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PRODUCTION MODEL USING AN ASYMMETRIC SIMPLE EXCLUSION PROCESS

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Abstract. *In manufacturing, improving throughput to secure management profit is important. We have been promoting a production flow system for improving productivity. Here we propose a lot-casting process, namely, the asymmetric simple exclusion process, to improve throughput. The main cause of production retention between processes is considered to be that different processes operate at different speeds, that is, the problem of working ability. Waiting-time constraints (idle time) occur between processes because of capacity differences. We use Burgers equation from statistical mechanics to analyze process retention. We propose that the nonlinear characteristics of Burgers equation correspond to an asynchronous state in the process. In contrast, the linear characteristics of Burgers equation correspond to the synchronous state of the process, that is, the state in which the throughput is maximized.*
Keywords: ASEP model, Burgers equation, Production density, Process retention

1. **Introduction.** The improvement of productivity and the evaluation method in the manufacturing industry have been discussed for many years [1, 2]. In the manufacturing industry, TOC (Theory of Constraints) was synonymous with basic productivity improvement. This is that an Israeli physicist, Dr. Erie Gold Rat has pioneered about 25 years ago. It is a methodology of system improvement [3]. Small fluctuations in an upstream subsystem appear as large fluctuations in the downstream (the so-called bullwhip effect) [4]. The bullwhip effect generates a large gap between the demand forecasts of the market and suppliers. Large fluctuations can be suppressed by the following mechanisms.

(1) Reducing the lead time, improving the throughput, and synchronizing the production process by the TOC.
(2) Sharing the demand information and performing mathematical evaluations.
(3) Analyzing the reduction and fluctuating demands of the subsystem (using nonlinear vibration theory).
(4) Basing the inventory management approach on stochastic demand.

When using manufacturing equipment, delays in one production step are propagated to the next. Hence, the use of manufacturing equipment itself may lead to delays. The improvement of production processes was presented that the "Synchronizing the production process method" was the most desirable in practice using the actual data in the production flow process based on the cash flow model by using the SDE of log-normal type [6]. In essence, we have proposed the best way, which is a synchronous method using the

Vasicek model for mathematical finance [5]. Then, the supply chain theme, which was a time delay in the production processes, was proposed for the throughput improvement based on a stochastic differential equation of log-normal type [4].

The synchronization method is superior for improving throughput in production processes, which is used by a production flow process [6]. The production flow process is utilized for production of high-mix low-volume equipments, which are produced through several stages in the production process. This method is good for producing specific control equipment such as semiconductor manufacturing equipment in our experience. Then, we have reported that the production flow process has nonlinear characteristics in our previous study [7].

Moreover, a working-time delay is propagated through the stages in the production process. Its delays are due to volatility in the model. Indeed, the actual data indicated that in the production flow process, the delays were propagated to the successive stages [8].

With respect to lead time, many aspects can potentially affect lead time. For example, from order products, the lead time from the start of development to the completion of a product is called the time-to-finish time, such as the work required preparation of the members for production equipments.

Moreover, several studies have focused on reducing customer lead times. In [9], the author addresses the problem of reducing the production lead time.

In [10], the authors propose a method that increases production efficiency and allows a greater diversity of consumer products to be produced. Their proposed approach shortens lead times and reduces the uncertainty in demand. Moreover, it captures the stochastic demand of customers and produces solutions by solving a nonlinear stochastic programming problem. In our previous report, we calculated the expected loss value from a lead-time function and verified a loss function with actual data [11]. Moreover, through theoretical analysis, we clarified that for improved productivity, minimizing volatility and fixed costs is important. A comparison between synchronous and asynchronous methods revealed a reduction of approximately 10% in the results when using synchronous throughput. A simple exclusion process is a non-equilibrium statistical mechanics model called a one-dimensional asymmetric simple exclusion process (ASEP). The ASEP is used in production lines to improve production efficiency. As an application method, the ASEP is used to optimize the production lot [12].

In this study, we use the ASEP to improve the efficiency of the production process. When applied as a model of a lot production system, the ASEP is fundamentally a nonlinear system represented by Burgers' equation. This indicates that the process transition probability plays an important role. From the experience of three test runs of PFP, we make the processes of each process identical and choose the appropriate value. We arrange the workers in each process appropriately. From the aforementioned procedure, a highly linear system is obtained that approaches a stationary system.

Dr. Nishinari proposed the ASEP model and the result of theoretical solution was consistent with the simulation result. The validity of the theoretical solution was confirmed. We apply the ASEP model to the actual production process, and also we represent the actual data compared with the production flow process (PFP). Comparing actual data on ASEP and PFP production efficiencies, ASEP was able to double the throughput and reduce production costs by 20%.

2. **Mathematical Model for ASEP Production Processes.** The ASEP is a non-equilibrium statistical mechanics model that is referred to an exclusive process. It is a

simple model in which numerous particles diffuse under the volume-exclusion interaction on a one-dimensional lattice.

2.1. **Burgers equation for production processes.** Figure 1 shows a one-lot-flowing model in ASEP. The terms α and β denote the input and output rates, respectively, and p denotes the transition probability from one stage to another.

