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CONSIDERATION OF AN IMPERFECT PRODUCTION PROCESS IN THE SUPPLY CHAIN OF AN INTEGRATED INVENTORY MODEL WITH POLYNOMIAL PRESENT VALUE AND DEPENDENT CRASHING COST

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Abstract. *Many studies have failed to give consideration to time value. However, consideration of the impact of inflation and speedy change of demand, shortening lead time and cost are the most important issues in the supply chain. In a real system, due to the deteriorating production process of the vendor and damage sustained during the transportation process from the vendor to the buyer, an arrival order batch for the buyer may contain some percentage of defective product. Therefore, we consider time value and develop an improved integrated inventory model with imperfect quality items and variable lead time which assumes dependent crashing cost is polynomial. It aims at the minimum present value of integrated inventory joint expected total cost over infinite time horizon. The goal of this paper is to derive simultaneously the optimal order quantity, the length of lead time and the number of lots which are delivered from the vender to the buyer. Finally, a numerical example is given to illustrate results.*
Keywords: Integrated inventory, Imperfect production, Time value, Present value

1. **Introduction.** Reducing lead time is one of the most important issues in the inventory system. Many inventory models assume that lead time is a crisp constant or a random variable, which is not subject to control. Tersine [1] proposed that lead time usually consists of the following components: order preparation, order transit, supplier lead time, delivery time and set-up time. Liao and Shyu [2] assumed that the demand follows normal distribution and the lead time consists of n components each having a different cost for reduced lead time. Later, Hariga and Ben-Daya [3] extended the Liao and Shyu's [2] model by considering both the lead time and the order quantity as decision variables. Ouyang et al. [4] referred to Hariga and Ben-Daya's [3] models, adding the stock-out cost and assumed that shortages are allowed.

Many inventory systems fail to give consideration to time value. The impacts of economic cycles such as inflation or deflation cause variation of the purchasing power of money. In Ding and Grubbström's [5] article, an approach in terms of a present value principle taking all payments into consideration is applied in order to derive the optimal solution. By applying the present value measure, the opportunity cost for the initial inventory build-up is incorporated automatically. Jaggi and Aggarwal [6] presented the economic ordering policies of deteriorating items in the presence of trade credit using a discounted cash-flows (DCF) approach. In Ben-Daya and Hariga's paper [7], they follow the discounted cash flow approach to develop two time varying lot sizing models for the inventory replenishment problem with linear trend in demand taking into account the effects of inflation and time value of money. Silver et al. [8] investigate the impact of inflation on the choice of replenishment quantities in the basic EOQ model.

A common unrealistic assumption of the above integrated inventory models is that all units produced or received products are perfect without any defects. This is not true in reality. In practice, it can often be observed that there are defective items being produced through unreliable production process. These items must be rejected, repaired, reworked, or if they have reached the customer, refunded. Thus quality related costs should be taken into account when giving consideration to the influence of process unreliability. Porteus [9] first incorporated the effect of defective items into the basic EOQ model and introduced the alternative of investing in-process quality improvement through reducing uncontrollable process quality parameters. Rosenblatt and Lee [10] also investigated the effect of an unreliable production process incorporated into the EPQ model. Their results showed a reduction in the lot size as the average percentage of imperfect quality items increases.

Lee and Rosenblatt [11] considered process inspection during production cycles so that changes which could lead to the process being out of control, could be inspected and restored earlier than classical EOQ models. Schwaller [12] extended the EOQ model by combining a known defective rate assumption in the incoming lots with fixed and variable screening costs incurred in finding and expelling the items. Zhang and Gerchak [13] considered a joint lot sizing and inspection policy in an EOQ model where a random proportion of items were defective. Recently, Ben-Daya and Raouf [14] examined the effect of defective items on production scheduling and established a mathematical model to illustrate the scheduling questions. Salameh and Jaber [15] examined a joint lot sizing and inspection policy under an EOQ model for items with imperfect quality. Their results showed that economic lot size quantity tends to increase as the average percentage of imperfect quality items increases. This contradicts the finding of Rosenblatt and Lee [10]. They also suggested that poor-quality items should be sold as a single batch at a discounted price prior to receiving the next shipment. Hayek and Salameh [16] studied an inventory operating policy under the condition that imperfect quality items would be reworked where shortages are allowed and backordered. Goyal and Cardenas-Barron [17] presented a simple approach to determine the economic production quantity for items with imperfect quality. Huang [18,19] added Salameh and Jaber's concept [15] into the integrated inventory model proposed by Ha and Kim [20], and developed an integrated vendor-buyer inventory policy for flawed items in a just-in-time manufacturing environment. Later, Chung [21] gave the necessary and sufficient condition for the existence of the optimal solution to complement and improve the solution procedure of Huang's [19] inventory model. Lo and Yang [22] studied imperfect environment in inventory models in 2008. We propose here extending Yang's [23] research which considered present value in traditional inventory model. To make our research more realistic, we incorporated consideration of an imperfect environment to derive optimal inventory cost.

