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NEW RESULTS OF H_∞ FILTERING FOR NEURAL NETWORK WITH TIME-VARYING DELAY

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Abstract. *A more effective Lyapunov functional has been constructed to investigate the H_∞ filtering problems for a class of neural networks with time-varying delay. By combining with some inequality technic or free-weighting matrix approach, the delay-dependent conditions have been proposed such that the filtering error system is globally asymptotically stable with guaranteed H_∞ performance. The time delay is divided into several subintervals; more information about time delay is utilized and less conservative results have been obtained. All results are expressed by the form of linear matrix inequalities, and the filter gain matrix can be determined easily by optimal algorithm. Examples and simulations have been provided to illustrate the less conservatism and effectiveness of the designed filter.*
Keywords: H_∞ filter design, Globally asymptotically stable, Linear matrix inequality (LMI), Neural networks, Time-varying delay

1. **Introduction.** The H_∞ filtering issue was introduced in [1] and its main aim is to design a signal estimator for given system such that L_2 gain of filtering error will be less than a prescribed level. Comparing with the traditional Kalman filtering, the H_∞ filtering can minimize the H_∞ norm of the transfer function between the noise and the estimation error, which has an advantage in dealing with external unknown noises. Since then, the H_∞ filtering technology has been extensively applied in diverse such as discrete-time system [2-4], fuzzy systems [5-7], Markvoian delay system [8,9], nonlinear stochastic system [10-12] and singular system [13].

At the same time, time delay is a natural phenomenon frequently encountered in various dynamic systems such as electronic, chemical systems, biological systems, economic and rolling mill systems, which is very often the main sources of instability, oscillation and poor performance. Therefore, H_∞ filtering research for systems with different time-delays such as independent-delay, dependent-delay, distributed-delay and discrete delay have attracted a number of researchers, and many important results have been reported, see [14-19] and the references therein.

For the time-varying delay case, it has been proved that delay-independent methods [14-16] are more conservative than delay-dependent methods, especially for small time-delays. Therefore, researchers have focused on designing the time-dependent H_∞ filters and the main concern is to reduce the conservatism of these conditions [20-25]. In the discussion above, a delay-partitioning approach was used to guarantee the reliable H_∞

filtering for discrete time-delay system with randomly occurred nonlinearities in [21], a switched Lyapunov function approach and free-weighted matrices were provided to guarantee H_∞ filtering for discrete time switched system with interval time-varying delay in [22], an average dwell time method was used to achieve filtering for discrete-time switched systems with constraint switching signal and interval time-varying delay in [23]. In order to overcome conservativeness, and void using both model transformation and bounding technique for cross term, a finite sum inequality approach has been proposed to research H_∞ filtering for the uncertain discrete-time system with time-delay in [24].

During the past several decades, many kinds of neural networks have been extensively investigated because of their successful applications in various scientific fields such as pattern recognition, image processing, associative memories, and fixed-point computations. A lot of research results about the stability analysis, passivity analysis, state estimation problems and H_∞ filtering for neural networks with time delay have been reported [25-28]. In the literatures discussions above, H_∞ and generalized H_2 filtering problem for delayed neural networks have been addressed in [28]; by employing a novel bounding technique and introducing slack variables, sufficient conditions such that the resulting filtering error system is globally exponentially stable with guaranteed H_∞ performance have been presented, and examples and simulations have shown the effectiveness of proposed method. While the conditions considered in [28], the limit to time derivative must be smaller than one or a positive constant, which does not allow the fast time-varying delay and will limit the application scope of results, see [31] and the references therein.

Motivated by the discussion above, in this paper, we investigate the delay-dependent H_∞ filter problem for a class of neural networks with time-varying delay. In order to reduce the possible conservatism, a new Lyapunov functional is constructed by using the delay decomposition idea [26]. By combining with some integral inequality technic [28] or free-weighting matrix approach [26,29], the time delay is divided into several subintervals, and more information about time delay is utilized and less conservatism has been obtained. The H_∞ filter is designed and H_∞ performance index γ is obtained by linear matrix inequalities (LMIs). Examples and simulations are presented to show the effectiveness and low conservatism of proposed methods.

The rest of this paper is organized as follows. The H_∞ filtering problems for delayed neural networks are formulated in Section 2. Sections 3 is dedicated to presenting delay-dependent criteria to ensure the existence of filters. Three examples and simulations are provided to illustrate the effectiveness and performance of the proposed approaches in Section 4, also some discussions and comparisons are given in Section 4. Finally, we draw some conclusions in Section 5.

Notation: Throughout this paper, if not explicit, matrices are assumed to have compatible dimensions. The notation $\mathrm{M} > (\geq, <, \leq) 0$ means that the symmetric matrix M is positive-definite (positive-semidefinite, negative, negative-semidefinite). $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix. The superscript "T" stands for the transpose of a matrix; the shorthand $diag\{\cdots\}$ denotes the block diagonal matrix; $\|\cdot\|$ represents the Euclidean norm for vector or the spectral norm of matrices. I refers to an identity matrix of appropriate dimensions, and $*$ means the symmetric terms. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2. **Problem Statement and Preliminaries.** Let us consider the following neural network with time delay and subject to noise disturbances delays:

$$\begin{cases} \dot{x}(t) = -Ax(t) + W_0 f(x(t)) + W_1 f(x(t-\tau(t))) + J + B_1 u(t), \\ y(t) = Cx(t) + Dx(t-\tau(t)) + B_2 u(t), \\ z(t) = Hx(t), \\ x(t) = \zeta(t), \quad \forall t \in [-2\tau, 0], \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector of the neural network associated with n neurons, $y(t) \in \mathbb{R}^m$ is the network measurement, $z(t) \in \mathbb{R}^p$, which needs to be estimated, is a linear combination of the states, and $u(t) \in \mathbb{R}^q$ is the noise input belongs to $L_2[0, \infty)$. $A = diag\{a_1, \dots, a_n\}$ with $|a_i| < 1$ $(i = 1, 2, \dots, n)$ is a diagonal matrix with positive entries. $f(x(t)) = [f(x_1(t)), f(x_2, (t)), \dots, f(x_n(t))]^T \in \mathbb{R}^n$ denotes the neuron activation function and $J = [J_1, J_2, \dots, J_n]^T \in \mathbb{R}^n$ is an external constant input vector. W_0, W_1 are the connection weight matrix and the delayed connection weight matrix, respectively. B_1, B_2, C, D and H are known real constant matrices with compatible dimensions. $\tau(t)$ denotes the transmission delay that satisfies

$$0 \le \tau(t) \le \tau, \quad -\mu \le \dot{\tau}(t) \le \mu, \tag{2}$$

where τ and μ are some scalars. $\zeta(t)$ is real-valued continuous initial condition on $[-2\tau, 0]$.

