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SIMPLE OPTIMAL PID TUNING METHOD BASED ON ASSIGNED ROBUST STABILITY –TRADE-OFF DESIGN BASED ON SERVO/REGULATION PERFORMANCE–

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Abstract. *The present paper investigates a design method for a one-degree-of-freedom control system with proportional-integral-derivative (PID) compensation. In the control system, the tracking performance and stability are in a trade-off relationship, and the servo performance and the regulation performance are also in a trade-off relationship depending on the design of the tracking performance. The present paper proposes a simple design method of the PID parameters. In the proposed system, the PID parameters are decided such that the tracking performance is optimized subject to the assigned robust stability. Furthermore, the tracking performance is seamlessly adjusted between the servo performance and the regulation performance. The effectiveness of the proposed method is demonstrated through numerical examples.*

Keywords: PID control, Sensitivity function, Robust stability, Servo performance, Regulation performance, Trade-off design

1. **Introduction.** The proportional-integral-derivative (PID) control [1, 2, 3] has been widely used in industry because its performance is adjusted intuitively based on user experiments, even if the control structure is simple. Furthermore, the meanings of the control parameters are clear, i.e., proportional, integral, and derivative compensation and denoted by P, I, and D, respectively. Since the control performance of the PID control is decided by the PID parameters, numerous studies have examined PID parameter tuning.

The present study discusses a design method for controlling a one-degree-of-freedom (1DOF) system. In a 1DOF system, trade-off design is necessary because multiple performance characteristics can change simultaneously based on the design of the controller. The trade-off relationship between the tracking performance and stability is well known. If the stability of a control system is insufficient, the control system will become unstable due to even a slight plant perturbation. On the other hand, poor tracking performance is obtained when stability is ensured too much.

Since the stability margin is adjusted using the sensitivity function, the stability margin is assigned by designing the maximum value of the sensitivity function [4]. Using the sensitivity function, the PID parameters are decided such that the tracking performance is optimized, in which case the assigned stability margin is achieved [5, 6, 7, 8]. In order to obtain intermediate performance for a servo with regulation optimization, Arrieta and Vilanova proposed an intermediate design method [9, 10]. Using this design method, the PID parameters are decided by selecting servo-optimized, regulation-optimized, or intermediate performance.

In the present study, we propose a new trade-off design method. In the proposed method, the tracking performance is optimized subject to the assigned stability margin. Moreover, the tracking performance is designed between the servo performance and regulation performance. As a result, the optimal PID parameters are decided seamlessly between the reference response and the disturbance response.

This paper is organized as follows. Section 2 presents a control system and the control objective, and Section 3 presents the proposed control system. In Section 4, we present and analyze numerical simulations using our design strategy. Concluding remarks are presented in Section 5.

2. **Problem Formulation.** Consider the PID control system illustrated in Figure 1. In this system, the controlled plant is a first-order plus dead-time model, which is given as follows:

$$P(s) = \frac{K}{Ts+1}e^{-Ls} \tag{1}$$

where K, T, and L are the gain, time-constant, and dead-time, respectively. The controller is represented by a PID control law as follows:

$$\begin{aligned} U(s) &= K_p\left\{\left(1+\frac{1}{T_i s}\right)E(s) - \left(\frac{T_d s}{T_d s/N+1}\right)Y(s)\right\} \\ E(s) &= R(s) - Y(s) \end{aligned} \tag{2}$$

where $U(s)$, $Y(s)$, and $R(s)$ are the control input, plant output, and reference input, respectively, and K_p, T_i, and T_d are the proportional gain, integral time, and derivative time, respectively. Moreover, N denotes the derivative filter constant and is set to 10, as is the usual practice in industry. This control law is rearranged as follows:

$$U(s) = C_r(s)R(s) - C_y(s)Y(s) \tag{3}$$

$$C_r(s) = K_p\left(1+\frac{1}{T_i s}\right) \tag{4}$$

$$C_y(s) = K_p\left(1+\frac{1}{T_i s}+\frac{T_d s}{T_d s/N+1}\right) \tag{5}$$

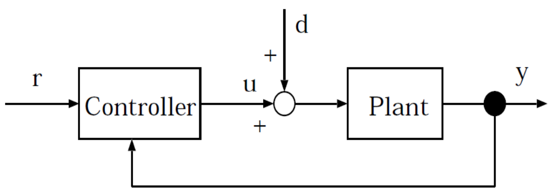

FIGURE 1. Block diagram of a PID control system

The tracking performance is evaluated using the integral absolute error (IAE), which is defined as follows:

