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LOW ORDER ROBUST v-GAP METRIC H_∞ LOOP SHAPING CONTROLLER SYNTHESIS BASED ON PARTICLE SWARM OPTIMIZATION

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Abstract. *The strong demand of the data storage capacity has been increasing significantly. According to the heat-assisted magnetic recording (HAMR) technology, the demand trend of hard disk drive (HDD) is predicted that the areal density will be achieved 10 Tbit/in^2 before the year 2020. High areal density results in a narrow track pitch which is quite sensitive to the external disturbance including the measured noise. This point is the benchmark problem of the high precision controller design for controlling the HDD servo mechanism. Moreover, the systematic uncertainties have to be taken into consideration in controller design procedure as well. The alternative robust v-gap metric related to H_∞ loop shaping is proposed in this paper to stabilize a voice coil motor in HDD under the uncertainty condition. The potential particle swarm optimization (PSO) is adopted to minimize the gap between the plant with H_∞ controller and the plant with specified 3 controller orders. Instead of using the conventional H_∞ controller with high order with a complicated structure, this paper applies the proposed lower controller order based on v-gap which is more appropriate implement in the actual application. The performance and robustness of both controllers are compared in the simulation studies. The results confirm the similar characteristics of both controllers in terms of performance tracking and disturbance rejection. Furthermore, the system stability index called stability margin with 0.472 and system perturbations testing condition also emphasizes the robustness and effectiveness of the proposed controller.*
Keywords: v-gap, H_∞ loop shaping, Robust controller, Voice coil motor, Hard disk drive

1. **Introduction.** Over the past two decades, the hard disk drive servo system is widely used throughout the world, especially an increase of areal density. The increase of areal density will be achieved 10 Tbit/in^2 following the industry roadmap. The consequence of the increasing of data areal density leads to a reduction of the gap between the data tracks pitch. A narrower track pitch causes the difficulty to precisely position the read/write head of voice coil motor (VCM) over the desired track. Currently, the areal density in hard disk industry hits over the 1 Tbit/in^2 [1]. For 10 Tbit/in^2 areal densities in the near future, the displacement of the data track is around 8 nm, while the three sigma value is about 1.16 nm [2]. Therefore, this very small displacement is one of the benchmark problems for controlling the high-density hard disk drive. In addition, the servo resonance modes caused by natural mechanism resonance are also the control problems. Thus, the disturbance caused by the repeatable runout (RRO) and non-repeatable runout (NRRO) have to be taken into consideration. Generally, the major source of RRO is generated by the un-concentricity of disk flutter, servo bearing, spindle vibrations, etc. The NRRO

is normally produced by air turbulence, and mechanical resonance. In case of resonance frequencies appearing in a servo hard disk, notch filters which are the most well-known and popular compensator [3] are utilized in this study to suppress the resonance effect. However, NRRO and RRO still are the main factors to degrade the performance and robustness of the hard disk. These disturbances also make the system nonlinearly [4]. To overcome these problems, many research papers have been proposed continuously. Modern controls such as fuzzy logic control, neuronal network, and adaptive control have been applied to accurately controlling the hard disk drive. In [5], a fuzzy control was applied with the switch mode PID and its saturation limitation. As shown in their results, fuzzy control is efficient to control the seeking mode positioning in hard disk drive accurately. Nevertheless, the uncertainties constraints are not added to be considered in their paper. Lai et al. [6] adapted the two neural networks to compensate the external vibration and the nonlinear pivot friction inside the hard disk drive. Their compensators were operated by following the signal of accelerometer sensing device. They claimed that their proposed controller increased the tracking performance and attenuated the effect of the sensing disturbance vibration. Herrmann et al. [8] utilized the adaptive neural network in their practical application. Their adaptive radial basis functions in neural network control were applied to compensating the resonance and voice-coil-motor (VCM) parameters changing effects. The results indicate that their technique can improve the servo bandwidth and suppress the resonance effect properly.

However, all techniques mentioned above do not take the systematic uncertainties and robustness criteria into their design processes. In order to stabilize the system, various references have presented the robust control and its applications. Robust control is a high potential technique to maintain the system performance and robustness. In the robust control, the system uncertainties can be modeled to many types of model. For instance, the polynomial static output feedback H-infinity control [9], linear matrix inequality (LMI) based H_∞ control [10], H_∞ robust loop shaping [11-14,22,23]. H_∞ loop shaping is one of the popular, simple and sensible techniques to design an effective controller against the uncertainties and disturbance criteria. Nonetheless, a number of the classical H_∞ loop shaping controller order is quite high, which leads to difficult implementation and needs complicated mathematical analysis. Therefore, this paper aims to bridge the gap between theory and application by synthesizing the low controller order based on the concept of v-gap metric with respect to H_∞ robust loop shaping method and particle swarm optimization (PSO). The evolutionary computation algorithms PSO is applied to solving the non-convex problem of the H_∞ robust controller design by searching optimal parameters of the proposed controller based on v-gap metric. The advantage of PSO is a high probability algorithm to achieve the global best solution in the problem search space which is unlike the other gradient techniques that normally trap the local minima. Furthermore, the performance of searching comparison between PSO and other searched techniques were discussed in [22,24]. The conventional H_∞ loop shaping controller, K_∞ is utilized to specify the appropriate weighting function and stabilize the nominal plant at the first process. Subsequently, the gap (distance) between the plant stabilized by the H_∞ controller ($W_1K_\infty W_2G_0$) and the plant stabilized by the proposed controller design with plant ($K(z)G_0$) of the effective v-gap metric is applied to being the fitness function of the particle swarm optimization (PSO). Finally, the gap related to the fitness function with the proposed specified controller is minimized to synthesize the robust controller with respect to the H_∞ characteristic. The objectives and motivations of this paper are: (1) to propose the alternative method to synthesize a robust controller with v-gap metric; (2) to apply the simple structure of the proposed controller to controlling hard disk drive servo system robustly and precisely.

