International Journal of Innovative Computing, Information and Control
Volume **11**, Number **1**, February **2015**

ICIC International ©2015 ISSN 1349-4198
pp. 363–374

A DIVIDE-CONQUER HEURISTIC SEARCH ALGORITHM FOR CUTTING SURPLUS STEEL PLATES OF STEEL AND IRON COMPANIES

Ziqiang Li[1], Rong Huang[1], Banglun Peng[2], Chinchen Chang[3,4,*] and Qiuzhen Wang[1]

[1]Key Laboratory of Intelligent Computing and Information Processing, Ministry of Education
Xiangtan University
Yanggutang Western Suburb, Xiangtan 411105, P. R. China
lzq@hotmail.com; 455583281@qq.com; wqz76@163.com

[2]Xiangtan Iron and Steel Group Co. Ltd
Xiangtan 411104, P. R. China
pengbl@mail.hnxg.com.cn

[3]Department of Information Engineering and Computer Science
Feng Chia University
No. 100, Wenhwa Rd., Seatwen, Taichung 40724, Taiwan
[*]Corresponding author: alan3c@gmail.com

[4]Department of Computer Science and Information Engineering
Asia University
No. 500, Lioufeng Rd., Wufeng, Taichung 41354, Taiwan

Received February 2014; revised June 2014

Abstract. *The cutting pattern problem of the surplus steel plates (SPCP problem) is difficult to solve. In this paper, a divide-conquer heuristic search approach (DHSA) is proposed for SPCP. Firstly, the related knowledge is extracted and used in transforming the SPCP problem into the special restricted rectangle packing problem (S2RP problem) for each group of surplus(es) and orders. Afterward, feasible candidate cutting schemes can be obtained through an exhaustive search based on the knowledge and proposed divide-conquer heuristic strategy for the S2RP problem. And the optimal one having passed through quality-monitoring can be quickly found from the sorted candidate schemes. Theoretical analysis and numerical experiments show that the proposed DHSA algorithm is superior to existing approaches, especially the MES-based human-computer interactive approach in performance for the SPCP problem.*
Keywords: Cutting problem, Rectangle packing problem, Heuristic method, Range size, Divide-conquer strategy

1. **Introduction.** Rectangle packing and cutting problems are often encountered in many fields of the production and real life, for example, the Ad layout [1] and rectangle plate cutting [2]. For the rectangle plate cutting, the right angle cutting is more common, which includes non-Guillotine cutting [3,4] and Guillotine cutting [5-7]. In this paper, we discuss a cutting pattern problem of surplus steel plates (surplus plates, SPCP problem) for steel and iron companies. According to related knowledge in Section 2, it belongs to the latter (i.e., the Guillotine cutting). Due to the NP-hard nature, many scholars have had deep discussion on the Guillotine cutting of the rectangle plate. So far, the existing effective algorithms can be classified into the following four major types.

(i) Linear programming. For example, according to the combination principle and theory of linear programming, Yuan [8] presented a cutting algorithm and its procedure for plate materials.

(ii) The heuristic methods. Haessler and Sweeney [9] proposed a heuristic method of cutting-stock one by one. Their idea is that after cutting-stock one stock sheet to be cut every time, numbers of ordered pieces are updated and this process is repeated until feasible cut-schemes of all stock sheets are obtained. Dolatabadia et al. [10] and Christoforos and Fleszar [11] presented two different heuristic methods for solving their restricted rectangle packing problems (2RP problems) based on the recursive idea, respectively; Wang [12] gave a Guillotine cutting method that constrained cutting patterns are generated by successive horizontal and vertical builds of ordered rectangular pieces.

(iii) The MES based human-computer interaction. Manufacture execution system (MES) was developed by Advanced Manufacturing Research, Inc (AMR, Inc.) in the early 1990s. In the last years, the MES based human-computer interactive (MHCI) approach has been used in designing SPCP for many steel and iron companies.

(iv) Intelligent algorithms (simulated annealing, genetic algorithms and ant colony optimization). Combining the improved BL heuristic with a pseudo-parallel agent-based system, Polyakovskya and M'Hallah [13] proposed a more effective hybrid algorithm for the complex Guillotine 2RP problem.

In addition, many intelligent methods are devised for the irregular polygon packing problem [14-16].

There exist the following four difficulties for solving the SPCP problem of large and middle iron and steel enterprizes.

(i) There are up to thousands of surplus plates and ten thousands of orders (an order includes several ordered pieces with the same size) in batch records. For handling the batch records, real-time calculation speed is required.

(ii) In batch records, there are up to hundreds of orders, each having the same sign and thickness as a surplus plate. Because of the uncertain quality-monitor result we have to exhaust all candidate cutting patterns to find the optimal ones from them for every surplus plate. Because the number of candidate cutting patterns of the surplus plate is increasing in an incredibly rapid speed, this probably results in a combinatorial explosion.

(iii) For the optimal cutting pattern of a surplus plate, it is required to satisfy ten rules (see Appendix 1) and to realize dynamic priorities of several given indexes (see Appendix 2).

(iv) There are some ordered pieces with variable size (including the length and/or width) in batch records.

Unfortunately, the existing algorithms do not simultaneously involve the above four difficulties. For example, it is very difficult to satisfy the requirement (i) for types (i), (iii) and (iv) of approaches and to obtain the best solution for the type (ii) of approaches. Therefore, an absence of a powerful heuristic search approach is a key obstacle to solving this problem.