$$J = \int_0^\infty |e(t)|dt \tag{6}$$
$$e(t) = r(t) - y(t)$$

Robust stability is guaranteed using the following sensitivity function:

$$S(s) = \frac{1}{1 + P(s)C_y(s)} \tag{7}$$

The relationships between the maximum value of the sensitivity function, M_s, and the gain g_m, and the phase margin, ϕ_m, respectively, are as follows [11]:

$$g_m \geq \frac{M_s}{M_s - 1} \tag{8}$$

$$\phi_m \geq 2 \arcsin \left(\frac{1}{2M_s} \right) \tag{9}$$

where M_s is defined as:

$$M_s \triangleq \max_\omega |S(j\omega)| = \max_\omega \frac{1}{|1 + P(j\omega)G_y(j\omega)|} \tag{10}$$

Although the stability margin is broadened with small M_s, since the relationship between the tracking performance and the stability margin is a trade-off relationship, the recommended range of M_s is from 1.4 to 2.0 [3]. The gain and the phase margins for $M_s = 1.4, 2.0$ are given as follows:

- $M_s = 1.4$: $g_m \geq 3.5$, $\phi_m \geq 41°$
- $M_s = 2.0$: $g_m \geq 2.0$, $\phi_m \geq 28°$

In order to achieve the assigned robust stability, the PID parameters are decided such that the desired value of M_s is obtained, i.e., the following constraint condition is satisfied [7]:

$$\left| M_s - M_s^d \right| = 0 \tag{11}$$

where M_s^d denotes the desired value of M_s. Therefore, the performance index Equation (6) is optimized such that the constraint is satisfied.

The objective of the present study is to obtain a simple design method of the PID parameters such that the performance function is optimized subject to the desired robust stability.

3. Controller Design Based on a Trade-off Relationship.

3.1. **Servo/regulation tuning.** The tuning points for the reference/disturbance optimization are shown in Figure 2. In this figure, J_r on the vertical axis indicates the evaluation of the reference response, and J_d on the horizontal axis is that of the disturbance response. Moreover, J_*^r denotes the evaluation on the reference response optimization design, and J_*^d denotes the evaluation on the disturbance response optimization design. Hence, the meanings of J_r^r, J_r^d, J_d^r, and J_d^d are as follows:

- J_r^r: Reference response evaluation on the reference response optimization design
- J_r^d: Reference response evaluation on the disturbance response optimization design
- J_d^r: Disturbance response evaluation on the reference response optimization design
- J_d^d: Disturbance response evaluation on the disturbance response optimization design

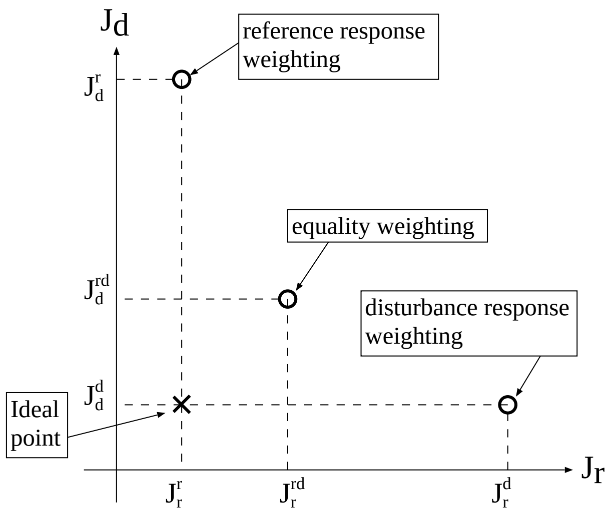

FIGURE 2. Servo/regulation and equality weighting optimization [9]

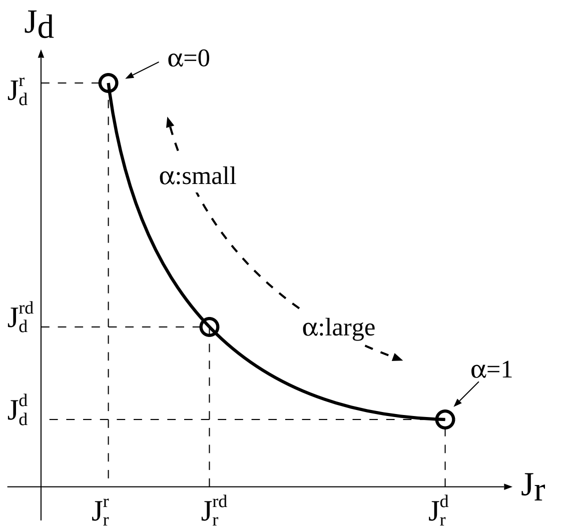

FIGURE 3. Seamless weighting optimization

On the other hand, J_*^{rd} denotes the optimization design of both the reference and disturbance responses, and hence, J_r^{rd} and J_d^{rd} are the reference and disturbance response evaluations on the optimization design of the reference and disturbance responses, respectively.