Figure 2 shows that the flow volume Q in the steady state changes its behavior in accordance with the magnitude relation of α, β, and p. It is determined as shown in Figure 2. In Figure 2, the flow volume is defined as follows [12]:

$$Q = \alpha\frac{p-\alpha}{p-\alpha^2}, \quad \text{in } A$$
$$Q = \beta\frac{p-\beta}{p-\beta^2}, \quad \text{in } B$$
$$Q = \frac{1-\sqrt{1-p}}{2}, \quad \text{in } C$$

According to our previous study, we obtain the Burgers equation as follows [13, 17]:

$$\frac{\partial S(t,x)}{\partial t} + S(t,x)\frac{\partial S(t,x)}{\partial x} = D\frac{\partial^2 S(t,x)}{\partial x^2} \tag{1}$$

Equation (1) denotes the phenomenon of turbulence in fluid dynamics. If we apply it to a production process, in the asynchronous state, the dynamic characteristics of the potential (production density) in the process are strictly nonlinear [17].

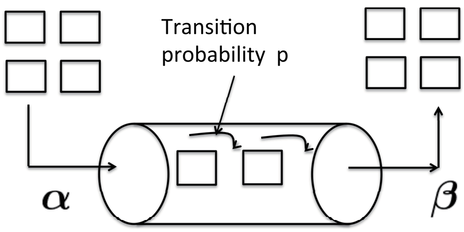

Figure 1. ASEP model

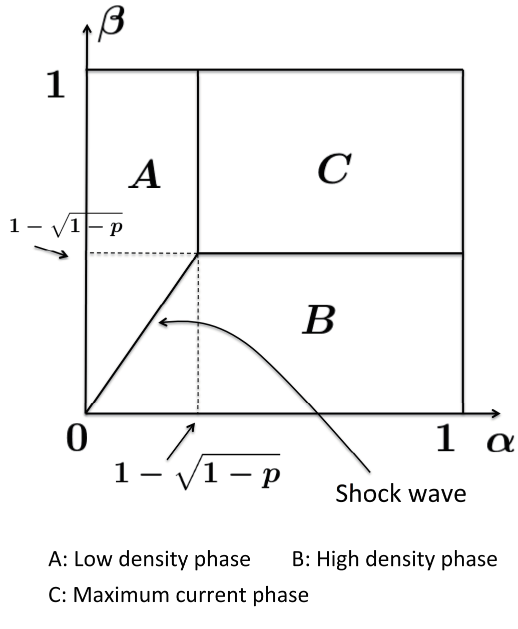

Figure 2. Phase diagram

We obtain the following after executing the Cole-Hopf transformation for Equation (1) [14]:

$$\frac{\partial \varphi(\tau,\xi)}{\partial \tau} = D\frac{\partial^2 \varphi(\tau,\xi)}{\partial \xi^2} \tag{2}$$

where $\varphi(\tau,\xi) \le 1$ and $\varphi(\tau,\xi)$ denotes a new production density function with a synchronous state and D denotes a production flow coefficient.

We obtain the solution $S(t,x)$ as follows [15]:

$$S(t,x) = -2D\frac{\partial}{\partial x}\ln\left[(4\pi Dt)^{-\frac{1}{2}}\int_{-\infty}^{\infty}\exp\left\{-\frac{(x-x^{'})^2}{4Dt} - \frac{1}{2D}\int_0^{x^{'}} S(0,x^{''})dx^{''}\right\}dx^{'}\right] \quad (3)$$

where the initial value $S(0,x)$ is assumed to be given, and $x^{'}$ and $x^{''}$ are first derivative and second derivative respectively.

From Equation (2), $\varphi(\tau,\xi)$ is derived at infinity as follows [15]:

$$\varphi(t,\xi) = \exp\left(-\int S(t,\xi)d\xi\right) \quad (4)$$

Equation (4) represents the fact that the spatial elements or components of the nonlinear function $S(t,\xi)$ are summed and that they decay exponentially. From the perspective of production processes, Equation (4) linearizes the nonlinear characteristics of workers and aligns them according to the production processes, linearize and synchronize by process improvement. In other words, Equation (4) converts a nonlinear function into a probability distribution with an exponential family of linear functions.

2.2. **Stochastic model of lot handling number.** Stochastic mode is derived as follows [16]:

$$dn(t) = \mu n(t)dt + \sigma n(t)dW(t) \quad (5)$$

where $n(t)$, μ, σ and $W(t)$ denote a lot handling number, trend, volatility and Wiener process respectively.

