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ACTUARIALLY FAIR DEPOSIT INSURANCE PREMIUM DURING A FINANCIAL CRISIS: A BARRIER-CAPPED BARRIER OPTION FRAMEWORK

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Abstract. *This paper examines loan-risk sensitive deposit insurance premiums with explicitly treating the early problems of bank lender and firm borrower in a financial crisis. In a bank-barrowing firm situation, the lending function creates the need to model bank equity value as a borrower-barrier-capped barrier option. This paper presents the following results. (i) A high barrier of the borrowing firm as well as the bank's barrier reduce bank interest margin, i.e., the spread between the loan rate and the deposit rate. Capped barrier as such makes the bank more prone to risk-taking. (ii) Deposit insurance increases the bank interest margin, implying that the insurance policy is effective to reduce bank risk-taking. (iii) Either the barrier increases the actuarially fair deposit insurance premium, indicating that the premium should be increased not only by the bank's barrier but also by the capped barrier from the borrowing firm. (iv) A good quality management of the bank will share the insurer's liabilities when the barrier of the borrowing firm increases.*

Keywords: Bank interest margin, Deposit insurance premium, Barrier, Capped-barrier

1. Introduction.

The recent financial crisis has been widely regarded as the worst financial crisis since the Great Depression. To prevent bank runs, deposit insurance schemes were extended in more than 20 countries, with coverage limits on deposits being raised considerably [1]. This may be understood that the goal of achieving and maintaining the soundness of financial institutions and markets has become a top priority for policymakers, and deposit insurance has been a part of safety nets in most countries [2]. Previous research on market-based evaluation of deposit insurance premiums has modeled the bank as a corporate firm with risky assets and insured liabilities and argues that deposit insurance is responsible for the increased risk taking activity in banks arising moral hazard.[1] However, Ivashina and Scharfstein [6] show that new loans to large corporate borrowers fell by 47% during the peak period of the financial crisis (fourth quarter of 2008) relative to the prior quarter and by 79% relative to the peak of the credit boom (second quarter of 2007). The ongoing argument in the literature concerning bank lending risks warrants an assessment of the extent to which deposit insurance affects bank risk-taking when both the bank and the borrowing firm face the problems of early closure during a financial crisis.

Lending operations involves acquiring costly information about an opaque corporate borrower in particular during a period of financial crisis. The bank anticipates credit risk

[1]See, for example, Garcia [3], Jeitschko and Jeung [4], and Davis and Obasi [5].

compensation from the corporate borrower. Under the circumstances, deposit insurance premium should be priced based on an explicit treatment the risk characteristics of bank loans related to the corporate borrower's equity return and operational risk. The broader contingent claims approach has found a natural application in pricing deposit insurance contracts on an actuarially fair basis, the premium that exactly covers the expected cost to the deposit insurance provider. In Merton [7], Ronn and Verma [8], and Episcopos [9], the liability to the Federal Deposit Insurance Corporation (FDIC) is a European put option written on the assets of the bank. However, these papers imply that the deposit insurer cannot opt for bank closure until the expiration of the insurance period. Acharya and Dreyfus [10], Mazumdar [11] Episcopos [12], for example, use path dependent, barrier options in some form to address the problem of early bank closure. In these papers on market-based estimates, the role of corporate borrower in distress is not modeled explicitly. The purpose of this paper is to model bank lending by creating the need to value bank equity as a "barrier-capped" barrier option and calculate loan-risk sensitive premiums. As we discuss further below, the barrier-capped setting can be motivated based on an argument of the explicit treatment of credit risk caused by the problem of early borrowing-firm closure in the spirit of Dermine and Lajeri [13], while the barrier setting can be motivated based on the problem of early bank closure in the spirit of Episcopos [12].

Our objective is to make a number of significant contributions to the literature as a result of the following expansion in methodology and scope. First, in regards to method, we propose for the liability of the deposit insurer by introducing a new framework not previously used in this context. We model an actuarially fair deposit insurance premium in the barrier option valuation outlined in Episcopos [12], explicitly integrating the risk characteristics of bank assets related to the problem of early borrowing-firm closure. This model is specifically designed to address early bank and borrower closure problems during a financial crisis period induced by a higher probability of hitting the bank's barrier as well as the borrowing-firm barrier before the expiration date. In regards to the scope, we propose a framework for barrier-capped barrier option valuation to price an actuarially fair deposit insurance premium with bank interest margin determination. The bank interest margin, that is the spread between the loan rate and the deposit rate, is one of the principal elements of bank net cash flows and earnings. Indeed, the bank interest margin is often used in the literature as a proxy for the efficiency of financial intermediation (see, for example, [14,15]). Therefore, identifying the determinants of bank interest margin explicitly linked to risk characteristics of a bank-borrowing firm situation would help to understand changing in bank efficiency and provide deposit insurance policy implications for the banking environment.

The results of this paper show how the capped barrier of the borrowing firm, the bank's barrier, and deposit insurance premium jointly determine the optimal bank interest margin. Based on the barrier-capped barrier option valuation on bank equity, we further model market-based estimates of deposit insurance premium for policy application purposes. We show that an increase in the capped barrier from the borrowing firm or the barrier from the bank increases the bank's lending scale at a reduced interest margin. Barriers as such make the bank more prone to risk-taking, thereby adversely affecting the stability of the banking system. An increase in the deposit insurance decreases the bank's lending scale at an increased margin. Deposit insurance as such makes the bank less prone to risk-taking. We also show that an increase in either the barrier increases the market-based estimates of deposit insurance premium. However, an increase in the firm's capped barrier decreases this premium when the bank has a high quality management, which partially absorbs the insurer's liability.

The recent financial crisis and the subsequent economic downturn have clearly shown the importance of banking stability. Deposit insurance has been widely adopted by countries to aid the stability of their banking systems since deposit insurance reduces self-fulfilling or information-driven depositor runs. However, the problem of early closure related to bank lending management makes the bank less prudent and more prone to risk-taking that adversely affects the stability of the banking system. This gives a rationale for bank regulation and supervision. We argue that the extent of early closure problems in the bank-borrowing firm situation is a key determinant of efficient deposit insurance premiums, and suggest that Basel III for a solution for lending and liquidity management goes into that direction.

The remaining parts of this paper are organized as follows. Section 2 briefly discusses related literature. Section 3 lays out the basic model of a bank-borrowing firm situation. Section 4 characterizes the optimal bank interest margin and the actuarially fair deposit insurance premium and further develops the comparative static results of the model. Section 5 presents a numerical analysis of the barrier effect on the market-based estimation of deposit insurance premiums. The final section concludes the paper.

2. **Related Literature.** Our theory of regulatory deposit insurance is related to four strands of the literature. The first is the literature on bank interest margins. The prevailing approach to analyzing bank interest margins has been the dealership model originated by Ho and Saunders [16]. Recent related studies include, for example, Maudos and de Guevara [17], Williams [18], Hawtrey and Liang [19] and Kasman et al. [14]. By contrast, Slovin and Sushka [20] use a firm-theoretical approach to examine the optimal bank interest margin. Related literature includes, for example, Zarruk and Madura [21], Wong [22,23], Lin [24], Lin and Jou [25] and Memmel and Schertler [15]. While we also examine the bank interest margin, our focus on the margin management under deposit insurance based on path-dependent, barrier-capped barrier option models shifts our analysis in a different direction.

The second strand is the lending risk treatment literature. Angbazo [26] incorporates the risk of loan defaults in the model, providing evidence of positive relationship between bank interest margin and the default risk. A result of Saunders and Schumacher [27] indicates that bank interest margin is affected by the volatility of interest rate. Maudos and de Guevara [17] show that interest rate, credit risk, and management quality are positively related to bank interest margin. Hawtrey and Liang [19] document the negative impact of managerial efficiency on bank interest margin, including risk aversion and interest rate volatility. The primary difference between our model and these papers is that we consider the effects of capped credit risk from the borrowing firm affecting the distribution of bank asset returns and do not assume managerial effort aversion.[2]

The third strand is the literature on bank equity valuation related to pricing of deposit insurance. The seminal work by Black and Scholes [28] on the valuation of options has led to an application in the banking literature of deposit insurance pricing [29-31]. In these papers, the equity of the bank is viewed as a call option on the bank's assets, and the deposit insurance liability is viewed as a put option. Davis and Obasi [5] also adopt an option-based approach and suggest that deposit insurance mainly affects bank risk through its relationship with profitability and asset quality. However, Bhattacharya et al. [32], and Episcopos [12] point out that the idea of early closure has deep roots in the banking crisis of the 1981-1993 period, during which protecting the insurance funds and the stability of the banking system are dominant issues. We propose a framework for

[2]Dermine and Lajeri [13] also consider credit risk affecting the distribution of bank asset returns; however, they assume that the loan market structure faced by the bank is perfectly competitive.

bank equity valuation based on a barrier-capped barrier option model and examine bank interest margin and deposit insurance relationship.