In a 1DOF system, since J_r and J_d are not independently optimized, the ideal point described in Figure 2 is not achieved. Therefore, the servo performance J_r and the regulation performance J_d must be mediated based on the trade-off between the reference optimization J^r and the disturbance optimization J^d [5, 6]. An intermediate tuning between J^r with J^d has been proposed [9, 10]. In this method, the performance index for the intermediate design is defined as follows:

$$J_{rd} = \sqrt{(J_r^{rd} - J_r^o)^2 + (J_d^{rd} - J_d^o)^2} \tag{12}$$

where J_r^o denotes the optimal evaluation of the reference response, and J_d^o denotes that of the disturbance response. Here, J_r^r is nearly equal to J_r^o, and J_d^d is nearly equal to J_d^o. The intermediate tuning J_{rd} corresponds to the equality weighting described in Figure 2.

In order to interpolate the trade-off between servo/regulation optimization and obtain a seamless design method, the proposed performance index is defined as follows:

$$J_{rd}^{\alpha} = \sqrt{\alpha(J_r^{rd} - J_r^o)^2 + (1-\alpha)(J_d^{rd} - J_d^o)^2} \quad (0 \leq \alpha \leq 1) \tag{13}$$

In Equation (13), $\alpha = 0$ corresponds to the disturbance response optimization, and $\alpha = 1$ corresponds to the reference response optimization. Moreover, Equation (13) with $\alpha = 0.5$ is comparable to Equation (12), and hence, the proposed performance index includes the conventional trade-off design method [9, 10]. The new trade-off design image using α is shown in Figure 3.

3.2. **Simple decision of PID parameters.**

3.2.1. *Optimal decision method.* In the present study, we propose a simple decision method of the PID parameters for a normalized system. The use of the transformation $\hat{s} = Ts$ provides a normalized form of the controlled plant and the PID compensators as follows:

$$\hat{P}(\hat{s}) = \frac{e^{-\tau\hat{s}}}{\hat{s}+1} \tag{14}$$

$$\hat{C}_r(\hat{s}) = \kappa_p\left(1 + \frac{1}{\tau_i\hat{s}}\right) \tag{15}$$

$$\hat{C}_y(\hat{s}) = \kappa_p\left(1 + \frac{1}{\tau_i\hat{s}} + \frac{\tau_d\hat{s}}{\tau_d\hat{s}/N+1}\right) \tag{16}$$

where the normalized PID parameters are defined as follows:

$$\kappa_p = K_pK,\ \tau_i = \frac{T_i}{T},\ \tau_d = \frac{T_d}{T},\ \tau = \frac{L}{T} \tag{17}$$

where κ_p, τ_i, τ_d, and τ are the normalized gain, normalized integral time, and normalized dead-time, respectively.

For the normalized control system, the normalized PID parameters optimized subject to the stability margin constraint, in which $M_s^d = 1.4$, 1.6, 1.8, and 2.0. In the present study, the optimal parameters are obtained numerically with the MATLAB function *fmincon*[1]. In Figure 4, the calculated κ_p for each τ, α, is plotted by a $\circ$ symbol, where τ is set to be from 0.2 to 1.2 in 0.1 increments, and α is also set to be from 0 to 1.0 in 0.1 increments. Moreover, τ_i and τ_d are plotted in Figure 5 and Figure 6, respectively. Using the obtained data, the decision rule of the normalized PID parameters is proposed. In the present study, the calculated normalized parameters are approximated by the following equations:

$$\kappa_p = a_0(\alpha) + a_1(\alpha)\tau^{a_2(\alpha)} \tag{18}$$
$$a_0(\alpha) = x_{00} + x_{01}\alpha,\ a_1(\alpha) = x_{10} + x_{11}\alpha,\ a_2(\alpha) = x_{20} + x_{21}\alpha$$

$$\tau_i = b_0(\alpha) + b_1(\alpha)\tau + b_2(\alpha)\tau^2 + b_3(\alpha)\tau^3 \tag{19}$$
$$b_0(\alpha) = y_{00} + y_{01}\alpha,\ b_1(\alpha) = y_{10} + y_{11}\alpha,\ b_2(\alpha) = y_{20} + y_{21}\alpha,\ b_3(\alpha) = y_{30} + y_{31}\alpha$$

$$\tau_d = c_0(\alpha) + c_1(\alpha)\tau \tag{20}$$
$$c_0(\alpha) = z_{00} + z_{01}\alpha,\ c_1(\alpha) = z_{10} + z_{11}\alpha$$

where the coefficients for each M_s^d are given in Table 1.