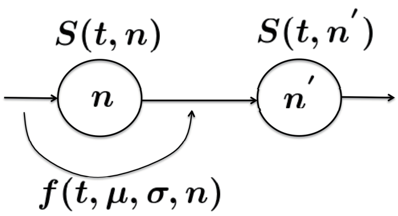

FIGURE 3. Change in the lot handling number and production density

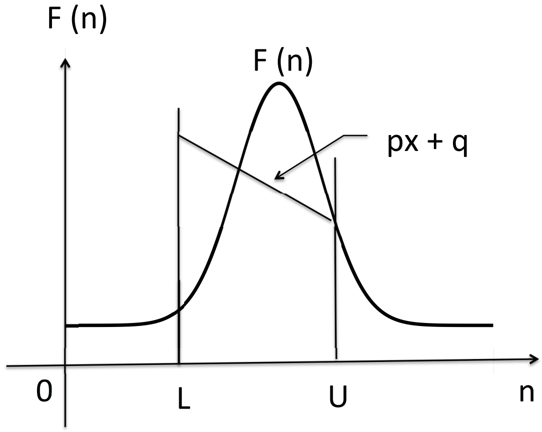

FIGURE 4. The expected processing function $F(n)$ and lot decreasing function $g(n)$

Figure 3 illustrates Equation (9), which is the Fokker-Planck equation. Figure 4 denotes the minimum lot (L) and maximum lot (U) using the expected processing function $F(n)$ and lot decreasing function $g(n)$. We can obtain the transition probability $f(n)$ from the Fokker-Planck equation:

$$f(n) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(n-\mu)^2}{2\sigma^2}\right\} \quad (6)$$

$$\frac{\partial P\left(t,n^{'}|t,n\right)}{\partial t} = \mathscr{L}_{FP}P\left(t,n^{'}|t,n\right) \quad (7)$$

$$\mathscr{L}_{FP} = -\frac{\partial}{\partial n} + \frac{D^2}{2}\frac{\partial^2}{\partial n^2} \quad (8)$$

where $P\left(t,n^{'}|t,n\right)$ denotes the transition probability density function, and the steady distribution of transition probability $f(n)$ is in accordance with a normal distribution [20].

According to Equations (6)-(8), the transition probability production density function $S\left(t,n^{'}|t,n\right)$ is derived as follows:

$$\frac{\partial S\left(t,n^{'}|t,n\right)}{\partial t} = \mathscr{L}_{FP} S\left(t,n^{'}|t,n\right) \tag{9}$$

where $\mathscr{L}_{FP}$ is derived as follows:

$$\mathscr{L}_{FP} \equiv \frac{\partial}{\partial n} + \frac{D^2}{2}\frac{\partial^2}{\partial n^2} \tag{10}$$

Definition 2.1. *Total lot processed number $G(n)$*

$$G(n) \equiv \bar{n} + F(n) \tag{11}$$

where $\bar{n}$ denotes a setting value. The function $F(n)$ denotes the expected function as follows [11]. When $y < L$, no production activity is taking place. When $y > U$, the quantity ordered exceeds the physical production limit. Therefore, we must reduce the demand and thus analyze $L \leq y \leq U$.

$$F(n) = \int_L^U (py + q) f(y) dy \; + \; C_0 \tag{12}$$

where let C_0 be a fixed cost. Please refer to our previous study for details [11].

1-stage processing time $T(n)$ is derived as follows:

$$T(n) \equiv \bar{N} + \frac{1}{G(n)} \tag{13}$$

where $\bar{N}$ and $1/G(n)$ denote a setting lead time and one lot transition time respectively.

From Equation (13), the expected total processing time T_M is derived as follows:

$$T_M = M \times \int_0^N T(n) dn = M \times \int_0^N \left(\bar{N} + \frac{1}{G(n)}\right) dn \tag{14}$$

where M denotes the total number of processes as 5×12.

3. Numerical Results.

3.1. Numerical simulation.

Table 1 gives four kinds of trend values and volatility values as parameters for Figures 6-9, Figures 10-13, Figures 14-17, Figures 18-21. For each of the four types of parameters, the representative each value of the trend value and volatility is selected. As is clear from the figure, the simulation result of the high trend value and the low volatility value is good. As a result, the four specific cases show case 4 > case 3 > case 2 > case 1.

- Expected processing rate function for normal probability distribution. We show Figures 6-9.
- Expected 1-lot transition time for normal probability distribution. We show Figures 10-13.
- 1-stage processing time for normal probability distribution. We show Figures 14-17.
- Total processing time for normal probability distribution. We show Figures 18-21.

Table 1. Numerical data for simulation

	case 1	case 2	case 3	case 4
μ	0.7	0.9	0.9	0.9
σ	0.2	0.2	0.05	0.1

With respect to Figures 6-9, the expected processing-rate function for a normal probability distribution is derived as follows:

$$F(n) = \int f(n)g(n)dn, \; G(n) = 5 + F(n) \tag{15}$$

where $g(n) = pn + q$ denotes a decreasing function ($p = -1$).

With respect to Figures 10-13, the expected one-lot transition time is derived as follows:

$$1 - \text{lot transition time} = \frac{1}{5 + F(n)} \tag{16}$$

With respect to Figures 14-17, the one-stage processing time for a normal probability distribution is derived as follows:

$$T(n) = 25 + \frac{1}{5 + F(n)} \tag{17}$$

where 25 denotes an average processing time (min).

