*International Journal of Innovative Computing, Information and Control*
Volume **13**, Number **5**, October **2017**  ISSN 1349-4198
pp. 1537–1547

# FORECASTING A STORES CREDIT PLAN USING THE MARKOV CHAINS

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Received March 2017; revised July 2017

Abstract. *This paper shows a model to forecast a stores credit plan using the Markov chains. The classical model uses the decision tree to forecast a stores credit plan, but this model is developed step by step for each period, because this model can be used for one or two stages and also depends on the number of decision making. The stores credit plan considers four categories: paid balances, unpaid balances, expired balances and loss balances. Also, a comparison is made between the two models, the proposed model (Markov chains) can be obtained in any period that one wishes to find, and the classic model (decision tree) is developed step by step for each period. The advantages of using the Markov chains and with the help of software can solve the transition probability matrix P for several decisions and multiple stages. Then, the Markov chains are the best option, since it is more economical in working time and also is more exact.*
**Keywords:** Markov chains, Forecasting a stores credit plan, Paid balances, Unpaid balances, Expired balances, Loss balances

1. **Introduction.** Markov chains have been widely studied and applied in brand switching problems and market share forecasting [1,2]. The Markov brand switching model studies customer loyalty and forecasts the brands, products, or service that a customer is likely to purchase next. As aggregation of individual customer choices, market shares of companies and their competitors can also be studied using similar approach. In the literature, most applications used a memory-less process, or first order Markov chain to model the probability that a customer switches from one provider to another over a period of time, and to forecast future market shares [3].

Markov chains are a particularly powerful and widely used tool for analyzing a variety of stochastic (probabilistic) systems over time.

Forecasting methodology is the most important and relevant in the field of management, including that for financial forecasting, production demand and supply forecasting, technology forecasting, and so on [4].

Selby and Porter proposed a general solution method for the automatic generation of decision (or classification) trees is investigated. The approach is to provide insights through in-depth empirical characterization and evaluation of decision trees for one problem domain, specifically, that of software resource data analysis [5].

Quinlan examined the techniques that represent knowledge about classified tasks in the form of decision trees. A sample of techniques is sketched, ranging from basic methods of constructing decision trees to ways of using them not categorically [6].

Esposito et al. make a comparative study of six well-known pruning methods with the aim of understanding their theoretical foundations, their computational complexity, and the strengths and weaknesses of their formulation [7]; Huarng and Yu applied a back propagation neural network to handle nonlinear forecasting problems [8]; Carriero et al. presented a model to forecasting exchange rates with a large Bayesian VAR [9].

The most closely-related works on the topic presented in this paper are as follows. Frydman et al. summarized methodology for testing the compatibility of discrete time stochastic processes-stationary and nonstationary Markov chains and an extension, the mover-stayer model-with longitudinal data from an unknown empirical process, and also applied this methodology to determining the suitability of these models to represent the payment behavior of a sample of retail revolving credit accounts [10]. Stanford develops algebraic methods for a general sensitivity analysis of the classic Cyert-Davidson-Thompson model of a system of accounts receivable, and uses the resulting analytical relationships to investigate the stability of estimates for the losses from doubtful accounts (bad debts) and the present value of interest income generated by charges to the accounts [11]. Hand and Henley investigated how credit scoring has developed in importance and to identify the key determinants in the construction of a scoring model, by means of a widespread review of different statistical techniques and performance evaluation criteria [12]. Foster and Stine predicted the onset of personal bankruptcy using least squares regression, and it uses stepwise selection to find predictors of these from a mix of payment history, debt load, demographics, and their interactions [13]. Thomas et al. presented a modelling for the loss given default (LGD), and the percentage of the defaulted amount of a loan that a lender will eventually lose is to model the collections process. This is particularly relevant for unsecured consumer loans where LGD depends both on a defaulter's ability and willingness to repay and the lender's collection strategy. When repaying such defaulted loans, defaulters tend to oscillate between repayment sequences where the borrower is repaying every period and non-repayment sequences where the borrower is not repaying in any period [14].