[1]Mathworks, Inc.

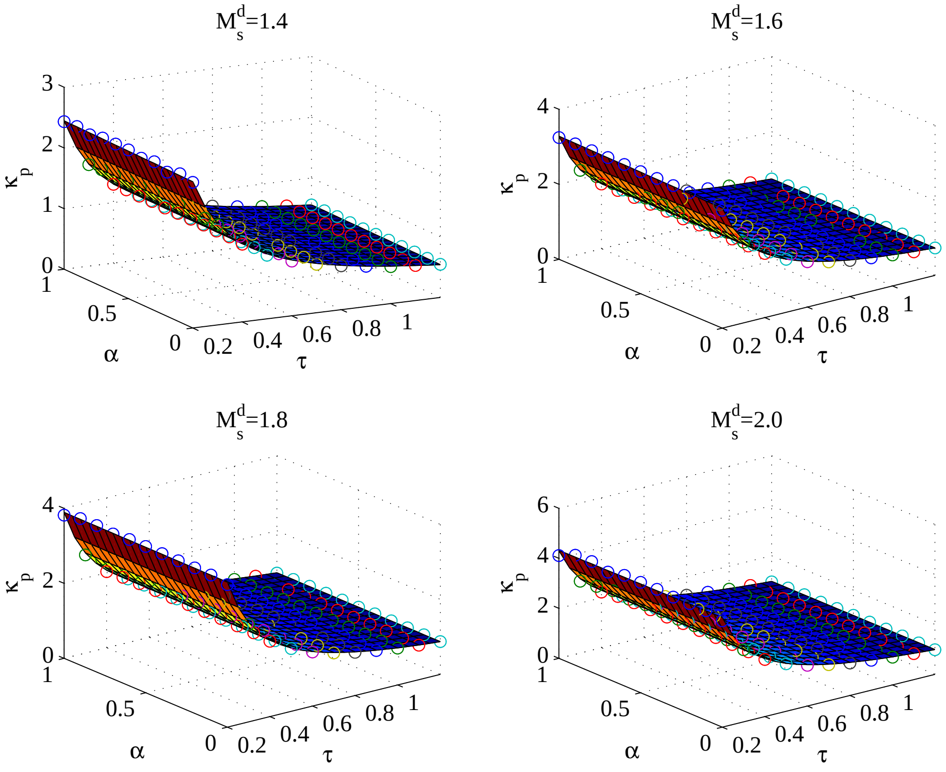

FIGURE 4. Optimal κ_p for τ and α and the approximated surface

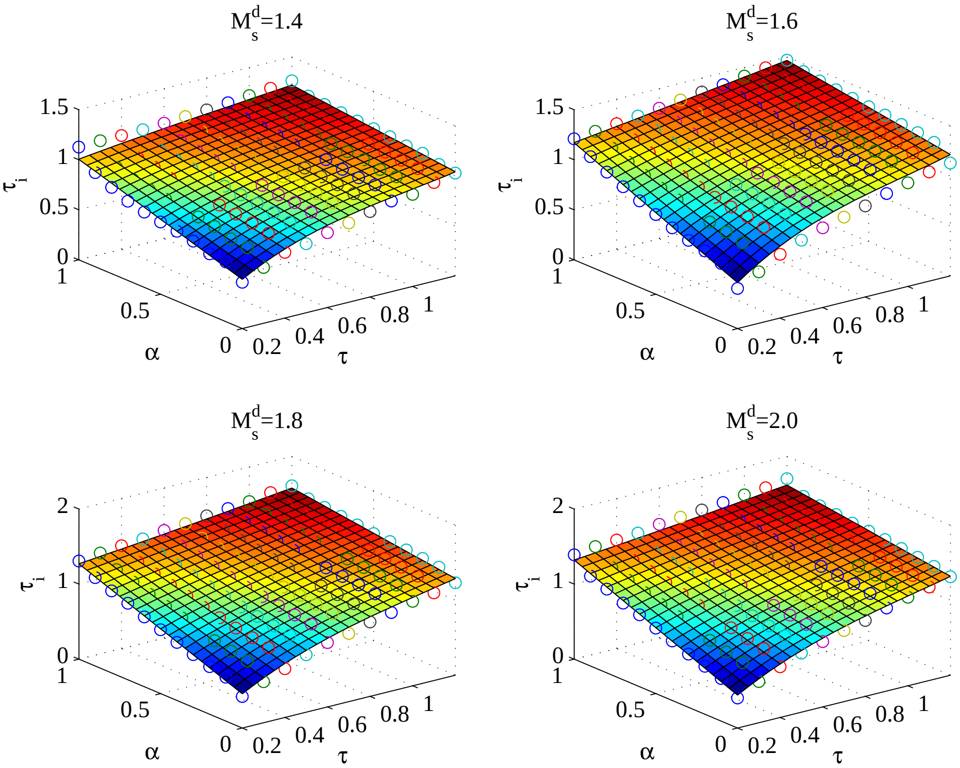

FIGURE 5. Optimal τ_i for τ and α and the approximated surface

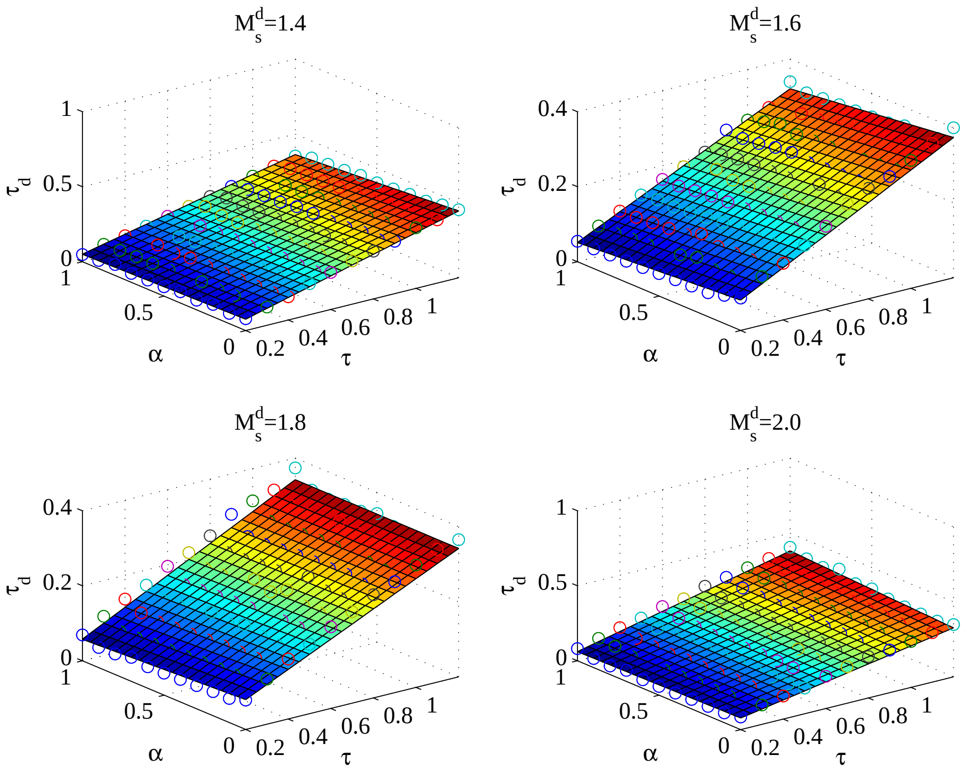

FIGURE 6. Optimal τ_d for τ and α and the approximated surface

TABLE 1. x_{ij}, y_{kl} and z_{mn} in Equation (18)-Equation (20) for M_s^d

M_s^d	1.4	1.6	1.8	2.0
x_{00}	0.1521	0.2484	0.3235	0.3635
x_{01}	0.04199	0.01711	−0.02680	−0.06098
x_{10}	0.4659	0.5794	0.6641	0.7586
x_{11}	−0.02869	−0.008122	0.03400	0.04427
x_{20}	−0.9807	−1.011	−1.0395	−1.035
x_{21}	−0.03788	−0.02278	0.02127	0.03574
y_{00}	0.2027	0.08962	0.1040	0.07031
y_{01}	0.7444	1.025	1.115	1.203
y_{10}	1.718	2.180	2.019	2.106
y_{11}	−1.399	−1.904	−1.764	−1.880
y_{20}	−1.435	−1.742	1.410	−1.440
y_{21}	1.304	1.759	1.410	1.507
y_{30}	0.4870	0.5874	0.4268	0.4591
y_{31}	−0.4421	−0.5908	−0.4268	−0.4738
z_{00}	0.001796	0.02239	0.02739	0.02729
z_{01}	−0.013732	−0.02483	−0.02678	−0.02991
z_{10}	0.3695	0.2929	0.2624	0.2490
z_{11}	−0.05715	−0.02371	0.02220	0.07068

3.2.2. *Confirmation of the assigned stability and the trade-off design.* The reliability of the proposed method is confirmed by two comparisons: a comparison of the optimized normalized parameters and the calculated normalized parameters using Equation (18) through Equation (20), and a comparison of the assigned M_s^d and the actual calculated M_s. The achieved trade-off performance is also shown, and the feature of the proposed method is shown.