With respect to Figures 18-21, the total processing time for a normal probability distribution is given by Equation (14).

3.2. **Actual test results.** We compare the one-lot flow process (ASEP) and the PFP in relation to throughput. From the results, we find that twice the throughput is achieved with the ASEP. In Table 2, σ, σ_k, and σ_s denote the total standard deviations of the consumption time, the time until completion of a single lot, and the time until completion of all lots, respectively.

Table 2. Total lead time of ASEP test 1 through 3 and volatility data

	ASEP test 1	ASEP test 2	ASEP test 3
Read time	1506	1506	1525
σ	0.42	0.36	0.43
σ_k	0.25	0.22	0.24
σ_s	0.16	0.15	0.14
σ_k-σ_s	0.09	0.07	0.1

Table 2 allows comparison of the lead times of PFP test runs 1-3. Terms σ, σ_k, and σ_s in Appendix B denote the total volatility including the consumption time, the volatility in completing a single lot, and the volatility in completing all lots, respectively. From Table 2, we observe that ASEP test 2 has small fluctuations in the process and is close to the stationary system. Subsequently, the order of ASEP tests 1 and 3 is set. As a result, ASEP test 2 is close to the maximum current phase.

With respect to the ASEP model, K1 to K5 are worker numbers, and there is no determined work order. The "consumption time" represents the in-process consumption time when continuous production is performed. The total lead time is the processing time of one lot, namely, the total time taken to go through steps 1-5 plus the time for the remaining 11 to pass through step 5. The theoretical value is derived as follows:

$$(5 \times 25) + (25 \times 11) = 400 \tag{18}$$

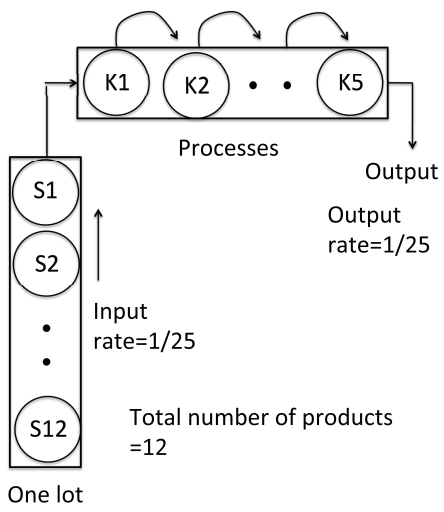

FIGURE 5. ASEP production model

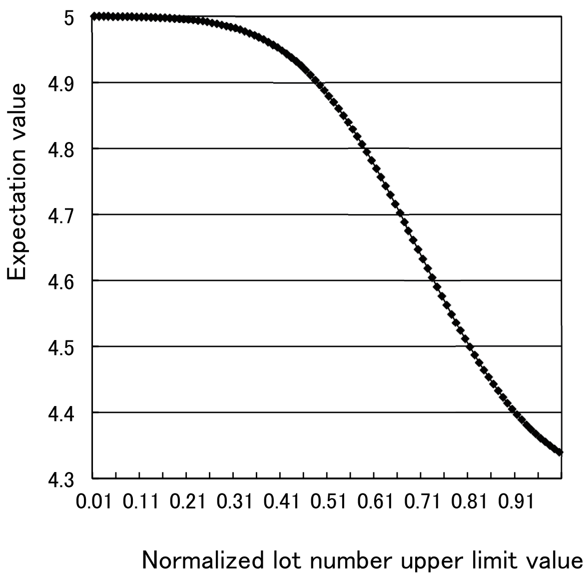

FIGURE 6. Expected processing rate function for normal probability distribution ($\mu = 0.7$, $\sigma = 0.2$)

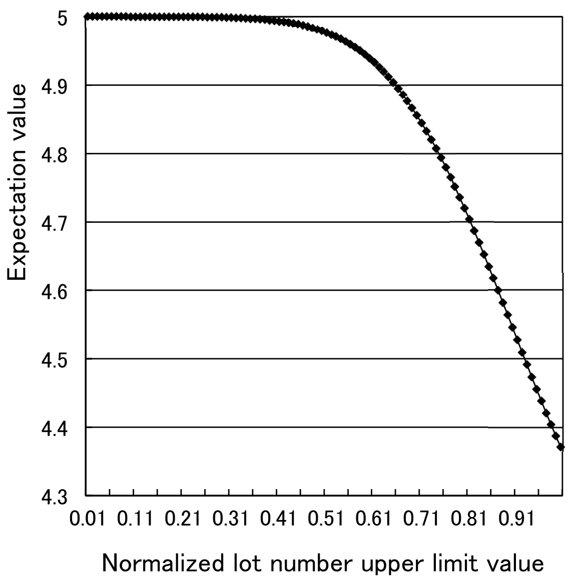

FIGURE 7. Expected processing rate function for normal probability distribution ($\mu = 0.9$, $\sigma = 0.2$)

From Table 2, ASEP test run 2 has small fluctuations in the process and is close to a stationary system. Next, it becomes ASEP test runs 1 and 3. As a result, ASEP test run 2 is close to the maximum distribution density phase.