The fourth strand of the literature to which our work is most directly related is that on conformity, particularly Maudos and de Guevara [17] and Davis and Obasi [5]. Other examples are Williams [18] and Hawtrey and Liang [19] concerning bank spread behavior, and Demirguc-Kunt and Detragiache [33] and Demirguc-Kunt et al. [2] concerning deposit insurance premium issues. The fundamental insight shared by these papers is that conformity is generated by a desire to distinguish oneself from the types whose attributes are not so desirable. This insight is an important aspect of bank interest margin management under deposit insurance as well, since both bank managers and regulators agree with the decisions to avoid being identified as untalented in estimating bank value related to deposit insurance policy. What distinguishes our work from the literature is our focus on the commingling of the assessment of firm investment with the assessment of bank lending and, in particular, the emphasis we put on the interaction between bank interest margin and conformity in the context of barrier and deposit insurance.

3. The Framework.

The framework is designed to capture the following characteristics of a bank-borrowing firm situation: (i) both the firm and the bank make decisions in a single period horizon with two dates, 0 and 1, $t \in [0,1]$; (ii) at $t = 0$, the firm is funding an investment with the bank's loans and its equity capital; (iii) at $t = 0$, the bank funds the loans and the risk-free liquid assets with insured deposits and equity capital; (iv) both the firm's assets and the bank's loans are subject to non-performance; so (v) we propose a framework for equity valuation based on a path-dependent option model. Note that we model such non-performance not only by the firm's expected investment returns as well as early closure problem but also by the bank's expected loan repayments from the borrowed firm as well as the possible early closure problem from its lending strategy. Accordingly, (ii) and (iii) imply that the model will have to incorporate two barrier options in which the lending function of the bank creates the need to model bank equity as a barrier-capped barrier option due to the explicit treatment of the borrowing firm's uncertainty.[3]

At $t = 0$, the firm has the following balance sheet:[4]

$$A = L + E \tag{1}$$

where $A > 0$ is the amount of invested assets, $L > 0$ is the quantity of the loans borrowed from the bank, and $E > 0$ is the stock of equity capital.

To address the problem of early closure, we adopt the path-dependent, barrier option formula of Merton [34] because equity can be knocked out whenever a legally binding barrier is breached. A direct implication of this formula is that firm equity will be priced as a down-and-out call (DOC) option [35]. According to Equation (1), the market value of firm equity is viewed as a DOC option on the market value of firm asset, $V_A = (1 + R_A)A > 0$ where R_A is the expected rate of return from the investment amount A. The market value of the investment is assumed to follow a geometric Brownian motion (GBM). Specifically, the dynamics of the firm's asset value follows:

$$dV_A = \mu_A V_A dt + \sigma_A V_A dW_A \tag{2}$$

where μ_A and σ_A are, respectively, the expected return and volatility of V_A, and W_A is a Wiener process. By Equation (2), one can express the closed form solution for the value

[3] It is well recognized that the capped barrier option model proposed by this paper becomes a capped call option model if the standard call framework for the bank's equity valuation is a good representation of reality due to superior quality of bank management. However, the explicit treatment of management quality is ignored in our model for simplicity.

[4] Dermine and Lajeri [13] develop a model of a firm with exactly this structure.

of DOC option as:[5]

$$S_A = [V_A N(a_1) - V_B e^{-R_L} N(a_2)] - \left[V_A \left(\frac{H_A}{V_A}\right)^{2\eta_A} N(b_1) - V_B e^{-R_L} \left(\frac{H_A}{V_A}\right)^{2\eta_A^{-2}} N(b_2) \right] \quad (3)$$

where

$$V_B = (1 + R_L)L$$
$$a_1 = \frac{1}{\sigma_A}\left(\ln \frac{V_A}{V_B} + R_L + \frac{\sigma_A^2}{2}\right), \quad a_1 = a_2 - \sigma_A$$
$$H_A = a_A V_B, \quad 0 \le a_A \le 1, \quad \eta_A = \frac{R_L}{\sigma_A^2} + \frac{1}{2}$$
$$b_1 = \frac{1}{\sigma_A}\left(\ln \frac{H_A^2}{V_A V_B} + R_L + \frac{\sigma_A^2}{2}\right), \quad b_2 = b_1 - \sigma_A$$

and where $V_B \equiv$ the book value of the firm's liabilities required on the discount loan rate issued by the bank and due at $t = 1$, where $R_L > 0$ is the loan rate set by the bank, $H_A \equiv$ the default barrier level, which is assumed to be proportional to the firm's liabilities by a barrier-to-debt ratio $\alpha_A(H_A = \alpha_A \alpha_B)$,[6] and $N(\cdot) \equiv$ the cumulative distribution function for a standard normal random variable.

The first term $[\cdot]$ on the right-hand side of Equation (3) is interpreted as the expected asset value and present value of the liability payment using the standard call option view of the firm. The barrier H_A can be viewed as the value of firm asset above which creditors cannot force dissolution. The omission of terms involving the barrier in the second term $[\cdot]$ of Equation (3) will have significant consequences especially when the likelihood of meeting the barrier is substantial. A related barrier option is the down-and-in call (DIC) that is activated only if the barrier is breached, denoted by the second term $[\cdot]$ of Equation (3). In the firm context, the DIC offers protection to bondholders by allowing them to "call in their chips" before asset value deteriorates further.

Using information about Equation (3), we apply Episcopos [12] and specify the value of the firm's limited liability when the firm defaults as:

$$\begin{aligned} P_A = V_B e^{-R_L} &- [V_B e^{-R_L} N(-a_2) - V_A N(-a_1)] \\ &+ \left[V_A \left(\frac{H_A}{V_A}\right)^{2\eta A} N(b_1) - V_B e^{-R_L} \left(\frac{H_A}{V_A}\right)^{2\eta A} N(b_2) \right] \end{aligned} \quad (4)$$

The first two terms on the right-hand side of Equation (4) represent the Merton [36] value of debt. The first term is the discounted value of the payment to the bank. The second term is the put value or the value of the fair payment needed in order to make payment risk free. The third term is the Brockman and Turtle [35] innovation on the standard contingent claims model and represents the value of the DIC. The bank would cash in on this option if it was able to jointly seize the assets of the firm when the firm's assets dropped to H_A.

At $t = 0$, the bank has the following balance sheet:[7]

$$L + B = D + K \quad (5)$$

[5]The DOC option valuation in general includes an additional term of the rebate paid to the firm's owners if the asset value reaches its barrier level. We follow Episcopos [12] and assume that the rebate is equal to zero for simplicity.

[6]The treatment of a barrier that is an exponential function of time is considered in Merton [34]. For simplicity, we follow Brockman and Turtle [35] and consider only the case of a constant barrier.

[7]Dermine and Lajeri [13] develop a model of a bank with exactly this structure.

where $L > 0$ is the amount of loans, $B > 0$ is the quantity of risk-free liquid assets, $D > 0$ is the volume of deposits, and $K > 0$ is the stock of equity capital.

The bank's loans belong to a single homogeneous class of fixed rate claims that mature at $t = 1$. The demand for loans is expressed as a downward-sloping demand function $L(R_L)$ with the conditions of $\partial L/\partial R_L < 0$ and $\partial^2 L/\partial R_L^2 < 0$ [22], where $R_L > 0$ is the loan rate set by the bank. Loans are risky in that they are subject to possible non-performance of the borrowing firm. The amount B of liquid assets held by the bank earns the security-market interest rate of $R > 0$. The assumption of $R_L > R$ meets a scope for asset substitution [37]. The total assets to be financed at $t = 0$ are $L + B$. They are financed partly by demandable deposits. At $t = 0$, the bank accepts D dollars of deposits. The bank is fully insured by the FDIC and it pays an insurance premium of $P > 0$ per dollar of deposits [21]. The bank provides depositors with a rate of return equal to the risk-free deposit interest rate $R_D > 0$ [38]. Equity capital K held by the bank is tied by regulation to be a fixed proportion q of the bank's deposits $K \geq qD$ where q is the required capital-to-deposits ratio [39]. The assumption of $R > R_D$ captures a scope for capital binding and thus Equation (5) can be restated as the form of $L + B = K(1/q + 1)$ [22].