The normalized parameters κ_p, τ_i, and τ_d calculated using Equation (18) through Equation (20) are shown as surfaces on Figure 4, Figure 5, and Figure 6, respectively. These surfaces are calculated by not only the preliminarily used τ and α for obtaining the $\circ$ symbols in Figure 4, Figure 5, and Figure 6, but also their interpolated values among the preliminarily used τ and α. Here, Figure 4, Figure 5, and Figure 6 show that the calculated surfaces are well approximated by the optimized points.

The obtained M_s corresponding to the assigned M_s^d is shown in Figure 7, where $0.2 \leq \tau \leq 1.2$ and $0 \leq \alpha \leq 1$. The maximum errors of the assigned M_s^d with respect to the actual obtained M_s are shown in Table 2. Since the errors are quite small, the assigned stability margin is achieved using the proposed design method.

The trade-off performance of the proposed method is confirmed. The trade-off relationship between J_r and J_d is shown in Figure 8, where τ is 1.2, and α is changed from 0 to 1.0 in 0.05 increments. This figure indicates that the trade-off design between J_r and J_d is accomplished by designing α.

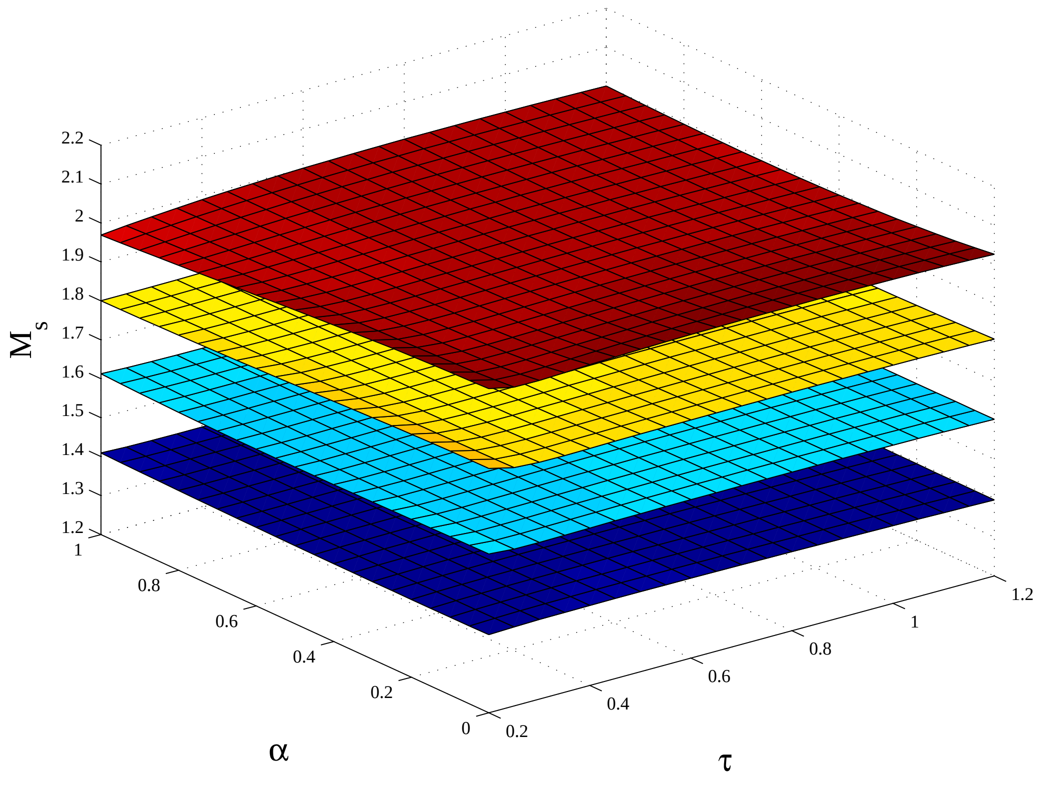

FIGURE 7. Obtained M_s for the assigned M_s^d

TABLE 2. Maximum error of $\left|M_s - M_s^d\right|$

M_s^d	1.4	1.6	1.8	2.0
$\max\left|M_s - M_s^d\right|$	0.0094	0.013	0.028	0.034

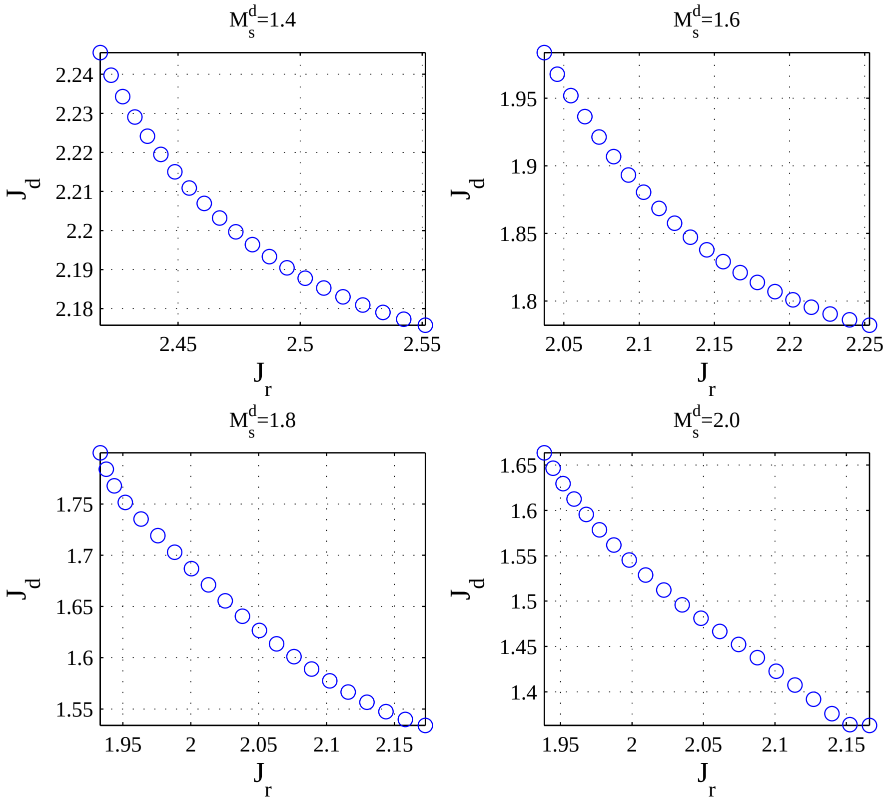

FIGURE 8. Trade-off between J_r and J_d with respect to α