In today's global competition environment, the length of lead time affects the customer service level, inventory investment in safety stock, and the competitive abilities of a business directly. Hence, the objective of this research is to derive an optimal inventory strategy that can minimize the present value of the joint expected total cost over infinite time horizon.

2. Notations and Assumptions.

The following notations and assumptions are used throughout the integrated vendor-buyer inventory model.

Notations:

- D Average demand per year,
- P Production rate,
- Q Order quantity of the purchaser,
- A Purchaser's ordering cost per order,
- S Vendor's set-up cost per set-up,
- L Length of lead time,
- C_V Unit production cost paid by the vendor,
- C_P Unit purchase cost paid by the purchaser,
- m An integer representing the number of lots in which the items are delivered from the vendor to the purchaser,
- r Annual inventory holding cost per dollar invested in stocks,
- i Interest rate per year that is compounded continuously,
- θ The out-of-control probability,
- g The cost of replacing a defective unit.

Assumptions:

(1) There are single-vendor and single-purchaser for a single product in this model.
(2) The product is manufactured with a finite production rate P, and $P > D$.
(3) The demand X during lead time L follows a normal distribution with mean μL and standard deviation $\sigma\sqrt{L}$.
(4) The reorder point (ROP) equals the sum of the expected demand during lead time and the safety stock, that is, ROP $= \mu L + k\sigma\sqrt{L}$, where $k > 0$ is the safety factor. Inventory is continuously reviewed.
(5) The lead time crashing cost determined by the length of lead time and satisfying $R(L) = CL^{-a}$ is polynomial, where C and a are positive constant. (This assumption resembled Chandra and Grabis' [24] research.)
(6) The number of perfect quality items is at least equal to the demand during the inspecting time interval.
(7) The vendor delivers imperfect quality items in a single batch at the end of the buyer's 100% inspection process.
(8) While the vendor is producing a lot, the process can go out-of-control with a given probability θ each time another unit is produced. The process is assumed to be in control at the beginning of the production process. Once the process is out-of-control, the process produces defective items and continues to do so until the entire lot is produced. (This assumption is in line with Porteus [9].)

3. **Mathematical Model.** Pan and Yang [25] have developed an integrated inventory vender-buyer model controllable lead time. Their model is shown to provide a lower total cost and shorter lead time compared with those of Banerjee [26] and Goyal [27], and is useful for practical inventory problems. We would like to extend the basic model to better fit realistic inventory problems. A time value concept for amending the purchaser's inventory model with variable lead time is proposed by Yang et al. [28]. In addition, it can often be observed that there are defective items being produced through unreliable production process. Here we derive this model in the same way as proposed by Yang and Pan [29] and Yang et al. [28]. In this model, we assumed the lead time crashing cost determined by the length of lead time is polynomial.

Based on the above notations and assumptions, the total expected cost for the purchaser is given by:

$$TRC_P = \text{ordering cost} + \text{holding cost} + \text{crashing cost}.$$

As we know, the expected ordering cost is given by A. From assumption (3), the expected net inventory at the beginning of the cycle is $Q + k\sigma\sqrt{L}$. In this study, the purchaser's replenishment cycle length is Q/D. Therefore, the expected average inventory level is $Q + k\sigma\sqrt{L} - Dt$ for $t \in [0, \frac{Q}{D}]$. So the inventory holding cost for the first cycle equals

$$\begin{aligned} & m\int_{i=0}^{\frac{Q}{D}} rC_P\left(Q + k\sigma\sqrt{L} - Dt\right)e^{-it}dt \\ & = \frac{mrC_P}{i}\left[\left(Q + k\sigma\sqrt{L}\right)\left(1 - e^{-\frac{Qi}{D}}\right) + Qe^{-\frac{Qi}{D}} + \frac{D}{i}\left(e^{-\frac{Qi}{D}} - 1\right)\right] \end{aligned}$$