This paper shows a model to forecast a stores credit plan using the Markov chains. The stores credit plan considers four categories: paid balances, unpaid balances, expired balances and loss balances. The classical model uses the decision tree to forecast a stores credit plan, but this model can be used for one or two stages and also depends on the number of decisions taken. The advantages of using the Markov chains and with the help of software can be solved for several decisions and multiple stages. Also, a numerical example is presented to observe the effectivity of the Markov chains against decision tree to forecast a stores credit plan.

The paper is organized as follows. Section 2 describes the specifications of the Markov chains that shows some of the equations for the general case, and two theorems and two definitions of the Markov chains are presented, also the transition probability matrix of state "**P**", the probability vector "**u**" and the number of steps "$n$". Section 3 presents an application of the Markov chains and the decision tree to forecast a stores credit plan with detailed techniques. Results and discussion are presented in Section 4. Conclusion (Section 5) completes the paper.

## 2. Specifications of the Markov Chains.

Markov chains are described as follows. We have a set of states, $S = \{s_1, s_2, \dots, s_r\}$. The process starts in one of these states and moves successively from one state to another. Each move is called a *step*. If the chain is currently in state $s_i$, then it moves to state $s_j$ at the next step with a probability denoted by $p_{ij}$, and this probability does not depend upon which states the chain was in before the current state.

The probabilities $p_{ij}$ are called *transition probabilities*. The process can remain in the state it is in, and this occurs with probability $p_{ii}$. An initial probability distribution, defined on $S$, specifies the starting state. Usually this is done by specifying a particular state as the starting state.

We consider the question of determining the probability that, given the chain is in state $i$ today; it will be in state $j$ two time periods from now. We denote this probability by $p_{ij}^{(2)}$. Using the transition matrix $\mathbf{P}$, we can write this product as $p_{11}p_{13}$. The other two events also have probabilities that can be written as products of entries of $\mathbf{P}$. Thus, we have

$$p_{ij}^{(2)} = p_{11}p_{13} + p_{12}p_{23} + p_{13}p_{33} \tag{1}$$

This equation should be a dot product of two vectors; we are dotting the first row of $\mathbf{P}$ with the third column of $\mathbf{P}$. This is just what is done in obtaining the 1, 3-entry of the product of $\mathbf{P}$ with itself. In general, if a Markov chain has $r$ states, then

$$p_{ij}^{(2)} = \sum_{k=1}^{r} p_{ik}p_{kj} \tag{2}$$

The following general theorem is easy to prove by using the above observation and induction.

**Theorem 2.1.** *Let* $\mathbf{P}$ *be the transition matrix of a Markov chain. The* $ij$*th entry* $p_{ij}^{(n)}$ *of the matrix* $\mathbf{P}^n$ *gives the probability that the Markov chain, starting in state* $s_i$*, will be in state* $s_j$ *after* $n$ *steps* [15,16]*.*

Then, the transition probability matrix $\mathbf{P}$ of state can be written as follows [4,17]:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1r} \\ p_{21} & p_{22} & \cdots & p_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ p_{r1} & p_{r2} & \cdots & p_{rr} \end{bmatrix} \tag{3}$$

For the matrix $\mathbf{P}$, some definitions are described as follows [4,17].

**Definition 2.1.** *If* $p_{ij} \geq 0$*, then state* $A_j$ *is accessible from state* $A_i$*.*

**Definition 2.2.** *If states* $A_i$ *and* $A_j$ *are accessible to each other, then* $A_i$ *communicates with* $A_j$*.*

Where $p_{ij}$ is the probability of transition from state $A_i$ to $A_j$ by one step, $M_{ij}$ is the transition times from state $A_i$ to $A_j$ by one step, and $M_i$ is the amount of data belonging to the $A_i$ state.

The transition probability matrix $\mathbf{P}$ reflects the transition rules of the system. For example, if the original data is located in the state $A_1$, and makes a transition into state $A_j$ with probability $p_{ij} \geq 0$, $j = 1, 2, \ldots, n$, then $p_{11} + p_{12} + \ldots + p_{1r} = 1$.