This paper is organized as follows. Section 2 details the HDD modeling and resonance modes. Section 3 describes the robust controller designed based on the v-gap metric concept and H_∞ loop shaping that apply the PSO to finding the optimal controller with specified controller structure. In Section 4, the simulation of the VCM actuator with the HDD parameters perturbation is described and the results between the proposed controller and conventional H_∞ loop shaping controller are compared. Finally, Section 5 summarizes and concludes the results and proposed concept design.

2. **Hard Disk Drive Modeling.** The hard disk drive actuator is an electromagnetic device as shown in Figure 1, which consists of the voice coil motor including its current driver. The principle model of voice coil motor can be analyzed as the electrical and the mechanical equivalence circuits. Based on the details described in [14], the transfer function of voice coil motor can be characterized as:

$$\frac{\theta(s)}{u(s)} = \frac{\frac{R_m k_a}{k_b}}{s(s\tau_e + 1) + (s\tau_a + 1)\left[s\tau_m + \frac{R_m}{(R_m+R_s)}\right]} = \frac{k_t k_a}{J}\frac{1}{s^2} \tag{1}$$

where θ, u, R_m, R_s, k_a, k_b, τ_a, τ_e and τ_m are the position of actuator in radian, the voltage input, VCM resistance, the parallel resistance, steady state gain, back emf force coefficient, amplifier, electrical and mechanical time constant, respectively.

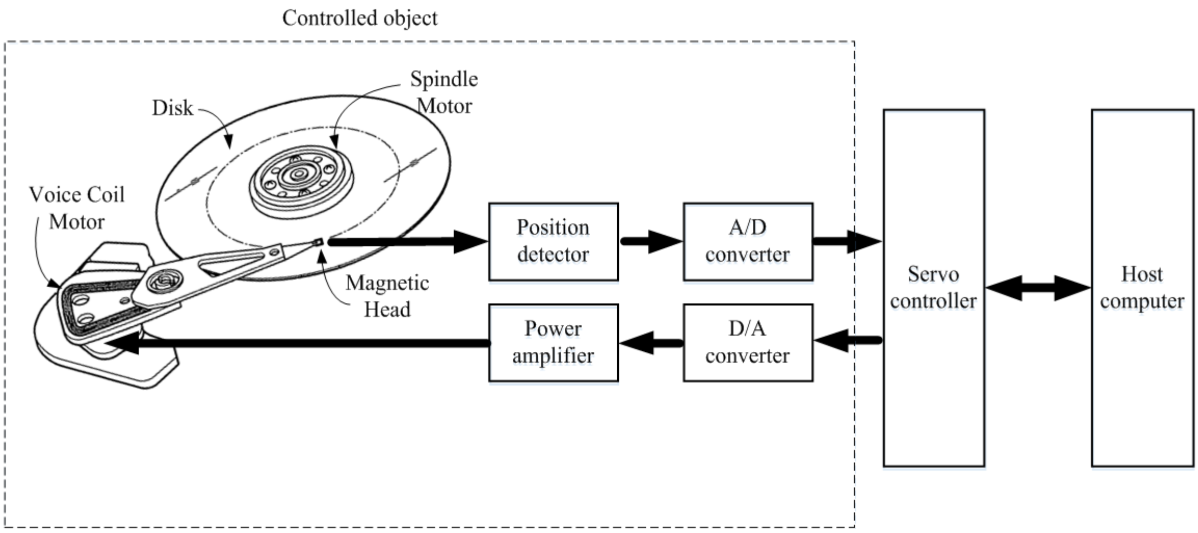


FIGURE 1. Schematic diagram of the head-positioning HDD

By the transformation method as detailed in [14] the characteristic of the VCM actuator model can be transformed to the tracks as:

$$G(s) = \frac{y(s)}{u(s)} = \left(\frac{k_t k_a}{J}\right)\left(l_{vcm}\frac{TPI}{2540}\right)\frac{1}{(s)^2} = k_v k_y \frac{1}{(s)^2} \tag{2}$$

The realistic model of hard disk drive including its resonance dynamic modeling can be written as:

$$G_{VCM}(s) = \frac{k_v k_y}{(s)^2} G_{r,i}(s) \tag{3}$$

where l_{vcm} denotes the length of the VCM actuator arm and TPI represents Track Per Inch of VCM model of hard disk drive, k_v is the acceleration constant, k_y is the position measurement gain. The ith resonance, $G_{r,i}(s)$ can be expressed as:

$$G_{r,i}(s) = \frac{a_i s^2 + b_i s + \omega_i^2}{s^2 + 2\xi_i \omega_i s + \omega_i^2} \tag{4}$$

where a_i, b_i, ξ_i and ω_i are coefficients of the ith resonance mode dynamic model. The model parameters of the HDD are shown in Table 1.

TABLE 1. The model data of the hard disk drive assembly [15]

Model data	**Value**	**Tolerance perturbation**
(m, k_t, k_v)	$(0.002, 20, 64.013)$	–
$(a_1, b_1, \varsigma_1, \omega_1)$	$(0.0000115, -0.00575, 0.05, 70)$	$(-, -, 50\%, 20\%)$
$(a_2, b_2, \varsigma_2, \omega_2)$	$(0, 0.0230, 0.005, 2200)$	$(-, -, 50\%, 20\%)$
$(a_3, b_3, \varsigma_3, \omega_3)$	$(0, 0.8185, 0.05, 4000)$	$(-, -, 50\%, 30\%)$
$(a_4, b_4, \varsigma_4, \omega_4)$	$(0.0273, 0.1642, 0.005, 9000)$	$(-, -, 50\%, 20\%)$

3. Robust Control Synthesized Based on v-Gap Metric and Particle Swarm Optimization.

3.1. $\boldsymbol{V}$-gap metric.