The lead time crashing cost is $R(L) = CL^{-a}$. Therefore, the total relevant cost for the purchaser at the first cycle is given by:

$$\begin{aligned} TRC_P(m, Q, L) = {} & mA + \frac{mrC_P}{i}\left[\left(Q + k\sigma\sqrt{L}\right)\left(1 - e^{-\frac{Qi}{D}}\right) + Qe^{-\frac{Qi}{D}} \right. \\ & \left. + \frac{D}{i}\left(e^{-\frac{Qi}{D}} - 1\right)\right] + mCL^{-a} \end{aligned} \tag{1}$$

The vender's total expected cost can be represented by:

$$TRC_V = \text{set-up cost} + \text{holding cost} + \text{defective cost}.$$

Since S is the set-up cost for the vender. The vendor manufactures mQ (where m is an integer) products with a finite production rate of P, in one setup but ships a quantity of Q to the buyer over m times. The inventory model for the vendor is shown in Figure 1. As the cycle length for the vender is mQ/D, the vender's average inventory can be evaluated as follows:

$$\begin{aligned} I_V & = \frac{\left\{\left[mQ\left(\frac{Q}{P} + (m-1)\frac{Q}{D}\right) - \frac{m^2Q^2}{2P}\right] - \left[\frac{Q}{D}(1 + 2 + \ldots + (m-1))Q\right]\right\}}{\frac{mQ}{D}} \\ & = \frac{Q}{2}\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2P}{D}\right], \quad \text{for } t \in \left(0, \frac{mQ}{D}\right) \end{aligned}$$

Hence, the inventory holding cost for the vender at the first cycle equals

$$\begin{aligned} & \int_{i=0}^{\frac{mQ}{D}} rC_V\frac{Q}{2}\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2P}{D}\right]e^{-it}dt \\ & = \frac{rC_V}{i}\left(1 - e^{-\frac{mQi}{D}}\right)\frac{Q}{2}\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2P}{D}\right] \end{aligned}$$

The vendor produces the item in the quantity mQ with a given probability of θ that the process can go out of control. Porteus [9] suggested the expected number of defective items in a run of size mQ can be evaluated as $m^2Q^2\theta/2$. Suppose g is the cost of replacing a defective unit, and the production quantity for the supplier in a lot of mQ. (Yang and Pan [29]).

Therefore, the total relevant cost for the vender at the first cycle is given by:

$$TRC_V(m, Q) = S + \frac{rC_V}{i}\left(1 - e^{-\frac{mQi}{D}}\right)\frac{Q}{2}\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2P}{D}\right] + \frac{gm^2Q^2\theta}{2} \tag{2}$$