Using information about the setting above, we also adopt the barrier option formula of Merton [34] as a tool to value the value of the bank's equity. In this context, the market value of the bank's equity is viewed as a DOC option on the market value of bank loan repayments V where the value is assumed to follow a GBM of the form:

$$dV = \mu V d_t + \sigma V dW \tag{6}$$

where μ and σ are, respectively, the expected return and volatility of V, and W is a Wiener process. The market value of the bank's equity S can be written as:[8]

$$S = [VN(c_1) - Ze^{-\delta}N(c_2)] - \left[V\left(\frac{H}{V}\right)^{2\eta} N(d_1) - Ze^{-\delta}\left(\frac{H}{V}\right)^{2\eta-2} N(d_2)\right] \tag{7}$$

where

$$Z = (1 + R_D)D + PD - (1 + R)B = (1 + R_D + P)\frac{K}{q} - (1 + R)\left[K\left(\frac{1}{q} + 1\right) - L\right]$$

$$\delta = R - R_D - P > 0$$

$$c_1 = \frac{1}{\sigma}\left(\ln\frac{V}{Z} + \delta + \frac{\sigma^2}{2}\right), \quad c_2 = c_1 - \sigma$$

$$H = \alpha Z, \quad 0 \leq \alpha \leq 1, \quad \eta = \frac{\delta}{\sigma^2} + \frac{1}{2}$$

$$d_1 = \frac{1}{\sigma}\left(\ln\frac{H^2}{VZ} + \delta + \frac{\sigma^2}{2}\right), \quad d_2 = d_1 - \sigma$$

and where $Z \equiv$ the promised future net-obligation payments required on the discount rate of δ, where the value of the net-obligation payments is the difference between the payments to depositors and the FDIC and the repayments from the liquid-asset investment, and $H \equiv$ the default barrier level, which is specified to be proportional to the bank's net obligations by a barrier-to-debt ratio, $H = aZ$. The interpretation of Equation (7) follows a similar argument as in the case of Equation (3). The firm term $[\cdot]$ on the right-hand side of Equation (7) can be identified as the standard call option value while the second term $[\cdot]$ can be identified as the DIC option value.

[8] Again, the rebate term in the DOC valuation is ignored in Equation (6) for the sake of simplicity.

Based on the settings of Equations (4) and (7), the bank-borrowing firm situation is captured by the value of the bank's loan repayments from the borrowing firm who funds an investment with bank loan, and also by the default barrier level faced by the bank partly caused by the barrier faced by the firm. In this case, the market value of the bank's underlying assets V in Equation (7) is specified as the capped form by $P_A : V = (1 + R_L)L - P_A$. The case analyzed in the naked DOC option valuation, when the term P_A in V is ignored, is a limit case of the capped DOC option valuation.

In this model, the liability of the deposit insurer (the FDIC) is modeled as a capped put on the asset of the borrowing firm. The implication of the insurance liability occurs because the capped put on the asset of the bank will only be exercised when the bank defaults, that is when the borrowing firm defaults and hands the borrowing firm's asset V_A to the bank [13]. The liability of the deposit insurer is valued as follows:[9]

$$\begin{aligned} Put = {} & Ze^{-\delta} - \left[Ze^{-\delta}N(-c_2) - VN(-c_1)\right] \\ & + \left[V\left(\frac{H}{V}\right)^{2\eta} N(d_1) - Ze^{-\delta}\left(\frac{H}{V}\right)^{2\eta-2} N(d_2)\right] \end{aligned} \tag{8}$$

The interpretation of Equation (8) follows a similar argument as in the case of Equation (4).

4. **Solution and Comparative Static Results.** The bank's objective is to set R_L to maximize the market value of the bank's equity, denoted by Equation (7), subject to $V = (1 + R_L)L - P_A$. Partially differentiating Equation (7) with respect to R_L, the first-order condition is given by:

$$\begin{aligned} \frac{\partial S}{\partial R_L} = {} & \left[\frac{\partial V}{\partial R_L}N(c_1) + V\frac{\partial N(c_1)}{\partial c_1}\frac{\partial c_1}{\partial R_L} - \frac{\partial Z}{\partial R_L}e^{-\delta}N(c_2) - Ze^{-\delta}\frac{\partial N(c_2)}{\partial c_2}\frac{\partial c_2}{\partial R_L}\right] \\ & - \left[\frac{\partial V}{\partial R_L}\left(\frac{H}{V}\right)^{2\eta} N(d_1) + V(2\eta)\left(\frac{H}{V}\right)^{2\eta-1}\left(\frac{1}{V}\frac{\partial H}{\partial R_L} - \frac{H}{V^2}\frac{\partial V}{\partial R_L}\right)N(d_1)\right. \\ & + V\left(\frac{H}{V}\right)^{2\eta}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial R_L} - \frac{\partial Z}{\partial R_L}e^{-\delta}\left(\frac{H}{V}\right)^{2\eta-2} N(d_2) \\ & - Ze^{-\delta}(2\eta - 2)\left(\frac{H}{V}\right)^{2\eta-3}\left(\frac{1}{V}\frac{\partial H}{\partial R_L} - \frac{H}{V^2}\frac{\partial V}{\partial R_L}\right)N(d_2) \\ & \left. - Ze^{-\delta}\left(\frac{H}{V}\right)^{2\eta-2}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial R_L}\right] = 0 \end{aligned} \tag{9}$$

A sufficient condition for an optimum is that $\partial^2 S/\partial R_L^2 < 0$. In Equation (9), the first term $[\cdot]$ can be interpreted as the marginal standard call option value of loan rate, while the second term $[\cdot]$ can be interpreted as the marginal DIC option value. The optimal loan rate is set for the equity maximization in Equation (7). We can further substitute the optimal loan rate to obtain the deposit insurance premium in Equation (8) staying on the optimization.

The optimal bank interest margin is given by the difference between the optimal loan rate obtained from the equilibrium condition of Equation (9) and the market deposit rate. Since the deposit rate is not a choice variable of the bank, examining the impact of parameters on the optimal bank interest margin is tantamount to examining that on the

[9] Dermine and Lajeri [13] develop a model of the deposit insurer's liability with exactly this structure. However, their analysis is limited to a perfectly competitive loan market.

optimal loan rate. Consider next the impact on the bank interest margin from changes in the firm barrier-to-debt ratio, the bank barrier-to-debt ratio, and deposit insurance premium. Implicit differentiation of Equation (9) with respect to α_A, α, and P yields, respectively:

$$\frac{\partial R_L}{\partial \alpha_A} = -\frac{\partial^2 S}{\partial R_L \partial \alpha_A} \Big/ \frac{\partial^2 S}{\partial R_L^2} \tag{10}$$

$$\frac{\partial R_L}{\partial \alpha} = -\frac{\partial^2 S}{\partial R_L \partial \alpha} \Big/ \frac{\partial^2 S}{\partial R_L^2} \tag{11}$$

$$\frac{\partial R_L}{\partial P} = -\frac{\partial^2 S}{\partial R_L \partial P} \Big/ \frac{\partial^2 S}{\partial R_L^2} \tag{12}$$

We further consider the impact on the actuarially fair deposit insurance premium from changes in the firm barrier-to-debt ratio and the bank barrier-to-debt ratio. Differentiating Equation (8) evaluated at the optimal loan rate with respect to α_A, and α yields:

$$\frac{dPut}{d\alpha_A} = \frac{\partial Put}{\partial \alpha_A} + \frac{\partial Put}{\partial R_L}\frac{\partial R_L}{\partial \alpha_A} \tag{13}$$

$$\frac{dPut}{d\alpha} = \frac{\partial Put}{\partial \alpha} + \frac{\partial Put}{\partial R_L}\frac{\partial R_L}{\partial \alpha} \tag{14}$$

The importance of the insurer contingent asset related to bank spread behavior is emphasized. Investors and banks are interested in the relationship between the implied FDIC closure policy and the value of their operations and investments. Insurers can study that relationship for more effective supervision in order to prevent bank failure. The same relationship would be useful to bank decision makers and policy groups to be more effective in communicating arguments related to deposit insurance. Therefore, quantifying the role of the barrier on bank contingent claims is important. Equations (10)-(12) and (13)-(14) demonstrate how responsive bank interest margin and deposit insurance pricing are to the capped barrier parameters in order to appreciate the significance of such barriers. The comparative static analyses with the added complexity of path dependents options do not always lead to clean-out results. Numerical analyses of Equations (10)-(14) will be presented in the following section to demonstrate economic intuition.