Remark 2.1. *In this paper, we assumed the time-varying delay is differentiable, and the time derivative can be positive or negative or zero, even bigger than one, so its result is more general than [28].*

Assumption 2.1. For $i \in \{1, 2, \dots, n\}$, $\forall x, y \in \mathbb{R}, x \neq y$, the neuron activation function $f_i(\cdot)$ is continuous, bounded and satisfies:

$$| f_i(x) - f_i(y) | \le l_i \mid x - y \mid, \tag{3}$$

with $L = diag\{l_1, l_2, \dots, l_n\}$.

In this paper, we consider the following full-order filter for the estimation of $z(t)$:

$$\begin{cases} \dot{\hat{x}}(t) = -A\hat{x}(t) + W_0 f(\hat{x}(t)) + W_1 f(\hat{x}(t-\tau(t))) + J + K(y(t) - \hat{y}(t)), \\ \hat{y}(t) = C\hat{x}(t) + D\hat{x}(t-\tau(t)), \\ \hat{z}(t) = H\hat{x}(t), \\ \hat{x}(0) = 0, \end{cases} \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state estimation, $\hat{y}(t) \in \mathbb{R}^m$, and $K \in \mathbb{R}^{n \times m}$ is the filter gain matrix to be determined.

Define the filter errors $e(t) = x(t) - \hat{x}(t)$ and $\tilde{z}(t) = z(t) - \hat{z}(t)$. Combining the neural networks (1) with (4), the filtering error system can be obtained as follows:

$$\begin{cases} \dot{e}(t) = -(A + KC)e(t) - KDe(t-\tau(t)) \\ \qquad\qquad +W_0\varphi(t) + W_1\varphi(t-\tau(t)) + (B_1 - KB_2)u(t), \\ \tilde{z}(t) = He(t), \\ \hat{x}(0) = 0, \end{cases} \tag{5}$$

where $\varphi(t) = f(x(t)) - f(\hat{x}(t))$, $\varphi(t-\tau(t)) = f(x(t-\tau(t))) - f(\hat{x}(t-\tau(t)))$.

The H_∞ filtering problem considered in this paper is now formulated as follows. Given a prescribed level of noise attenuation $\gamma > 0$, design a suitable filter such that the filtering error system (5) has a H_∞ performance γ for $\tau(t)$ satisfying:

(i) the error system (5) with $u(t) = 0$ is globally asymptotically stable and

(ii) the H_∞ performance $\|\tilde{z}(t)\|_2 < \gamma\|u(t)\|_2$ is guaranteed under zero-initial conditions for all nonzero $u(t) \in L_2[0,\infty)$, where $\|\tilde{z}(t)\|_2 = \sqrt{\int_0^\infty \tilde{z}^T(t)\tilde{z}(t)dt}$, and $\|u(t)\|_2 = \sqrt{\int_0^\infty u^T(t)u(t)dt}$.

At first, we give the following lemma which will be used frequently in the proof of our main results.

Lemma 2.1. [25] *For any constant symmetric positive defined matrix $J \in R^{m\times m}$, scalar η and the vector function $\nu : [0,\eta] \to R^m$, the following inequality holds:*

$$\eta \int_0^\eta \nu^T(s)J\nu(s)\mathrm{d}s \geq \left(\int_0^\eta \nu(s)\mathrm{d}s\right)^T J \left(\int_0^\eta \nu(s)\mathrm{d}s\right)$$

Lemma 2.2. [26] *For given proper dimensions constant matrix Φ_1, Φ_2 and Φ_3, where $\Phi_1^T = \Phi_1$ and $\Phi_2^T = \Phi_2 > 0$, we have $\Phi_1 + \Phi_3^T\Phi_2^{-1}\Phi_3 < 0$ such that only and only if*

$$\begin{bmatrix} \Phi_1 & \Phi_3^T \\ * & -\Phi_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Phi_2 & \Phi_3 \\ * & \Phi_1 \end{bmatrix} < 0$$

Lemma 2.3. [26] *For given function $\mu_1 \leq \dot{\tau}(t) \leq \mu_2$, there exist nonnegative function $\lambda_1(t) \geq$ and $\lambda_2(t) \geq$ satisfying $\lambda_1(t) + \lambda_2(t) = 1$, such that the following equation holds*

$$\dot{\tau}(t) = \mu_1\lambda_1(t) + \mu_2\lambda_2(t)$$

Lemma 2.4. [26] *For any real vectors a, b and any matrix $Q > 0$ with appropriate dimensions, it follows that $\pm 2a^Tb \leq a^TQa + b^TQ^{-1}b$.*

3. **Main Results.** In this section, firstly, we show the filtering error system (5) with $u(t) = 0$ is globally asymptotically stable. When $u(t) = 0$, the system (5) becomes the form as follows:

$$\begin{cases} \dot{e}(t) = -(A+KC)e(t) - KDe(t-\tau(t)) + W_0\varphi(t) + W_1\varphi(t-\tau(t)), \\ \tilde{z}(t) = He(t), \\ \hat{x}(0) = 0 \end{cases} \tag{6}$$

Then we have the following Theorem 3.1.

