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TRACKING CONTROL FOR OMNIDIRECTIONAL REHABILITATIVE TRAINING WALKER WITH CENTER OF GRAVITY SHIFT AND FAULT INPUT

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Abstract. *This paper investigates a trajectory-tracking-control method for an omnidirectional rehabilitative training walker (ODW) subjected to center-of-gravity shift and faulty inputs. The tracking performance and reliability of the ODW are treated simultaneously. The motion of the ODW under a center-of-gravity shift is described by a stochastic model. On the basis of the stochastic model, we developed a trajectory-tracking controller with high fault tolerance and exponential stability of the tracking error system. If the design parameters are appropriately chosen, the mean absolute error becomes arbitrarily small. Simulation results demonstrate the feasibility and effectiveness of the proposed method.*

Keywords: Center-of-gravity shift, Tracking control, Faulty input, Omnidirectional walker

1. **Introduction.** An omnidirectional rehabilitative training walker (ODW) [1,2], which provides walking rehabilitation and support for people with walking impairments, is being developed. This walker allows omnidirectional movement, including not only forward and backward motions but also right and left motions, oblique motions, rotations, and combinations of these motions. The training programs are stored in the walker enabling accurate rehabilitation without the presence of physical therapists.

Safety and performance are important requirements of many mechanical systems and are especially demanded in rehabilitative training robots. Hence, the design of controllers that simultaneously satisfy these two requirements is a growing field [3]. In the classical control approach, any faulty information (from the input items or sensor) will degrade the system performance and may even destabilize the system. To overcome these disadvantages, researchers have developed many effective control approaches that tolerate faulty information while maintaining stability and satisfactory performance. Fault-tolerant control can be generally classified into passive fault-tolerant control, active fault-tolerant control, and redundant fault-tolerant control [4]. Controllers in passive fault-tolerant control are designed to be robust against a class of presumed faults, which limits their fault tolerance capabilities. Active fault-tolerant control systems actively react to the faults by switching the control parameters and maintaining real-time fault detection and diagnosis, i.e., fault detection, isolation, and estimation [5]. However, active fault-tolerant control systems largely depend on the estimation of the fault value, which limits their applicability. For these reasons, redundant fault-tolerant control has attracted significant attention. Increasing numbers of mechanical systems are now embedded with hardware redundancy, for example, actuator or sensor redundancy.

As is well known, robotic systems in practical working environment often encounter random disturbances and random uncertainties, which deteriorate their performance in an uncertain manner. Thus, many methods have been proposed to solve this problem, for instance, a robust tracking-control method for robot manipulators with input disturbances is proposed [6] and an adaptive controller based on a stochastic Lagrange dynamic model is investigated [7]. However, the above stochastic models and control approaches consider only the random noise from input channels. In practice, mechanical systems are affected by many unknown system parameters such as the variable arm-length of a robot manipulator [6] and the center-of-gravity shift of a rehabilitative training walker [8]. When the system parameters change at random, ensuring a safe and reliable operation with accurate tracking performance is a non-trivial design problem. Simultaneous correction of the center-of-gravity shift and faulty inputs to an ODW (using stochastic theory and redundant fault-tolerant technology, respectively) would significantly advance robotic research, but has not been previously reported. From both theoretical and practical engineering perspectives, many problems are worthy of investigation by this approach.

Motivated by the above observations, we investigate an ODW under center-of-gravity shifts and faulty inputs. The main contributions of this paper are summarized below.

(i) Correcting the center-of-gravity shift is a major challenge in many mechanical systems. By changing the random parameters into random disturbances, we construct a reasonable stochastic model that describes the motion of an ODW subjected to center-of-gravity shifts and faulty inputs.

(ii) On the basis of the stochastic model, we propose a trajectory-tracking control for the ODW with shifted center-of-gravity and faulty input. To ensure exponential stability of the tracking error system, we adopt the redundant fault tolerance approach. The mean absolute error (and its derivative) becomes arbitrarily small with appropriate tuning of the design parameters.

(iii) During the simulation, the ODW is subjected to realistic noises and fault inputs. The effectiveness of the proposed method is confirmed in the tracking performance. The control input and the mean absolute error show the appropriateness of the designed parameters.