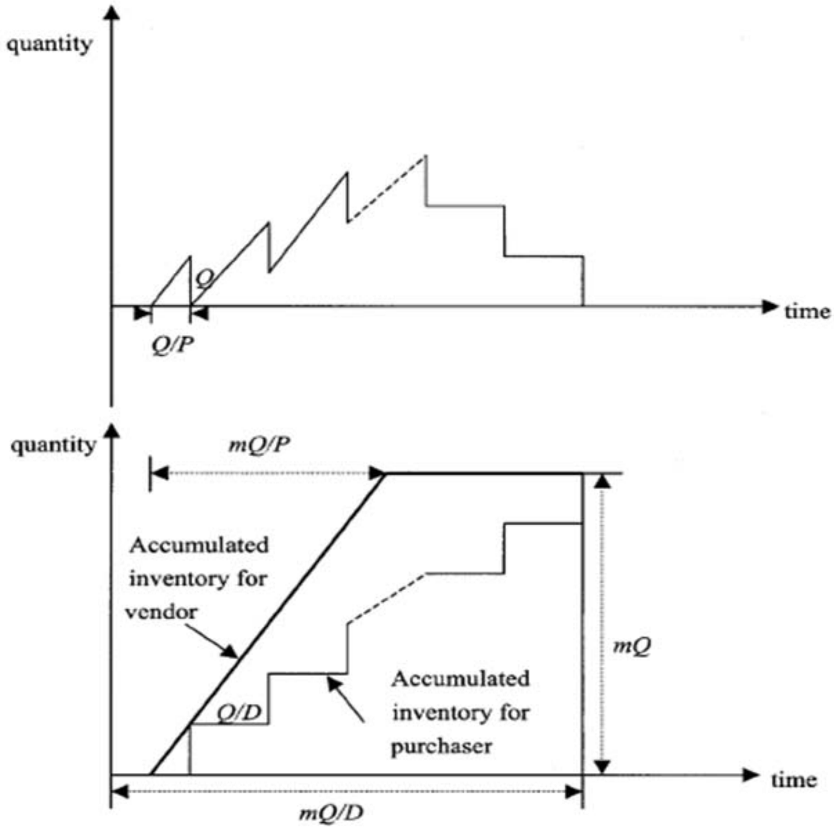

FIGURE 1. The inventory model for the vendor

Therefore, the joint expected total relevant cost for the first cycle $JTRC(m, Q, L)$ is presented as follows

$$JTRC(m, Q, L) = TRC_P(m, Q, L) + TRC_V(m, Q)$$

We adopt the time value concepts approach of Yang et al. [28]. Therefore, the present value of the joint expected total relevant cost over infinite time horizon, $PVC(m, Q, L)$, is given by

$$\begin{aligned} &PVC(m, Q, L) \\ =& \frac{1}{1 - e^{-\frac{mQi}{D}}}[JTRC(m, Q, L)] \\ =& \frac{1}{1 - e^{-\frac{mQi}{D}}}\left\{mA + \frac{mrC_P}{i}\left[\left(Q + k\sigma\sqrt{L}\right)\left(1 - e^{-\frac{Qi}{D}}\right) + Qe^{-\frac{Qi}{D}} + \frac{D}{i}\left(e^{-\frac{Qi}{D}} - 1\right)\right]\right. \\ &\left. + mCL^{-a} + S + \frac{rC_V}{i}\left(1 - e^{-\frac{mQi}{D}}\right)\frac{Q}{2}\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2P}{D}\right] + \frac{gm^2Q^2\theta}{2}\right\} \end{aligned} \quad (3)$$

4. **Solution Procedure.** It is necessary to find the minimum value of the present value of the expected total relevant cost $PVC(m, Q, L)$. For fixed Q and m, taking the first and second partial derivation of $PVC(m, Q, L)$ with respect to L, we have

$$\frac{\partial PVC(m, Q, L)}{\partial L} = \frac{1}{1 - e^{-\frac{mQi}{D}}}\left[\frac{mrC_P}{2i\sqrt{L}}k\sigma\left(1 - e^{-\frac{Qi}{D}}\right) - maCL^{-a-1}\right] \quad (4)$$

and

$$\frac{\partial^2 PVC(m,Q,L)}{\partial^2 L} = \frac{1}{1-e^{-\frac{mQi}{D}}}\left[\frac{mrC_P}{4i\sqrt{L^3}}k\sigma\left(1-e^{-\frac{Qi}{D}}\right) - ma(a+1)CL^{-a-2}\right] \tag{5}$$

Because (5) >0, hence, for fixed Q and m, $PVC(m,Q,L)$ is a convex function in L. Thus, there exists a unique value of L which minimizes $PVC(m,Q,L)$. L can be obtained by solving the equation $\partial PVC(m,Q,L)/\partial L = 0$ in (4), and is given by

$$L = \left(\frac{2iac}{rC_P k\sigma\left(1-e^{-\frac{Qi}{D}}\right)}\right)^{\frac{1}{a+\frac{1}{2}}} \tag{6}$$

Let $\left(\frac{2iac}{rC_P k\sigma}\right)^{\frac{1}{a+\frac{1}{2}}} = B$. Substituting (6) into (3), we obtain

$$\begin{aligned} &PVC(m,Q)\\ &= \frac{1}{1-e^{-\frac{mQi}{D}}}\left\{mA + \frac{mrC_P}{i}\left[\left(Q + k\sigma B^{\frac{1}{2}}\left(1-e^{-\frac{Qi}{D}}\right)^{\frac{-1}{2a+1}}\right)\left(1-e^{-\frac{Qi}{D}}\right)\right.\right.\\ &\quad \left. + Qe^{-\frac{Qi}{D}} + \frac{D}{i}\left(e^{-\frac{Qi}{D}}-1\right)\right] + mCB^{-a}\left(1-e^{-\frac{Qi}{D}}\right)^{\frac{2a}{2a+1}} + S\\ &\quad \left. + \frac{rC_V}{i}\left(1-e^{-\frac{mQi}{D}}\right)\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2P}{D}\right] + \frac{gm^2Q^2\theta}{2}\right\} \end{aligned} \tag{7}$$