**Theorem 2.2.** *Let* $\mathbf{P}$ *be the transition matrix of a Markov chain, and let* $\mathbf{u}$ *be the probability vector which represents the starting distribution. Then the probability that the chain is in state* $s_i$ *after* $n$ *steps is the* $i$*th entry in the vector* [4,17]*:*

$$\mathbf{u}^{(n)} = \mathbf{u}\mathbf{P}^n \tag{4}$$

We note that if we want to examine the behavior of the chain under the assumption that it starts in a certain state $s_i$, we simply choose $\mathbf{u}$ to be the probability vector with the $i$th entry equal to 1 and all other entries equal to 0.

3. **Application to a Stores Credit Plan.** A general accounting office of commercial stores large has a credit plan in its stores. The accounts of each month are classified into four categories: paid balances, unpaid balances, expired balances and loss balances. The paid balances are those that have not balance to pay in the month. The unpaid balances are those that do not owe balances in the previous month, but have been charged the purchases made in the month. The expired balances are those that have a balance that has remained unpaid for more than a month, but less than three. Finally, the loss balances are those that have a balance with more than three months of expiration and that are not expected to be able to collect.

From the store's records, it has been determined that 60% of the accounts with unpaid balances are paid to the following month, 30% remain in the same category and 10% becomes in expired balances. It has also been determined that 40% of overdue accounts are converted into unpaid balances, 30% are paid, 20% remain overdue and 10% are canceled as loss balances. Once an account reaches the loss category, it is canceled. Similarly, once an account goes pass to the settled account category, that money is no longer part of the accounts receivable.

If there are currently 100,000 dollars of accounts receivable in the paid balances category, 50,000 dollars in the unpaid balances category, 20,000 dollars in the expired balances category and 5000 dollars in the loss balances category, what amount will be in each category until the end of each month in the following 7 months?

The summary of the information is shown in Table 1.

TABLE 1. Probabilities information

| Accounts type | Paid balances | Unpaid balances | Expired balances | Loss balances |
|---|---|---|---|---|
| Paid balances | 1.00 | 0.00 | 0.00 | 0.00 |
| Unpaid balances | 0.60 | 0.30 | 0.10 | 0.00 |
| Expired balances | 0.30 | 0.40 | 0.20 | 0.10 |
| Loss balances | 0.00 | 0.00 | 0.00 | 1.00 |

3.1. **Proposed model (Markov chains).** From Table 1, the transition probability matrix $\mathbf{P}$ of state can be written as

$$\mathbf{P} = \begin{bmatrix} \cdot & s_1 & s_2 & s_3 & s_4 \\ s_1 & 1.00 & 0.00 & 0.00 & 0.00 \\ s_2 & 0.60 & 0.30 & 0.10 & 0.00 \\ s_3 & 0.30 & 0.40 & 0.20 & 0.10 \\ s_4 & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}$$

And vector $\mathbf{u}$ is:

$$\mathbf{u} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ \dfrac{100{,}000}{175{,}000} & \dfrac{50{,}000}{175{,}000} & \dfrac{20{,}000}{175{,}000} & \dfrac{5{,}000}{175{,}000} \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ 0.5714 & 0.2857 & 0.1143 & 0.0286 \end{bmatrix}$$

where $s_1$ is the state 1 that is the paid balances category, $s_2$ is the state 2 that is the unpaid balances category, $s_3$ is the state 3 that is the expired balances category, and $s_4$ is the state 4 that is the loss balances category.