4. **Numerical Examples.** Consider the following controlled plant:

$$P(s) = \frac{2.5}{19.6s + 1} e^{-4.9s} \tag{21}$$

where the reference input is set as a unit step function, and the control input is disturbed by the signal generated by a unit step function after 150s.

4.1. **Performance comparison for $\boldsymbol{\alpha}$.** The simulation results with respect to $M_s^d = 1.4$, 1.6, 1.8, and 2.0 are shown in Figure 9 through Figure 12, where α is set to 0, 0.25, 0.5, 0.75, and 1.0. These figures show that the control performance improves as M_s^d increases. Furthermore, the trade-off design between the tracking performance and the regulation performance is adjusted by tuning α.

4.2. **Performance comparison for $\boldsymbol{M_s^d}$.** The robust stability for M_s^d is confirmed. The controlled plant is changed to $\tilde{P}(s)$ after 200s.

$$\tilde{P}(s) = \frac{5.1}{21.5s + 1} e^{-6.2s} \tag{22}$$

Using $\alpha = 0.25$, the PID parameters are designed based on $P(s)$ for $M_s^d = 1.4$, 1.6, 1.8, 2.0, respectively. The simulation results are shown in Figure 13. It can be seen that the transient response is superior with large M_s^d, whereas the robust margin is large with small M_s^d.

5. **Conclusions.** In the present study, we proposed a simple tuning method of a PID control system. In the proposed method, the trade-off between the tracking performance and the regulation performance is adjusted seamlessly. Since the robustness for plant