The remainder of this paper is organized as follows. Section 2 formulates the stochastic ODW model under center-of-gravity shifts and faulty inputs. Sections 3 and 4 present the design of the trajectory-tracking controller and the stability analysis, respectively. Section 5 presents the simulation results, and Section 6 concludes the paper.

2. **Stochastic Model of the ODW with Center-of-Gravity Shifts and Faulty Inputs.** Figures 1 and 2 display the ODW and the coordinate settings and structure of the ODW, respectively.

In Figure 2, $\Sigma(x, O, y)$ and $\Sigma(x', C, y')$, respectively, refer to the absolute and translated coordinate systems. The variables are defined as follows:

v: Speed of the ODW

v_i $(i = 1, 2, 3, 4)$: Speed of an omniwheel

f_i: Force on each omniwheel

G: Center-of-gravity of the walker

r_0: Distance between G and the center-of-gravity when the ODW is loaded

α: Angle between the x' axis and the direction of v

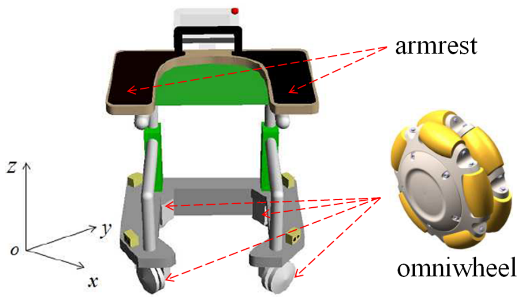

FIGURE 1. ODW and omniwheel

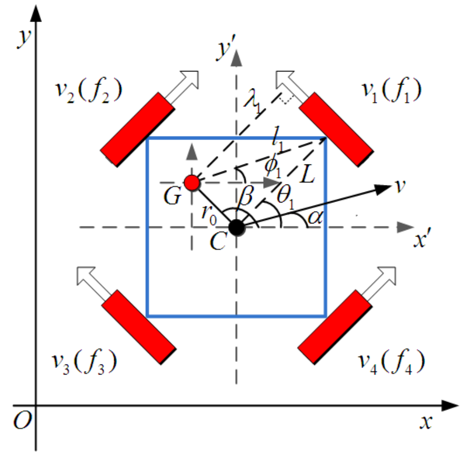

FIGURE 2. Structure of ODW

β: Angle between the x' axis and r_0
L: Distance from the center of the ODW to each omniwheel
l_i: Distance from the center-of-gravity to the center of each omniwheel
λ_i: Vertical distance from the center-of-gravity to each omniwheel
θ_i: Angle between the x' axis and the position of each omniwheel
ϕ_i: Angle between the x' axis and l_i
The dynamic model is borrowed from [9]:

$$M_0 K \ddot{X}(t) + M_0 \dot{K} \dot{X}(t) = B(\theta) u(t), \tag{1}$$

where

$$M_0 = \begin{bmatrix} M+m & 0 & 0 \\ 0 & M+m & 0 \\ 0 & 0 & I_0 + m r_0^2 \end{bmatrix}, \quad K = \begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix},$$

$$\begin{matrix} p = \dfrac{1}{2}[(\lambda_1 - \lambda_3)\sin\theta + (\lambda_2 - \lambda_4)\cos\theta] \\ q = \dfrac{1}{2}[(\lambda_2 - \lambda_4)\sin\theta - (\lambda_1 - \lambda_3)\cos\theta] \end{matrix}, \quad X(t) = \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix},$$

$$B(\theta) = \begin{bmatrix} -\sin\theta_1 & \sin\theta_2 & \sin\theta_3 & -\sin\theta_4 \\ \cos\theta_1 & -\cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ \lambda_1 & -\lambda_2 & -\lambda_3 & \lambda_4 \end{bmatrix}, \quad \begin{matrix} \lambda_1 = l_1\cos(\theta_1 - \phi_1) \\ \lambda_2 = l_2\cos(\theta_2 - \phi_2) \\ \lambda_3 = l_3\cos(\theta_3 - \phi_3) \\ \lambda_4 = l_4\cos(\theta_4 - \phi_4) \end{matrix}, \quad u(t) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}.$$