Substituting the matrix $\mathbf{P}$, the vector $\mathbf{u}$ and the value of $\boldsymbol{n}$ to each time period into Equation (4) is obtained:

$$\begin{aligned}\mathbf{u}^{(1)} &= \begin{bmatrix} 0.5714 & 0.2857 & 0.1143 & 0.0286 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.60 & 0.30 & 0.10 & 0.00 \\ 0.30 & 0.40 & 0.20 & 0.10 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \\ &= \begin{bmatrix} 0.7771 & 0.1314 & 0.0514 & 0.0400 \end{bmatrix}\end{aligned}$$

where $\mathbf{u}^{(1)}$ is the probability vector after one month.

$$\begin{aligned}\mathbf{u}^{(2)} &= \begin{bmatrix} 0.5714 & 0.2857 & 0.1143 & 0.0286 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.60 & 0.30 & 0.10 & 0.00 \\ 0.30 & 0.40 & 0.20 & 0.10 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}^2 \\ &= \begin{bmatrix} 0.8714 & 0.0600 & 0.0234 & 0.0451 \end{bmatrix}\end{aligned}$$

where $\mathbf{u}^{(2)}$ is the probability vector after two months.

$$\begin{aligned}\mathbf{u}^{(3)} &= \begin{bmatrix} 0.5714 & 0.2857 & 0.1143 & 0.0286 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.60 & 0.30 & 0.10 & 0.00 \\ 0.30 & 0.40 & 0.20 & 0.10 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}^3 \\ &= \begin{bmatrix} 0.9145 & 0.0274 & 0.0107 & 0.0475 \end{bmatrix}\end{aligned}$$

where $\mathbf{u}^{(3)}$ is the probability vector after three months.

$$\begin{aligned}\mathbf{u}^{(4)} &= \begin{bmatrix} 0.5714 & 0.2857 & 0.1143 & 0.0286 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.60 & 0.30 & 0.10 & 0.00 \\ 0.30 & 0.40 & 0.20 & 0.10 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}^4 \\ &= \begin{bmatrix} 0.9341 & 0.0125 & 0.0049 & 0.0486 \end{bmatrix}\end{aligned}$$

where $\mathbf{u}^{(4)}$ is the probability vector after four months.

$$\begin{aligned}\mathbf{u}^{(5)} &= \begin{bmatrix} 0.5714 & 0.2857 & 0.1143 & 0.0286 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.60 & 0.30 & 0.10 & 0.00 \\ 0.30 & 0.40 & 0.20 & 0.10 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}^5 \\ &= \begin{bmatrix} 0.9430 & 0.0057 & 0.0022 & 0.0490 \end{bmatrix}\end{aligned}$$

where $\mathbf{u}^{(5)}$ is the probability vector after five months.

$$\begin{aligned}\mathbf{u}^{(6)} &= \begin{bmatrix} 0.5714 & 0.2857 & 0.1143 & 0.0286 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.60 & 0.30 & 0.10 & 0.00 \\ 0.30 & 0.40 & 0.20 & 0.10 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}^6 \\ &= \begin{bmatrix} 0.9471 & 0.0026 & 0.0010 & 0.0493 \end{bmatrix}\end{aligned}$$

where $\mathbf{u}^{(6)}$ is the probability vector after six months.

$$\mathbf{u}^{(7)} = \begin{bmatrix} 0.5714 & 0.2857 & 0.1143 & 0.0286 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.60 & 0.30 & 0.10 & 0.00 \\ 0.30 & 0.40 & 0.20 & 0.10 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}^7$$
$$= \begin{bmatrix} 0.9490 & 0.0012 & 0.0005 & 0.0494 \end{bmatrix}$$

where $\mathbf{u}^{(7)}$ is the probability vector after seven months.

Table 2 presents a summary of the probabilities by each category.

TABLE 2. Summary of the probabilities by each category

| Time period | Paid balances | Unpaid balances | Expired balances | Loss balances |
|---|---|---|---|---|
| Zero month | 0.5714 | 0.2857 | 0.1143 | 0.0286 |
| After one month | 0.7771 | 0.1314 | 0.0514 | 0.0400 |
| After two months | 0.8714 | 0.0600 | 0.0234 | 0.0451 |
| After three months | 0.9145 | 0.0274 | 0.0107 | 0.0475 |
| After four months | 0.9341 | 0.0125 | 0.0049 | 0.0486 |
| After five months | 0.9430 | 0.0057 | 0.0022 | 0.0490 |
| After six months | 0.9471 | 0.0026 | 0.0010 | 0.0493 |
| After seven months | 0.9490 | 0.0012 | 0.0005 | 0.0494 |

3.2. **Classical model (Decision tree).** Figure 1 shows the probabilities by means of the decision tree of the current state until after the seventh month.