Theorem 3.1. *For given constant α, β, τ, ρ and diagonal matrices L_1, L_2, the error system (6) is said to be asymptotically stable, if there exist symmetric positive definite matrices $P > 0$, $Q_i > 0$ $(i = 1,2,3,4)$, $R > 0$, and two diagonal matrices S_1, S_2 such that the following linear matrix inequalities (LMIs) hold*

$$\begin{bmatrix} \Theta_i & \tau\Phi^T \\ \tau\Phi & \tau(-2P+\rho R) \end{bmatrix} < 0, \quad i = 1,2, \tag{7}$$

where $\Phi = [-(PA+YC)\ 0\ -YD\ 0\ 0\ PW_0\ PW_1]$,

$$\Theta(\dot{\tau}(t)) = \begin{bmatrix} \Theta_{11} & \frac{1}{\alpha\tau}R & -YD & 0 & 0 & \Theta_{16} \\ * & \Theta_{22} & \frac{1}{(1-\alpha)\tau}R & 0 & 0 & 0 \\ * & * & \Theta_{33} & \frac{1}{\beta\tau}R & 0 & 0 \\ * & * & * & \Theta_{44} & \frac{1}{(1-\beta)\tau}R & 0 \\ * & * & * & * & \Theta_{55} & 0 \\ * & * & * & * & * & \Theta_{66} \end{bmatrix}$$

$\Theta_{11} = -PA - A^TP - YC - C^TY^T + LS_1L + Q_1 - \frac{1}{\alpha\tau}R$, $\Theta_{16} = [PW_0\ PW_1]$,
$\Theta_{22} = (1-\alpha\dot{\tau}(t))(Q_2 - Q_1) - \frac{1}{\alpha\tau}R - \frac{1}{(1-\alpha)\tau}R$,
$\Theta_{33} = (1-\dot{\tau}(t))(Q_3 - Q_2) + LS_2L - \frac{1}{(1-\alpha)\tau}R - \frac{1}{\beta\tau}R$,

$\Theta_{44} = (1-(1-\beta))\dot{\tau}(t)(Q_4 - Q_3) - \frac{1}{(1-\beta)\tau}R - \frac{1}{\beta\tau}R,$
$\Theta_{55} = -Q_4 - \frac{1}{(1-\beta)\tau}R,$
$\Theta_{66} = diag\{Q_5 - S_1, -(1-\dot{\tau}(t))Q_5 - S_2\},$
where Θ_1 and Θ_2 are defined as: replacing $\dot{\tau}(t)$ in $\Theta(\dot{\tau}(t))$ by μ and $-\mu$ respectively. Moreover, the estimator gain matrix is given by $K = P^{-1}Y$.

Proof: Choose the following Lyapunov-Krasovskii functional candidate as:

$$V(t) = \sum_{i=1}^{3} V_i(t) \tag{8}$$

where

$$V_1(t) = e(t)^T P e(t),$$

$$V_2(t) = \int_{-\tau}^{0}\int_{t+s}^{t} \dot{e}^T(\theta)R\dot{e}(\theta)\mathrm{d}\theta\mathrm{d}s,$$

$$\begin{aligned} V_3(t) = &\int_{t-\alpha\tau(t)}^{t} e^T(s)Q_1e(s)\mathrm{d}s + \int_{t-\tau(t)}^{t-\alpha\tau(t)} e^T(s)Q_2e(s)\mathrm{d}s \\ &+ \int_{t-\delta(t)}^{t-\tau(t)} e^T(s)Q_3e(s)\mathrm{d}s + \int_{t-\tau}^{t-\delta(t)} e^T(s)Q_4e(s)\mathrm{d}s \\ &+ \int_{t-\tau(t)}^{t} \varphi^T(s)Q_5\varphi(s)\mathrm{d}s, \end{aligned}$$

where P, Q_k ($k = 1, 2, \ldots, 4,$) and R are positive definite matrices, $\delta(t) = \tau(t) + \beta(\tau - \tau(t))$, and $0 < \alpha < 1$, $0 < \beta < 1$.

Differentiating $V(t)$ with respect to t along error system (6), we can obtain that

$$\begin{aligned} \dot{V}_1(t) = &2e^T(t)P\dot{e}(t) \\ = &2e^T(t)P[-(A+KC)e(t) - KDe(x(t-\tau(t))) \\ &+ W_0\varphi(x(t)) + W_1\varphi(x(t-\tau(t)))] \end{aligned} \tag{9}$$

$$\begin{aligned} \dot{V}_2(t) = &\tau[-(A+KC)e(t) - KDe(x(t-\tau(t))) + W_0\varphi(t) + W_1\varphi(t-\tau(t))]^T R \\ &\times [-(A+KC)e(t) - KDe(x(t-\tau(t))) + W_0\varphi(t) + W_1\varphi(t-\tau(t))] \\ &- \int_{t-\alpha\tau(t)}^{t} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s - \int_{t-\tau(t)}^{\alpha\tau(t)} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s \\ &- \int_{t-\delta(t)}^{t-\tau(t)} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s - \int_{t-\tau}^{t-\delta(t)} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{V}_3(t) = &e^T(t)Q_1e(t) + (1-\alpha\dot{\tau}(t))e^T(t-\alpha\tau(t))(Q_2 - Q_1)e(t-\alpha\tau(t)) \\ &+ (1-\dot{\tau}(t))e^T(t-\tau(t))(Q_3 - Q_2)e(t-\tau(t)) \\ &+ (1-(1-\beta)\dot{\tau}(t))e^T(t-\delta(t))(Q_4 - Q_3)e(t-\delta(t)) \\ &- e^T(t-\tau)Q_4e(t-\tau) + \varphi^T(t)Q_5\varphi(t) \\ &- (1-\dot{\tau}(t))\varphi(t-\tau(t))^T(t)Q_5\varphi(t-\tau(t)) \end{aligned} \tag{11}$$

By virtue of Lemma 2.1, we can get that

$$\begin{aligned}-\int_{t-\alpha\tau(t)}^{t}\dot{e}^T(s)R\dot{e}(s)\mathrm{d}s &\le -\frac{1}{\alpha\tau(t)}\left(\int_{t-\alpha\tau(t)}^{t}\dot{e}(s)\mathrm{d}s\right)^T\times R\int_{t-\alpha\tau(t)}^{t}\dot{e}(s)\mathrm{d}s\\ &\le \frac{1}{\alpha\tau(t)}[e^T(t)-e^T(t-\alpha\tau(t))]R[e(t)-e(t-\alpha\tau(t))],\end{aligned}\tag{12}$$