Here, M is the mass of the ODW, m is the equivalent mass of the user (which causes an inertial mass mr_0^2), and I_0 is the inertial mass of the ODW. f_1, f_2, f_3, and f_4 are the input forces, and λ_1, λ_2, λ_3, λ_4 and r_0 are random parameters associated with the center-of-gravity shifts. θ is the angle between the x' axis and the position of the first omniwheel. Denoting $\theta = \theta_1$, the angular positions of the other wheels are $\theta_2 = \theta + \frac{\pi}{2}$, $\theta_3 = \theta + \pi$, and $\theta_4 = \theta + \frac{3\pi}{2}$.

Using the above method and the idea proposed by Chang et al. [10], the stochastic model under a center-of-gravity shift is expressed as

$$\begin{aligned} dX(t) &= v(t)dt \\ dv(t) &= M_1^{-1} B^*(\theta) u(t) dt + M_1^{-1} N(\theta) \Sigma dw, \end{aligned} \tag{2}$$

where

$$v(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix}, \quad M_1 = \begin{bmatrix} M+m & 0 & 0 \\ 0 & M+m & 0 \\ 0 & 0 & I_0 \end{bmatrix},$$

$$B^*(\theta) = \begin{bmatrix} -\sin\theta_1 & \sin\theta_2 & \sin\theta_3 & -\sin\theta_4 \\ \cos\theta_1 & -\cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ L & L & L & L \end{bmatrix},$$

$$N(\theta) = \begin{bmatrix} -(M+m)\sin\theta & \dot{\theta}^2(M+m)\sin\theta & -\dot{\theta}^2(M+m)\cos\theta & -(M+m)\cos\theta & 0 \\ (M+m)\cos\theta & -\dot{\theta}^2(M+m)\cos\theta & -\dot{\theta}^2(M+m)\sin\theta & -(M+m)\sin\theta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Here, w is a five-dimensional independent standard Wiener process. $M_1^{-1}N\Sigma dw$ depicts the influence of the center-of-gravity shift, and $\Sigma/2\pi$ is the power spectral density of the white noise.

The wheels of rehabilitative walking robots are inevitably damaged by working under poor conditions or incorrect operations. Given that the ODW tolerates faults to some extent, it should be governed by a redundant fault tolerance controller. The fault controller should enable training of the ODW when a wheel breakdown occurs. Actuator faults are handled by a uniform actuator fault model [11]. After separating the redundant actuator as

$$u(t) = (I - \rho^*)u^*(t), \tag{3}$$

we factorize the redundant controller $u^*(t)$ and the matrix $B^*(\theta)$ as follows:

$$u^*(t) = \left[u_i^{*^T}(t) \quad \Delta u_i^*(t)\right]^T \tag{4}$$

$$B^*(\theta) = [B_i^*(\theta) \quad \Delta B_i^*(\theta)]. \tag{5}$$

The redundant actuator is then separated as

$$u(t) = \left[u_i^{*^T}(t) \quad (1-\rho_i)\Delta u_i^*(t)\right]^T, \tag{6}$$

where $\rho^* = diag\{\rho_1, \rho_2, \rho_3, \rho_4\}$. $0 < \rho_i \le 1$ $(i = 1, 2, 3, 4)$ is a constant (here, the index i denotes the ith faulty actuator), and $u_i^*(t)$ and $\Delta u_i^*(t)$ are the input forces from the normal and faulty actuators respectively, with corresponding control matrices $B_i^*(\theta)$ and $\Delta B_i^*(\theta)$, respectively. When $0 < \rho_i < 1$, the ith actuator loses its effectiveness, i.e., the ith controller loses a proportion ρ_i of its force. The state $\rho_i = 1$ denotes outage of the ith actuator, i.e., zero force is input to the ith controller.