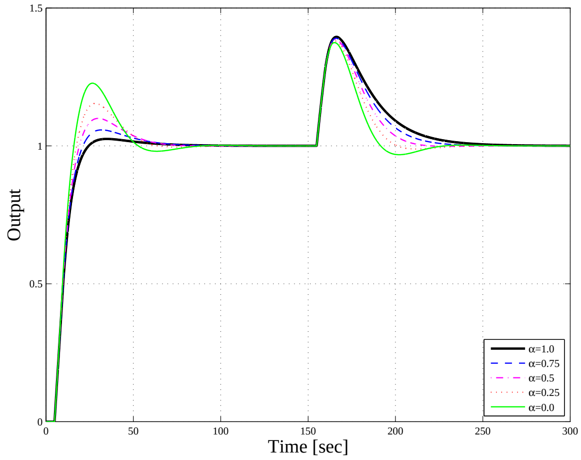

FIGURE 9. Transient responses on $M_s^d = 1.4$

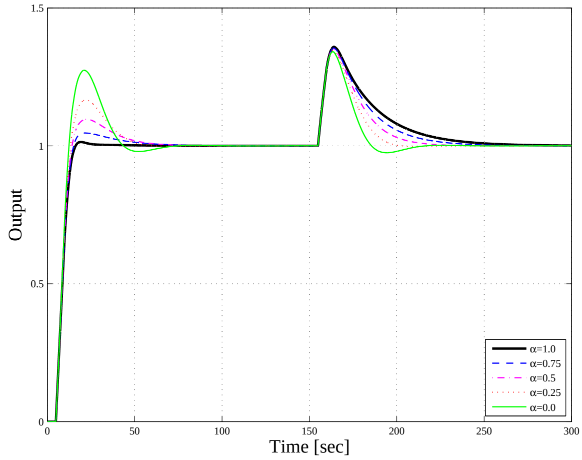

FIGURE 10. Transient responses on $M_s^d = 1.6$

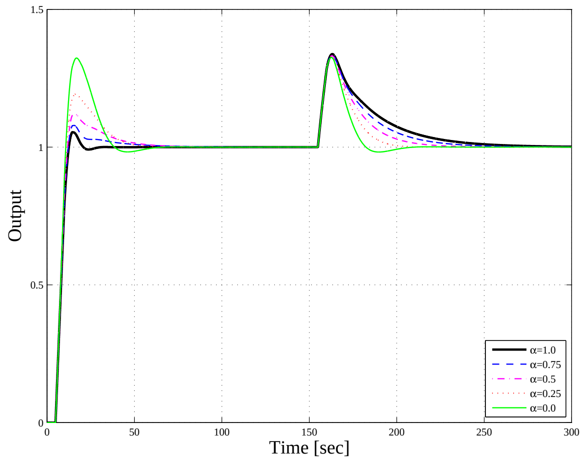

FIGURE 11. Transient responses on $M_s^d = 1.8$

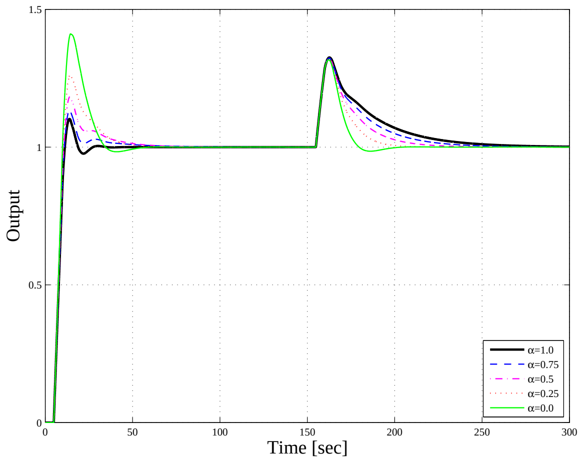

FIGURE 12. Transient responses on $M_s^d = 2.0$

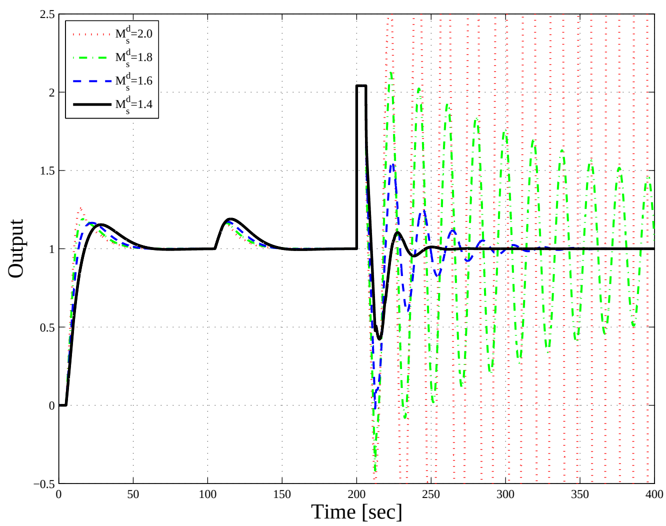

FIGURE 13. Transient responses for each M_s^d ($\alpha = 0.25$)

perturbation is assigned using the sensitivity function, the robust stability is designed based on the accuracy of the plant model. Our future work is an extension for a second-order system.

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