For fixed m, to obtain the optimal order quantity of the purchaser Q, by taking the first partial derivation of $PVC(m,Q)$ in (7) with respect to Q and setting the result to be zero, we have

$$\begin{aligned} &\frac{\partial PVC(m,Q)}{\partial Q}\\ &= \frac{1}{1-e^{-\frac{mQi}{D}}}\left\{\frac{mrC_P}{i}\left[\left(\frac{2a}{2a+1}k\sigma B^{\frac{1}{2}}\left(1-e^{-\frac{Qi}{D}}\right)^{\frac{-1}{2a+1}}\right)\left(\frac{i}{D}e^{-\frac{Qi}{D}}\right) + \left(1-e^{-\frac{Qi}{D}}\right)\right]\right.\\ &\quad + \frac{2am}{2a+1}CB^{-a}\left(1-e^{-\frac{Qi}{D}}\right)^{\frac{-1}{2a+1}}\left(\frac{i}{D}e^{-\frac{Qi}{D}}\right)\\ &\quad \left. + \frac{rC_V}{2i}\left(1-e^{-\frac{mQi}{D}}\right)\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2P}{D}\right] + gm^2Q\theta\right\}\\ &\quad - \frac{1}{1-e^{-\frac{mQi}{D}}}\left(\frac{mi}{D}e^{-\frac{mQi}{D}}\right)\left\{mA + \frac{mrC_P}{i}\left[Q + k\sigma B^{\frac{1}{2}}\left(1-e^{-\frac{Qi}{D}}\right)^{\frac{2a}{2a+1}}\right.\right.\\ &\quad \left.\left. + \frac{D}{i}\left(e^{-\frac{Qi}{D}}-1\right)\right] + CB^{-a}\left(1-e^{-\frac{Qi}{D}}\right)^{\frac{2a}{2a+1}} + S + \frac{gm^2Q^2\theta}{2}\right\} = 0 \end{aligned} \tag{8}$$

Next we need to check the second-order condition for concavity, that is

$$\begin{aligned} &\frac{\partial^2 PVC(m,Q)}{\partial^2 Q}\\ &= \frac{1}{1-e^{\frac{-mQi}{D}}}\left\{\frac{mrC_P}{i}\left\{\frac{2a}{2a+1}kaB^{\frac{1}{2}}\left[\frac{-1}{2a+1}\left(1-e^{\frac{-Qi}{D}}\right)^{\frac{-2a-2}{2a+1}}\left(\frac{i}{D}e^{\frac{-Qi}{D}}\right)^2\right.\right.\right. \end{aligned}$$

$$
\begin{aligned}
&-\left(1-e^{-\frac{Qi}{D}}\right)^{\frac{-1}{2a+1}}\left(\frac{i^2}{D^2}e^{\frac{-Qi}{D}}\right)\Bigg]+\frac{i}{D}e^{\frac{-Qi}{D}}\Bigg\}\\
&+\frac{2am}{2a+1}CB^{-a}\left[\frac{-1}{2a+1}\left(1-e^{\frac{-Qi}{D}}\right)^{\frac{-2a-2}{2a+1}}\left(\frac{i}{D}e^{\frac{-Qi}{D}}\right)^2\right.\\
&\left.-\left(1-e^{\frac{-Qi}{D}}\right)^{\frac{-1}{2a+1}}\left(\frac{i^2}{D^2}e^{\frac{-Qi}{D}}\right)\right]+gm^2\theta\Bigg\}\\
&-\frac{2}{\left(1-e^{\frac{-mQi}{D}}\right)^2}\left(\frac{mi}{D}e^{\frac{-mQi}{D}}\right)\left\{\frac{mrC_P}{i}\left[\frac{2a}{2a+1}kaB^{\frac{1}{2}}\left(\frac{i}{D}e^{\frac{-Qi}{D}}\right)\left(1-e^{\frac{-Qi}{D}}\right)^{\frac{-1}{2a+1}}\right.\right.\\
&\left.+\left(1-e^{\frac{-Qi}{D}}\right)\right]+\frac{2am}{2a+1}CB^{-a}\left(\frac{i}{D}e^{\frac{-Qi}{D}}\right)\left(1-e^{\frac{-Qi}{D}}\right)^{\frac{-1}{2a+1}}+gm^2Q\theta\Bigg\}\\
&+\frac{1+e^{\frac{-mQi}{D}}}{\left(1-e^{\frac{-mQi}{D}}\right)^2}\left(\frac{m^2i^2}{D^2}e^{-\frac{mQi}{D}}\right)\left\{mA+\frac{mrC_P}{i}\left[Q+kaB^{\frac{1}{2}}\left(1-e^{\frac{-Qi}{D}}\right)^{\frac{-1}{2a+1}}\right.\right.\\
&\left.+\frac{D}{i}\left(e^{\frac{-Qi}{D}}-1\right)\right]+mCB^{-a}\left(1-e^{\frac{-Qi}{D}}\right)^{\frac{2a}{2a+1}}+S+\frac{gm^2Q^2\theta}{2}\Bigg\}
\end{aligned} \tag{9}
$$