5. **Numerical Exercises.** Starting from a set of assumptions of $R = 3.50\%$, $R_D = 2.50\%$, $D = 300$, $q = 8.00\%$, $\sigma_A = 0.20$, we first calculate Equation (10).[10] In the first case reported in Table 1, we set P equal to a fixed premium of 0.23%, where the premium per dollars of deposits is imposed on U.S. commercial banks [39], and $\alpha = 0.44$. We let $(R_L\%, L)$ fluctuate between (4.5, 300) and (5.1, 279) due to $\partial R_L/\partial L < 0$ and $\partial^2 R_L/\partial L^2 < 0$ as mentioned previously, and let α_A increase from 0.50 to 0.90.[11] In addition, we let $(R_A\%, A) = (R_A\%, L + E)$ with $E = 20$ change from (4.8, 320) to (5.4, 299), where we assume $R_A > R_L$ due to borrowing cost consideration of the firm. The horizontal axis of the bundles of $(R_L\%, L)$ and $(R_A\%, A)$ indicates that (i) the firm uses total borrowed funds plus equity capital for its investment, for example, $320 = 300 + 20$ in the first column, and $319 = 299 + 20$ in the second column, and (ii) the loan demand faced by the bank is derived from the firm's expected returns of investment, that implies

[10] The mean value of asset volatility is 0.2904 with a corresponding standard deviation of 0.2608 in the empirical findings of Brockman and Turtle [35]. We assume that the asset volatility σ_A is 0.20 in our numerical exercise based on the empirical findings above.

[11] In the empirical findings of Brockman and Turtle [35], the average barrier estimates by year (1989-1998) is ranged from 0.5900 with a corresponding standard derivation of 0.2227 in the year of 1993 to 0.8395 with a corresponding standard deviation of 0.1405 in the year of 1990. Accordingly, the barrier range used in our numerical exercises is assumed to be from 0.50 to 0.90.

$\partial L/\partial R_L < 0$ and $\partial A/\partial R_A < 0$ due to the limitation of the firm's external financing.[12] The condition of $R_L > R = 3.5\%$ indicates asset substitution of the bank's earning-asset portfolio [37]. The condition of $R = 3.5\% > R_D = 2.5\%$ implies a case of capital binding of the balance-sheet constraint in Equation (5) [22]. The specification of $q = 8.00\%$ is interpreted as the capital adequacy requirement consistent with the Basel [39]. The conditions and specifications presented above can give an intuition roughly approaching a real state of a hypothetical bank.

First of all, we consider the impact on the bank's loan rate (and thus on the bank's interest margin) from changes in the firm's barrier based on Equation (10). The findings are summarized in Table 1.

In Table 1, we have the consistent result of $S > 0$ observed from the first panel. In the second panel, we observe the consistent result of $\partial^2 S/\partial R_L^2 < 0$, confirming the validness of the second-order condition of Equation (7). The sign of Equation (10) is governed by its numerator. According to the observations presented in the third panel, we have the result of $\partial^2 S/\partial R_L \partial\alpha_A > 0$ when α_A is low and A is high, while $\partial^2 S/\partial R_L \partial\alpha_A < 0$ when α_A is high and A is low. As a result, we have $\partial R_L/\partial\alpha_A > 0$ when α_A is low and A is high, while $\partial R_L/\partial\alpha_A < 0$ when α_A is high and A is low observed from the last panel.

Intuitively, as the bank faces an increase in a relatively low level of the firm barrier-to-debt ratio associated with high investment, it must now provide a return to a larger lending risk base. One way the bank may attempt to augment its total returns is by shifting its investments to the liquid-asset market and away from its loan portfolio. If loan demand is relatively rate-elastic, a less loan portfolio is possible at an increased margin or spread. However, if the bank faces an increase in a relatively high level of the firm barrier-to-debt ratio associated with low investment, an increase in the ratio increases the loan amount held by the bank at a reduced margin. Barrier as such makes the bank more prudent and less prone to loan risk-taking only up to a certain threshold, and less prudent and more prone to loan risk-taking when the barrier is high.

As pointed out by Brockman and Turtle [35], firms with high asset variability, high operating leverage, or high financial leverage are likely to exhibit a higher probability of hitting the barrier before the expiration date than firms without such characteristics. This paper explicitly integrates such characteristics with bank interest margin determination. We argue that an aggressive lending strategy to the riskier borrower who has a higher barrier may have resulted in increased risk taking and poor performance. This result is consistent with the empirical findings of Maudos and de Guevara [17] that bank interest margin is positively related to the high barrier level resulted from interest rate and credit risk or inferior quality of management. On the contrary, our result is consistent with the empirical finding of Hawtrey and Liang [19] that bank interest margin is negatively related to the low Barrier level resulted from interest rate volatility. Accordingly, we provide a formal illustration of bank interest margin decision mechanism, which allows distinguishing between high and low probability of early closure of the borrowing firm.

In Table 2, we consider the impact on bank interest margin from changes of the bank's barrier-to debt ratio, $0.40 \leq \alpha \leq 0.47$, at a constant level of the firm's barrier-to debt ratio, $\alpha_A = 0.50$. We have the result of $S > 0$ observed from the first panel and the condition of $\partial^2 S/\partial R_L^2 < 0$ in the second panel confirms the validness of the second-order condition for bank equity maximization denoted by Equation (9). In the third panel, the result of $\partial^2 S/\partial R_L \partial\alpha < 0$ demonstrates the quantitative result of the numerator of Equation (11). Accordingly, we have the result of $\partial R_L/\partial\alpha < 0$ observed from the last

[12]Results to be derived from our model do not extend to the case where the borrowing firm can raise external finance.

TABLE 1. Impact on bank interest margin from changes in the firm's barrier*

	$(R_L\%, L)$ $(R_A\%, A) = (R_A\%, L+E)$, $E = 20$						
	(4.5, 300)	(4.6, 299)	(4.7, 297)	(4.8, 294)	(4.9, 290)	(5.0, 285)	(5.1, 279)
α_A	(4.8, 320)	(4.9, 319)	(5.0, 317)	(5.1, 314)	(5.2, 310)	(5.3, 305)	(5.4, 299)
	S						
0.50	50.2041	50.5068	50.3026	49.5660	48.2479	46.2745	43.5448
0.55	50.2041	50.5068	50.3026	49.5660	48.2479	46.2745	43.5448
0.60	50.2041	50.5068	50.3026	49.5660	48.2479	46.2745	43.5448
0.65	50.2041	50.5067	50.3026	49.5659	48.2478	46.2745	43.5448
0.70	50.2017	50.5044	50.3003	49.5636	48.2455	46.2722	43.5425
0.75	50.1643	50.4673	50.2634	49.5268	48.2087	46.2354	43.5054
0.80	49.8297	50.1353	49.9326	49.1960	47.8768	45.9014	43.1689
0.85	47.9457	48.2642	48.0657	47.3254	45.9952	44.0026	41.2475
0.90	40.3888	40.7545	40.5668	39.8029	38.4172	36.3396	33.4728
	$\partial^2 S/\partial R_L^2$						
0.50	–	−0.5069	−0.5325	−0.5815	−0.6552	−0.7564	–
0.55	–	−0.5069	−0.5325	−0.5815	−0.6552	−0.7564	–
0.60	–	−0.5069	−0.5325	−0.5815	−0.6552	−0.7564	–
0.65	–	−0.5069	−0.5325	−0.5815	−0.6552	−0.7564	–
0.70	–	−0.5069	−0.5325	−0.5815	−0.6552	−0.7564	–
0.75	–	−0.5070	−0.5326	−0.5816	−0.6553	−0.7564	–
0.80	–	−0.5084	−0.5339	−0.5826	−0.6562	−0.7571	–
0.85	–	−0.5170	−0.5419	−0.5898	−0.6625	−0.7625	–
0.90	–	−0.5534	−0.5762	−0.6218	−0.6919	−0.7892	–
	$\partial^2 S/\partial R_L \partial \alpha_A$						
$0.50 \sim 0.55$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$0.55 \sim 0.60$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$0.60 \sim 0.65$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$0.65 \sim 0.70$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$0.70 \sim 0.75$	0.0003	0.0002	0.0001	−0.0000	−0.0001	−0.0001	
$0.75 \sim 0.80$	0.0026	0.0012	−0.0000	−0.0011	−0.0020	−0.0027	
$0.80 \sim 0.85$	0.0129	0.0042	−0.0038	−0.0109	−0.0172	−0.0226	
$0.85 \sim 0.90$	0.0471	0.0108	−0.0236	−0.0555	−0.0849	−0.1116	
	$\partial R_L/\partial \alpha$						
$0.50 \sim 0.55$	–	0.0000	0.0000	0.0000	0.0000	0.0000	–
$0.55 \sim 0.60$	–	0.0000	0.0000	0.0000	0.0000	0.0000	–
$0.60 \sim 0.65$	–	0.0000	0.0000	0.0000	0.0000	0.0000	–
$0.65 \sim 0.70$	–	0.0000	0.0000	0.0000	0.0000	0.0000	–
$0.70 \sim 0.75$	–	0.0004	0.0002	−0.0000	−0.0001	−0.0002	–
$0.75 \sim 0.80$	–	0.0024	−0.0000	−0.0019	−0.0030	−0.0035	–
$0.80 \sim 0.85$	–	0.0082	−0.0069	−0.0185	−0.0260	−0.0297	–
$0.85 \sim 0.90$	–	0.0194	−0.0409	−0.0893	−0.1228	−0.1415	–

*Parameter values, unless stated otherwise, $R = 3.50\%$, $R_D = 2.50\%$, $D = 300$, $q = 8.00\%$, $P = 0.23\%$, $\sigma_A = 0.20$ and $\alpha = 0.44$.

panel that the bank with a high probability of hitting the barrier, e.g., with a high asset variability, reduces its interest margin. The interpretation of this result follows a similar argument as in the case of a change in the higher level of the firm barrier-to-the bank debt ratio. Our result observed from Table 2 is consistent with the empirical finding of Williams [18], and Hawtrey and Liang [19].

