023.10	100	119
15	8	22	455.1	164	121
16	7	8	182.6	290	125
17	6	7	221.3	290	123
18	1	19	583.9	164	118
19	5	18	452	229	122
20	6	16	794.7	100	115
21	7	15	717.7	100	116
22	8	14	655.6	258	127
23	14	15	165.5	100	112
24	15	16	252.1	100	124
25	17	16	331.5	100	116
26	18	17	500	204	121
27	17	21	579.9	164	120
28	19	23	842.8	100	113
29	20	21	792.6	100	121
30	14	20	846.3	184	125
31	9	11	164	258	126
32	21	23	427.9	100	116
33	16	18	379.2	100	138
34	24	1	158.2	368	117

3.3. **Efficiency evaluation.** Assume that the value range of α is 0 to 0.5, and increases at the step of 0.1. Calculate the input matrix and output matrix of each initial failure node in different tolerance parameters. Figure 3 shows that the efficiency of WDN nodes calculated based on the method in this paper. The abscissa is node number while the ordinate is node efficiency calculated according to the input and output matrix based on DEA models in different tolerance parameter conditions. A smaller efficiency value signifies more serious consequence caused by cascading failures, i.e., the node is critical in WDN. Based on Figure 3, the ranking of node efficiency in WDN is as follows: $1 > (2, 6) > (3, 7, 8, 9) > 18 > 11 > 19 > 14 > 17 > 10 > 22 > 13 > 21 > 23 > (4, 5, 12, 15, 16, 20)$. Node 1 is the most critical node. Node 1 is directly connected to water reservoir. Its invalidation can cause the failure of the whole WDN. Then, Nodes 2 and 6 are the

TABLE 3. The cascading failure process of each node in WDN

Node	Cascading failure process	Node	Cascading failure process
1	1 → (1-23)	13	13 → (12)
2	2 → (3-4, 9-16) → (7-8)	14	14 → (15-16, 20-21, 23) → (17)
3	3 → (9-16, 20-23) → (4, 8, 17)	15	15 → (-)
4	4 → (-)	16	16 → (-)
5	5 → (3-4, 6-23)	17	17 → (20-21, 23) → (15)
6	6 → (4, 7-8, 10, 12-17, 20-23) → (9)	18	18 → (13, 15-17, 20-21, 23) → (10, 12, 14, 22) → (8, 11) → (9)
7	7 → (8, 10, 12-17, 20-23) → (9, 11)	19	19 → (20-21, 23)
8	8 → (10, 12-17, 20-23) → (9, 11)	20	20 → (-)
9	9 → (10-13, 20-23) → (14-17) → (8)	21	21 → (23)
10	10 → (12-13, 20)	22	22 → (12-13) → (10)
11	11 → (10, 12-13, 20, 22) → (9, 21, 23) → (15, 17)	23	23 → (-)
12	12 → (-)		

Notes: "→" means behavior that triggers secondary failures. "()" represents nodes that become failure at the same time.