The "No Free Lunch Theorem" indicates that without combining the knowledge of this problem with its optimization algorithms, performances of these algorithms are equivalent [17]. By obtaining the knowledge from the known information and packing scheme diagraphs of a weighted circle packing problem (WCP problem), Li et al. [18] proposed a knowledge-based heuristic particle swarm optimization approach with the adjustment strategy. Li's experimental results show that both its computational efficiency and solution quality are obviously improved for the WCP problem. In this paper, obtaining the knowledge from the SPCP problem itself and designer's experience, we consider a divide-conquer heuristic search algorithm (DHSA). Its divide-conquer heuristic strategy to solve the 2RP problem will be superior to that of [10] (2012) in performance, and compared with the MHCI approach, both its rolling yield of the surplus plates and its design efficiency will be significantly increased. In addition, the optimal cutting scheme of each surplus plate obtained by this algorithm will also satisfy a dynamic priority of some given indexes (such as the rolling yield, and delivery).

The rest of this paper is organized as follows. Section 2 is the related knowledge and definitions; dividing and filtering are in Section 3; calculating candidate cutting patterns of each surplus plate is in Section 4; Section 5 is to propose the DHSA algorithm to obtain the optimal cutting pattern of each surplus plate; the experiment and analysis are in Section 6; the summary of this paper is in Section 7.

2. Related Knowledge and Definitions.

After analyzing the SPCP problem and summarizing the prior design experience of designers with respect to this problem, we have obtained the following knowledge.

(1) For each cutting pattern designed, it is required that the length direction and width direction of each ordered piece are consistent with those of the surplus plate, respectively.

(2) For each ordered piece of the cutting pattern, its sign and thickness must be the same as those of the surplus plate, respectively.

(3) A surplus plate can be cut one time along the length direction at most and be cut eight times along the width direction at most.

(4) When selecting the ordered piece to design the cutting pattern, we take the strategy of "the large size first with its width and length in turn".

(5) The rule set C in Appendix 1 consists of the attribute rule set $C_1\{(1)\text{-}(6),\ (10)\}$ and mode rule set $C_2\{(7)\text{-}(9)\}$, where the mode is defined by Definition 2.2.

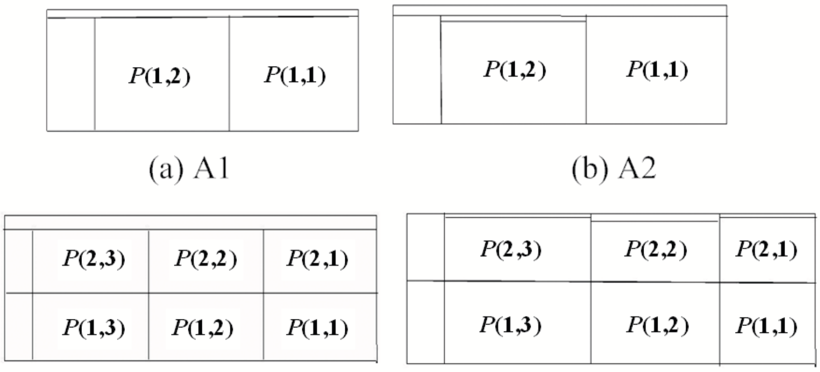
Definition 2.1. *Let S_p and S_{od} denote the area of a surplus plate and the sum of areas of all ordered pieces of its cutting pattern, respectively, then the quotient $S_{od}/S_p \times 100\%$ is called the rolling yield of the surplus plate with respect to the cutting pattern.*

Definition 2.2. *Suppose that a surplus plate can only be cut into a row of ordered pieces which satisfy the knowledge (2). If the widths of the ordered pieces are the same, the cutting mode is denoted by "A1" (see Figure 1(a)); otherwise, it is denoted by "A2" (see Figure 1(b)). Suppose that a surplus plate can be cut into two rows of ordered pieces which satisfy the knowledge (2). If the widths of ordered pieces in each row are the same and the lengths of two ordered pieces in each column are the same, the cutting mode is denoted by "S1" (see Figure 1(c)); otherwise, it is denoted by "S2" (see Figure 1(d)).*

3. Grouping and Filtering Out.

According to the knowledge (2) and (4) in Section 2, thousands of surplus plates and ten thousands of orders are sorted in descending order with respect to their sign, thickness, width and length in turn, respectively (it can be implemented by the nested sorting select statement of the used database languish). Using cursor data blocks of the database system, we can quickly obtain and store them into the surplus plate list $\boldsymbol{L}_1$ and order list $\boldsymbol{L}_2$, respectively. Then by Algorithm 1 we quickly group them in the sign and thickness and filter out orders which do not satisfy any rule in the set C_1 (see Appendix 1) for each group, respectively.

Let N denote the number of groups; $\boldsymbol{PL}_i$ and $\boldsymbol{OL}_i$ be the surplus plate list and order list of the i-th group ($i = 1, 2, \ldots, N$); n_i and m_i be the number of surplus plates and the number of orders of the i-th group, respectively. Suppose that both two pointers p_1 and q_1 point to the head of the list $\boldsymbol{L}_1$, and both two pointers p_2 and q_2 point to the head of the list $\boldsymbol{L}_2$. The pseudo-code of Algorithm 1 (grouping and filtering out) is as follows:



(a) A1 (b) A2

(c) S1 (d) S2

FIGURE 1. Four cutting modes

Initialize $N = 0$, $n_i = 0$, $m_i = 0$ $(i = 1, 2, \dots, N)$;
While ($p_1 \neq$ null and $p_2 \neq$ null) {
While (both p_1.next and p_2.next $\neq$ null) {
If ((p_1.sign>p_2.sign) or (p_1.sign=p_2.sign and p_1.thick > p_2.thick)) $p_1 = p_1$.next;
Else If (p_1.sign=p_2.sign and p_1.thick=p_2.thick) break;
Else $p_2 = p_2$.next;
$q_1 = p_1$; $q_2 = p_2$; N++; i++;
do {$p_1 \Rightarrow PL_i$; $p_1 = p_1$.next; n_i++;
} while ($p_1 \neq$ null and p_1.sign $= q_1$.sign and p_1.thick $= q_1$.thick);
do {if (p_2 satisfies rules in $\boldsymbol{C}_1$ and p_2.length $\leq q_1$.length and p_2.width $\leq q_1$.width)
$p_2 \Rightarrow OL_i$; p_2= p_2.next; m_i++;
} while ($p_2 \neq$ null and p_2.sign $= q_2$.sign and p_2.thick $= q_2$.thick);
}

After grouping and filtering out, the SPCP problem can be transformed into a special restricted rectangle packing problem (an S2RP problem) of each group of the orders and surplus(es) according to the knowledge (1) and (3) in Section 2.