The v-gap metric was introduced into the robust control literature by Vinnicombe. The good tutorial overview can be found in [16,17]. The v-gap metric is utilized to measure the distance between two linear time invariant (LTI) of the interested feedback plants for indexing the similarity performance of both systems. The definition of v-gap metric that calculates the upper bound on the distance between two systems (G_0 and G_1) can be described as:

Given G_0, $G_1 \in \Re^{p\times q}$

$$\delta_v(G_0, G_1) = \begin{cases} \left\| \begin{bmatrix} -\tilde{M}_1 & \tilde{N}_1 \end{bmatrix} \begin{bmatrix} N_0 \\ M_0 \end{bmatrix} \right\|_\infty, & \begin{array}{l} \text{if } \det\Phi(s) \neq 0 \ \forall\omega \\ \text{and wno } \det\Phi(s) = 0 \end{array} \\ 1, & \text{otherwise} \end{cases} \tag{5}$$

where $G_i = N_i M_i^{-1} = \tilde{M}_{i1}^{-1}\tilde{N}_i$, are normalized right (left) coprime factorizations of the interested plants and wno represents the winding number in the standard Nyquist D-contour [16,17].

The calculated v-gap value always lies between 0 and 1. Small value (relative to 1) implies that the close loop gain of both systems will be similar. In the other words, high value of v-gap indicates that the plants are far apart. Moreover, the input and output of both systems must be the same. The theorem that associates with the v-gap metric to measure the system performance is the following:

Theorem 3.1. (Theorem 2.4 detailed in [17]): *Any two plants G_0, G_1 with m inputs n outputs and a controller K with n inputs and m outputs,*

$$|b_{G_0,K} - b_{G_1,K}| \leq \delta_v(G_0, G_1) \tag{6}$$

where

$$b_{G_i,K_i} := \left\| \begin{bmatrix} I \\ K_i \end{bmatrix} (I + G_i K_i)^{-1} [I \;\; G_i] \right\|_\infty^{-1} = \left\| \begin{bmatrix} I \\ G_i \end{bmatrix} (I + K_i G_i)^{-1} [I \;\; K_i] \right\|_\infty^{-1} \tag{7}$$

The following interpretation is described in [17]: "The v-gap is an effective measurement of the important difference between the two systems, in terms of closed-loop behavior when both are controlled by the same, near unity-gain, feedback compensator. When the feedback compensator to be used is not of near unity-gain at all frequencies, it is necessary to weight the system concerned (by the controller, or the expected shape of the controller – as characterized by the weights used in the H_∞ loop-shaping design procedure for example), for such an interpretation to be meaningful."

3.2. **Robust fixed-structure procedure based on v-gap metric and PSO.** In order to achieve the robust performance of the H_∞ loop shaping procedure, the pre-weight (W_1) for decreasing the effect of disturbance with high gain at low frequency and post-weight (W_2) for rejecting the noise with low gain at high frequency must be properly selected. Following the details described in [14,19,20], the selected weights can be evaluated by using the concept of Riccati Equation (8). The result of Riccati is $\gamma_{\min}$ which is the inverse proportion of the maximum stability margin $\varepsilon_{\max}$. Based on the suggestion of McFarlane and Glover in [11], the $\varepsilon_{\max}$ should be more than 0.25 in order to properly maintain the system stability under uncertainties and disturbance perturbations. Thereafter, the selected weights are utilized to shape the nominal plant of HDD actuator as: $G_S = W_1G_0W_2$, where G_0 represents the nominal plant. The uncertainty of the interested hard disk drive is evaluated by using the concept of left coprime factorization, which is separated into the normalized nominator N and denominator M factors as illustrated in Figure 2. The inverse maximum stability margin index $\gamma_{\min}$ can be calculated as follows.

$$\gamma_{\min} = \varepsilon_{\max}^{-1} = 1 - \left\{\|[N \quad M]\|_H^2\right\}^{0.5} = (1 - \lambda_{\max}(XZ))^{0.5} \tag{8}$$

where $\|\cdot\|_H$ represents the Hakel norm of system, $\lambda_{\max}$ is the maximum eigenvalue of matrix X multiplied by matrix Z, while X and Z are unique positive definite solutions to algebraic Riccati equation.

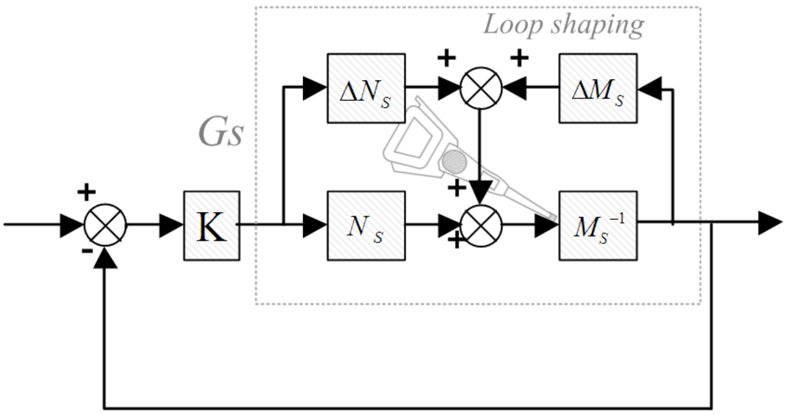


FIGURE 2. Controlled diagram of feedback loop with system uncertainty based on left coprime factorization

After achieving the desired $\varepsilon_{\max}$ regarding the appropriate weight selection, the H_∞ controller (K_∞) is synthesized by solving the following inequality equation of the shaped plant, $W_1G_0W_2$. The H_∞ norm of disturbance to state can be expressed as:

$$\begin{aligned}\|T_{zw}\|_\infty &= \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + W_1G_0W_2K_\infty)^{-1} M_S^{-1} \right\|_\infty \\ &= \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + W_1G_0W_2K_\infty)^{-1} [I \quad W_1G_0W_2] \right\|_\infty < \frac{1}{\varepsilon}\end{aligned} \tag{9}$$