Combining (5) and (6) with (2), the stochastic model of the ODW with center-of-gravity shifts and faulty actuators are obtained as

$$\begin{aligned} &dX(t) = v(t)dt \\ &dv(t) = M_1^{-1}B_i^*(\theta)u_i^*(t)dt + M_1^{-1}\Delta B_i^*(\theta)(1-\rho_i)\Delta u_i^*(t)dt + M_1^{-1}N(\theta)\Sigma dw. \end{aligned} \tag{7}$$

Assumption 2.1. *As the angular velocity $\dot{\theta}$ is bounded by practical constraints, there exists a constant h such that*

$$2\dot{\theta}^4(M+m)^2 + 2(M+m)^2 + 1 \le h. \tag{8}$$

Assumption 2.2. *As $\Delta u_i^*(t)$ and $\Delta B_i^*(\theta)$ are bounded, $M_1^{-1}\Delta B_i^*(\theta)(1-\rho_i)\Delta u_i^*(t)$ are also bounded. Thus, there exists a three-dimensional constant vector W such that*

$$M_1^{-1}\Delta B_i^*(\theta)(1-\rho_i)\Delta u_i^*(t) \le W. \tag{9}$$

3. **Design of the Tracking Controller.** The trajectory tracked by the rehabilitative training program in the ODW is designed by a therapist. Therefore, the quality of the training is enhanced as the tracking accuracy improves. With respect to a reference signal $X_d(t) \in C^2(R^n)$, the tracking error is defined as

$$\begin{aligned} e_1(t) &= X(t) - X_d(t) \\ e_2(t) &= \dot{e}_1(t) + \alpha e_1(t) = v(t) - \dot{X}_d + \alpha e_1(t). \end{aligned} \tag{10}$$

Combining (10) with the stochastic model (7), the error system becomes

$$\begin{aligned} de_1(t) &= [e_2(t) - \alpha e_1(t)]dt \\ de_2(t) &= M_1^{-1} B_i^*(\theta) u_i^*(t)dt + M_1^{-1} \Delta B_i^*(\theta) \Delta u_i^*(t)dt - \ddot{X}_d dt \\ &\quad + \alpha e_2(t)dt - \alpha^2 e_1(t)dt + M_1^{-1} N(\theta) \Sigma dw, \end{aligned} \tag{11}$$

where the parameter α will be designed later.

We next define the Lyapunov function

$$V(t) = \frac{1}{2} e_1^T(t) Q_1^2 e_1(t) + \frac{1}{2} e_2^T(t) Q_2^2 e_2(t), \tag{12}$$

where $Q_1 = diag\{q_{11}, q_{12}, q_{13}\} > 0$ and $Q_2 = diag\{q_{21}, q_{22}, q_{23}\} > 0$.

The infinites generator of $V(t)$ along the system $\left(e_1^T(t), e_2^T(t)\right)^T$ satisfies

$$\begin{aligned} LV(t) = e_1^T Q_1^2 (e_2 - \alpha e_1) + e_2^T Q_2^2 \Big(& M_1^{-1} B_i^*(\theta) u_i^*(t) + M_1^{-1} \Delta B_i^*(\theta)(1 - \rho_i) \Delta u_i^*(t) \\ & - \ddot{X}_d + \alpha e_2(t) - \alpha^2 e_1(t) \Big) + \frac{1}{2} Tr\left\{ \Sigma^T N^T(\theta) M_1^{-1} Q_2^2 M_1^{-1} N(\theta) \Sigma \right\}. \end{aligned} \tag{13}$$

By Young's inequality, we have

$$e_1^T Q_1^2 e_2 \le \frac{\varepsilon_1^2}{2} e_1^T Q_1^2 e_1 + \frac{1}{2\varepsilon_1^2} e_2^T Q_1^2 e_2 \tag{14}$$

$$e_2^T Q_2^2 M_1^{-1} \Delta B_i^*(\theta)(1 - \rho_i) \Delta u_i^*(t) \le e_2^T Q_2^2 W \le \frac{\varepsilon_2^2}{2} e_2^T Q_2^2 e_2 + \frac{\lambda_{\max Q_2}^2}{2\varepsilon_2^2} W^T W, \tag{15}$$