In Table 3, we consider various deposit insurance premium per dollar of deposits at the constant barrier levels of $\alpha_A = 0.50$ and $\alpha = 0.44$. In the first panel, we have the result of $S > 0$. In the second panel, we have the result of $\partial^2 S/\partial R_L^2 < 0$, confirming the validness of the second-order condition. In the third panel, we have the observed result

TABLE 2. Impact on bank interest margin from changes in the bank's barrier*

α_A	$(R_L\%, L)$ $(R_A\%, A) = (R_A\%, L+E)$, $E = 20$						
	(4.5, 300) (4.8, 320)	(4.6, 299) (4.9, 319)	(4.7, 297) (5.0, 317)	(4.8, 294) (5.1, 314)	(4.9, 290) (5.2, 310)	(5.0, 285) (5.3, 305)	(5.1, 279) (5.4, 299)
	S						
0.40	78.4351	78.8192	78.9240	78.1432	78.1432	77.1170	75.5393
0.41	71.5824	71.9489	71.9688	71.6146	70.8341	69.5525	67.6731
0.42	64.5611	64.9075	64.8499	64.3623	63.3956	61.8773	59.7106
0.43	57.4259	57.7508	57.0056	57.0056	55.8614	54.1143	51.6657
0.44	50.2041	50.5068	49.5660	49.5660	48.2479	46.2745	43.5448
0.45	42.9092	43.1892	42.0539	42.0539	40.5625	38.3630	35.3512
0.46	35.5474	35.8044	34.4744	34.4744	32.8092	30.3824	27.0867
0.47	28.1221	28.3559	26.8300	26.8300	24.9900	22.3343	18.7525
	$\partial^2 S/\partial R_L^2$						
0.40	–	−0.2793	−0.3104	−0.3694	−0.4511	−0.5516	–
0.41	–	−0.3466	−0.3741	−0.4263	−0.5011	−0.5978	–
0.42	–	−0.4040	−0.4299	−0.4792	−0.5516	−0.6483	–
0.43	–	−0.4565	−0.4820	−0.5305	−0.6030	−0.7015	–
0.44	–	−0.5069	−0.5325	−0.5815	−0.6552	−0.7564	–
0.45	–	−0.5564	−0.5825	−0.6325	−0.7081	−0.8123	–
0.46	–	−0.6057	−0.6325	−0.6839	−0.7617	−0.8689	–
0.47	–	−0.6553	−0.6828	−0.7356	−0.8157	−0.9261	–
	$\partial^2 S/\partial R_L \partial \alpha$						
0.40 ~ 0.41	−0.0176	−0.0849	−0.1486	−0.2054	−0.2554	−0.3016	
0.41 ~ 0.42	−0.0201	−0.0775	−0.1333	−0.1862	−0.2367	−0.2872	
0.42 ~ 0.43	−0.0215	−0.0740	−0.1261	−0.1774	−0.2288	−0.2820	
0.43 ~ 0.44	−0.0222	−0.0725	−0.1231	−0.1740	−0.2262	−0.2811	
0.44 ~ 0.45	−0.0227	−0.0722	−0.1222	−0.1733	−0.2262	−0.2821	
0.45 ~ 0.46	−0.0230	−0.0724	−0.1224	−0.1738	−0.2273	−0.2839	
0.46 ~ 0.47	−0.0232	−0.0728	−0.1231	−0.1748	−0.2289	−0.2861	
	$\partial R_L/\partial \alpha$						
0.40 ~ 0.41	–	−0.2450	−0.3971	−0.4819	−0.5097	−0.5045	–
0.41 ~ 0.42	–	−0.1919	−0.3102	−0.3886	−0.4292	−0.4430	–
0.42 ~ 0.43	–	−0.1621	−0.2615	−0.3344	−0.3794	−0.4020	–
0.43 ~ 0.44	–	−0.1431	−0.2311	−0.2992	−0.3453	−0.3716	–
0.44 ~ 0.45	–	−0.1298	−0.2098	−0.2739	−0.3194	−0.3473	–
0.45 ~ 0.46	–	−0.1195	−0.1935	−0.2541	−0.2984	−0.3268	–
0.46 ~ 0.47	–	−0.1111	−0.1802	−0.2377	−0.2806	−0.3089	–

*Parameter values, unless stated otherwise, $R = 3.50\%$, $R_D = 2.50\%$, $D = 300$, $q = 8.00\%$, $P = 0.23\%$, $\sigma_A = 0.20$ and $\alpha_A = 0.50$.

of $\partial^2 S/\partial R_L \partial P > 0$. As a result, we have $\partial R_L/\partial P > 0$ in Equation (12) presented in the last panel. An increase in the premium paid for deposit insurance increases the bank's interest margin. Basically, increases in the cost of deposit insurance encourage the bank to shift investments from its loan portfolio to other earning assets. In an imperfect loan market, the bank must increase the size of its spread in order to decrease the amount of loans, thereby reducing the risk taking. Demirguc-Kunt et al. [2], and Demirguc-Kunt and Detragiache [33] report that an increase in deposit insurance premium is associated with a reduction in the level of bank asset risk. Thus, our result provides an alternative explanation for this empirical observation.

It is of interest to study the effects of the firm's barrier and the bank's barrier on the actuarially fair deposit insurance premium. The former is identified by Equation (13),

TABLE 3. Impact on bank interest margin from changes in deposit insurance*

	$(R_L\%, L)$ $(R_A\%, A) = (R_A\%, L+E)$, $E = 20$						
$P\%$	(4.5, 300) (4.8, 320)	(4.6, 299) (4.9, 319)	(4.7, 297) (5.0, 317)	(4.8, 294) (5.1, 314)	(4.9, 290) (5.2, 310)	(5.0, 285) (5.3, 305)	(5.1, 279) (5.4, 299)
	S						
0.23	50.2041	50.5068	50.3026	49.5660	48.2479	46.2745	43.5448
0.25	53.9229	54.2342	54.0809	53.4383	52.2598	50.4752	47.9882
0.27	57.5864	57.9063	57.8027	57.2523	56.2104	54.6103	52.3605
0.29	61.1955	61.5240	61.4692	61.0088	60.1006	58.6810	56.6629
0.31	64.7511	65.0880	65.0809	64.7089	63.9314	62.6881	60.8966
0.33	68.2539	68.5991	68.6389	68.3532	71.4182	66.6328	65.0625
0.35	71.7048	72.0582	72.1439	71.9427	67.7036	70.5159	69.1619
0.37	75.1045	75.4661	75.5968	75.4783	75.0761	74.3386	73.1957
	$\partial^2 S/\partial R_L^2$						
0.23	–	−0.5069	−0.5325	−0.5815	−0.6552	−0.7564	–
0.25	–	−0.4647	−0.4892	−0.5359	−0.6061	−0.7024	–
0.27	–	−0.4236	−0.4469	−0.4914	−0.5582	−0.6497	–
0.29	–	−0.3833	−0.4055	−0.4479	−0.5115	−0.5984	–
0.31	–	−0.3439	−0.3651	−0.4054	−0.4658	−0.5483	–
0.33	–	−0.3054	−0.3256	−0.3639	−0.4212	−0.4995	–
0.35	–	−0.2677	−0.2869	−0.3233	−0.3777	−0.4518	–
0.37	–	−0.2309	−0.2492	−0.2837	−0.3353	−0.4054	–
	$\partial^2 S/\partial R_L \partial P$						
0.23 ∼ 0.25		0.0087	0.0508	0.0941	0.1397	0.1887	0.2427
0.25 ∼ 0.27		0.0086	0.0498	0.0921	0.1366	0.1845	0.2372
0.27 ∼ 0.29		0.0085	0.0488	0.0901	0.1336	0.1804	0.2318
0.29 ∼ 0.31		0.0084	0.0478	0.0882	0.1307	0.1764	0.2265
0.31 ∼ 0.35		0.0083	0.0468	0.0864	0.1279	0.1724	0.2213
0.35 ∼ 0.37		0.0082	0.0459	0.0845	0.1251	0.1686	0.2162
0.37 ∼ 0.39		0.0082	0.0450	0.0827	0.1223	0.1648	0.2112
	$\partial R_L/\partial P$						
0.23 ∼ 0.25	–	0.1093	0.1923	0.2606	0.3114	0.3456	–
0.25 ∼ 0.27	–	0.1175	0.2061	0.2780	0.3306	0.3651	–
0.27 ∼ 0.29	–	0.1273	0.2223	0.2984	0.3528	0.3873	–
0.29 ∼ 0.31	–	0.1390	0.2417	0.3225	0.3787	0.4130	–
0.31 ∼ 0.35	–	0.1534	0.2653	0.3514	0.4094	0.4430	–
0.35 ∼ 0.37	–	0.1714	0.2946	0.3869	0.4463	0.4784	–
0.37 ∼ 0.39	–	0.1947	0.3320	0.4312	0.4915	0.5210	–