The synthesized feedback controller K is obtained as:

$$K = W_1K_\infty W_2 \tag{10}$$

As known, the H_∞ controller (K) has a complicate structure and high controller order. Therefore, the controller applied in this paper is specified with 3 controller orders (3 poles 3 zeros). The optimal controller parameters are found by using PSO algorithm to synthesize the robust controller with respect to the H_∞ loop shaping technique. The specified controller structure and set of controller parameters are the following:

$$K(z) = \frac{z_{m1}s^3 + z_{m2}s^2 + z_{m3}s^1 + z_{m4}}{z_{n1}s^3 + z_{n2}s^2 + z_{n3}s^1 + z_{n4}} \tag{11}$$

The set of coefficients in the proposed controller is:

$$z = [z_{m1}, z_{m2}, z_{m3}, z_{m4}, z_{n1}, z_{n2}, z_{n3}, z_{n4}] \tag{12}$$

In order to synthesize the proposed robust controller related to H_∞ loop shaping technique of this paper, the distance between both closed loop systems is applied to being the cost function of the PSO searching tool. Two systems applied in this paper are: (1) $W_1K_\infty W_2G_0$ and (2) $K(z)G_0$. The cost function (J_{cost}) of PSO based on the proposed v-gap method is expressed as:

$$Fitness\ [f^n] = J_{Cost}^{-1} = \left\{ \begin{array}{ll} [\delta_v(W_1K_\infty W_2G_0, K(z)G_0)]^{-1} & \text{if } K \text{ stabilized the } G_0 \\ 0.0001 & \text{Otherwise} \end{array} \right\} \tag{13}$$

The high potential PSO is applied to searching the optimal coefficients in proposed controller $K(z)$ in order to minimize the gap (distance) between both interested systems in the closed loop behavior. The optimal minimum gap implies that the nominal plant with synthesized controller is similar to the characteristic of the nominal plant with the H_∞ loop shaping controller. The details of proposed technique can be summarized as follows.

Step 1. Specify the controller parameter structure $K(z)$ in Equation (12), while z is the set of the controller coefficients, which are referred as a 'particle' in the PSO technique.

Step 2. Initialize the parameter sets of PSO in the 1st iteration, such as population size, maximum and minimum velocities and momentum.

Step 3. Generate a swarm movement of the first iteration randomly, and find the fitness value of each particle. The inverse of cost function in Equation (13) is applied as the fitness function (*Fitness* $[f^n]$) of the PSO optimization.

Step 4. Update the inertia weight (J), position and velocity of each particle as:

$$J = J_{\max} - \left(\frac{J_{\max} - J_{\min}}{i_{\max}}\right) i \tag{14}$$

$$v_{i+1} = Q_{vi} + \alpha_1[\gamma_{1i}(P_b - p_i)] + \alpha_2[\gamma_{2i}(U_b - p_i)] \tag{15}$$

Update the position (p) and velocity (v) of each particle.

$$p_{i+1} = p_i + v_{i+1} \tag{16}$$

where α_1, α_2 are the specified acceleration coefficients and γ_{1i}, γ_{2i} are the numbers by random search and U_b is the best fitness value (Global best) of each former iteration.

Step 5. Return to step 4, if the current iteration is less than the maximum iteration. If the current iteration reaches the maximum iteration, then stop. The particle that achieves the maximum fitness value is the answer of this optimization.

Figure 3 shows the flowchart of the proposed controller design. The chart starts from shaping the plant with appropriate selected weights, and then PSO is applied to finding the optimal parameters of the proposed fixed structure controller with respect to the minimum gap between both systems. Finally, the process is stopped when the PSO achieves the optimal best solution and/or the maximum iteration.