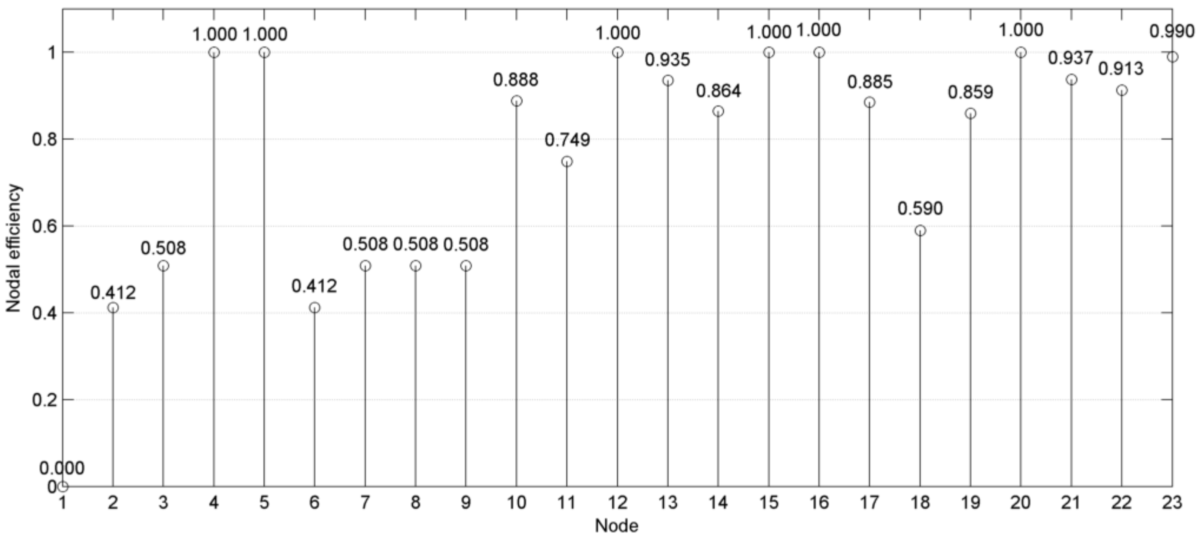

FIGURE 3. Efficiency of WDN nodes

secondary critical nodes which failure will lead to the efficiency of WDN decreased by 60%. Nodes 3, 6, 7, 8 and 9 are the third critical nodes in WDN. Its invalidation will reduce the efficiency of the whole WDN by 50%. Redundancy should be increased, or supplementary enhancement measures should be adopted to improve the anti-disaster ability of nodes.

The efficiency values of Nodes 2 and 6, Nodes 3, 7, 8 and 9 are the same (i.e., 0.412 and 0.508, respectively). They have the same comparison foundation, and the relative efficiency can be compared by their performance with different tolerance parameters. With the relative efficiency, the difference of their critical degree can be obtained. In the pervious, each node is a DMU. Here, the tolerance condition is assumed to be DMU. The relative efficiency reaches 1.0 when tolerance parameter α increases to its maximum.