Terms K1-K5 in Tables 11-16 denote the number of workers. There is no determined order of workers. With respect to the lead time, we obtain the following equation according to Minemura and Nishinari [12]:

$$T = \left[\left(\frac{1}{\frac{1}{25}}\right) + \left(\frac{1}{\frac{1}{25}}\right)\right] + \left(\frac{11}{\frac{1}{25}}\right) + \left(\frac{5}{\frac{1}{25}}\right) \tag{19}$$

where $\frac{1}{25}$ denotes a transition rate. The first and second terms denote an input/output time.

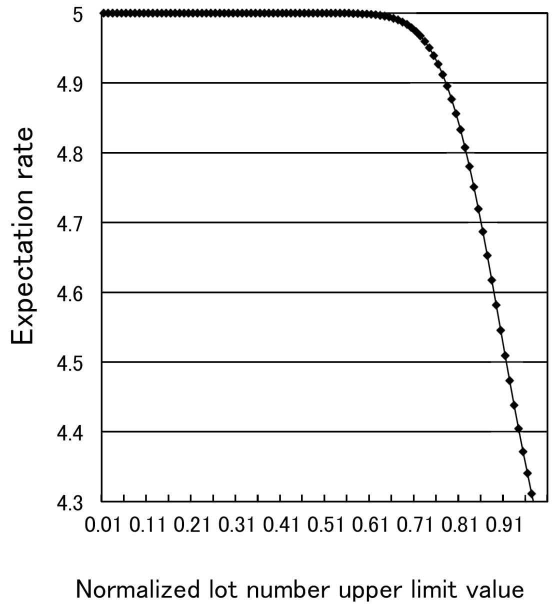

Figure 8. Expected processing rate function for normal probability distribution ($\mu = 0.9$, $\sigma = 0.1$)

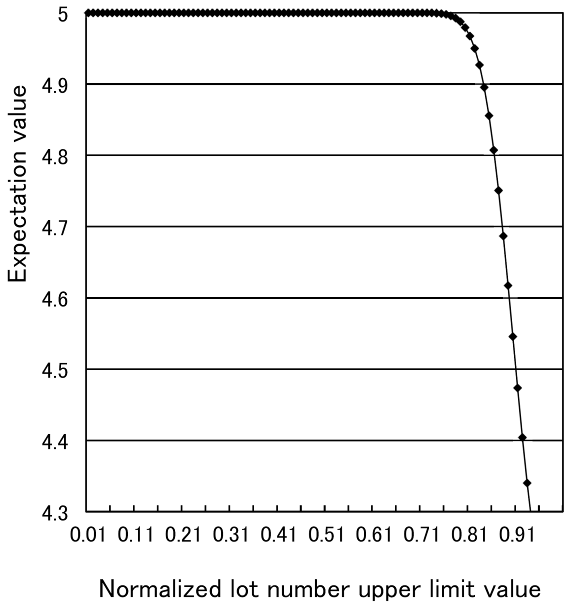

Figure 9. Expected processing rate function for normal probability distribution ($\mu = 0.9$, $\sigma = 0.05$)

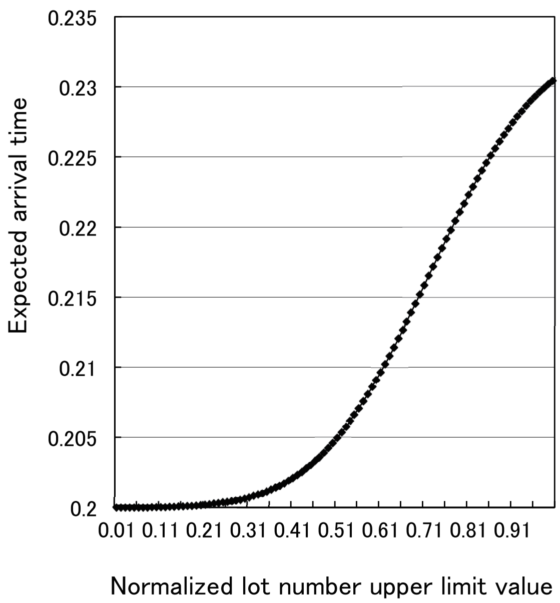

Figure 10. Expected 1-lot transition time for normal probability distribution ($\mu = 0.7$, $\sigma = 0.2$)

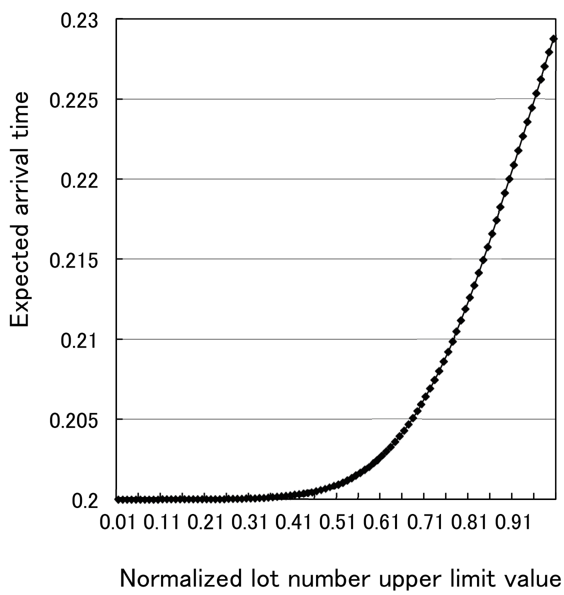

Figure 11. Expected 1-lot transition time for normal probability distribution ($\mu = 0.9$, $\sigma = 0.2$)