4. **Candidate Cutting Patterns.** For obtaining candidate cutting patterns of each surplus plate, we will build a mathematical model of the S2RP problem and propose a divide-conquer and heuristic strategy in Sections 4.1 and 4.2, respectively.

4.1. **Mathematical model of the S2RP problem.** The models of [3,7,9] are to pack given different-sized ordered pieces on several surplus plates such that the number of consumed surplus plates is the least. [10] is to pack maximum profit subset of "small" rectangles into a unique "large" rectangle. It belongs to the two-dimensional knapsack problem.

The S2RP problem is similar to that of [10] and its surplus plate and ordered piece are responding to the "large" rectangle and "small" rectangle of [10] respectively. However, there are the following differences for them. (i) Each packing scheme of the S2RP problem must satisfy Knowledge (1), (3) and is one of four modes "A1", "A2", "S1" and "S2" of Definition 2.2. (ii) The objective of the S2RP problem is to calculate all the packing schemes of the surplus plate rather than the optimal one for the "large" rectangle.

Let L and W denote the length and width of the "large" rectangle R, respectively, and a set $\boldsymbol{OL} = \{od_i(l_i, w_i, b_i),\ i = 1, 2, \dots, m\}$, where l_i ($l_i \in [l_{i,\min}, l_{i,\max}]$), w_i ($w_i \in [w_{i,\min}, w_{i,\max}]$) and b_i denote the length and width and number of i-th kind of "small" rectangles, respectively. Set $\boldsymbol{I} = \{1, 2, \ldots, m\}$. Let $[L/l_i]$ be the integer part of L/l_i and $h_i = \min\{b_i,\ [L/l_i]\}$ ($h_i \geq 1$, $i = 1, 2, \ldots, m$), then the mathematical model of the S2RP problem can be described as follows: from the set $\boldsymbol{OL}$, find the packing scheme set $\boldsymbol{Y} = \{\boldsymbol{X}(l_i,\ w_i,\ a_{1i} + a_{2i}),\ i = t_1, t_2, \ldots, t_q \mid q \in \boldsymbol{I}$ and $\{t_1, t_2, \ldots, t_q\} \subseteq \boldsymbol{I}\}$ of the rectangle R and $\boldsymbol{X}$ satisfies Formulas (1)-(4).

$$0 \leq a_{1i} + a_{2i} \leq h_i, \quad i = t_1, t_2, \ldots, t_q \tag{1}$$

$$\sum\nolimits_{u=t_1}^{t_q} a_{1u} l_u \leq L, \quad \sum\nolimits_{v=t_1}^{t_q} a_{2v} l_v \leq L \tag{2}$$

$$\max\{w_u | a_{1u} \neq 0 \text{ and } u = t_1, t_2, \ldots, t_q\} + \max\{w_v | a_{2v} \neq 0 \text{ and } v = t_1, t_2, \ldots, t_q\} \leq W \tag{3}$$

$$l_{f_j,\min} \leq l(j) \leq l_{f_j,\max} \text{ for } j = 1, 2, \ldots, \ \sum\nolimits_{v=t_1}^{t_q} a_{2v} \tag{4}$$

Formula (1) denotes that the number of "small" rectangles with the same length and width is not larger than both the maximum number allowed for the "large" rectangle and its maximum number. Formula (2) means that the sum of lengths of "small" rectangles in each row is not larger than the length of the "large" rectangle. Formula (3) indicates the sum of maximal widths of ordered pieces in each column is not larger than the width of the "large" rectangle. Formula (4) shows that there exists the common length ranges for two "small" rectangles in each column, where $l(j)$ denotes the length of "small" rectangle in the j-th column of the first row (see Figure 2).

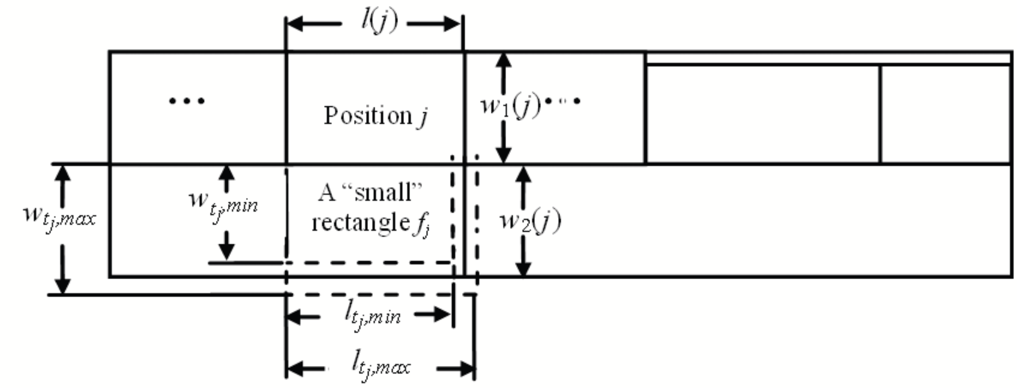


FIGURE 2. Packing j-th column of the 2nd row and handling of the range size in j-th column

4.2. **Divide-conquer and heuristic strategy.** Scholars proposed some different and efficient methods [6-12] based on models of the respective 2RP problems. However, it is difficult to fit the S2RP problem for them due to different problems and models. Considering its speciality, we suggest a divide-conquer and heuristic strategy.