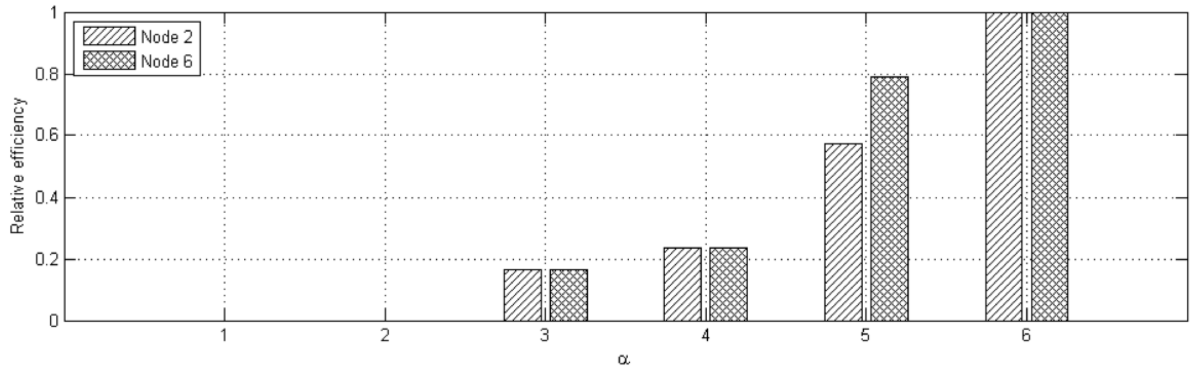

FIGURE 4. Relative efficiency of Nodes 2 and 6

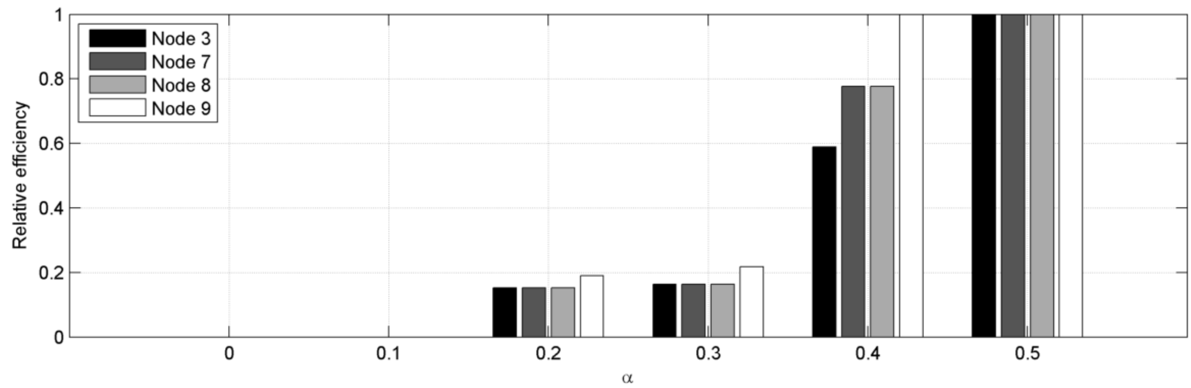

FIGURE 5. Relative efficiency of Nodes 3, 7, 8 and 9

Figure 4 shows the relative efficiency of Nodes 2 and 6. The relative efficiency of Nodes 3, 7, 8 and 9 is shown in Figure 5.

It can be seen from Figure 4, when $\alpha = 0, 0.1, 0.3$, and 0.5, the relative efficiencies of Node 2 and Node 6 are the same. However, When $\alpha = 0.4$, the relative efficiency of Node 6 is larger than that of Node 2, which means that WDN can retain more service after Node 6 fails. It indicates that the loss caused by the failure of Node 6 is lower than the loss caused by the failure of Node 2. Therefore, the critical degree of Nodes 2 and 6 is 2 > 6.

It can be seen from Figure 5, as $\alpha = 0$ and $\alpha = 0.1$, the efficiencies of every node are all 0. The parameter selection of tolerance is too small, leading to the small capacity. The relative efficiency of Node 9 is higher than the other three nodes as $\alpha = 0.2$ and $\alpha = 0.3$, and it reaches 1.0 as $\alpha = 0.4$. Therefore, it is obvious that the damage with initial failure in Node 9 is lighter than other nodes. The relative efficiencies of Node 7 and Node 8 are the same with different α. The relative efficiency of Node 3 is the same as Node 7 and Node 8 as $\alpha = 0.2$ and $\alpha = 0.3$. However, Node 3's relative efficiency reduces 24% as $\alpha = 0.4$, compared with Node 7 and Node 8 on the same condition. Hence, the critical degree of Nodes 3, 7, 8 and 9 is 3 > (7, 8) > 9.

With the analyses of node efficiencies and relative efficiencies, the ranking of nodes in WDN is 1 > 2 > 6 > 3 > (7, 8) > 9 > 18 > 11 > 19 > 14 > 17 > 10 > 22 > 13 > 21 > 23 > (4, 5, 12, 15, 16, 20).

3.4. **Comparison.** The proposed method is compared with node degree [28], node betweenness [29] and flow entropy [30]. The results are shown in Table 4. The rankings in parentheses indicate the node critical. For the first two methods, the larger values

Table 4. Comparison with node degree, node betweenness and flow entropy

	1	2	3	4	5	6	7	8	9	10	11	12
The proposed method	0 (1)	0.412 (2)	0.508 (4)	1.000 (23)	1.000 (23)	0.412 (3)	0.508 (5)	0.508 (5)	0.508 (7)	0.888 (13)	0.749 (9)	1.000 (23)
Node degree	0.088 (1)	0.059 (3)	0.059 (3)	0 (23)	0.059 (3)	0.088 (1)	0.059 (3)	0.059 (3)	0.059 (3)	0.029 (13)	0.059 (3)	0 (23)
Node betweenness	0.091 (4)	0.058 (8)	0.062 (7)	0 (23)	0.078 (5)	0.099 (3)	0.107 (1)	0.103 (2)	0.066 (6)	0.025 (15)	0.021 (17)	0 (23)
Flow entropy	0 (1)	0 (1)	0 (1)	1.083 (23)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0.374 (16)	0 (1)	0.597 (19)
	13	14	15	16	17	18	19	20	21	22	23	
The proposed method	0.935 (15)	0.864 (11)	1.000 (23)	1.000 (23)	0.885 (12)	0.590 (8)	0.859 (10)	1.000 (23)	0.937 (16)	0.913 (14)	0.990 (17)	
Node degree	0.029 (13)	0.059 (3)	0.029 (13)	0 (23)	0.059 (3)	0.029 (13)	0.059 (3)	0.029 (13)	0.029 (13)	0.029 (13)	0 (23)	
Node betweenness	0.025 (15)	0.058 (8)	0.012 (19)	0 (23)	0.037 (11)	0.041 (10)	0.021 (17)	0.033 (12)	0.033 (12)	0.033 (12)	0 (23)	
Flow entropy	0.495 (18)	0 (1)	0.676 (20)	1.028 (22)	0 (1)	0.071 (15)	0 (1)	0 (1)	0.390 (17)	0 (1)	0.681 (21)	

mean that the nodes are more vulnerable. The results are sorted in increase, where the prior rank implies the critical nodes. The smaller values in flow entropy method mean vulnerable. The result is sorted in decrease. Nodes with the same rankings represent equal importance.