The results identify $PVC(m,Q)$ is a convex function in Q for fixed m. Therefore, it is reduced to find a local optimal solution in local minimum.

Summarizing the above arguments, we establish the following algorithm to obtain the optimal values of m, Q, L.

Step 1. Set $m=1$.
Step 2. Determine Q by solving (8).
Step 3. If there exists a Q and satisfying (9), we could determine L by (6). Then $(Q^{(m)},L^{(m)})$ is the optimal solution for given m.
Step 4. Get $PVC\left(m,Q^{(m)},L^{(m)}\right)$ by (3).
Step 5. If $PVC(m,Q^{(m)},L^{(m)})\le PVC(m-1,Q^{(m-1)},L^{(m-1)})$ then set $m=m+1$ and repeat Steps 2-4, otherwise go to Step 6.
Step 6. Set $PVC(m^*,Q^*,L^*)=PVC(m-1,Q^{(m-1)},L^{(m-1)})$. Then (m^*,Q^*,L^*) is the optimal solution.

5. **Example Instructions.** To illustrate the results of the proposed models, consider an inventory system with the following data (Yang and Pan [29]): $D=1000$ unit/year, $P=3200$ unit/year, $A=\$25$/order, $S=\$400$/set-up, $C_P=\$25$/unit, $C_V=\$20$/unit, $r=0.2$, $k=2.33$, $\sigma=7$ unit/week, $\theta=0.0002$, $g=\$15$ per defective unit and the lead time crashing cost $R(L)=1000\text{L}^{-3}$.

Following the algorithm in Section 4 we have the optimal number of lots $m^*=3$, optimal lead time $L^*=6.21$ weeks, optimal order quantity of the purchaser $Q^*=124$ units and the optimal PVC = \$28422.7. The solution procedure is shown as Table 1 and Figure 2.

For this example, we use the sensitivity analysis (shown in Table 2 and Table 3) to illustrate the proposed model.

Furthermore, three of these findings are worth summarizing:

(1) When the out-of-control probability θ increases, the optimal expected number of products per run m^*Q^* decreases and the integrated PVC increases.
(2) When the standard deviation of demand per week σ increases, the optimal order quantities of the purchaser Q^* and the integrated PVC increase.

TABLE 1. Summary of the computation results

m	Q	L (weeks)	PVC		
			Purchaser	Vender	Integrated
1	304	4.82	10562.1	19888.6	30450.6
2	173	5.65	8126.5	20502.1	28628.6
3	124	6.21	7599.4	20823.4	28422.7*
4	97	6.66	7583.1	21051.7	28634.8
5	81	7.01	7770.0	21238.0	29008.0

*The minimum present value of the joint expected total relevant cost.