$$\begin{aligned}-\int_{t-\tau(t)}^{t-\alpha\tau(t)}\dot{e}^T(s)R\dot{e}(s)\mathrm{d}s &\le -\frac{1}{(1-\alpha)\tau(t)}\left(\int_{t-\tau(t)}^{t-\alpha\tau(t)}\dot{e}(s)\mathrm{d}s\right)^T R\int_{t-\tau(t)}^{t-\alpha\tau(t)}\dot{e}(s)\mathrm{d}s\\ &\le -\frac{1}{(1-\alpha)\tau}[e^T(t-\alpha\tau(t))-e^T(t-\tau(t))]R[e(t-\alpha\tau(t))-e(t-\tau(t))],\end{aligned}\tag{13}$$

$$\begin{aligned}-\int_{t-\delta(t)}^{t-\tau(t)}\dot{e}^T(s)R\dot{e}(s)\mathrm{d}s &\le -\frac{1}{\alpha\tau(t)}\left(\int_{t-\delta(t)}^{t-\tau(t)}\dot{e}(s)\mathrm{d}s\right)^T R\int_{t-\delta(t)}^{t-\tau(t)}\dot{e}(s)\mathrm{d}s\\ &\le -\frac{1}{\beta\tau}[e^T(t-\tau(t))-e^T(t-\delta(t))]R[e(t-\tau(t))-e(t-\delta(t))],\end{aligned}\tag{14}$$

$$\begin{aligned}-\int_{t-\tau}^{t-\delta(t)}\dot{e}^T(s)R\dot{e}(s)\mathrm{d}s &\le -\frac{1}{(1-\beta)(\tau-\tau(t))}\left(\int_{t-\tau}^{t-\delta(t)}\dot{e}(s)\mathrm{d}s\right)^T R\int_{t-\tau}^{t-\delta(t)}\dot{e}(s)\mathrm{d}s\\ &\le -\frac{1}{(1-\beta)\tau}[e^T(t-\delta(t))-e^T(t-\tau)]R[e(t-\delta(t))-e(t-\tau)]\end{aligned}\tag{15}$$

In addition, it can be deduced from Assumption 2.1 that there exist two positive diagonal matrices S_1 and S_2 such that the following inequations hold:

$$\begin{aligned}\varphi^T(t)S_1\varphi(t) &= [f(x(t))-f(\hat{x}(t))]^T S_1[f(x(t))-f(\hat{x}(t))]\\ &\le e^T(t)LS_1Le(t)\end{aligned}\tag{16}$$

Therefore, we can obtain that

$$e^T(t)LS_1Le(t)-\varphi^T(t)S_1\varphi(t)\ge 0\tag{17}$$

By the same way, the following inequality can be obtained

$$e^T(t-\tau(t))LS_2Le(t-\tau(t))-\varphi^T(t-\tau(t))S_2\varphi(t-\tau(t))>0\tag{18}$$

By substituting (9)-(11) into $\dot{V}(t)$, adding the left side of (17)-(18) into the right side of $\dot{V}(t)$, then using (12)-(15), we can get that

$$\dot{V}(t)\le\xi^T(t)[\Theta(\dot{\tau}(t))+\tau\Omega^T R\Omega]\xi(t),\tag{19}$$

where

$$\Theta(\dot{\tau}(t))=\begin{bmatrix}\Theta_{11} & \frac{1}{\alpha\tau}R & -PKD & 0 & 0 & \Theta_{16}\\ * & \Theta_{22} & \frac{1}{(1-\alpha)\tau}R & 0 & 0 & 0\\ * & * & \Theta_{33} & \frac{1}{\beta\tau}R & 0 & 0\\ * & * & * & \Theta_{44} & \frac{1}{(1-\beta)\tau}R & 0\\ * & * & * & * & \Theta_{55} & 0\\ * & * & * & * & * & \Theta_{66}\end{bmatrix}$$

$\xi^T(t)=[e^T(t)\ e^T(t-\alpha\tau(t))\ e^T(t-\tau(t))\ e^T(t-\delta(t))\ e^T(t-\tau)\ \varphi^T(t)\ \varphi^T(t-\tau(t))]$,
$\Omega=[-(A+KC)\ 0\ -KD\ 0\ 0\ W_0\ W_1]$.

By Lemma 2.3, there exist nonnegative functions $\lambda_1(t)$ and $\lambda_2(t)$ satisfying $\lambda_1(t) + \lambda_2(t) = 1$ such that

$$\Theta(\dot{\tau}(t)) = \lambda_1(t)\Theta_1 + \lambda_2(t)\Theta_2 \tag{20}$$

where Θ_1 and Θ_2 are defined as: replacing $\dot{\tau}(t)$ in $\Theta(\dot{\tau}(t))$ by μ and $-\mu$ respectively.

Substituting (20) into (19), then (19) can be rewritten as

$$\dot{V}(t) \le \lambda_1(t)\xi^T(t)[\Theta_1 + \tau\Omega^T R\Omega]\xi(t) + \lambda_2(t)\xi^T(t)[\Theta_2 + \tau\Omega^T R\Omega]\xi(t). \tag{21}$$

Therefore, the following matrix inequality holds

$$\Theta_i + \tau\Omega^T R\Omega < 0, \quad i = 1, 2. \tag{22}$$

We can get that $\dot{V}(t) < 0$, which implies that error system (6) is asymptotically stable. By Lemma 2.2, (22) are equivalent to the following matrix inequality

$$\begin{bmatrix} \Theta_i & \tau\Omega^T R \\ \tau R\Omega & -\tau R \end{bmatrix} < 0, \quad i = 1, 2. \tag{23}$$

Pre- and post multiplying (23) by $diag\{I, I, I, I, I, I, PR^{-1}\}$ and $diag\{I, I, I, I, I, I, R^{-1} P\}$, respectively, we can easily obtain the following inequality:

$$\begin{bmatrix} \Theta_i & \tau\Omega^T P \\ \tau P\Omega & -\tau P^T R^{-1} P \end{bmatrix} < 0, \quad i = 1, 2. \tag{24}$$

By using the fact $-PR^{-1}P \le -2P + \rho R$ and introducing the new variable $PK = Y$, it is clear that LMIs (7) can guarantee the asymptotic stability of the error system (6). This completes the proof. □

Remark 3.1. *The new Lyapunov function proposed in Theorem 3.1 is based on the decomposition of delay interval $[-\tau, 0]$ into four subinterval, which are $[-\alpha\tau(t), 0]$, $[-\tau(t), -\alpha\tau(t)]$, $[-\delta(t), -\tau(t)]$ and $[-\tau, -\tau(t)]$.*

Remark 3.2. *In order to convert nonlinear matrix inequality into LMI, the fact $-PR^{-1}P \le -2\ P + \rho R$ is used in [27]. It can be concluded that the proposed method in [27] is more conservative than those in [29]. In Theorem 3.1, the fact $-PR^{-1}P \le -2\ P + R$ is used to convert (24) into LMIs. It is obvious that the adjustable parameter ρ is introduced in Theorem 3.1, which brings much flexibility in reducing the conservatism.*

Next, we will establish the H_∞ performance for the filtering error system (5).