The procedure by the decision tree is as follows:

1) The probabilities of the current state are obtained for each category (see vector $\mathbf{u}$ used in Markov chains).

2) The probability of the current state is multiplied by the probability that it will occur for each one.

3) The probabilities are summed for the same category, i.e., all the probabilities obtained in 1, 2, 3 and 4, respectively, and these are the probabilities presented after the first month.

4) This procedure is repeated for the following time periods.

- The probabilities after the first month for each category are:

The probability of the paid balances is: $0.5714 + 0.1714 + 0.0343 + 0.0000 = 0.7771$
The probability of the unpaid balances is: $0.0000 + 0.0857 + 0.0457 + 0.0000 = 0.1314$
The probability of the expired balances is: $0.0000 + 0.0286 + 0.0229 + 0.0000 = 0.0515$
The probability of the loss balances is: $0.0000 + 0.0000 + 0.0114 + 0.0286 = 0.0400$

- The probabilities after the second month for each category are:

The probability of the paid balances is: $0.7771 + 0.0788 + 0.0155 + 0.0000 = 0.8714$
The probability of the unpaid balances is: $0.0000 + 0.0394 + 0.0206 + 0.0000 = 0.0600$
The probability of the expired balances is: $0.0000 + 0.0131 + 0.0103 + 0.0000 = 0.0234$
The probability of the loss balances is: $0.0000 + 0.0000 + 0.0052 + 0.0400 = 0.0452$

- The probabilities after the third month for each category are:

The probability of the paid balances is: $0.8714 + 0.0360 + 0.0070 + 0.0000 = 0.9144$
The probability of the unpaid balances is: $0.0000 + 0.0180 + 0.0094 + 0.0000 = 0.0274$
The probability of the expired balances is: $0.0000 + 0.0060 + 0.0047 + 0.0000 = 0.0107$
The probability of the loss balances is: $0.0000 + 0.0000 + 0.0023 + 0.0452 = 0.0475$