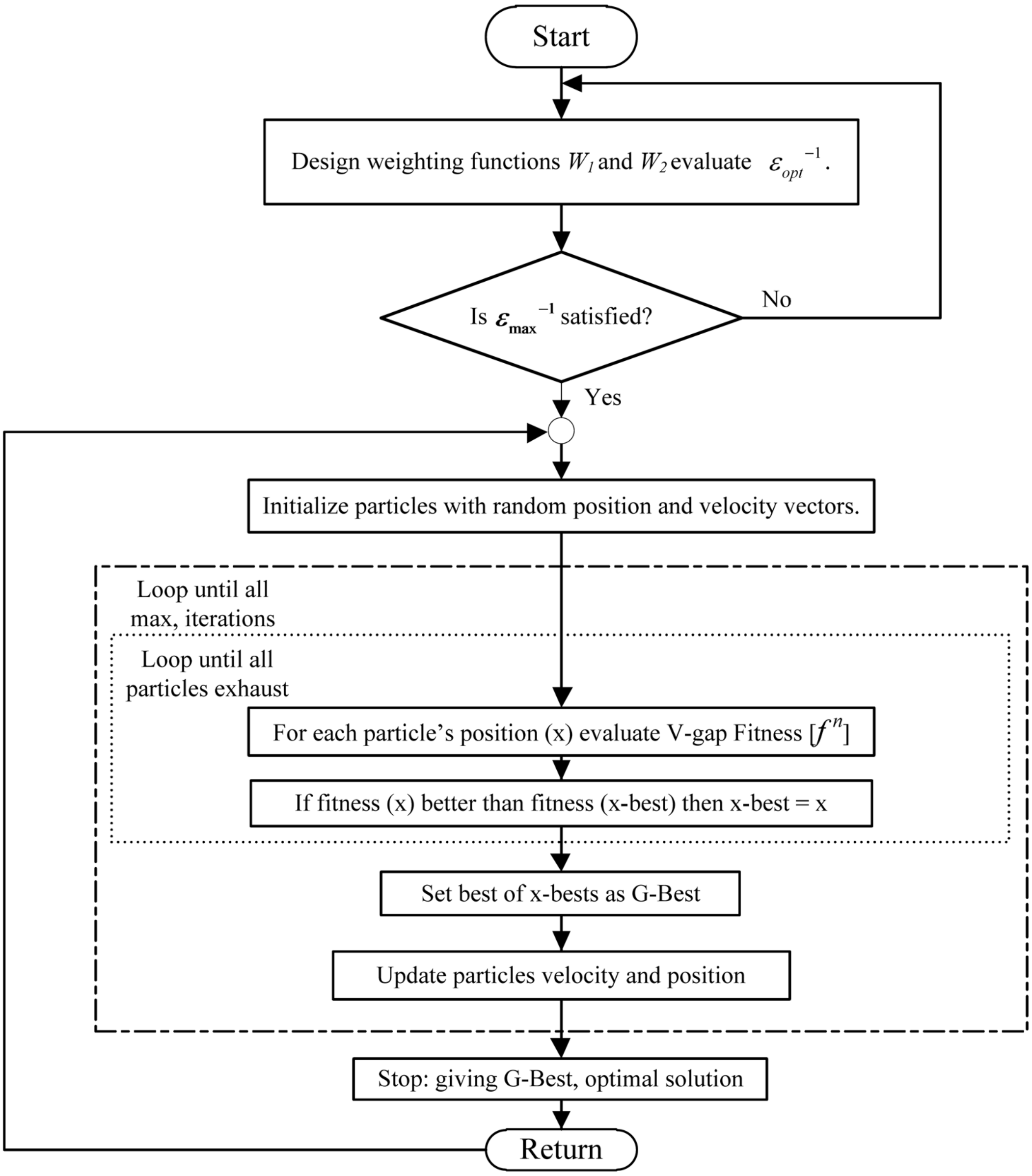


FIGURE 3. A control flowchart of the proposed robust controller design based on v-gap metric

4. **Simulation Results.** The position control in voice coil motor (VCM) of hard disk assembly (HDA) is utilized to examine the performance of the proposed controller design. In this paper, the principle VCM and four resonant dynamic models characteristic of HDA with the 12th order described in Table 1 can be constructed as follows:

$$G_{\substack{VCM\& \\ NOTCH}}(s) = \left\{ \frac{\begin{array}{c} 1.544 \times 10^{09}s^{11} + 1.055 \times 10^{15}s^{10} + 1.175 \times 10^{19}s^{9} \\ +6.247 \times 10^{24}s^{8} + 2.878 \times 10^{28}s^{7} + 1.097 \times 10^{34}s^{6} \\ +2.575 \times 10^{37}s^{5} + 5.934 \times 10^{42}s^{4} + 4.539 \times 10^{45}s^{3} \\ +7.838 \times 10^{50}s^{2} + 3.525 \times 10^{52}s + 1.505 \times 10^{56} \end{array}}{\begin{array}{c} s^{13} + 53557s^{12} + 8.625 \times 10^{09}s^{11} + 3.1 \times 10^{14}s^{10} + 2.338 \times 10^{19}s^{9} \\ +5.26 \times 10^{23}s^{8} + 2.163 \times 10^{28}s^{7} + 2.78 \times 10^{32}s^{6} \\ +7.439 \times 10^{36}s^{5} + 3.647 \times 10^{40}s^{4} + 7.816 \times 10^{44}s^{3} \\ +5.095 \times 10^{46}s^{2} + 1.51 \times 10^{51}s + 1.881 \times 10^{51} \end{array}} \right\} \tag{17}$$

Based on the well-known concept of McFarlane and Glover, the hard disk drive plant of the proposed design needs to be shaped with high performance weights to achieve a high at low-frequency gain with W_1 and a low at high-frequency gain with W_2 of the open loop frequency response for rejecting the effect of disturbance and measurement noise. Furthermore, the crossover frequency of the shaped plant should be high to increase the servo bandwidth including the tracking performance. The selected weighting function of the proposed controller design can be selected as:

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} \dfrac{0.0081s + 56.95}{s + 5900} \\ \dfrac{s + 39300}{s + 217500} \end{bmatrix} \tag{18}$$

In order to indicate the system robustness of the proposed loop shaping controller design, Riccati equation provides the maximum stability margin $\varepsilon_{\max} = 0.612$, which guarantees the performance of selected weights for maintaining the system stability under uncertain constraints. Based on these weights, the H_∞ loop shaping controller is synthesized, and applied to being the fitness function of PSO to minimize the gap between both systems. The initialized PSO parameters are selected as follows: the number of particles $= 24$, an acceleration coefficient $= 2.1$, the minimum and maximum of velocities and weight inertias are $[0.6, 1.8]$ and $[0.3, 0.9]$, respectively. Figure 4 illustrates the convergence fitness function curve of PSO algorithm. The parameters of the specified controller structure with order 3 in Equation (11) are optimized. After that, the final proposed controller based on v-gap metric is obtained as:

$$K(z) = \frac{0.2653s^3 + 707.3s^2 + 3561s + 3157}{167.8s^3 + 4.055 \times 10^5 s^2 + 2.788 \times 10^5 s + 8.713 \times 10^5} \tag{19}$$