With respect to applying the ASEP model to a one-lot flow process, the mathematical model is a nonlinear system represented by the Burgers' equation, which means that the process transition probability plays an important role. Based on our PFP, we assume the following [4, 6]:

- Based on the results of the previous PFP test runs 1-3, make the processing time of each process the same and set it to an appropriate value.
- Arrange the workers in each process appropriately.

As mentioned previously, it is very close to a system that is both linear and stationary.

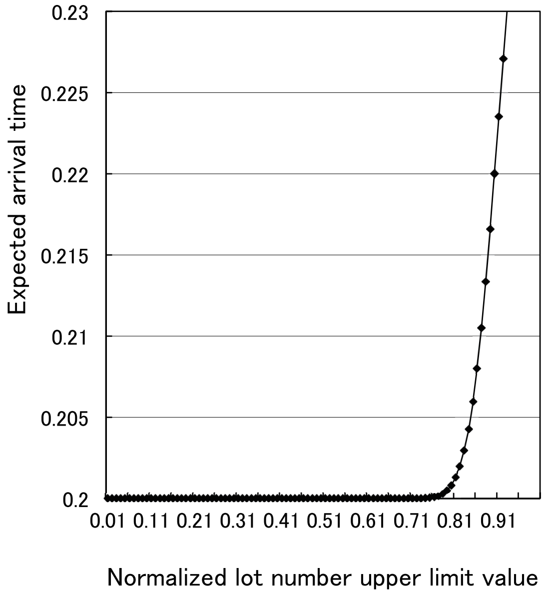

FIGURE 12. Expected 1-lot transition time for normal probability distribution ($\mu = 0.9$, $\sigma = 0.05$)

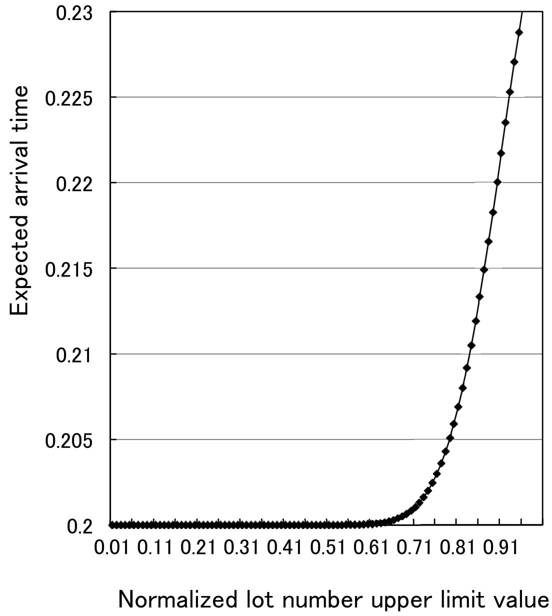

FIGURE 13. Expected 1-lot transition time for normal probability distribution ($\mu = 0.9$, $\sigma = 0.1$)

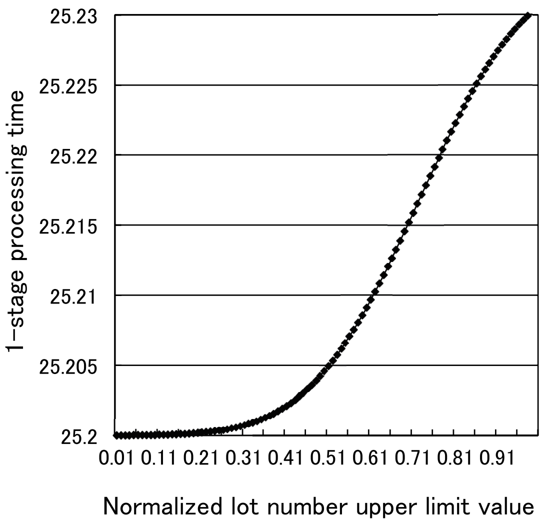

FIGURE 14. 1-stage processing time for normal probability distribution ($\mu = 0.7$, $\sigma = 0.2$)