First, we consider its special case: $l_i = l_{i,\min}$ and $w_i = w_{i,\min}$ ($i = 1, 2, \ldots, m$). Suppose that $\boldsymbol{X} \in \mathbf{Y}$. According to Knowledge (3) and Knowledge (4), we decompose $\boldsymbol{X}$ into $\boldsymbol{X}_1 = \{(l_i, w_i, a_{1i}),\ i = t_1, t_2, \ldots, t_q \mid q \in \boldsymbol{I}$ and $\{t_1, t_2, \ldots, t_q\} \subseteq \boldsymbol{I}\}$ and $\boldsymbol{X}_2 = \{(l_i, w_i, a_{2i}),\ i = t_1, t_2, \ldots, t_q \mid q \in \boldsymbol{I}$ and $\{t_1, t_2, \ldots, t_q\} \subseteq \boldsymbol{I}\}$, respectively.

Let $l_{\min} = \min\{l_1, l_2, \ldots, l_m\}$, d be the maximal number of "small" rectangles in $\boldsymbol{X}_1$, then $d = [L/l_{\min}]$, $q \leq d$ and, when $b_i = 1$ ($i = 1, 2, \ldots, m$) there exist $\sum_{k=1}^{d} c_m^k$ possible combinations for $\boldsymbol{X}_1$. For each non negative integer sequence a_{1t_1}, a_{1t_2}, $\ldots$, a_{1t_q}, if it satisfies Formula (2) and $a_{1t_j} \leq h_{t_j}$ ($j = 1, 2, \ldots, q$), then $\boldsymbol{X}_1\{(l_i, w_i, a_{1i}),\ i = t_1, t_2, \ldots, t_q \mid q \in \boldsymbol{I}$ and $\{t_1, t_2, \ldots, t_q\} \subseteq \boldsymbol{I}\}$ is one packing sub-scheme of R. So, after traversing each combination in turn by a backtrack strategy, all packing sub-schemes of R can be found.

After finding the packing scheme $\boldsymbol{X}_1$ of R, we update set $\boldsymbol{OL}$, i.e., $b_i = b_i - a_{1i}$ for $i = t_1, t_2, \ldots, t_q$. According to the Knowledge (4) we pack a "small" rectangle of the set $\boldsymbol{OL}$ into the position j ($j = 1, 2, \ldots, \sum_{k=t_1}^{t_q} a_{1k}$) of $\boldsymbol{X}_2$ in turn (see Figure 2), until all its positions are packed or no ordered piece is suitable for it. Set $\boldsymbol{X} = \boldsymbol{X}_1 + \boldsymbol{X}_2$ and test whether it satisfied rules in the set $\boldsymbol{C}_2$. In this way, we can construct all packing schemes of R which satisfy rules in the set $\boldsymbol{C}_2$ and Formulas (1)-(4). These schemes, whose rolling yields $\geq \eta$ (a given threshold value), are taken as better cutting patterns of the surplus plate.

4.3. **How to relax combination explosion.** Experimental results show that there exist a few candidate cutting patterns which fail to pass the quality-monitoring (see Section 5.1) and/or the test of at least one rule in the set $\boldsymbol{C}_2$. Therefore, in order to find the optimal and quality-monitoring-qualified cutting pattern for each surplus plate. It is necessary to do the exhaustive search for all packing schemes of the corresponding "large" rectangle. However, when $m = 50$, its maximal number $\prod_{i=1}^{m} h_i$ of packing sub-schemes, such as $\boldsymbol{X}_1$, is lager than 2^{50}. Obviously, the combination explosion will occur probably.

By analyzing sizes of surplus plates and ordered pieces of the large steel and iron company in China for past years, we can know that $0 < d < 5$, and when $m = 50$, $d = 4$, $\sum_{k=1}^{d} c_m^k = 252875 \ll 2^{50}$.

Therefore, we can effectively alleviate the combination explosion by estimating the maximal number of "small" rectangles of the sub-scheme such as $\boldsymbol{X}_1$.

4.4. **Handling the range size.** After obtaining all better packing schemes of the surplus plate based on the mathematical model with the fixed size, we test whether they satisfy the following both Formulas (5) and (6) for each scheme $\boldsymbol{X}$.

$$L - \sum_{k=t_1}^{t_q} a_{1k} l_k = 0, \tag{5}$$

$$W - (W^1 + W^2) = 0. \tag{6}$$

In Formula (6), $W^1 = \max\{w_u | a_{1u} \neq 0 \text{ and } u = t_1, t_2, \ldots, t_q\}$, $W^2 = \max\{w_v | a_{2v} \neq 0 \text{ and } v = t_1, t_2, \ldots, t_q\}$. For the scheme $\boldsymbol{X}$ which satisfies both Formulas (5) and (6), we directly consider it as $\boldsymbol{X}_+$; otherwise, as shown in Figure 2, suppose that the 1st and 2nd ordered pieces in the j-th ($j = 1, 2, \ldots, \sum_{k=t_1}^{t_q} a_{1k}$) column in $\boldsymbol{X}$ belong to orders τ_j and f_j, respectively, and the widths of two "small" rectangles in the j-th column after handling are denoted by $w_1(j)$ and $w_2(j)$, respectively.