For simplicity, firstly, we define the following matrix

$$\bar{\Theta}(\dot{\tau}(t)) = \begin{bmatrix} \bar{\Theta}_{11} & \bar{\Theta}_{12} & \bar{\Theta}_{13} & \bar{\Theta}_{14} & \bar{\Theta}_{15} & \bar{\Theta}_{16} \\ * & \bar{\Theta}_{22} & \bar{\Theta}_{23} & \bar{\Theta}_{24} & \bar{\Theta}_{25} & 0 \\ * & * & \bar{\Theta}_{33} & \bar{\Theta}_{34} & \bar{\Theta}_{35} & 0 \\ * & * & * & \bar{\Theta}_{44} & \bar{\Theta}_{45} & 0 \\ * & * & * & * & \bar{\Theta}_{55} & 0 \\ * & * & * & * & * & \bar{\Theta}_{66} \end{bmatrix}, \tag{25}$$

where
$\bar{\Theta}_{11} = -PA - A^TP - YC - C^TY^T + LS_1L + Q_1 + H^TH + M_1 + M_1^T$, $\bar{\Theta}_{12} = M_2^T - M_1 - N_1$,
$\bar{\Theta}_{22} = (1 - \alpha\dot{\tau}(t))(Q_2 - Q_1) - M_2 - M_2^T + N_2 + N_2^T$,
$\bar{\Theta}_{13} = -YD + N_1 + U_1 + M_3^T$, $\bar{\Theta}_{14} = M_4^T - U_1 + V_1$,
$\bar{\Theta}_{15} = M_5^T - V_1$, $\bar{\Theta}_{23} = -M_3^T + N_3^T - N_2 + U_2$,
$\bar{\Theta}_{24} = -M_4^T + N_4^T - U_2 + V_2$, $\bar{\Theta}_{25} = -M_5^T + N_5^T - V_2$,
$\bar{\Theta}_{33} = (1 - \dot{\tau}(t))(Q_3 - Q_2) + LS_2L - N_3 - N_3^T + U_3 + U_3^T$, $\bar{\Theta}_{34} = -M_4^T + U_4^T - U_3 + V_3$,
$\bar{\Theta}_{35} = -N_5^T + U_5^T - V_3$, $\bar{\Theta}_{16} = [PW_0\ PW_1\ PB_1 - YB_2)]$,
$\bar{\Theta}_{44} = (1 - (1 - \beta))\dot{\tau}(t)(Q_4 - Q_3) - U_4 - U_4^T + V_4 + V_4^T$, $\bar{\Theta}_{45} = -U_5^T + V_5^T - V_4$,

$\bar{\Theta}_{55} = -Q_4 - U_5 - U_5^T$,
$\bar{\Theta}_{66} = diag\{Q_5 - S_1, -(1-\dot{\tau}(t))Q_5 - S_2, -\gamma^2 I\}$.
So for the error system (5), we have the following Theorem 3.2.

Theorem 3.2. *For given constant α, β, τ, ρ and diagonal matrices L_1, L_2, the error system (5) is said to be asymptotically stable, if there exist symmetric positive definite matrices $P > 0$, $Q_i > 0$ $(i = 1, 2, 3, 4)$, $R > 0$ and two diagonal matrices S_1, S_2 such that the following linear matrix inequalities (LMIs) hold*

$$\begin{bmatrix} \bar{\Theta}_i & \tau\prod^T & \alpha\tau M^T & (1-\alpha)\tau N^T \\ * & \bar{\Theta}_{22} & 0 & 0 \\ * & * & -\alpha\tau R & 0 \\ * & * & * & -(1-\alpha)\tau R \end{bmatrix} < 0, \quad i = 1, 2, \tag{26}$$

$$\begin{bmatrix} \bar{\Theta}_i & \tau\prod^T & \beta\tau U^T & (1-\beta)\tau V^T \\ * & \bar{\Theta}_{22} & 0 & 0 \\ * & * & -\beta\tau R & 0 \\ * & * & * & -(1-\beta)\tau R \end{bmatrix} < 0, \quad i = 1, 2, \tag{27}$$

where
$\bar{\Theta}_{22} = \tau(-2P + \rho R)$,
$\prod = [-(PA + YC)\ 0\ -YD\ 0\ 0\ PW_0\ PW_1\ PB_1 - YB_2]$,
$\bar{\Theta}_1$ *and* $\bar{\Theta}_2$ *are defined as: replacing* $\dot{\tau}(t)$ *in* $\overline{\Theta}(\dot{\tau}(t))$ *by* μ *and* $-\mu$ *respectively.*

Proof: Choose the same Lyapunov functional as Theorem 3.1, and follow the same line to that of Theorem 3.1, we have

$$\begin{aligned} \dot{V}_1(t) &= 2e^T(t)P\dot{e}(t) \\ &= 2e^T(t)P[-(A+KC)e(t) - KDe(x(t-\tau(t))) \\ &\quad + W_0\varphi(x(t)) + W_1\varphi(x(t-\tau(t))) \\ &\quad + (B_1 - KB_2)u(t)] \end{aligned} \tag{28}$$

$$\begin{aligned} \dot{V}_2(t) &= \tau[-(A+KC)e(t) - KDe(x(t-\tau(t))) \\ &\quad + W_0\varphi(t) + W_1\varphi(t-\tau(t)) + (B_1 - KB_2)u(t)]^T R \\ &\quad \times [-(A+KC)e(t) - KDe(x(t-\tau(t))) \\ &\quad + W_0\varphi(t) + W_1\varphi(t-\tau(t)) + (B_1 - KB_2)u(t)] \\ &\quad - \int_{t-\alpha\tau(t)}^{t} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s - \int_{t-\tau(t)}^{\alpha\tau(t)} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s \\ &\quad - \int_{t-\delta(t)}^{t-\tau(t)} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s - \int_{t-\tau}^{t-\delta(t)} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s \end{aligned} \tag{29}$$