where $\lambda_{\max Q_2}$ is the maximum eigenvalue of Q_2 and $\varepsilon_i > 0$ $(i = 1, 2)$ are design parameters. Furthermore, by considering the definition of the Frobenius norm, the norm compatibility, Equation (8), and Young's inequality, we have

$$\begin{aligned} & \frac{1}{2} Tr\left\{ \Sigma^T N^T(\theta) M_1^{-1} Q_2^2 M_1^{-1} N(\theta) \Sigma \right\} \\ \le & \frac{1}{2} \|Q_2\|_F^2 \left\|M_1^{-1}\right\|_F^2 \|N(\theta)\|_F^2 \|\Sigma\|_F^2 \\ \le & \frac{1}{2} \|Q_2\|_F^2 \left\|M_1^{-1}\right\|_F^2 \left[2\dot{\theta}^4 (M + m)^2 + 2(M + m)^2 + 1 \right] \|\Sigma\|_F^2 \\ \le & \frac{h}{2} \|Q_2\|_F^2 \left\|M_1^{-1}\right\|_F^2 \|\Sigma\|_F^2 . \end{aligned} \tag{16}$$

Substituting (14)-(16) into (13), we obtain

$$\begin{aligned} LV \le & -\alpha e_1^T Q_1^2 e_1 + \frac{\varepsilon_1^2}{2} e_1^T Q_1^2 e_1 + \frac{1}{2\varepsilon_1^2} e_2^T Q_1^2 e_2 + e_2^T Q_2^2 M_1^{-1} B_i^*(\theta) u_i^*(t) + \frac{\varepsilon_2^2}{2} e_2^T Q_2^2 e_2 \\ & + \frac{\lambda_{\max Q_2}^2}{2\varepsilon_2^2} W^T W - e_2^T Q_2^2 \ddot{X}_d + \alpha e_2^T Q_2^2 e_2(t) - \alpha^2 e_2^T Q_2^2 e_1(t) \\ & + \frac{h}{2} \|Q_2\|_F^2 \left\|M_1^{-1}\right\|_F^2 \|\Sigma\|_F^2 . \end{aligned} \tag{17}$$

The tracking controller and its parameter are designed as follows:

$$\begin{aligned} u_i^*(t) = {B_i^*}^{-1}(\theta) M_1 Q_2^{-2} \Big(& -\frac{c_2}{2} Q_2^2 e_2(t) - \frac{1}{2\varepsilon_1^2} Q_1^2 e_2(t) \\ & - \frac{\varepsilon_2^2}{2} Q_2^2 e_2(t) + Q_2^2 \ddot{X}_d - \alpha Q_2^2 e_2(t) - \alpha^2 Q_2^2 e_1(t) \Big) \end{aligned} \tag{18}$$

$$\alpha = \frac{\varepsilon_1^2}{2} + \frac{c_1}{2}, \tag{19}$$

where $c_j > 0$ $(j = 1, 2)$ are time-invariant designed parameters. Substituting (18) and (19) into (17), we have

$$LV \le -\frac{c_1}{2} e_1^T(t) Q_1^2 e_1(t) - \frac{c_2}{2} e_2^T(t) Q_2^2 e_2(t) + d \le -cV + d, \tag{20}$$

where $c = \min\{c_1, c_2\}$ and $d = \frac{\lambda^2_{\max Q_2}}{2\varepsilon_2^2} W^T W + \frac{h}{2} \|Q_2\|_F^2 \left\|M_1^{-1}\right\|_F^2 \|\Sigma\|_F^2$.

4. **Stability Analysis.**

Theorem 4.1. *For any appropriately designed stochastic ODW model with center-of-gravity shifts and faulty inputs (7) and given a reference signal $X_d(t) \in C^2(R^n)$, the closed-loop system $\left(e_1^T(t), e_2^T(t)\right)^T$ of the controller has a unique solution on $[t_0, \infty)$ and is exponentially stable for initial values $e_1(t_0) \in R^n$ and $e_2(t_0) \in R^n$. The tracking errors $e_1(t)$ and $\dot{e}_1(t)$ satisfy*

$$\lim_{t\to\infty} E\,|e_1| \le \sqrt{2}\, \|Q_1\|_F^{-1} \left(\frac{d}{c}\right)^{\frac{1}{2}} \tag{21}$$

$$\lim_{t\to\infty} E\,|\dot{e}_1| \le \sqrt{2}(1+\alpha) \left(\|Q_1\|_F^{-1} + \left\|Q_2^{-1}\right\|_F^{-1}\right) \left(\frac{d}{c}\right)^{\frac{1}{2}}. \tag{22}$$

Moreover, the right-hand sides of (21) and (22) can be made arbitrarily small by choosing appropriate design parameters.