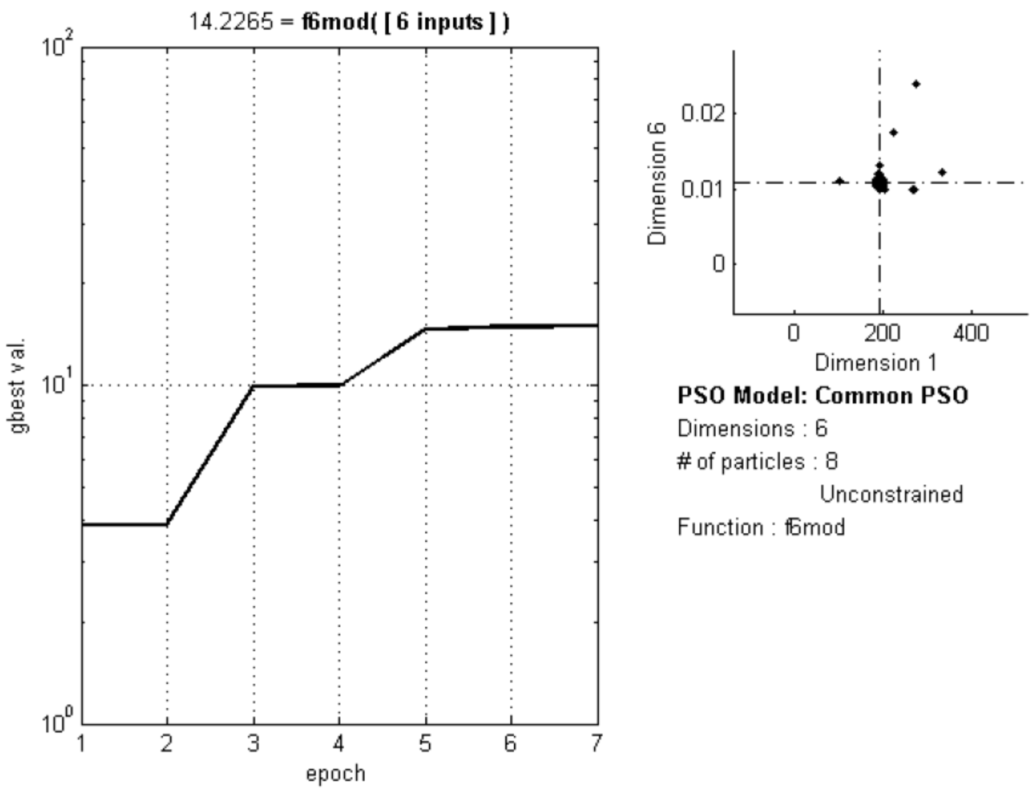


Figure 4. PSO solution convergence of proposed v-gap metric

A conventional H_∞ loop shaping controller can be synthesized as:

$$K_{H\infty}(s) = \left\{ \frac{\begin{matrix} 0.008235s^{12} + 2354s^{11} + 1.526 \times 10^{8}s^{10} + 1.109 \times 10^{13}s^{9} \\ 5.046 \times 10^{17}s^{8} + 1.28 \times 10^{22}s^{7} + 3.468 \times 10^{26}s^{6} \\ +4.877 \times 10^{30}s^{5} + 5.868 \times 10^{34}s^{4} + 5.366 \times 10^{38}s^{3} \\ +1.844 \times 10^{42}s^{2} + 1.807 \times 10^{44} + 3.484 \times 10^{47} \end{matrix}}{\begin{matrix} s^{12} + 4.721 \times 10^{5}s^{11} + 6.461 \times 10^{10}s^{10} + 3.04 \times 10^{15}s^{9} \\ 2.483 \times 10^{20}s^{8} + 5.426 \times 10^{24}s^{7} + 1.818 \times 10^{29}s^{6} \\ +2.383 \times 10^{33}s^{5} + 3.083 \times 10^{37}s^{4} + 2.756 \times 10^{41}s^{3} \\ +8.697 \times 10^{44}s^{2} + 9.01 \times 10^{46} + 1.638 \times 10^{50} \end{matrix}} \right\} \tag{20}$$

After the optimization process, which PSO is applied, finished, the close loop gap between the plant with H_∞ loop shaping and the proposed controller is 14.2265. The inverse of this value equals 0.0703, which is very small gap of both compared systems; therefore, the robust performance of both systems is almost similar. Additionally, the stability margin and the time performance of H_∞ loop shaping and proposed technique are compared in Table 2.

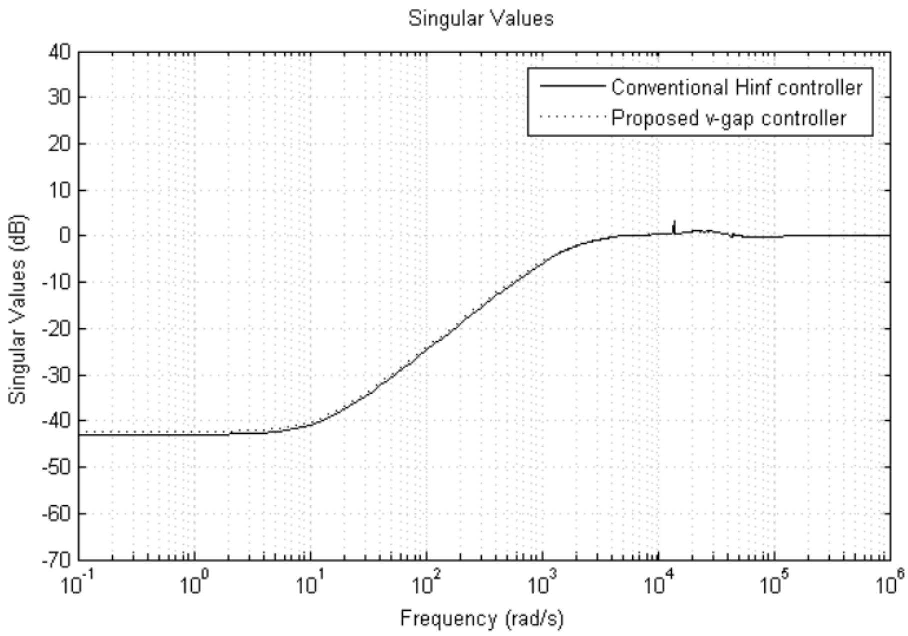
The step responses of both techniques are illustrated in Figure 6. This result indicates that the settling time and overshoot of both techniques are almost the same. In addition, the similar disturbance rejection response of the two controllers shown in Figure 5 also guarantees the system stability and robustness of both systems.