Using the Leibniz-Newton formular, for any appropriately dimensional matrices M, N, U, and V, we can obtain

$$2\bar{\xi}^T(t)M^T\left[e(t) - e(t-\alpha\tau(t)) - \int_{t-\alpha\tau(t)}^{t} \dot{e}(s)\mathrm{d}s\right] = 0, \tag{30}$$

$$2\bar{\xi}^T(t)N^T\left[e(t-\alpha\tau(t)) - e(t-\tau(t)) - \int_{t-\tau(t)}^{t-\alpha\tau(t)} \dot{e}(s)\mathrm{d}s\right] = 0, \tag{31}$$

$$2\bar{\xi}^T(t)U^T\left[e(t-\tau(t))-e(t-\delta(t))-\int_{t-\delta(t)}^{t-\tau(t)}\dot{e}(s)\mathrm{d}s\right]=0, \tag{32}$$

$$2\bar{\xi}^T(t)V^T\left[e(t-\delta(t))-e(t-\tau)-\int_{t-\tau}^{t-\delta(t)}\dot{e}(s)\mathrm{d}s\right]=0, \tag{33}$$

where
$\bar{\xi}^T(t)=[e^T(t)\ e^T(t-\alpha\tau(t))\ e^T(t-\tau(t))\ e^T(t-\delta(t))\ e^T(t-\tau)]$,
$M=[M_1^T\ M_2^T\ M_3^T\ M_4^T\ M_5^T]$, $N=[N_1^T\ N_2^T\ N_3^T\ N_4^T\ N_5^T]$,
$U=[U_1^T\ U_2^T\ U_3^T\ U_4^T\ U_5^T]$, and $V=[V_1^T\ V_2^T\ V_3^T\ V_4^T\ V_5^T]$.
By Lemma 2.4, we can get that

$$-2\bar{\xi}^T(t)M^T\int_{t-\alpha\tau(t)}^{t}\dot{e}(s)\mathrm{d}s\le\alpha\tau(t)\bar{\xi}^T(t)M^TR^{-1}M\bar{\xi}(t)+\int_{t-\alpha\tau(t)}^{t}\dot{e}^T(s)R\dot{e}(s)\mathrm{d}s, \tag{34}$$

$$\begin{aligned}-2\bar{\xi}^T(t)N^T\int_{t-\tau(t)}^{t-\alpha\tau(t)}\dot{e}(s)\mathrm{d}s\le{}&(1-\alpha)\tau(t)\bar{\xi}^T(t)N^TR^{-1}N\bar{\xi}(t)\\&+\int_{t-\tau(t)}^{t-\alpha\tau(t)}\dot{e}^T(s)R\dot{e}(s)\mathrm{d}s,\end{aligned} \tag{35}$$

$$-2\bar{\xi}^T(t)U^T\int_{t-\delta(t)}^{t-\tau(t)}\dot{e}(s)\mathrm{d}s\le\beta\tau(t)\bar{\xi}^T(t)U^TR^{-1}U\bar{\xi}(t)+\int_{t-\delta(t)}^{t-\tau(t)}\dot{e}^T(s)R\dot{e}(s)\mathrm{d}s, \tag{36}$$

$$\begin{aligned}-2\bar{\xi}^T(t)U^T\int_{t-\tau}^{t-\delta(t)}\dot{e}(s)\mathrm{d}s\le{}&(1-\beta)(\tau-\tau(t))\bar{\xi}^T(t)V^TR^{-1}V\bar{\xi}(t)\\&+\int_{t-\tau}^{t-\delta(t)}\dot{e}^T(s)R\dot{e}(s)\mathrm{d}s\end{aligned} \tag{37}$$

Define

$$J(t)=\int_0^\infty\left[\hat{z}^T(t)\hat{z}^T(t)-\gamma^2u^T(t)u(t)\right]\mathrm{d}t \tag{38}$$

Under the zero-initial condition, we can obtain that $V(t)\mid_{t=0}=0$ and $V(t)>0$, then for any nonzero $u(t)\in L_2[0,\infty)$, the following inequality hold:

$$\begin{aligned}J(t)&\le\int_0^\infty\left[\hat{z}^T(t)\hat{z}^T(t)-\gamma^2u^T(t)u(t)\right]dt+V(t)\mid_{t\to\infty}-V(t)\mid_{t=0}\\&=\int_0^\infty\left[\hat{z}^T(t)\hat{z}^T(t)-\gamma^2u^T(t)u(t)+\dot{V}(t)\right]dt\end{aligned} \tag{39}$$

By substituting (11), (28)-(33) into (39), utilizing (34)-(37), we can get

$$\begin{aligned}&\hat{z}^T(t)\hat{z}^T(t)-\gamma^2u^T(t)u(t)+\dot{V}(t)\\&\le\eta^T(t)[\bar{\Theta}(\dot{\tau}(t))+\tau\bar{\Omega}^TR\bar{\Omega}+\alpha\tau(t)M^TR^{-1}M\\&\quad+(1-\alpha)\tau N^TR^{-1}N+\beta\tau(t)U^TR^{-1}U\\&\quad+(1-\beta)(\tau-\tau(t))V^TR^{-1}V]\eta(t)\end{aligned}$$

$$
\begin{aligned}
&= \theta_1(t)\eta^T(t)[\bar{\Theta}(\dot{\tau}(t)) + \tau\bar{\Omega}^T R\bar{\Omega} \\
&\quad + \alpha\tau M^T R^{-1} M + (1-\alpha)\tau N^T R^{-1} N]\eta(t) \\
&\quad + \theta_2(t)\eta^T(t)[\bar{\Theta}(\dot{\tau}(t)) + \tau\bar{\Omega}^T R\bar{\Omega} \\
&\quad + \beta\tau U^T R^{-1} U + (1-\beta)\tau V^T R^{-1} V]\eta(t)
\end{aligned}
\tag{40}
$$

where $\bar{\Theta}(\dot{\tau}(t))$ is defined in (25), $\theta_1(t) = \frac{\tau(t)}{\tau} \geq 0$, $\theta_2(t) = \frac{\tau-\tau(t)}{\tau} \geq 0$,
$\eta^T(t) = [\bar{\xi}^T(t)\ \varphi^T(t)\ \varphi^T(t-\tau(t))\ u^T(t)]$,
$\bar{\Omega} = [-(A+KC)\ 0\ -KD\ 0\ 0\ W_0\ W_1\ B_1 - KB_2]$.
From Lemma 2.3, we can obtain the following equation

$$\bar{\Theta}(\dot{\tau}(t)) = \lambda_1(t)\bar{\Theta}_1 + \lambda_2(t)\bar{\Theta}_2, \tag{41}$$

where $\lambda_1(t) > 0$, $\lambda_2(t) > 0$ and $\lambda_1(t) + \lambda_2(t) = 1$.