*Parameter values, unless stated otherwise, $R = 3.50\%$, $R_D = 2.50\%$, $D = 300$, $q = 8.00\%$, $\sigma_A = 0.50$ and $\alpha = 0.44$.

while the latter is identified by Equation (14) in our model. The findings of the former effect are summarized in Table 4.

In Table 4, we have the result of $Put > 0$ observed from the first panel. The result of the second panel indicates the direct effect of Equation (13) that is consistently positive in sign. The direct effect captures the change in Put due to an increase in α_A, holding the optimal bank interest margin constant. This is unambiguously positive because an increase in α_A makes loans more risky to grant. In response to this, the FDIC has an incentive to increase the actuarially fair deposit insurance premium, ceteris paribus. The results of the third panel indicate the indirect effect of Equation (13) that is positive in sign when the α_A level is relatively low and is negative when the α_A level is relatively high. The indirect effect arises because an increase in the barrier level decreases the bank's profits by $L(R_L)$ in every possible state.

TABLE 4. Impact on the fair value of deposit insurance from changes in the firm's barrier*

	$(R_L\%, L)$ $(R_A\%, A) = (R_A\%, L+E)$, $E = 20$						
	(4.5, 300)	(4.6, 299)	(4.7, 297)	(4.8, 294)	(4.9, 290)	(5.0, 285)	(5.1, 279)
α_A	(4.8, 320)	(4.9, 319)	(5.0, 317)	(5.1, 314)	(5.2, 310)	(5.3, 305)	(5.4, 299)
	put						
0.50	151.0816	151.4608	153.3904	156.9021	162.0552	168.9382	177.6720
0.55	151.0816	151.4608	153.3904	156.9021	162.0552	168.9382	177.6720
0.60	151.0816	151.4608	153.3904	156.9021	162.0552	168.9382	177.6720
0.65	151.0818	151.4609	153.3905	156.9022	162.0553	168.9383	177.6721
0.70	151.0852	151.4643	153.3938	156.9055	162.0586	168.9416	177.6754
0.75	151.1393	151.5180	153.4471	156.9585	162.1113	168.9940	177.7275
0.80	151.6233	151.9983	153.9249	157.4345	162.5862	169.4684	178.2015
0.85	154.3388	154.6964	156.6124	160.1177	165.2707	172.1586	180.9007
0.90	165.0900	165.3856	167.2718	170.7771	175.9569	182.8950	191.7067
	$\partial Put/\partial \alpha_A$						
0.50 ~ 0.55	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.55 ~ 0.60	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.60 ~ 0.65	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.65 ~ 0.70	0.0034	0.0034	0.0033	0.0033	0.0033	0.0033	0.0032
0.70 ~ 0.75	0.0541	0.0537	0.0533	0.0530	0.0527	0.0524	0.0522
0.75 ~ 0.80	0.4840	0.4804	0.4777	0.4760	0.4749	0.4743	0.4740
0.80 ~ 0.85	2.7156	2.6981	2.6875	2.6833	2.6845	2.6902	2.6992
0.85 ~ 0.90	10.7511	10.6892	10.6595	10.6594	10.6862	10.7365	10.8061
	$(\partial Put/\partial R_L)(\partial R_L/\partial \alpha_A)$						
0.50 ~ 0.55	–	0.0000	0.0000	0.0000	0.0000	0.0000	–
0.55 ~ 0.60	–	0.0000	0.0000	0.0000	0.0000	0.0000	–
0.60 ~ 0.65	–	0.0000	0.0000	0.0000	0.0000	0.0000	–
0.65 ~ 0.70	–	0.0001	0.0001	0.0001	0.0000	0.0000	–
0.70 ~ 0.75	–	0.0008	0.0005	−0.0001	−0.0009	−0.0015	–
0.75 ~ 0.80	–	0.0047	−0.0001	−0.0097	−0.0208	−0.0308	–
0.80 ~ 0.85	–	0.0157	−0.0243	−0.0955	−0.1792	−0.2595	–
0.85 ~ 0.90	–	0.0367	−0.1433	−0.4624	−0.8517	−1.2465	–
	$dPut/d\alpha_A$						
0.50 ~ 0.55	–	0.0000	0.0000	0.0000	0.0000	0.0000	–
0.55 ~ 0.60	–	0.0000	0.0000	0.0000	0.0000	0.0000	–
0.60 ~ 0.65	–	0.0001	0.0001	0.0001	0.0001	0.0001	–
0.65 ~ 0.70	–	0.0034	0.0034	0.0033	0.0033	0.0033	–
0.70 ~ 0.75	–	0.0541	0.0535	0.0526	0.0516	0.0507	–
0.75 ~ 0.80	–	0.4824	0.4758	0.4652	0.4535	0.4432	–
0.80 ~ 0.85	–	2.7033	2.6590	2.5889	2.5110	2.4396	–
0.85 ~ 0.90	–	10.6961	10.5161	10.2238	9.8847	9.5596	–

*Parameter values, unless stated otherwise, $R = 3.50\%$, $R_D = 2.50\%$, $D = 300$, $q = 8.00\%$, $P = 0.23\%$, $\sigma_A = 0.20$ and $\alpha = 0.44$.

As usual, the sign of this indirect effect is indeterminate. However, our numerical exercises provide us with a hunch that the two different results above are obtained. The rationale is the role played by $\partial R_L/\partial \alpha_A$ (shown in Table 1) in the indirect effect. Since the positive indirect effect reinforces the positive direct effect when the α_A level is low and the negative indirect effect is insufficiently large to offset the positive direct effect when the α_A level is high, we have the result of $dPut/d\alpha_A$ shown in the last panel of Table 4: an increase in the firm barrier-to-the bank debt ratio increases the fair deposit insurance premium.

TABLE 5. Impact on the fair value of deposit insurance from changes in the bank's barrier*