Set $r_1 = L - \sum_{k=t_1}^{t_q} a_{1k} l_k$, then $r_1 > 0$. If $r_1 \geq \Delta l(1)$, where $\Delta l(1) = \min(l_{\tau_1,\max} - l_{\tau_1}, l_{f_1,\max} - l_{\tau_1})$, then $l(1) = l_{\tau_1} + \Delta l(1)$ and $r_1 = r_1 - \Delta l(1)$. Repeat the above process, $\Delta l(j) = \min(l_{\tau_j,\max} - l_{\tau_j}, l_{f_j,\max} - l_{\tau_j})$ and $l(j) = l_{\tau_j} + \Delta l(j)$ and $r_1 = r_1 - \Delta l(j)$ for $j = 2, 3, \ldots$, until $j = \sum_{i=t_1}^{t_q} a_{1i}$ or $\exists$ a positive integer k ($k \leq \sum_{k=t_1}^{t_q} a_{1k}$), $r_1 < \Delta l(k)$. For the latter, $l(j) = l_{\tau_j} + \Delta l(j)$ for $j = 1, 2, \ldots, k-1$, but $l(j) = l(j) + r_1$ for $j = k$.

Set $r_2 = W - (W^1 + W^2)$. For $k = 1, 2, \ldots, \sum_{k=1}^{m} a_{1k}$, if $r_2 \geq w_{\tau_k,\max} - w_{\tau_k,\min} > 0$, $w_1(k) = w_{\tau_k,\max}$; otherwise, $w_1(k) = w_{\tau_k} + r_2$. Calculating W^1 and r_2, respectively, we repeat the above process for "small" rectangles in the 2nd row if $r_2 \neq 0$.

So, the scheme $\boldsymbol{X}$ is further improved into $\boldsymbol{X}_+$, which is taken as the candidate cutting pattern of the surplus plate.

5. The Optimal Cutting Pattern.

5.1. Dynamic priority and quality-monitor. After obtaining the cutting pattern $\boldsymbol{X}_+$ of a surplus plate by Algorithm 2, we compare its performance indexes with those of cutting patterns in $\boldsymbol{L}_3$ one by one in turn. If their two values corresponding to the current performance index are the same, then we compare their next performance index values. So, its appropriate position sorted in descending order with respect to given indexes is found in $\boldsymbol{L}_3$ and it is inserted into the position. Candidate cutting patterns in $\boldsymbol{L}_3$ are quality-monitored in turn, and the first one quality-monitored successfully is taken as the optimal cutting pattern $\boldsymbol{X}_{opt}$ of the surplus plate.

Indexes, such as the rolling yield, and their corresponding values (the smaller the value is, the higher the priority is) are stored into a database table (see Appendix 2). As long as we interchange their values in the table, their priorities will also be changed correspondingly after renew sorting. In this way, dynamic priorities of several indexes are achieved.

5.2. Cutting mode. For two packing sub-scheme $\boldsymbol{X}_k$ $(k = 1, 2)$ of $\boldsymbol{X}_{opt}$, we define $W^k = \max\{w_i | a_{ki} \neq 0,\ i = t_1, t_2, \ldots, t_q\}$, $w^k = \min\{w_i | a_{ki} \neq 0,\ i = t_1, t_2, \ldots, t_q\}$. While $W^2 = 0$, if $W^1 = w^1$, the cutting mode of $\boldsymbol{X}_{opt}$ is A1; otherwise, its cutting mode is A2. While $W^2 > 0$, if $W^2 = w^2$, its cutting mode is S1; otherwise, its cutting mode is S2.

5.3. The proposed algorithm. After synthetizing the above discuss, we represent DH SA for the SPCP problem. Let L_3 and L_4 denote the candidate cutting pattern and the optimal cutting pattern lists respectively. Other symbolics have been defined in Sections 2-4. According to the above discussion, the pseudo-code of the proposed DHSA (Algorithm 3) is as follows:

Algorithm 2 Constructing candidate patterns

```
void Candidate_Pattern (d, m, PL, OL, L3)
{  Define X, X1, X2, j;
   For each combination of j orders (j = 1, 2, ..., d)
    {  Construct X1 = {(li, wi, a1i), i = u1, u2, ..., uj};
       OL = OL − X1;
       Generate X2 = {(lj, wj, a2j), j = v1, v2, ..., vj};
       X = X1 + X2; OL = OL − X2;
       If (X satisfies rules in C2 and its rolling yield ≥ η)
       {  Calculate X+;
          Insert X+ to L3 in descending order of indexes;
       }
    }
}
```

Algorithm 3 DHSA

```
Construct two lists L1 and L2;
Call Algorithm 1 to divide L1 and L2 into PLi and OLi
Calculate mi and ni for i = 1, 2, ..., N;
If (N = 0) exit (0)
For (i = 1; i ≤ N; i++)
{  k = 1;
   While (k ≤ ni)
   { Calculate dk of k-th surplus plate of PLi by OLi;
      Candidate_Pattern (dk, mi, PLi, OLi, L3);
```

$\boldsymbol{P} = \boldsymbol{L}_3$;
While ($P \neq$ null)
{ If (quality-monitor (P) = 1) break;
Else $P = P$.next;
}
Calculate its cutting mode and rolling-yield;
Insert P into $\boldsymbol{L}_4$; update $\boldsymbol{OL}_i$; empty $\boldsymbol{L}_3$; k++;
}
}
All surplus plates in $\boldsymbol{L}_4$ are stored into the table;
Free $\boldsymbol{L}_1 - \boldsymbol{L}_4$, P, $\boldsymbol{PL}_i$ and $\boldsymbol{OL}_i$ for $i = 1, 2, \ldots, N$.