Therefore, by using the same method as Theorem 3.1, we can get the following matrix inequalities

$$\bar{\Theta}_i + \tau\bar{\Omega}^T R\bar{\Omega} + \alpha\tau M^T R^{-1} M + (1-\alpha)\tau N^T R^{-1} N < 0, \quad i = 1, 2, \tag{42}$$

$$\bar{\Theta}_i + \tau\bar{\Omega}^T R\bar{\Omega} + \beta\tau U^T R^{-1} U + (1-\beta)\tau V^T R^{-1} V < 0, \quad i = 1, 2. \tag{43}$$

By Lemma 2.2, (42) and (43) are equivalent to the following matrix inequalities:

$$\begin{bmatrix} \bar{\Theta}_i & \tau\bar{\Omega}^T & \alpha\tau M^T & (1-\alpha)\tau N^T \\ * & -\tau R & 0 & 0 \\ * & * & -\alpha\tau R & 0 \\ * & * & * & -(1-\alpha)\tau R \end{bmatrix} < 0, \quad i = 1, 2, \tag{44}$$

$$\begin{bmatrix} \bar{\Theta}_i & \tau\bar{\Omega}^T & \beta\tau U^T & (1-\beta)\tau V^T \\ * & -\tau R & 0 & 0 \\ * & * & -\beta\tau R & 0 \\ * & * & * & -(1-\beta)\tau R \end{bmatrix} < 0, \quad i = 1, 2, \tag{45}$$

respectively.

Pre- and post multiply (44) and (45) by $diag\{I, I, I, I, I, I, I, I, PR^{-1}, I, I\}$ and $diag\{I, I, I, I, I, I, I, I, R^{-1}P, I, I\}$, respectively. By introducing the new variable $PK = Y$, then using the fact $-PR^{-1}P \leq -2P + \rho R$, (26) and (27), we can obtain that

$$\tilde{z}^T(t)\tilde{z}^T(t) - \gamma^2 u^T(t)u(t) + \dot{V}(t) < 0, \tag{46}$$

so we can get

$$J(t) = \int_0^\infty [\tilde{z}^T(t)\tilde{z}^T(t) - \gamma^2 u^T(t)u(t)]dt < 0, \tag{47}$$

and the error system (5) is said to be asymptotically stable with performance γ. This completes the proof.

Remark 3.3. *Recently, the free-weighting matrix approach is used in many literatures about time delay to reduce the conservatism, a lot of examples have proved that the free-weighting matrix approach is a good method in reducing the conservatism, see [26,30]. The free-weighting matrix approach is introduced in this article, so the less conservatism can be expected.*

4. **Numerical Example.** In this section, two numerical examples with simulation results have been provided to demonstrate the low conservatism of the proposed H_∞ filtering design approaches.

Example 4.1. *Consider the delayed neural network with parameters as follows [28]:*

$$A = \begin{bmatrix} 0.76 & 0 \\ 0 & 1.32 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 0.2 & -0.5 \\ -0.4 & 1.2 \end{bmatrix}, \quad W_1 = \begin{bmatrix} -0.5 & 0.2 \\ -0.2 & 0.5 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix},$$

$$B_2 = 0.2, \quad B_1 = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}, \quad J = \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0.5 & 0 \end{bmatrix}.$$

Choose the same neuron activation function given by $f(x) = 0.3(| x + 1 | - | x - 1 |)$ *with* $L = diag\{0.6, 0.6\}$.

Just like [28], we set noise disturbance $u(t) = 0.01e^{-0.0006t}\sin(0.02t)$, $t > 0$ *and* $\tau = 1$. *By using the Matlab LMI Control box, solving the LMIs (26) and (27), the filter gain matrix can be obtained as*

$$K = \begin{bmatrix} -1.0000 \\ 1.0000 \end{bmatrix}. \tag{48}$$

At the same time, the optimal H_∞ performance index $\gamma_{\min}$ can reach 0.0001. Compare with the H_∞ performance index $\gamma_{\min} = 0.8991$ in [28], it is obvious that our result has reduced the conservatism.

Then we set $\gamma = 0.8991$, $\alpha = 0.6$, $\rho = 0.5$, and $\beta = 0.5$, the allowable upper bounds of τ, which guaranteeing the error system (5) stable are listed in Table 1. From Table 1 we can see that in [28], the maximum value of time delay is 1.300. However, by Theorem 3.2 in our article, when we take $\mu = 1.2 > 1$, the maximum value of time delay can reach 1.5232; what is more, when increasing the value μ and decreasing the value ρ, we can get much bigger value of the τ. For example, when we set $\rho = 0.05$, $\beta = 0.8$, $\alpha = 0.2$, $\mu = 18$ and $\gamma = 3$, the allowable upper bounds of τ can reach 2.5585.