Proof: For practical purposes, the inertia matrix M_1 is symmetric and positive-definite. Therefore, M_1 is also smooth, implying that the local Lipschitz condition holds in M_1 and in the functions $u_i^*(t)$ and $B_i^*(\theta)$. Therefore, the *closed-loop system* $\left(e_1^T(t), e_2^T(t)\right)^T$ also satisfies the local Lipschitz condition. From (12) and (20), and Lemma 1 in [7], there exists a unique strong solution to the *closed-loop system* $\left(e_1^T(t), e_2^T(t)\right)^T$ on $[t_0, \infty)$ for initial values $e_1(t_0) \in R^n$ and $e_2(t_0) \in R^n$, and the *closed-loop system* $\left(e_1^T(t), e_2^T(t)\right)^T$ is exponentially stable.

Moreover, multiplying the inequality (20) by $e^{ct} > 0$, we find that

$$e^{ct} \left(LV(x,t) + cV(x,t)\right) \le e^{ct} d_c. \tag{23}$$

Hence, by Lemma 3.3.1 in [12] and integrating (23) from t_0 to t, we have

$$E\left(e^{ct} V(x,t)\right) \le e^{ct_0} V(x_0, t_0) + E \int_{t_0}^{t} e^{cs} d_c \cdot ds \quad \forall t \ge t_0. \tag{24}$$

From (12), we can deduce that

$$\begin{aligned} E\,|e_1(t)| \le & \sqrt{2}\, \|Q_1\|_F^{-1} e^{\frac{1}{2}c(t_0 - t)} \left(\frac{1}{2} e_1^T(t_0) Q_1^2 e_1(t_0) + \frac{1}{2} e_2^T(t_0) Q_2^2 e_2(t_0)\right)^{\frac{1}{2}} \\ & + \|Q_1\|_F^{-1} \left(\frac{2d}{c}\right)^{\frac{1}{2}} \end{aligned} \tag{25}$$

$$E\left|e_2(t)\right| \le \sqrt{2}\left\|Q_2\right\|_F^{-1} e^{\frac{1}{2}c(t_0-t)}\left(\frac{1}{2}e_1^T(t_0)Q_1^2e_1(t_0)+\frac{1}{2}e_2^T(t_0)Q_2^2e_2(t_0)\right)^{\frac{1}{2}} + \left\|Q_2\right\|_F^{-1}\left(\frac{2d}{c}\right)^{\frac{1}{2}}. \quad (26)$$

From (25) and (26) and acknowledging that $|\dot{e}_1| \le |e_2| + \alpha|e_1| \le (1+\alpha)(|e_2|+|e_1|)$, it follows that (21) and (22) hold. Also noting that $c = \min\{c_1, c_2\}$ and $d = \frac{\lambda^2_{\max Q_2}}{2\varepsilon_2^2}W^TW + \frac{h}{2}\|Q_2\|_F^2\left\|M_1^{-1}\right\|_F^2\|\Sigma\|_F^2$, the right-hand side of (21) and (22) can be made sufficiently small by choosing the independent quantities c_1, c_2, $\|Q_1\|_F$, and $\|Q_2\|_F$.

5. **Simulation Results.** This section verifies the proposed trajectory-tracking-control algorithm by simulating an ODW under center-of-gravity shifts and faulty inputs.