TABLE 2. Performance and robustness comparison

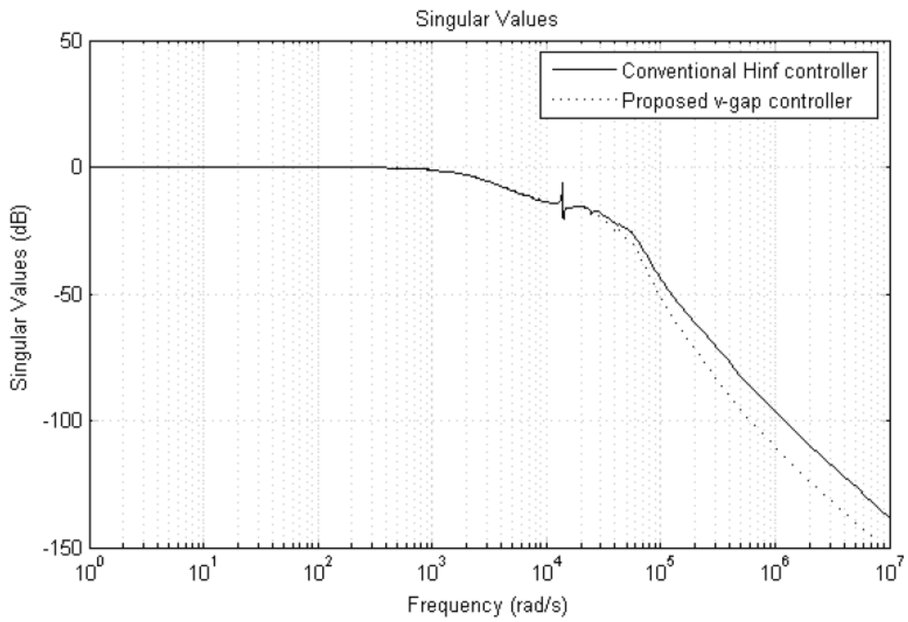
Procedure	**Stability margin (ε)**	**Settling time**	**% Overshoot**	**Order**
H_∞ loop shaping	0.612	0.0027	0%	14
Proposed v-gap technique	0.4782	0.0021	0.08%	3

Although, the stability margin of H_∞ is greater than the proposed technique, the order of the conventional H_∞ controller is higher and more complicated. Therefore, the proposed controller design gains more advantages than that of the conventional H_∞ controller in terms of simple structure while the stability margin is acceptable. Table 2 illustrates the value of ε which is equal to 0.4782, indicating that the designed controller is suitable for the specified open loop shaping method and also guarantees the hard disk system performance. In order to analyze the robustness of the proposed controller based on v-gap metric design, the tolerance perturbation at 20-50% of the resonance mode coefficient of HDD shown in Table 1 is utilized. Figure 7 shows the open loop singular value of the 16-case perturbations compared to the nominal HDD plant, while the step responses of the 16 perturbed cases and the nominal plant are shown in Figure 8. The step response performance of the whole cases is almost similar. Even though the system is internally perturbed by the effect of parameter variations, these results substantiate the robust performance of the proposed design.

5. **Conclusion.** V-gap metric is utilized in this paper to measure the gap between two interested systems (the nominal plant with H_∞ loop shaping controller and the nominal plant with proposed controller). The very small gap implies that both systems share quite similar behavior. According to the concept of H_∞ loop shaping, the H_∞ controller (K_∞) is synthesized for stabilizing the system performance under the uncertainties and perturbations. The strong advantages of the K_∞ and v-gap metric are combined to be the fitness function of the proposed alternative v-gap based robust H_∞ loop shaping controller synthesized method. PSO algorithm is adapted to search the optimal parameters of the



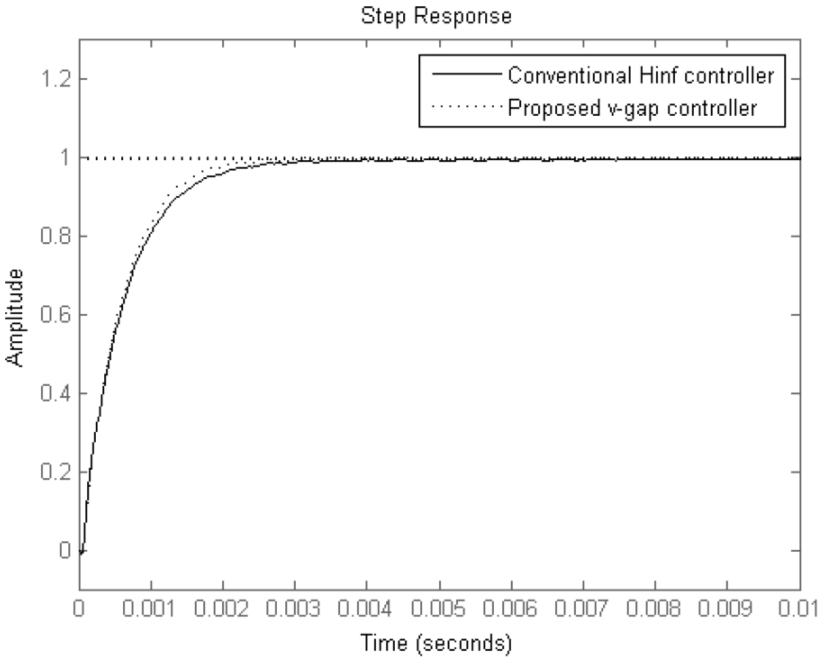
(a)