TABLE 1. Allowable upper bounds of τ with different values of μ

μ	0.2	0.5	0.8	1.2
[28]	1.300	1.300	1.300	1.300
Theorem 3.2	1.5345	1.5320	1.5276	1.5232

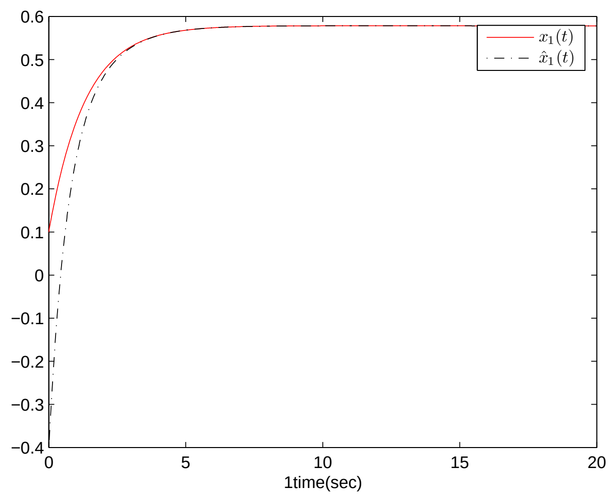

FIGURE 1. Response of the true $x_1(t)$ and its estimation $\hat{x}_1(t)$

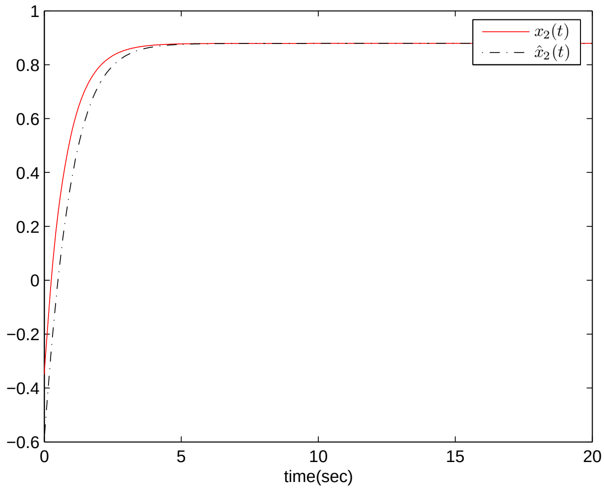

FIGURE 2. Response of the true $x_2(t)$ and its estimation $\hat{x}_2(t)$

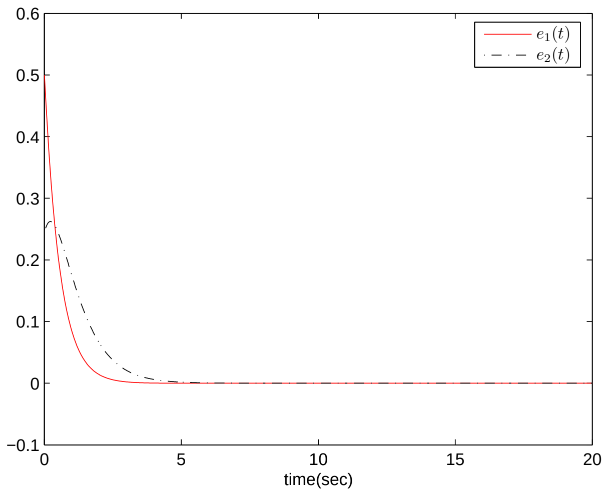

FIGURE 3. Response of the filtering error state $e(t)$ of H_∞

On the other hand, by choosing the parameters in Example 4.1, and using the Matlab software, we can get the state estimate and filtering error simulation results. The simulation results are shown in Figures 1-3, where Figure 1 and Figure 2 show the true states x_1, x_2 and their estimations, respectively, and Figure 3 shows the responses of the filtering error $e(t)$. The simulation results confirm the effectiveness of Theorem 3.2 for the H_∞ filtering design of the delayed neural networks.

Remark 4.1. *The introduction of parameter ρ is also an important technic to reduce the possible conservatism. What is more, the restrictive requirements that time derivative must be smaller than one or a positive constant are no longer needed.*

Remark 4.2. *In order to utilize the new methods, a new modified Lyapunov functional which including two adjusting parameters is constructed, and the delay derivative is considered and assumed to be bounded. When the lower bound of delay derivative is unknown* *or time-varying delay is not differentiable, the corresponding results are also given by using the modified Lyapunov functionals. The H_∞ filter is designed in terms of linear matrix inequalities (LMIs), which can be easily solved by the Matlab LMI toolbox.*

Example 4.2. *Consider a delayed neural network with parameters as follows [28]:*

$$A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\ W_0=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix},\ B_1=\begin{bmatrix}-0.1\\0.1\\0.1\end{bmatrix},\ W_1=\begin{bmatrix}0.1&0&0.2\\0&-0.1&1\\-3&-0.5&0\end{bmatrix},$$

$$H=\begin{bmatrix}1&-1&0\\0&-1&0\\0&0&1\end{bmatrix},\ C=\begin{bmatrix}1&0&0\end{bmatrix},\ D=\begin{bmatrix}1&1&0\end{bmatrix},\ B_2=0.1,\ J=\begin{bmatrix}0.5\\-1\\-0.2\end{bmatrix}.$$

The neuron activation function is of the form $f(x)=tanh(x)$ with $L=I$ and $\tau=0.1$. When we set $\rho=0.5$, $\beta=0.4$, $\alpha=0.8$, and $\mu=2.2$. By the Matlab LMI Control Toolbox, we find the solution to the LMIs (26) and (27) and get the the filter gain matrix as follows:

$$K=\begin{bmatrix}2.6930\\-1.3041\\-0.1824\end{bmatrix}$$

with the optimal performance H_∞ index $\gamma_{\min}=1.199$. Compare with $\gamma_{\min}=6.9402$ in [28], our result has been improved greatly.

Remark 4.3. *In [28], the slack variables are introduced to reduce the conservatism, but the time derivative introduced must be smaller than one or a positive constant, which does not allow the fast time-varying delay and will limit the application scope of results, while in our article, the free weight matrix and inequality technic are used, when we take $\mu=2.2$ and $\mu=-2.2$, respectively, a less conservative results have been obtained, and the time derivative that must be smaller than one is no longer needed.*

5. Conclusions.

In this paper, a more effective Lyapunov functional has been developed to investigate the H_∞ filtering problems for a class of neural networks with time delay. By combining with some inequality technic or free-weighting matrix approach, the delay-dependent conditions have been proposed such that the filtering error system is globally asymptotically stable with guaranteed H_∞ performance. It has been also shown that the filter gain matrix can be determined by solving LMIs. Finally, two examples and simulations have been provided to illustrate the effectiveness and low conservatism of the designed filter.

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