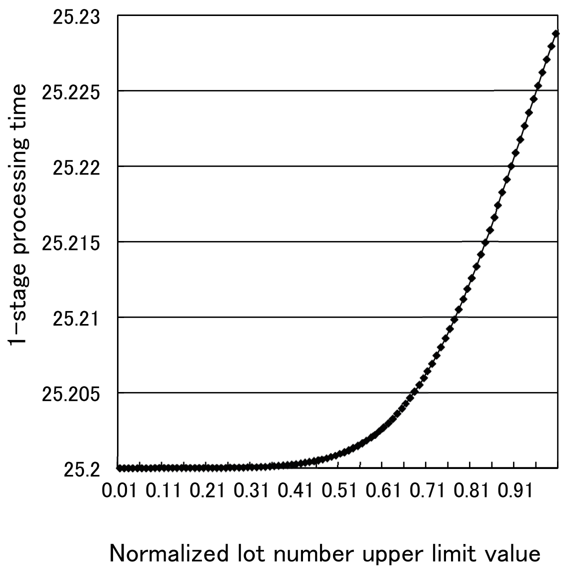

FIGURE 15. 1-stage processing time for normal probability distribution ($\mu = 0.9$, $\sigma = 0.2$)

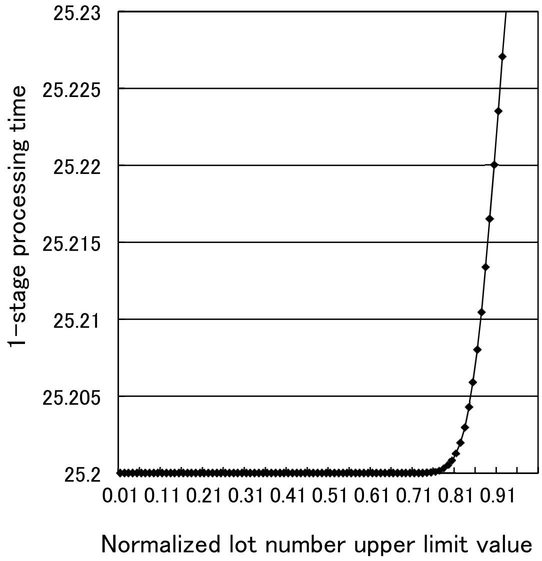

FIGURE 16. 1-stage processing time for normal probability distribution ($\mu = 0.9$, $\sigma = 0.05$)

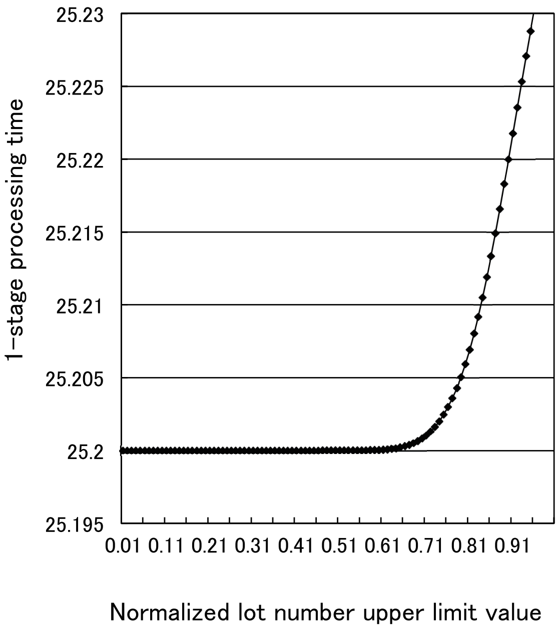

FIGURE 17. 1-stage processing time for normal probability distribution ($\mu = 0.9$, $\sigma = 0.1$)

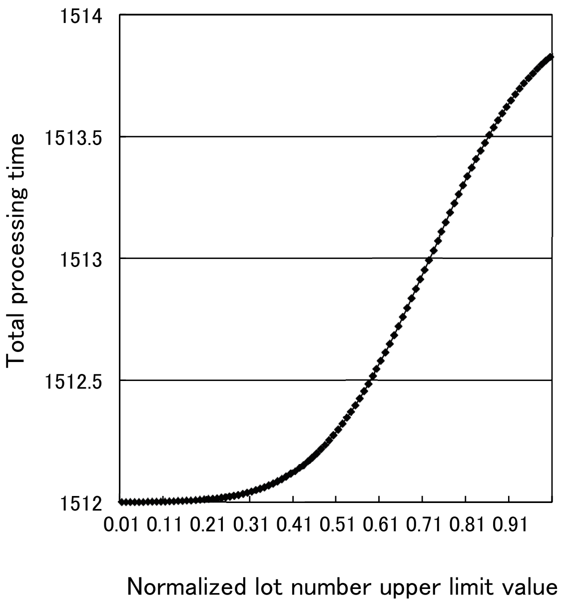

FIGURE 18. Total processing time for normal probability distribution ($\mu = 0.7$, $\sigma = 0.2$)