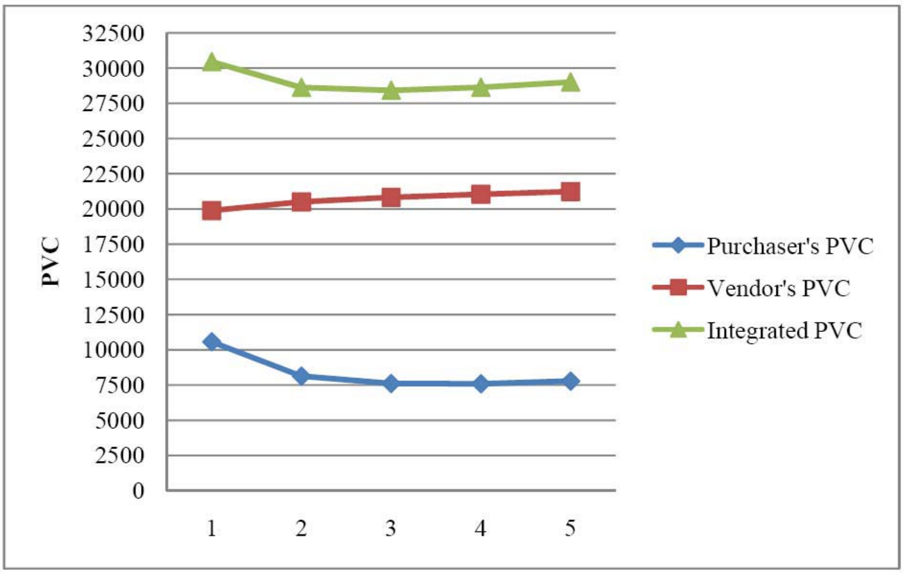

FIGURE 2. The optimal solutions

TABLE 2. Sensitive analysis of θ

θ	m^*	Q^*	m^*Q^*	L^*	Integrated PVC
0.0002	3	124	372	6.21	28422.7
0.0003	3	112	336	6.39	31112.7
0.0004	2	146	292	5.93	33464.8
0.0005	2	137	274	6.04	35615.9
0.0006	2	129	258	6.14	37634.0
0.0007	2	122	244	6.24	39541.3
0.0008	2	117	234	6.31	41354.1
0.0009	2	112	224	6.39	43085.1
0.0010	2	108	216	6.46	44745.3

*The minimum present value of the joint expected total relevant cost.

(3) When the average demand per year D increases, the optimal expected numbers of products per run m^*Q^* and the integrated PVC increase.

A more detailed understanding of this relationship can be gained from Table 2 and Table 3. Note that the increases in the out-of-control probability θ, the standard deviation of demand per week σ and the average demand per year D will lead to the increase in the integrated PVC.

TABLE 3. Sensitive analysis of σ and D

σ	D	m^*	Q^*	L^*	Integrated PVC
7	1000	3	124	6.21	28422.7
14	1000	3	125	5.08	30370.2
21	1000	3	126	4.52	32174.7
28	1000	2	178	3.77	33891.8
35	1000	2	179	3.53	35393.8
7	600	2	147	5.12	20976.6
7	1000	3	124	6.21	28422.7
7	1400	3	136	6.66	35398.4
7	1800	3	145	7.02	42150.3
7	2200	3	152	7.34	48774.7

*The minimum present value of the joint expected total relevant cost.