The node degree method is consistent with the proposed method. However, the results of all nodes can be divided into four types, i.e., 0.088, 0.059, 0.029 and 0. It is difficult to rank nodes effectively. Node 1 is not the critical node in node betweenness method. Node 1 connects directly to the water reservoir, and its failure will lead to the collapse of the entire WDN. Hence, the results are inconsistent in actual. Node degree and node betweenness method belong to the topology-based methods. However, WDNs are designed with redundancy to resist disaster. It is difficult to identify critical nodes only considering topology.

The flow entropy method evaluates flexibility and redundancy in WDNs. From the results, there are a number of nodes with the same values. It is not able to recognize the critical nodes which lead to large-scale cascading failures.

It can be seen that the critical nodes need to be evaluated with topology and hydraulic structure. The proposed method adopts the flow-based method to simulate cascading dynamics in WDNs, which obtains more accurate simulation results.

4. **Conclusion.** The tasks of ensuring the safety of water supply and identifying the critical nodes in water distribution networks (WDNs) are significant to improve the ability of resistance to sudden disasters. It is also meaningful in guaranteeing the stable operation of urban infrastructure. The paper defined the input indexes and output indexes under cascading failures. The cascading dynamic model is used to simulate the cascading failure process with different tolerance parameters. The DEA model is used to assess the efficiency of each node. A smaller efficiency value indicates a more critical node. Its invalidation may significantly reduce service functions of WDN.

Case study shows that the method can simulate the process of cascading failures in WDN, and points out secondary failure nodes that change along with time steps. Also, the method in this paper can identify the critical nodes in WDN, and demonstrate the node critical ranking. To the node with the same efficiency, the method can further analyze the relative efficiency with different tolerance parameters. Based on critical ranking, future research can focus on protective measures, which can be added to against disaster and improve the security of the whole WDN.

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REFERENCES

[1] G. Zhu, X. Wang, M. Tian and J. Long, Cascading failures of interdependent modular small-world networks, *Modern Physics Letters B*, vol.30, no.18, 2016.
[2] J. Wang, Y. Li and Q. Zheng, Cascading load model in interdependent networks with coupled strength, *Physica A: Statistical Mechanics & Its Applications*, vol.430, pp.242-253, 2015.
[3] R. Sitzenfrei, M. Mair, M. Möderl and W. Rauch, Cascade vulnerability for risk analysis of water infrastructure, *Water Science & Technology*, vol.64, no.9, pp.1885-1891, 2011.
[4] Q. Shuang, M. Zhang and Y. Yuan, Performance and reliability analysis of water distribution systems under cascading failures and the identification of crucial pipes, *PLoS ONE*, vol.9, no.2, 2014.
[5] Q. Shuang, Y. Yuan, M. Zhang and Y. Liu, A cascade-based emergency model for water distribution network, *Mathematical Problems in Engineering*, vol.2015, 2015.
[6] Y. Zhang, N. Yang and U. Lall, Modeling and simulation of the vulnerability of interdependent power-water infrastructure networks to cascading failures, *Journal of Systems Science and Systems Engineering*, vol.25, no.1, pp.102-118, 2016.
[7] B. Zhuang, K. Lansey and D. Kang, Resilience/availability analysis of municipal water distribution system incorporating adaptive pump operation, *Journal of Hydraulic Engineering*, vol.139, pp.527-537, 2013.
[8] F. Zheng, A. R. Simpson, A. C. Zecchin and J. W. Deuerlein, A graph decomposition-based approach for water distribution network optimization, *Water Resources Research*, vol.49, no.4, pp.2093-2109, 2013.
[9] M. Herrera, E. Abraham and I. Stoianov, A graph-theoretic framework for assessing the resilience of sectorised water distribution networks, *Water Resources Management*, vol.30, no.5, pp.1685-1699, 2016.
[10] J. M. Torres, L. Duenas-Osorio, Q. Li and A. Yazdani, Exploring topological effects on water distribution system performance using graph theory and statistical models, *Journal of Water Resources Planning & Management*, vol.143, no.1, 2016.
[11] J. W. Wang and L. L. Rong, Cascade-based attack vulnerability on the US power grid, *Safety Science*, vol.47, no.10, pp.1332-1336, 2009.
[12] J. W. Wang and L. L. Rong, Robustness of the western United States power grid under edge attack strategies due to cascading failures, *Safety Science*, vol.49, no.6, pp.807-812, 2011.
[13] J. J. Wu, H. J. Sun and Z. Y. Gao, Cascading failures on weighted urban traffic equilibrium networks, *Physica A: Statistical Mechanics & Its Applications*, vol.386, no.1, pp.407-413, 2007.
[14] A. Yazdani and P. Jeffrey, Complex network analysis of water distribution systems, *Chaos An Interdisciplinary Journal of Nonlinear Science*, vol.21, no.1, 2011.
[15] M. Ouyang, Review on modeling and simulation of interdependent critical infrastructure systems, *Reliability Engineering and System Safety*, vol.121, no.1, pp.43-60, 2014.
[16] J. L. Winkler, A. M. Dueñas-Osorio, R. Stein and D. Subramanian, Interface network models for complex urban infrastructure systems, *Journal of Infrastructure Systems*, vol.17, no.4, pp.138-150, 2011.
[17] N. Nojima and H. Kameda, Cross-impact analysis for lifeline interaction, *Proc. of Lifeline Earthquake Engineering in Los Angeles*, pp.629-638, 1991.
[18] L. Dueñas-Osorio and S. M. Vemuru, Cascading failures in complex infrastructure systems, *Structural Safety*, vol.31, no.2, pp.157-167, 2009.