6. Experiments and Analysis.

6.1. Experiments.

Environment: The data of Experiment 1 derive from [10]. Our algorithm is coded in C++ language and run on a 1.83 GHz Pentium(R) Dual-Core machine with 1 GB memory. The data of Experiments 2-4 derive from the large steel and iron company in China, whose run environment is the unix operation system, MES application software, tuxedo middleware, oracle data-base and Pro C language. The computer is an 8CPU IBM p570 with 32GB memory. Numbers of the surplus plates and ordered pieces of three experiments are shown in Table 2 respectively. We take threshold $\eta = 0.80$ and regard the rolling yield as the first priority index in the following Experiments 2-4.

Experiment 1. The data of the experiment derive from [10], whose gcuts 1-7 are the same as the SPCP problem in the size of d. After removing Formula (4), Algorithm 2 is revised to find the optimal solution for the gcuts 1-7 and their computational results are shown in Table 2. Other data in Table 2 are taken from [10]. It can be known from Table 1 that, the overall areas of our algorithm are greater than or equal to those of the gcut 1 and gcuts 5-7, are close to those of gcuts 2-4, and the computational efficiency is higher than that of [10].

Table 1. The performance comparison of algorithms in [10] and the proposed DHSA for Example 1

Problem	size	A1 [10]		A2 [10]		The revised Algorithm 2	
		$z_1(S_{od})$	T_1	$UB_2(S_{od})$	T_2	S_{od}	T/s
gcut 1	10	48368	2.89	48368	0.1	48368	0.0027
gcut 2	20	59307	5.56	59307	7.58	59249	0.0029
gcut 3	30	60241	6.50	61070	T.L.	59827	0.0609
gcut 4	50	60942	12.95	61379	T.L.	60590	0.1641
gcut 5	10	195582	4.35	195582	0.03	200359	0.0001
gcut 6	20	235305	7.13	235305	0.02	237797	0.0006
gcut 7	30	238974	11.06	238974	113.0	242034	0.0080

Experiment 2. There are 421 surplus plates and 18869 ordered pieces in the surplus plate table and order table, respectively. We take the MHCI approach and the proposed DHSA to successfully solve the optimal cutting patterns (denoted by N_y in Figure 3(a)) of 186 and 269 surplus plates, respectively. Their numbers (denoted by N_o in Figure 3(a)) of ordered pieces packed, computation times (denoted by T_c in Figure 3(a)) and average rolling yields (denoted by Avg in Figure 3(a)) are shown in Table 1 and Figure 3(a). From Table 1 and Figure 3(a), we can see that the overall number of ordered pieces to be cut by the proposed DHSA is 45% more than that of the MHCI approach; the computational

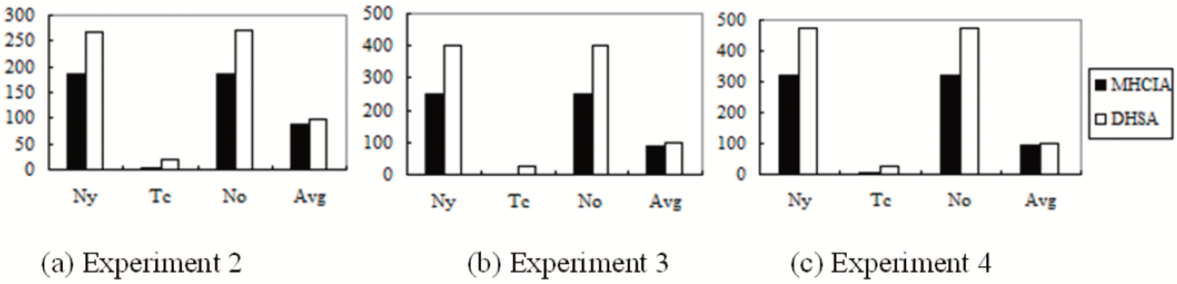


Figure 3. The performance comparison of DSHA and MCHI for Experiments 2-4

Table 2. The performance comparison of algorithms in [10] and the proposed DHSA for Example 1

Experimental number		2	3	4
Data	Number of surplus plates	421	859	1307
	Number of ordered pieces	18869	161778	14062
MHCIA	Number of surplus plates with the optimal cutting pattern	186	249	320
	Cost time/day	2 (day)	2.5 (day)	3 (day)
	Number of ordered pieces to be cut	186	249	320
	the average rolling yield	0.8758	0.9021	0.9341
DHSA	Number of surplus plates with the optimal cutting pattern	269	399	475
	Cost time/Second	19(s)	24(s)	27(s)
	Number of ordered pieces to be cut	271	400	476
	The average rolling yield	0.9716	0.9762	0.9794

efficiency of the proposed DHSA is at least three magnitudes faster than that of the MHCI approach; the average rolling yield of the DHSA is at least 10.1% higher than that of the proposed MHCI approach.

Experiment 3. There are 859 surplus plates and 16178 ordered pieces in the surplus plate table and order table, respectively. We take the MHCI approach and the proposed DHSA to successfully solve the optimal cutting patterns of 249 and 399 surplus plates, respectively. Their numbers of ordered pieces packed, computation times and average rolling yields are shown in Table 1 and Figure 3(b). From Table 1 and Figure 3(b), we can see that the overall number of ordered pieces to be cut by the proposed DHSA is 61% more than that of the MHCI approach; the computational efficiency of the proposed DHSA is at least three magnitudes faster than that of the MHCI approach; the average rolling yield of the proposed DHSA is at least 7.4% higher than that of the MHCI approach.

Experiment 4. There are 1037 surplus plates and 14062 ordered pieces in the surplus plate table and order table, respectively. We take the MHCI approach and the proposed DHSA to successfully solve the optimal cutting patterns of 320 and 475 surplus plates, respectively. Their numbers of ordered pieces packed, computation times and average rolling yields are shown in Table 1 and Figure 3(c). From Table 1 and Figure 3(c), we can see that the overall number of ordered pieces to be cut by the proposed DHSA is 49% more than that of the MHCI approach; the computational efficiency of the proposed DHSA is at least three magnitudes faster than that of the MHCI approach; the average rolling yield of the proposed DHSA is at least 4.5% higher than that of the MHCI approach.