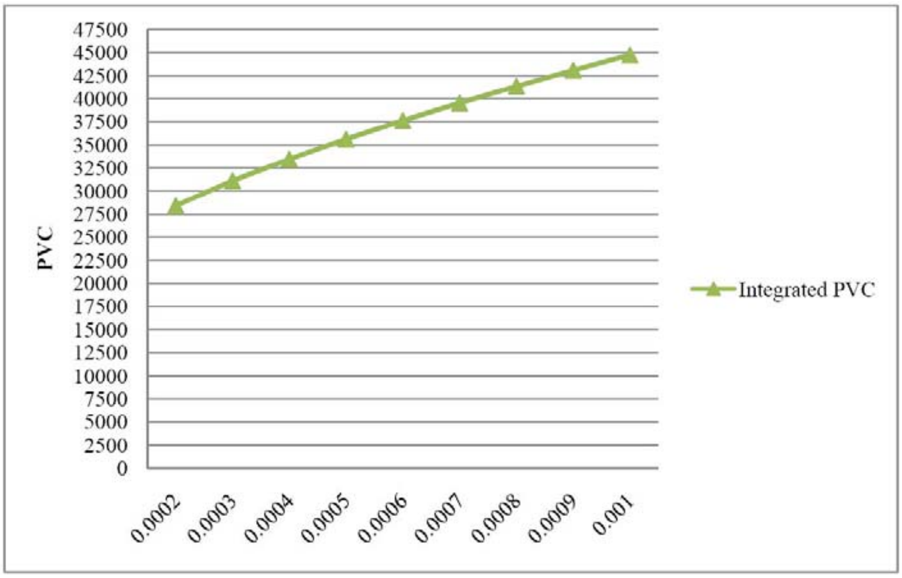

FIGURE 3. Sensitive analysis of θ

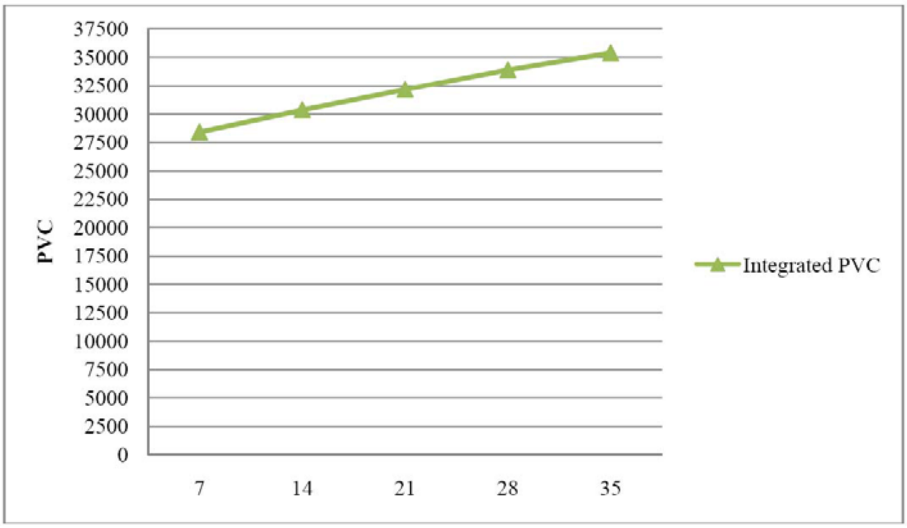

FIGURE 4. Sensitive analysis of σ

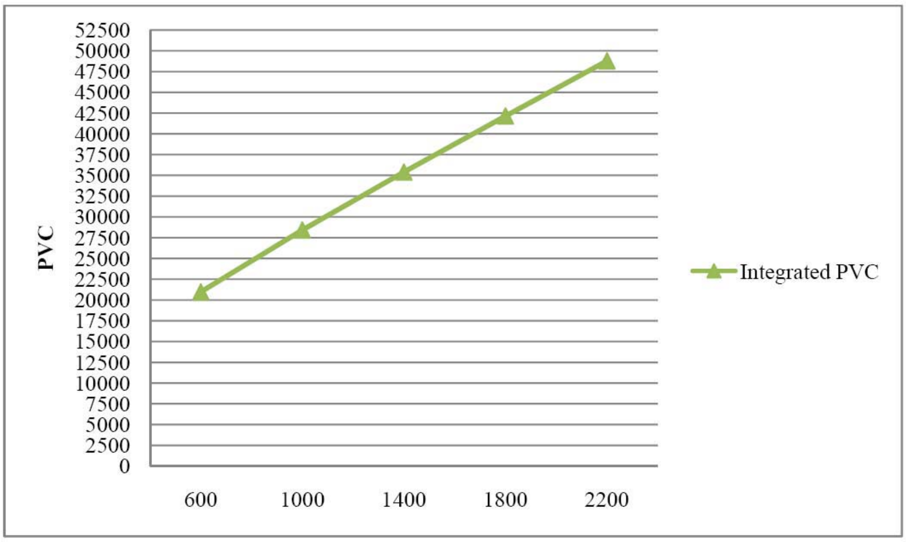

FIGURE 5. Sensitive analysis of D

6. **Conclusions.** Uncertainty of production process is inherent in real supply chain inventory systems. This chapter investigates a production/inventory situation in which producing process would go out of control with variable lead time which assumed dependent crashing cost is polynomial. Moreover, the time effect is too critical to ignore. There has thus far been relatively little research on this issue. For the reasons above, in this model, we employ Yang et al.'s [28] concept of the time value to formulate the optimal present value of integrated inventory joint expected total cost over infinite time horizon and then derive the corresponding optimal order quantity, the length of lead time and the number of lots which are delivered from the vender to the buyer.

Finally, the numerical example available in the Yang and Pan [29] demonstrates the solution algorithm. In addition, the decision-makers can use the solutions derived from the integrated inventory model to perform sensitivity analysis, so as to examine the effects of the interest rate per year.

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