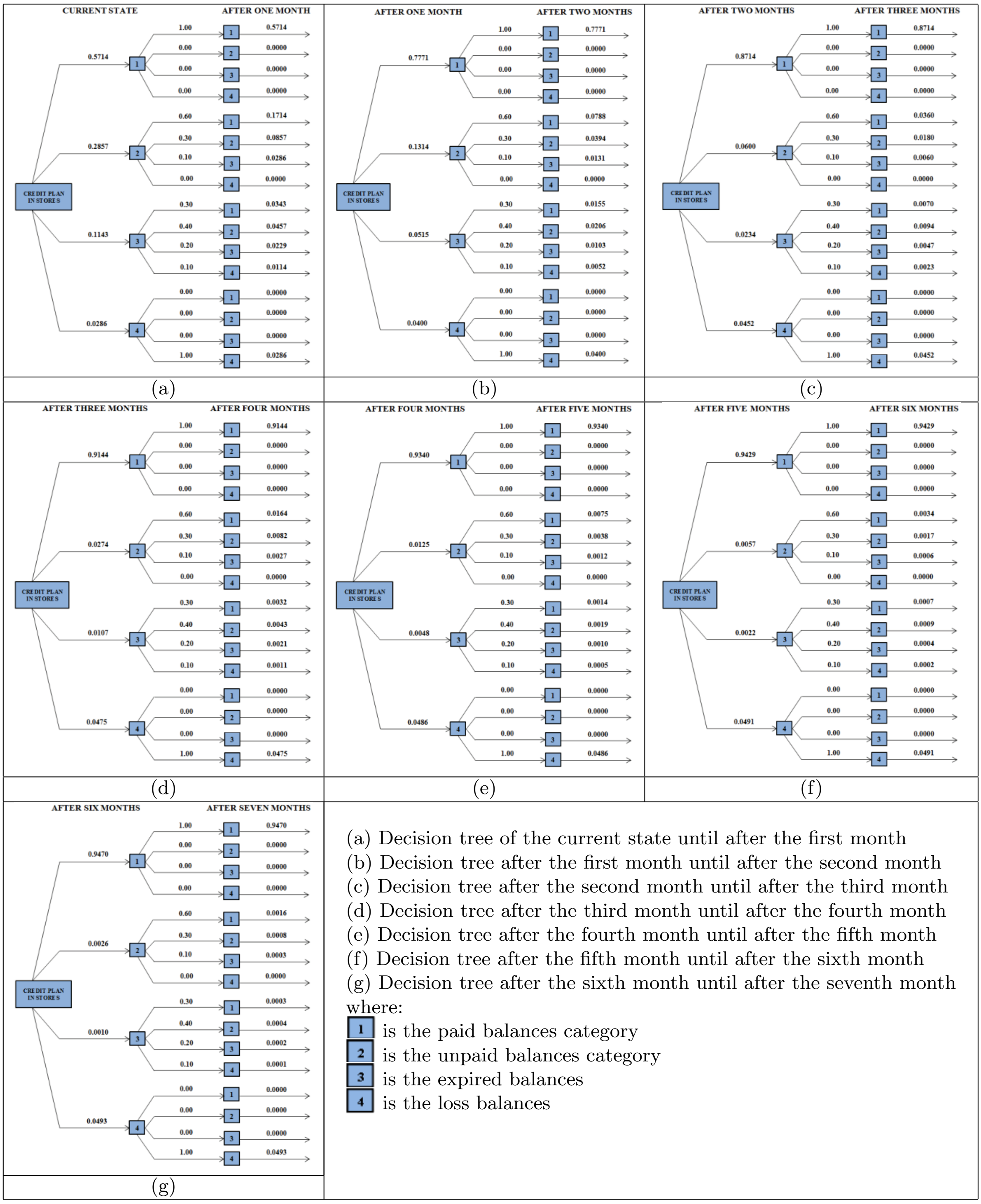


FIGURE 1. Decision tree of the current state until after the seven months

- The probabilities after the fourth month for each category are:

The probability of the paid balances is: $0.9144 + 0.0164 + 0.0032 + 0.0000 = 0.9340$
The probability of the unpaid balances is: $0.0000 + 0.0082 + 0.0043 + 0.0000 = 0.0125$
The probability of the expired balances is: $0.0000 + 0.0027 + 0.0021 + 0.0000 = 0.0048$
The probability of the loss balances is: $0.0000 + 0.0000 + 0.0011 + 0.0475 = 0.0486$

- The probabilities after the fifth month for each category are:

The probability of the paid balances is: 0.9340 + 0.0075 + 0.0014 + 0.0000 = 0.9429
The probability of the unpaid balances is: 0.0000 + 0.0038 + 0.0019 + 0.0000 = 0.0057
The probability of the expired balances is: 0.0000 + 0.0012 + 0.0010 + 0.0000 = 0.0022
The probability of the loss balances is: 0.0000 + 0.0000 + 0.0005 + 0.0486 = 0.0491

- The probabilities after the sixth month for each category are:

The probability of the paid balances is: 0.9429 + 0.0034 + 0.0007 + 0.0000 = 0.9470
The probability of the unpaid balances is: 0.0000 + 0.0017 + 0.0009 + 0.0000 = 0.0026
The probability of the expired balances is: 0.0000 + 0.0006 + 0.0004 + 0.0000 = 0.0010
The probability of the loss balances is: 0.0000 + 0.0000 + 0.0002 + 0.0491 = 0.0493

- The probabilities after the seventh month for each category are:

The probability of the paid balances is: 0.9470 + 0.0016 + 0.0003 + 0.0000 = 0.9489
The probability of the unpaid balances is: 0.0000 + 0.0008 + 0.0004 + 0.0000 = 0.0012
The probability of the expired balances is: 0.0000 + 0.0003 + 0.0002 + 0.0000 = 0.0005
The probability of the loss balances is: 0.0000 + 0.0000 + 0.0001 + 0.0493 = 0.0494

4. **Results and Discussion.** For the proposed model (Markov chains) was used the DERIVE software, and for the classical model (Decision tree) was used a calculator, and the two models obtain the same results. Table 3 shows a summary of the results obtained for each time period. Table 4 presents the contribution of the other accounts to the paid balances, for example, the unpaid balances contribute a 60% to the paid balances, the expired balances contribute the 30% to the paid balances, and the loss balances contribute 0% to the paid balances. Table 5 shows the contribution of the other accounts to the unpaid balances, for example, the 30% remain in the same category of unpaid balances,

TABLE 3. Summary of the results in dollars

| Time period | Paid balances | Unpaid balances | Expired balances | Loss balances |
|---|---|---|---|---|
| Zero month | 100,000.00 | 50,000.00 | 20,000.00 | 5,000.00 |
| After one month | 136,000.00 | 23,000.00 | 9,000.00 | 7,000.00 |
| After two months | 152,500.00 | 10,500.00 | 4,100.00 | 7,900.00 |
| After three months | 160,030.00 | 4,790.00 | 1,870.00 | 8,310.00 |
| After four months | 163,465.00 | 2,185.00 | 853.00 | 8,497.00 |
| After five months | 165,031.90 | 996.70 | 389.10 | 8,582.30 |
| After six months | 165,746.65 | 454.65 | 177.49 | 8,621.21 |
| After seven months | 166,072.69 | 207.39 | 80.96 | 8,638.96 |

TABLE 4. Contribution of the other accounts to the paid balances in dollars

| Time period | Unpaid balances | Expired balances | Total by period | Total accumulated |
|---|---|---|---|---|
| Zero month | – | – | 100,000.00 | 100,000.00 |
| After one month | 30,000.00 | 6,000.00 | 36,000.00 | 136,000.00 |
| After two months | 13,800.00 | 2,700.00 | 16,500.00 | 152,500.00 |
| After three months | 6,300.00 | 1,230.00 | 7,530.00 | 160,030.00 |
| After four months | 2,874.00 | 561.00 | 3,435.00 | 163,465.00 |
| After five months | 1,311.00 | 255.9 | 1,566.90 | 165,031.90 |
| After six months | 598.02 | 116.73 | 714.75 | 165,746.65 |
| After seven months | 272.79 | 53.25 | 326.04 | 166,072.69 |

Table 5. Contribution of the other accounts to the unpaid balances in dollars

| Time period | Unpaid balances | Expired balances | Remnant in unpaid balances by period |
|---|---|---|---|
| Zero month | – | – | 50,000.00 |
| After one month | 15,000.00 | 8,000.00 | 23,000.00 |
| After two months | 6,900.00 | 3,600.00 | 10,500.00 |
| After three months | 3,150.00 | 1,640.00 | 4,790.00 |
| After four months | 1,437.00 | 748.00 | 2,185.00 |
| After five months | 655.50 | 341.20 | 996.70 |
| After six months | 299.01 | 155.64 | 454.65 |
| After seven months | 136.39 | 71.00 | 207.39 |

Table 6. Contribution of the other accounts to the expired balances in dollars

| Time period | Unpaid balances | Expired balances | Remnant in expired balances by period |
|---|---|---|---|
| Zero month | – | – | 20,000.00 |
| After one month | 5,000.00 | 4,000.00 | 9,000.00 |
| After two months | 2,300.00 | 1,800.00 | 4,100.00 |
| After three months | 1,050.00 | 820.00 | 1,870.00 |
| After four months | 479.00 | 374.00 | 853.00 |
| After five months | 218.50 | 170.60 | 389.10 |
| After six months | 99.67 | 77.82 | 177.49 |
| After seven months | 45.46 | 35.50 | 80.96 |