6.2. **Time complexity analysis.** The proposed DHSA mainly includes three steps: (i) grouping and filtering out; (ii) calculating all candidates cutting patterns of each group; (iii) obtaining the optimal cutting patterns of each group.

(i) Time complexities of reading data, grouping and filtering out all are $O(n+m)$.

(ii) Suppose that after grouping and filtering out, the number of orders of the i-th group is m_i ($i = 1, 2, \ldots, N$). For calculating $\boldsymbol{X}_+$ of the surplus plate j ($j = 1, 2, \ldots, n_i$) of i-th group, let d_{ij} be the maximal number of "small" rectangles in its sub-scheme, such as $\boldsymbol{X}_1$, the number of its sub-schemes searched by Algorithm 2 is $O(\sum_{k=1}^{d_{ij}} c_{m_i}^k)$. That is to say, for obtaining candidate packing schemes of the i-th group, it is essential to construct $O(\sum_{j=1}^{n_i} \sum_{k=1}^{d_{ij}} c_{m_i}^k)$ schemes. So, it needs to construct $O(G)$ schemes for N groups, where $G = \sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{d_{ij}} c_{m_i}^k$. Suppose that $m_{\max} = \max(m_1, m_2, \ldots, m_N)$, $d_{\max} = \max(d_{ij},\ i = 1, 2, \ldots, N,\ j = 1, 2, \ldots, m_i)$ and $m_{\max} \gg d_{\max}$. The number of schemes is approximately $O(nC_{m_{\max}}^{d_{\max}})$ or $O(n(m_{\max})^{d_{\max}})$, where $n = \sum_{i=1}^{N} n_i$. Therefore, both $m_{\max}$ and $d_{\max}$ is two key elements to impact the time complexity of the 2nd step of the proposed DHSA.

Although the time complexity of the insertion sort method is $O(G_1^2)$, where G_1 is an overall number of the candidate cutting patterns, the time of inserting them into L_3 is very less. This is because that G_1 is two or three orders magnitude less than $O(G)$.

When $d_{\max} < 5$ and $m_{\max} = 50$, $G < 252875n$. So, there exists no problem of the combination explosion for the 2nd step of the proposed DHSA. For the step, best case is that there is only one surplus steel plate and one order with one ordered piece in each group, whose time complexity is $O(n)$. In addition, while $d_{\max} < 5$, it is impossible to occur the combinatorial explosion for the cause of the larger number of ordered pieces of the order.

(iii) For quality-monitoring (the function of MES), the best case is that the first candidate cutting pattern in $\boldsymbol{L}_3$ passes the quality-monitoring for each surplus plate and its time complexity is $O(n)$. The time complexity is $O(G)$ for the worst case. Statistical data show that the consuming time of quality-monitoring is 3/5 of whole consuming time. In order to reduce the quality-monitoring time, if an ordered piece of a candidate cutting pattern of a surplus plate fails to pass the quality-monitoring, then all the candidate cutting patterns which include the kind of ordered pieces are deleted. So, the time of quality-monitoring can be massively decreased.

The results of the above four experiments and theoretical analysis show that owing to a fusion of the knowledge obtained and divide-conquer heuristic strategy, the proposed DHSA can fit four difficulties of this problem, by which the optimal cutting pattern of each surplus can be found quickly.

7. **Conclusions.** This paper suggests a divide-conquer heuristic search approach for the cutting pattern problem of surplus plates of the steel and iron company. For the proposed DHSA, quickly grouping and filtering can be implemented by calling Algorithm 1. The knowledge-based divide-conquer and heuristic search of Algorithm 2 is an exhaustive search for all candidate cutting patterns but generates no combination explosion and makes the cutting pattern of each surplus plate have the optimal rolling yield. By changing priority values corresponding to indexes in a table, dynamic priority of the proposed DHSA (Algorithm 3) is simply implemented. In addition, the proposed DHSA involves only arithmetic and comparison operation except for quality-monitoring. Therefore, the proposed DHSA has better performance. The related key technologies are introduced in detail in the paper. Experiment 1 on seven examples in [10] shows that our approach achieves better solutions on three ones, the same solution on one and near solutions

on three ones in shorter time. Experiments 2-4 show that compared with the MHCI design approach, the proposed DHSA algorithm improves the calculation efficiency and rolling yield of the surplus plates. The above results fully validate the feasibility and effectiveness of the proposed DHSA algorithm. Simultaneously we also know that the study on a feasible and effective approach for the SPCP problem with a larger $d_{\max}$ is our future work.

Acknowledgment. This work was supported by the National Natural Science Foundation of China (Grant No. 61272294) and the National Science and Technology Supporting Projects (Grant No. 2012BAF10B04) and Research Foundation of Education Bureau of Hunan Province, China (Grant No. 11A120) and the construct program of the key discipline in Hunan province.