	$(R_L\%, L)$ $(R_A\%, A) = (R_A\%, L+E)$, $E = 20$						
	(4.5, 300)	(4.6, 299)	(4.7, 297)	(4.8, 294)	(4.9, 290)	(5.0, 285)	(5.1, 279)
α	(4.8, 320)	(4.9, 319)	(5.0, 317)	(5.1, 314)	(5.2, 310)	(5.3, 305)	(5.4, 299)
	Put						
0.40	116.1924	116.4539	118.0816	121.1049	125.5788	131.5844	139.2317
0.41	124.9053	125.1962	126.8991	130.0439	134.6862	140.9089	148.8245
0.42	133.6238	133.9442	135.7224	138.9888	143.8002	150.2413	158.4271
0.43	142.3487	142.6985	144.5522	147.9409	152.9225	159.5836	168.0422
0.44	151.0816	151.4608	153.3904	156.9021	162.0552	168.9382	177.6720
0.45	159.8246	160.2331	162.2389	165.8746	171.2005	178.3074	187.3190
0.46	168.5795	169.0174	171.0999	174.8604	180.3606	187.6933	196.9852
0.47	177.3485	177.8156	179.9753	183.8615	189.5374	197.0978	206.6725
	$\partial Put/\partial\alpha$						
$0.40 \sim 0.41$	8.7129	8.7423	8.8175	8.9389	9.1074	9.3245	9.5928
$0.41 \sim 0.42$	8.7186	8.7480	8.8233	8.9449	9.1140	9.3324	9.6027
$0.42 \sim 0.43$	8.7249	8.7543	8.8298	8.9521	9.1223	9.3423	9.6150
$0.43 \sim 0.44$	8.7329	8.7623	8.8382	8.9612	9.1327	9.3546	9.6299
$0.44 \sim 0.45$	8.7429	8.7723	8.8486	8.9725	9.1453	9.3692	9.6470
$0.45 \sim 0.46$	8.7550	8.7843	8.8610	8.9858	9.1601	9.3859	9.6662
$0.46 \sim 0.47$	8.7689	8.7982	8.8754	9.0011	9.1768	9.4045	9.6873
	$(\partial Put/\partial R_L)(\partial R_L/\partial\alpha)$						
$0.40 \sim 0.41$	–	−0.4171	−1.2489	−2.2371	−3.1715	−3.9938	–
$0.41 \sim 0.42$	–	−0.3412	−1.0132	−1.8699	−2.7644	−3.6266	–
$0.42 \sim 0.43$	–	−0.3004	−0.8863	−1.6657	−2.5274	−3.4000	–
$0.43 \sim 0.44$	–	−0.2762	−0.8115	−1.5420	−2.3765	−3.2455	–
$0.44 \sim 0.45$	–	−0.2603	−0.7628	−1.4590	−2.2701	−3.1297	–
$0.45 \sim 0.46$	–	−0.2488	−0.7278	−1.3976	−2.1882	−3.0364	–
$0.46 \sim 0.47$	–	−0.2399	−0.7005	−1.3489	−2.1213	−2.9577	–
	$dPut/d\alpha$						
$0.40 \sim 0.41$	–	8.4004	7.6900	6.8703	6.1530	5.5990	–
$0.41 \sim 0.42$	–	8.4820	7.9317	7.2441	6.5680	5.9760	–
$0.42 \sim 0.43$	–	8.5294	8.0658	7.4565	6.8149	6.2150	–
$0.43 \sim 0.44$	–	8.5620	8.1497	7.5907	6.9781	6.3843	–
$0.44 \sim 0.45$	–	8.5883	8.2097	7.6864	7.0991	6.5173	–
$0.45 \sim 0.46$	–	8.6122	8.2581	7.7624	7.1977	6.6298	–
$0.46 \sim 0.47$	–	8.6355	8.3006	7.8278	7.2833	6.7296	–

*Parameter values, unless stated otherwise, $R = 3.50\%$, $R_D = 2.50\%$, $D = 300$, $q = 8.00\%$, $P = 0.23\%$, $\sigma_A = 0.20$ and $\alpha_A = 0.50$.

The intuition is very straightforward. The liability of the FDIC is increased by the largely increased liability of the insured bank when the capped barrier from the borrowing firm is increased. This is also highlighted by Dermine and Lajeri [13] who argue that credit risk affects the distribution of bank asset returns so that the standard Merton [36] methodology that has been used to provide market-based estimation of deposit insurance premiums needs to be adapted. Unlike Dermine and Lajeri [13], we have developed a comprehensive model that integrates the risk considerations of the borrowing-firm barrier with loan rate-setting behavioral mode to provide alternative explanations for the fair value of deposit insurance estimation.

Next, the findings of the latter effect are summarized in Table 5. In Table 5, we have the result of $Put > 0$ observed from the first panel. The second panel indicates the positive direct effect $\partial Put/\partial\alpha > 0$ in Equation (14). The indirect effect in Equation (14) presented in the third panel is consistently negative in sign. Since the negative indirect effect is insufficient to offset the positive direct effect to give an overall positive response of an increase in Put, we conclude that an increase in the bank's barrier level will increase the fair deposit insurance premium. The interpretation of this result follows a similar argument as in the case of a change in α_A. Basically, increases in the bank's barrier level encourage the bank to shift investments to its loan portfolio at a reduced margin and away from other earning assets, but a reduced margin decreases the liability of the FDIC (denoted by the negative indirect effect). In addition, an increase in the bank's barrier largely increases in the liability of the FDIC. Accordingly, we have the result presented in the last panel of Table 5. Acharya et al. [41] report a positive relationship between the actuarially fair deposit insurance premium and individual bank failure risk for the premium schemes that have been used in the United States. Our result provides an alternative explanation for this empirical observation.

In Table 6, we consider a special case where $\alpha = 0$. This is the case demonstrating that the bank has a good quality of management without the problem of early closure even though its borrowing firm has such problem. Under the circumstances, Equation (7) becomes a capped call option valuation. We have the result of $\partial Put/\partial \alpha_A < 0$ when α_A is low observed from the last panel of Table 6. This indicates that an increase in the firm barrier-to-debt ratio decreases the liability of the insurance due to the good quality management of the bank. Maudos and de Guevara [17] include management quality as an explicit component of bank interest margin, and conclude that bank interest margin is positively related to management quality. An increase in the bank interest margin results in decreasing the loan amount held by the bank and thus decreasing the liability of the insurer. Our result is largely supported by the empirical finding of Maudos and de Guevara [17].

Much of the theoretical literature on deposit guarantees has focused on their pricing and the feasibility of risk-adjusted insurance premiums. This work contributes to the existing literature by exploring the aspects of the actuarially fair deposit insurance premium and bank early closure problem relationship. In Figure 1, the four diagrams on the left-hand side are obtained from Table 4, respectively, under the condition of $\alpha \neq 0$, while those on the right-hand side are obtained from Table 6 under the condition of $\alpha = 0$. The former can be interpreted as a low quality management of the bank, while the latter can be interpreted as a high quality management. We present the following results. (i) The actuarially fair deposit insurance premium in the case of $\alpha \neq 0$ is consistently higher than that in the case of $\alpha = 0$. This explains an important role played by the high quality management of the bank, which can share the liability burden of the insurer. (ii) The capped barrier from the borrowing firm increases the fair value of deposit insurance premium under the condition of $\alpha \neq 0$, while that reduces the fair value under the condition of $\alpha = 0$, ceteris paribus. Again, this direct effect indicates the importance of bank management related to the problem of early closure. (iii) An increase in the capped barrier inconsistently changes the fair value of deposit insurance in both the case of $\alpha \neq 0$ and $\alpha = 0$ since this capped barrier may or may not reduce the bank's interest margin as mentioned in Table 1. This explains the considerable role played by the borrowing firm related to the explicit consideration of credit risk realized by the bank. (iv) When the capped barrier from the borrowing firm is at a low level, the overall response of the fair deposit insurance premium to an increase in the capped barrier is positive when $\alpha \neq 0$ and is negative when $\alpha = 0$. From a normative standpoint, we suggest that lending decisions with explicit considerations of borrower default risk and bank management quality related to bankruptcy are crucial to price the actuarially fair deposit insurance premium.

TABLE 6. Impact on the fair value of deposit insurance from changes in the firm's barrier when the bank's barrier equals zero*