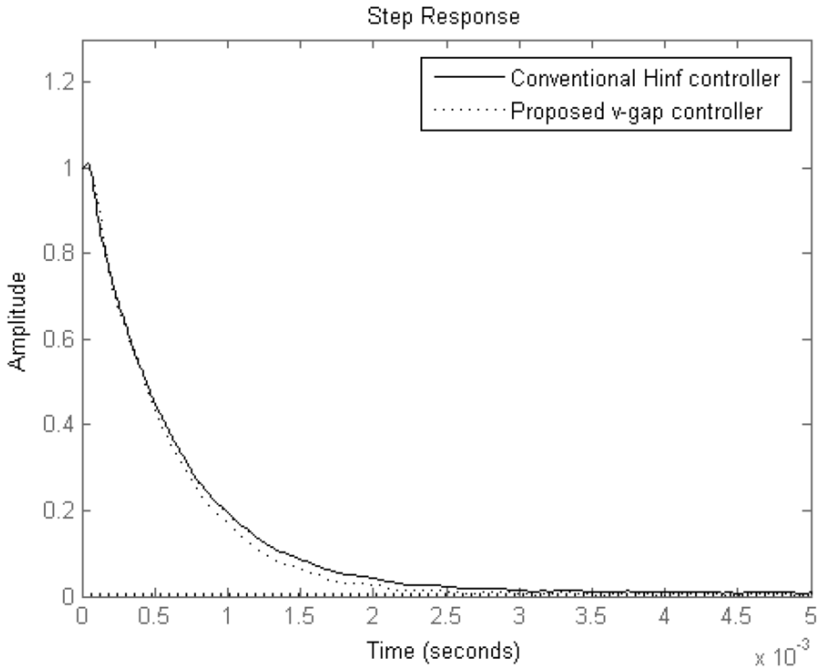
(b)

FIGURE 5. (a) Singular value of the sensitivity and (b) singular value of the complementary sensitivity frequency response between the conventional H_∞ controller with plant (solid line) and the proposed v-gap metric controller with plant (dot line)

proposed specified controller the 3rd order in the designed fitness function. The optimal result found in this paper shows that the gap between both controllers is very small with 0.0703. It explains that behaviors of both systems are almost the same. In addition, the time and frequency simulated results also substantiate the similarity of both controllers. However, the order of the proposed controller is less than the conventional H_∞ controller, which is more appropriate to apply to an industrial application than controller that has high order and complicated structure, such as H_∞ controller. Furthermore, in order



(a)



(b)

FIGURE 6. (a) Step responses of proposed v-gap metric compared to H_∞ controller, (b) disturbance rejection of proposed v-gap metric compared to H_∞ controller

to emphasize the effectiveness of the proposed controller, the 16 cases of the internal perturbation are utilized. The results confirm the effectiveness of the proposed controller that it can maintain the robustness and system stability even the parameters of the system are changed. This proposed concept will be applied to the dual stage actuator or muti-input multi-output (MIMO) system in the future research.

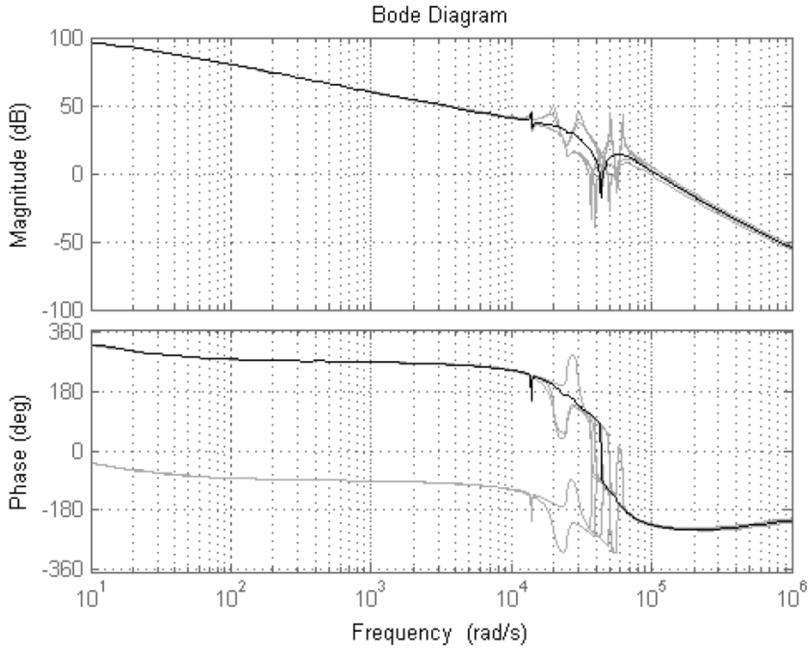


FIGURE 7. The open loop singular value responses of the normal model (black line) compared to the 16 cases perturbation of HDA (gray line)

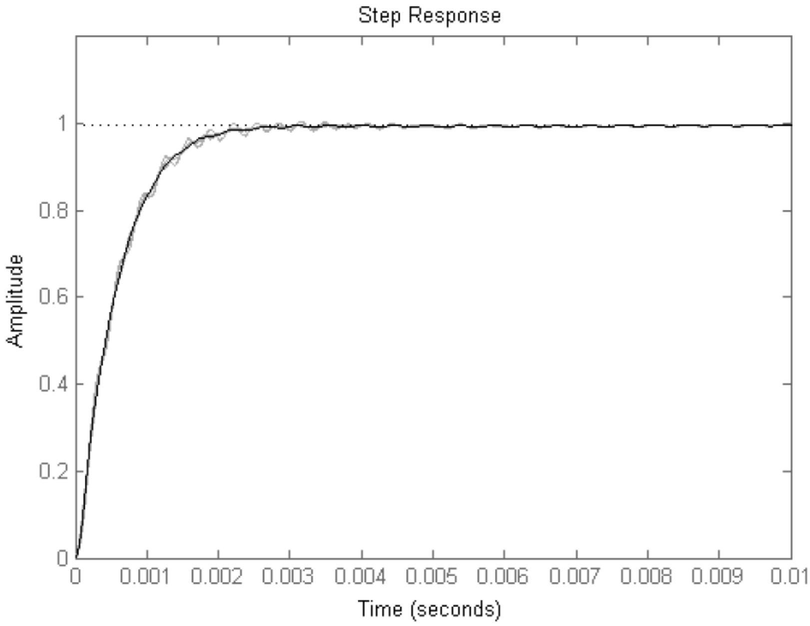


FIGURE 8. Step responses of the normal model (black line) compared to the 16 cases perturbation of HDA (gray line)

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