REFERENCES

[1] C.-Y. Wang, D.-Y. Zhao and J.-G. Xiao, Study on the layout problem of business advertisements, *J. Control and Decision*, vol.18, no.5, pp.559-562, 2003 (in Chinese).
[2] Y.-D. Cui, C.-L. Zhang and Y.-A Zhao, Continued fractions and branch-and-bound algorithm for generating cutting patterns with equal rectangles, *Journal of Computer Aided Design & Computer Graphics*, vol.16, no.2, pp.252-256, 2004.
[3] Y.-R. Li, C.-P. Yan and F. Liu, An interpolation algorithm for constrained two-dimensional non-guillotine cutting stock, *Journal of Chongqing University*, vol.25, no.10, pp.43-45, 2002 (in Chinese).
[4] S. Khebbache, C. Prins and A. Yalaoui, Iterated local search algorithm for the constrained two-dimensional non-guillotine cutting problem, *Journal of Industrial and Systems Engineering*, vol.2, no.3, pp.164-179, 2008.
[5] D. Fayard, M. Hifi and V. Zissimopoulos, An efficient approach for large-scale two-dimensional guillotine cutting stock problems, *Journal of the Operational Research Society*, vol.49, no.12, pp.1270-1277, 1998.
[6] J. E. Beasley, Algorithms for unconstrained two-dimensional guillotine cutting, *Journal of the Operational Research Society*, vol.36, no.4, pp.297-306, 1985.
[7] R. Macedo, C. Alves and J. M. Valério de Carvalho, Arc-flow model for the two-dimensional guillotine cutting stock problem, *J. Computers & Operations Research*, vol.37, no.6, pp.991-1001, 2010.
[8] Z.-L. Yuan, The method and procedure for the optimization cutting of plate materials, *Journal of Tianjin University*, vol.20, no.3, pp.103-110, 1987 (in Chinese).
[9] R. W. Haessler and P. E. Sweeney, Cutting stock problems and solution procedures, *European Journal of Operational Research*, vol.54, no.5, pp.141-150, 1991.
[10] M. Dolatabadia, A. Lodib and M. Monaci, Exact algorithms for the two-dimensional guillotine knapsack, *J. Computers & Operations Research*, vol.39, no.1, pp.48-53, 2012.
[11] C. Christoforos and K. Fleszar, A constructive bin-oriented heuristic for the two-dimensional bin packing problem with guillotine cuts, *J. Computers & Operations Research*, vol.38, no.10, pp.1443-1451, 2011.
[12] P.-Y. Wang, Two algorithms for constrained two-dimensional cutting stock problems, *J. Operations Research*, vol.31, no.3, pp.573-586, 1983.
[13] S. Polyakovskya and R. M'Hallah, An agent-based approach to the two-dimensional guillotine bin packing problem, *European Journal of Operational Research*, vol.192, no.3, pp.767-781, 2009.
[14] J. Błażewicz, P. Hawryluk and R. Walkowiak, Using a tabu search approach for solving the two-dimensional irregular cutting problem, *J. Annals of Operations Research*, vol.41, no.4, pp.313-325, 1993.
[15] V. Petridis, S. Kazarlis and A. Bakirtzis, Varying fitness functions in genetic algorithm constrained optimization: The cutting-stock and unit commitment problems, *IEEE Transactions on Systems, Man, and Cybernetics – PART B: CYBERNETICS*, vol.28, no.5, pp.629-640, 1998.
[16] Y. Chen, M. Tang, R. Tong and J. Dong, Packing of polygons using genetic simulated annealing algorithm, *Journal of Computer Aided Design & Computer Graphics*, vol.15, no.5, pp.598-609, 2003 (in Chinese).
[17] D. Wolpert and W. Macready, No free lunch theorems for optimization, *IEEE Transactions on Evolutionary Computation*, vol.1, no.1, pp.67-82, 1997.

[18] Z.-Q. Li, Z.-J. Tian, Y.-F. Xie, R. Huang and J.-Y. Tan, A knowledge-based heuristic particle swarm optimization approach with the adjustment strategy for the weighted circle packing problem, *J. Computer and Mathematics with Applications*, vol.66, no.10, pp.1758-1769, 2013.

Appendix 1 Rule Set *C*. The cutting rules of surplus plates are as follows: (in order to keep the secret of the company, keywords have already been replaced by X-XXXXX, similarly hereinafter).

(1) The X-th bit of the xx code of the surplus plate is equal to the X-th bit of the XX code of every packing ordered piece;

(2) When the XXXX of the ordered piece is not equal to 0, XXXXX codes of all the packing ordered pieces of the cutting pattern of each surplus plate are the same;

(3) The XXXX code of the surplus plate is equal to the XXXXX code of every ordered piece packed on it plus XXXX and the serial number xx of the surplus plate is equal to the XXXX code of each ordered piece plus XXXX (XXXX);

(4) The XX code of the surplus plate is equal to the XX code of the ordered piece plus XXXX and the XX number of the surplus plate is equal to the XX code of the ordered piece plus the XXXX (XXXX); but except for the case that the XX code of the surplus plate is not equal to the xx code of the ordered piece and the XX code of the ordered piece is not equal to X, if the XX code of the surplus plate is equal to X and the XX code of the ordered piece is not equal to X, then allow packing the order piece(s) on the surplus plate;

(5) The XXXX code of the surplus plate is equal to the XX mark of the ordered piece;

(6) When the judgment standard XX of the surplus plate is equal to X, the XX standard(s) of the surplus plate and the ordered piece are the same and the XX rank of the surplus plate is not larger than the XX rank of the ordered piece, not allow packing the ordered piece on it;

(7) When the XXXX identification of the ordered piece is not equal to X, only the surplus plate can be cut with the X mode;

(8) When the ordered piece with XXXX code is not equal to X, only the surplus plate can be cut with X mode;

(9) When the XXXXX of the order is equal to X, only the surplus plate can be cut with XX and XX mode;

(10) When the X-th bit of XX code of the surplus plate is equal to X, and the XX way code of the ordered piece is equal to X, if the XX of the surplus plate is equal to X, then they cannot pack on the surplus plate.

Appendix 2 The Definition of Priority. Priorities of several performance indicators such as the rolling yield are given in the following table, their default value are 1, 2, . . . , respectively.

Appendix Table 1 Priorities of several indexes

a[0]	Rolling yield	1
a[1]	XXXX	2
a[2]	XXX	3
a[3]	XX	4
a[4]	XX	5
a[5]	XX	6
. . .	. . .	. . .