	$(R_L\%, L)$ $(R_A\%, A) = (R_A\%, L+E)$, $E = 20$						
	(4.5, 300)	(4.6, 299)	(4.7, 297)	(4.8, 294)	(4.9, 290)	(5.0, 285)	(5.1, 279)
α_A	(4.8, 320)	(4.9, 319)	(5.0, 317)	(5.1, 314)	(5.2, 310)	(5.3, 305)	(5.4, 299)
	Put						
0.50	25.1354	25.2611	25.2834	25.2003	25.0102	24.7113	24.3019
0.55	25.1354	25.2611	25.2834	25.2003	25.0102	24.7113	24.3019
0.60	25.1354	25.2611	25.2834	25.2003	25.0102	24.7113	24.3019
0.65	25.1354	25.2611	25.2833	25.2003	25.0102	24.7113	24.3019
0.70	25.1348	25.2604	25.2827	25.1997	25.0096	24.7107	24.3014
0.75	25.1242	25.2499	25.2723	25.1895	24.9998	24.7013	24.2924
0.80	25.0296	25.1559	25.1795	25.0986	24.9113	24.6158	24.2106
0.85	24.5069	24.6356	24.6650	24.5930	24.4179	24.1379	23.7512
0.90	22.5575	22.6931	22.7404	22.6977	22.5630	22.3346	22.0107
	$\partial Put/\partial\alpha_A$						
$0.50 \sim 0.55$	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000
$0.55 \sim 0.60$	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000
$0.60 \sim 0.65$	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000
$0.65 \sim 0.70$	−0.0007	−0.0007	−0.0007	−0.0006	−0.0006	−0.0006	−0.0006
$0.70 \sim 0.75$	−0.0106	−0.0105	−0.0104	−0.0101	−0.0098	−0.0095	−0.0090
$0.75 \sim 0.80$	−0.0945	−0.0940	−0.0928	−0.0909	−0.0885	−0.0854	−0.0817
$0.80 \sim 0.85$	−0.5228	−0.5203	−0.5146	−0.5056	−0.4934	−0.4780	−0.4594
$0.85 \sim 0.90$	−1.9493	−1.9425	−1.9246	−1.8953	−1.8549	−1.8033	−1.7405
	$(\partial Put/\partial R_L)(\partial R_L/\partial\alpha_A)$						
$0.50 \sim 0.55$	–	0.0000	0.0000	0.0000	0.0000	0.0000	–
$0.55 \sim 0.60$	–	0.0000	−0.0000	0.0000	0.0000	0.0000	–
$0.60 \sim 0.65$	–	0.0000	−0.0000	0.0000	0.0000	0.0000	–
$0.65 \sim 0.70$	–	0.0000	−0.0007	0.0004	0.0002	0.0001	–
$0.70 \sim 0.75$	–	0.0005	−0.0116	0.0057	0.0032	0.0021	–
$0.75 \sim 0.80$	–	0.0049	−0.1100	0.0565	0.0313	0.0206	–
$0.80 \sim 0.85$	–	0.0614	−0.8858	0.5576	0.3065	0.2020	–
$0.85 \sim 0.90$	–	6.2635	−24.8064	31.7536	16.7963	10.9525	–
	$dPut/d\alpha_A$						
$0.50 \sim 0.55$	–	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000	–
$0.55 \sim 0.60$	–	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000	–
$0.60 \sim 0.65$	–	−0.0000	−0.0000	−0.0000	−0.0000	−0.0000	–
$0.65 \sim 0.70$	–	−0.0006	−0.0014	−0.0003	−0.0004	−0.0004	–
$0.70 \sim 0.75$	–	−0.0099	−0.0218	−0.0041	−0.0063	−0.0069	–
$0.75 \sim 0.80$	–	−0.0878	−0.2009	−0.0320	−0.0541	−0.0611	–
$0.80 \sim 0.85$	–	−0.4531	−1.3914	0.0642	−0.1714	−0.2574	–
$0.85 \sim 0.90$	–	4.3389	−26.7017	29.8987	14.9931	9.2120	–

*Parameter values, unless stated otherwise, $R = 3.50\%$, $R_D = 2.50\%$, $D = 300$, $q = 8.00\%$, $P = 0.23\%$ and $\sigma_A = 0.20$.

6. **Conclusion.** The objective of the paper is to show that bank lending and credit risk create a specific stochastic process for the asset of a bank when both the bank and the borrower face bankruptcy problems under deposit insurance. Bank equity is equivalent to a barrier-capped barrier option and the leverage relevant for the insurer is the deposits to borrower investment. It has been shown that (i) the bank interest margin is negatively related to the firm's barrier when the barrier is high associated with a low investment, (ii) the bank interest margin is negatively related to the bank's barrier, (iii) the bank interest margin is positively related to deposit insurance premium, (iv) the market-based estimates of deposit insurance premium is positively related to the firm's barrier and

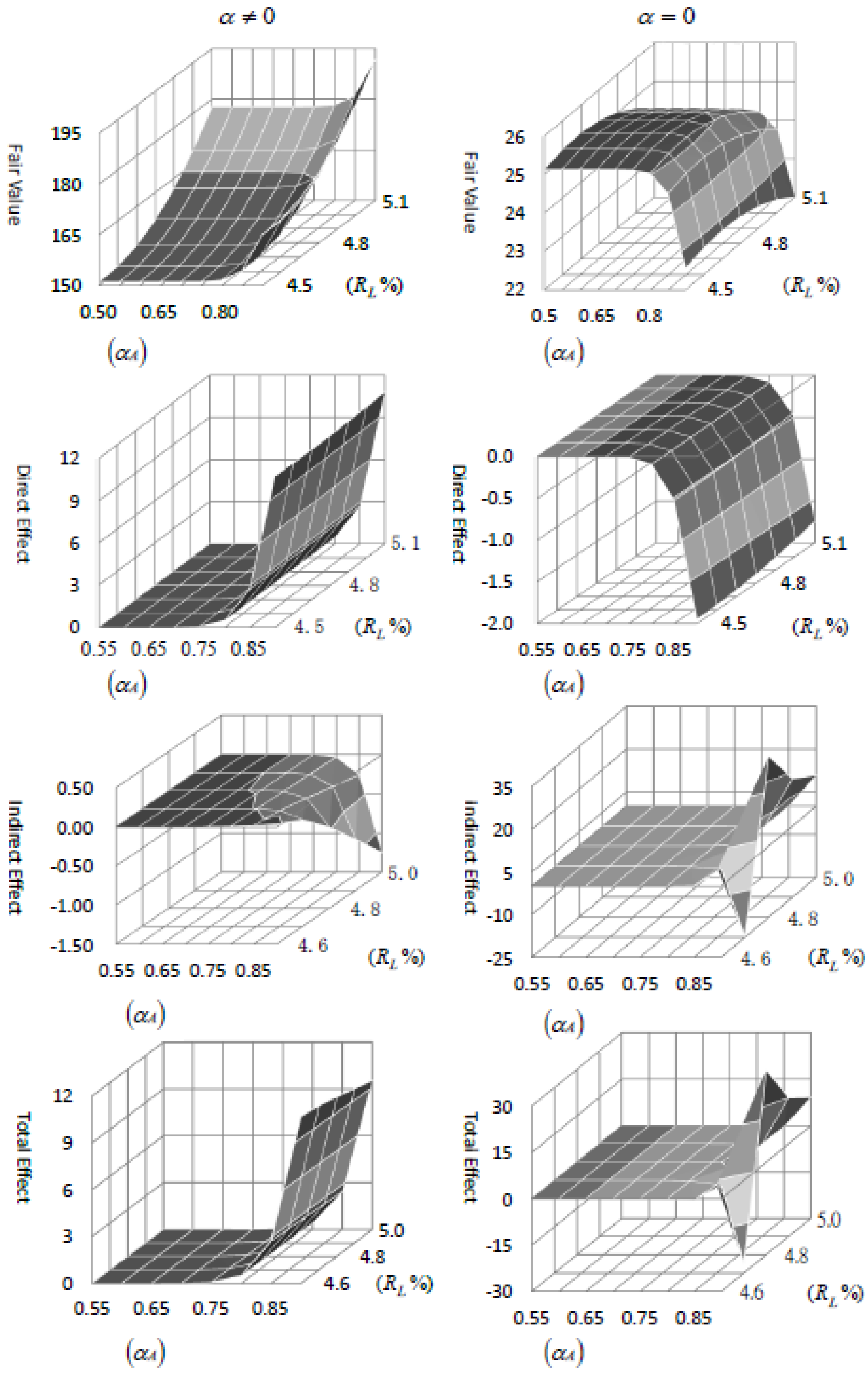

FIGURE 1. The actuarially fair value of deposit insurance premiums: $\alpha \neq 0$ vs. $\alpha = 0$

the bank's barrier, and (v) the market-based estimates of deposit insurance premium is negatively related to the firm's barrier when the bank has a good quality management without the early closure problem.

A corollary of the paper is that regulatory deposit insurance premiums should be related to loan-to-value ratio policy of the bank. Indeed, depositors are protected not only by the equity of the bank but also by the equity of the borrowing firm, in particular, explicitly taking early closure problem into account. As mentioned earlier, the recent Basel III for a solution for liquidity goes into that direction [42]. If accepted, the Basel would consider the effective riskiness of loans and loan-to-value ratio with barrier structures.

One issue that has not been addressed is the lending contract for renegotiated loans in the bank-borrowing firm case. In particular, is it the case that the results of this paper also apply to the renegotiated case? In a very simple rational expectation framework, the answer is yes. Since the borrowing firm and the bank are assumed rational, the loan amount associated with the optimal bank interest margin exists in a rational expectation equilibrium. In such a world, renegotiations need never occur as participants would not have entered other lending contracts in the first place. Of course, in a world without such strict rational expectation requirements, other factors would affect the bank-borrowing situation. For example, strategic considerations may play an important role, as would more critical problems of information asymmetries. Such concerns are beyond the scope of this paper and so are not addressed here. What this paper does demonstrate, however, is the important role played by barrier structures related to early closure problems in affecting bank default risk and the insurer's liability, thereby the stability of the banking system.

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