Table 7. Contribution of the other accounts to the lost accounts in dollars

| Time period | Expired balances | Total by period | Total accumulated |
|---|---|---|---|
| Zero month | – | 5,000.00 | 5,000.00 |
| After one month | 2,000.00 | 2,000.00 | 7,000.00 |
| After two months | 900.00 | 900.00 | 7,900.00 |
| After three months | 410.00 | 410.00 | 8,310.00 |
| After four months | 187.00 | 187.00 | 8,497.00 |
| After five months | 85.30 | 85.30 | 8,582.30 |
| After six months | 38.91 | 38.91 | 8,621.21 |
| After seven months | 17.75 | 17.75 | 8,638.96 |

the 40% of expired balances are converted into unpaid balances, and the loss balances do not contribute. Table 6 presents the contribution of the other accounts to the expired balances, for example, the 10% of unpaid balances are converted into expired balances, the 20% remain in the same category of expired balances, and the loss balances do not contribute. Table 7 shows the contribution of the other accounts to the loss balances, for example, the paid balances and the unpaid balances do not contribute to the loss balances, and the expired balances contribute the 10% to the loss balances.

Obtained results are as follows. Table 3 presents that the paid balances and the loss balances are increased month to month, and the unpaid balances and the expired balances decreased with time. Table 4 shows the accounts that contribute to the paid balances that are the unpaid balances and the expired balances. Table 5 presents the accounts that contribute to the unpaid balances that are the unpaid balances remnants and the expired balances. Table 6 presents the accounts that contribute to the expired balances that are the unpaid balances and the expired balances remnants. Table 7 shows the accounts that contribute to the loss balances that are the expired balances.

The two models show the same results, so the proposed model (Markov chains) is valid and does not need to be done in stages like the classic model that is done in stages. The proposed model can be obtained in the desired stage without going through each of the stages as the classical model, for example, if we desired to find the stage 7 by the classical model obtained as is shown in this paper (see Figure 1), the proposed model is found by Equation (4), considering $n = 7$.

The slight differences in probabilities between the two models are: the proposed model takes account of all the decimals since it is done by means of the software, and the classic model considers four decimals since it is done manually.

5. **Conclusions.** This paper shows a model to forecast a stores credit plan using the Markov chains. The stores credit plan considers four categories: paid balances, unpaid balances, expired balances and loss balances. The classical model uses the decision tree to forecast a stores credit plan, but this model can be used for one or two stages and also depends on the number of decisions taken. The advantages of using the Markov chains and with the help of software can be solved for several decisions and multiple stages. Also, a numerical example is presented to validate the effectivity of the Markov chains against decision tree to forecast a stores credit plan.

The main conclusions of this paper are as follows.

1) The proposed model (Markov chains) is validated by means of the classical model (Decision tree) to forecast a stores credit plan, because the results are the same.

2) The proposed model (Markov chains) converges faster than the classical model (Decision tree).

3) The proposed model (Markov chains) can be used for several decisions and multiple stages, and the classical model (Decision tree) is limited.

Then, the Markov chains are the best option, since it is more economical in working time and also is more exact.

The advantages of Markov-type models may be summarized as follows.

1) Markov models are relatively easy to derive (or infer) from successional data.

2) The Markov model does not require deep insight into the mechanisms of dynamic change, but it can help to indicate areas where such insight would be valuable and hence act as both a guide and stimulant to further research.

3) The basic transition matrix summarizes the essential parameters of dynamic change in a way that is achieved by very few other types of model.

4) The results of the analysis of Markov models are readily adaptable to graphical presentation and, in this form, are frequently more readily presented to, and understood by, resource managers and decision-makers.

5) The computational requirements of Markov models are modest, and can easily be met by small computers, or, for small numbers of states, by simple calculators.

6) The proposed model can be obtained in the desired stage without going through each of the stages as the classical model.

Suggestions for future research used the Markov-type models: a) applications to multi-product demand estimation to obtain better prediction rules; b) applications to credit rating due to the practical importance and relevance of risk analysis of credit portfolios.

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