To rigorously evaluate the tracking performance, we assume that the walker follows a cycloidal path. The random parameters of the center-of-gravity shift were given by $r_0 = 0.1(1+\sin t)$ m, $\lambda_1 = L - r_0\sin t$ m, $\lambda_2 = L + r_0\cos t$ m, $\lambda_3 = L + r_0\sin t$ m, and $\lambda_4 = L - r_0\cos t$ m. The physical ODW parameters in the simulation were $M = 58$ kg, $m = 70$ kg, $L = 0.4$ m, and $I_0 = 27.7$ kg.m^2 and the design parameters were $c_1 = 1.3$, $c_2 = 0.02$, $\varepsilon_1 = 0.95$, $\varepsilon = 0.03$, $Q_1 = diag\{1.4, 1.3, 2.1\}$, and $Q_2 = diag\{1.4, 1.5, 0.9\}$. Assuming the second input faults, the control matrix is calculated as follows:

$$B_2^*(\theta) = \begin{bmatrix} -\sin\theta & -\sin\theta & \cos\theta \\ \cos\theta & \cos\theta & \sin\theta \\ L & -L & L \end{bmatrix},$$

$$B_2^{*^{-1}}(\theta) = \begin{bmatrix} -\dfrac{\sin\theta+\cos\theta}{2} & -\dfrac{\sin\theta-\cos\theta}{2} & \dfrac{1}{2L} \\ -\dfrac{\sin\theta-\cos\theta}{2} & \dfrac{\sin\theta+\cos\theta}{2} & -\dfrac{1}{2L} \\ \cos\theta & \sin\theta & 0 \end{bmatrix}.$$

The path X_d and the fault value are described by

$$\begin{cases} x_d = 2(0.3t - \sin(0.3t)) \\ y_d = 2(0.3 - \cos(0.3t)) \\ \theta_d = \dfrac{\pi}{4} \end{cases} \quad 0 \le t \le 120s; \qquad \rho_2 = \begin{cases} 0.5 & 0 \le t \le 60s \\ 1.0 & 60 \le t \le 120s \end{cases}.$$

The simulation results are shown and discussed below.