[19] A. E. Motter and Y. C. Lai, Cascade-based attacks on complex networks, *Physical Review E*, vol.66, no.6, 2002.

[20] J. M. Wagner, U. Shamir and D. H. Marks, Water distribution reliability-simulation methods, *Journal of Water Resource and Planning Management*, vol.114, no.3, pp.276-294, 1988.

[21] A. Ostfeld, D. Kogan and U. Shamir, Reliability simulation of water distribution systems – Single and multiquality, *Urban Water*, vol.4, no.1, pp.53-61, 2002.

[22] O. Giustolisi and T. M. Walski, Demand components in water distribution network analysis, *Journal of Water Resources Planning & Management*, vol.138, no.4, pp.356-367, 2012.

[23] D. Laucelli, L. Berardi and O. Giustolisi, Assessing climate change and asset deterioration impacts on water distribution networks: Demand-driven or pressure-driven network modeling?, *Environmental Modelling & Software*, vol.37, no.11, pp.206-216, 2012.

[24] C. Kao, Efficiency decomposition in network data envelopment analysis with slacks-based measures, *European Journal of Operational Research*, vol.45, no.3, pp.1-6, 2014.

[25] A. Charnes, W. W. Cooper and E. Rhodes, Measuring the efficiency of decision making units, *European Journal of Operational Research*, vol.2, no.6, pp.429-444, 1978.

[26] J. Liu and G. Yu, Iterative methodology of pressure-dependent demand based on EPANET for pressure-deficient water distribution analysis, *Journal of Water Resources Planning & Management*, vol.139, no.1, pp.34-44, 2013.

[27] O. Giustolisi, D. Savic and Z. Kapelan, Pressure-driven demand and leakage simulation for water distribution networks, *Journal of Hydraulic Engineering*, vol.134, no.5, pp.626-635, 2008.

[28] S. V. Buldyrev, N. W. Shere and G. A. Cwilich, Interdependent networks with identical degrees of mutually dependent nodes, *Physical Review E*, vol.83, no.2, 2011.

[29] N. Kourtellis, T. Alahakoon, R. Simha and R. Tripathi, Identifying high betweenness centrality nodes in large social networks, *Social Network Analysis and Mining*, vol.3, no.4, pp.899-914, 2013.

[30] T. T. Tanyimboh and Y. Setiadi, Joint layout, pipe size and hydraulic reliability optimization of water distribution systems, *Engineering Optimization*, vol.40, no.8, pp.729-747, 2008.