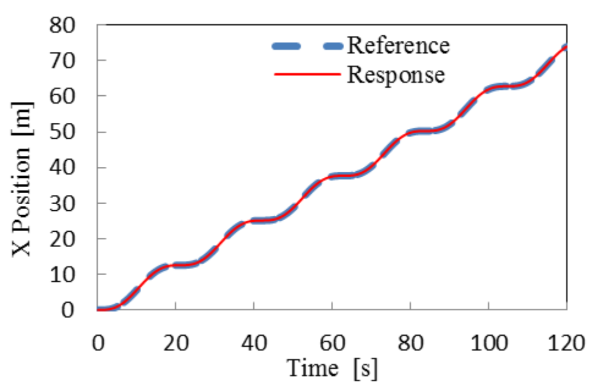

FIGURE 3. Tracking performance of the x position

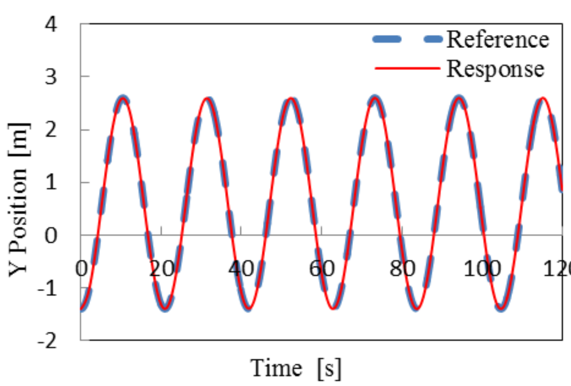

FIGURE 4. Tracking performance of the y position

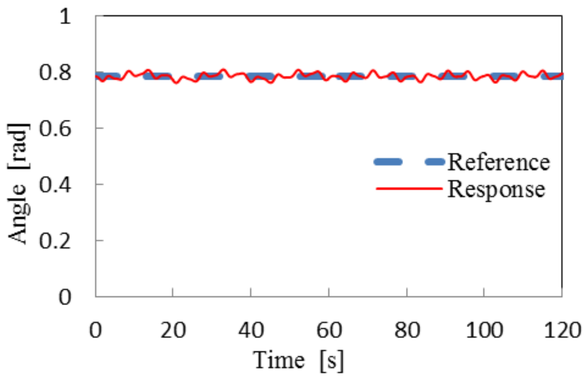

Figure 5. Tracking performance of the angle

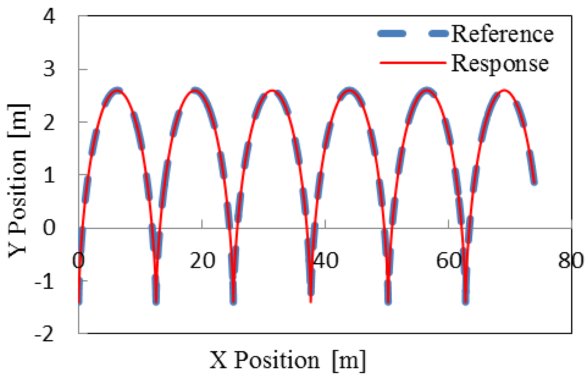

Figure 6. Tracking of the cycloidal path

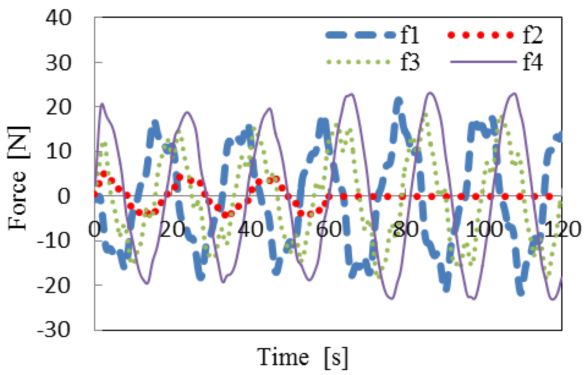

Figure 7. Control inputs

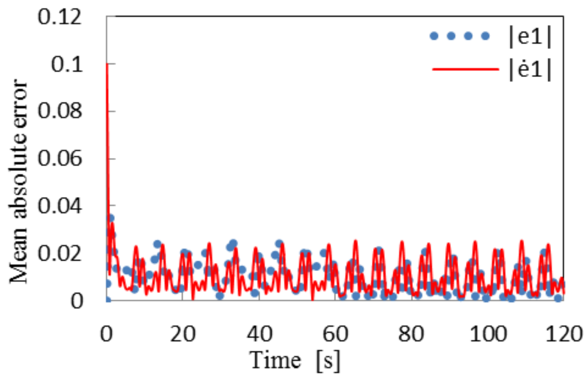

Figure 8. Mean absolute errors

Figures 3, 4 and 5 plot the tracking performance of the ODW in terms of the x position, y position and orientation angle, respectively, and Figure 6 displays the cycloidal path tracking result. Clearly, the closed-loop system realizes an exponentially stable behavior, and the controller achieves effective trajectory tracking of the ODW (18). Therefore, the stochastic ODW model (7) successfully corrects the center-of-gravity shift and faulty input. The control inputs (see Figure 7) confirm that f_2 contributes only half of the expected force during the first 60 s. Thereafter, f_2 vanishes for all time, but its loss is compensated by an increase in the other force inputs. Moreover, when the design parameters are appropriately chosen, the mean absolute errors become arbitrarily small (see Figure 8). These simulation results demonstrate the effectiveness of the stochastic model (7) and the tracking controller (18) in correcting center-of-gravity shifts and faulty inputs in ODWs.

6. Conclusions.

We proposed a tracking-control method based on a stochastic model of an ODW subjected to center-of-gravity shifts and faulty inputs. Exploiting the redundant input of the ODW, we first separated the fault item from the previous control input. We then described the motion of the disturbed ODW by a reasonable stochastic model. Thirdly, we designed a trajectory-tracking controller that guarantees exponential stability and input-fault tolerance of the tracking error system. Finally, we demonstrated the effectiveness of our proposed controllers in simulation studies. Ultimately, we plan to simultaneously resolve center-of-gravity shifts and faulty inputs in other mechanical systems. These generalizations of our model will be attempted in future work.

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