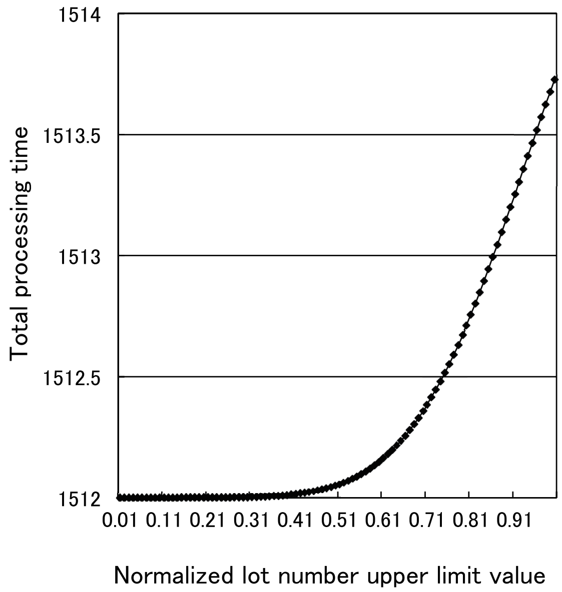

FIGURE 19. Total processing time for normal probability distribution ($\mu = 0.9$, $\sigma = 0.2$)

From Table 3, we observed that the ASEP model achieves a cost that is roughly 20% ($2000/2400 \approx 0.83$) lower than the PFP one, as well as achieving twice the throughput. We represent the PFP data of Table 3 by using Appendix A.

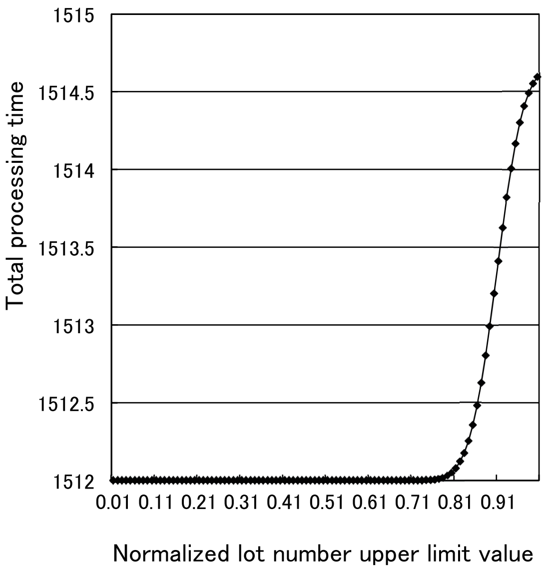

FIGURE 20. Total processing time for normal probability distribution ($\mu = 0.9$, $\sigma = 0.05$)

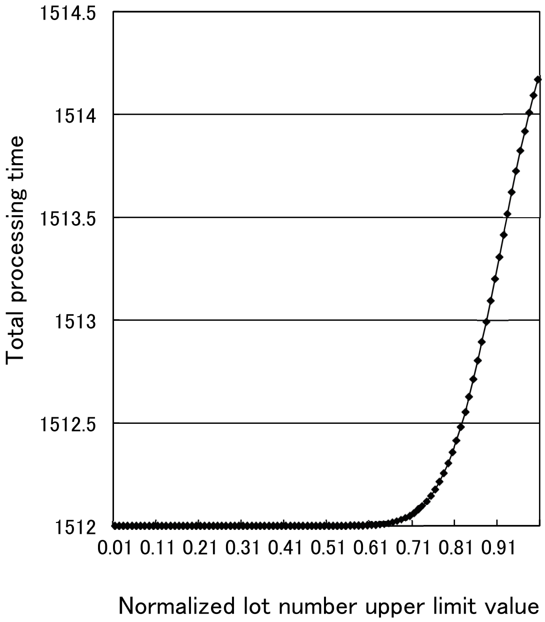

FIGURE 21. Total processing time for normal probability distribution ($\mu = 0.9$, $\sigma = 0.1$)

TABLE 3. Comparison of PFP and ASEP

	PFP	ASEP
Theoretical lead time	$3 \times 120 + 40 = 400$	$25 \times 12 + 4 \times 25 = 400$
Production cost	3 Cycle $\times$ 400 day $\times$ 2 = 2400	5 Cycle $\times$ 400 day $\times$ 1 = 2000
Throughput	6/day	12/day

4. **Conclusions.** We evaluated the ASEP as being effective for improving production efficiency because of the number of lots in the production line. Therefore, we applied the ASEP method to our production processes. Twice the throughput was obtained compared to the conventional PFP method, and the production cost was reduced by 20%. From here on, we intend to adopt the ASEP method for production. When the difficulty level of each process is high, the comparison of throughput between ASEP and PFP will be handed over to the next research.

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Appendix A. Analysis of the Test-run Results (PFP).

- (Test-run1) Because the throughput of each process (S1-S6) is asynchronous, the overall process throughput is asynchronous. In Table 5, we list the manufacturing time (min) of each process. In Table 6, we list the volatility in each process performed by the workers. Finally, Table 5 lists the target times. The theoretical throughput is obtained as $3 \times 199 + 2 \times 15 = 627$ (min). In addition, the total working time in stage S3 is 199 (min), which causes a bottleneck. In Figure 22, we plot the measurement data listed in Table 5, which represents the total working time of each worker (K1-K9). In Figure 23, we plot the data contained in Table 5, which represents the volatility of the working times.
- (Test-run2) Set to synchronously process the throughput. The target time listed in Table 7 is 500 (min), and the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 8 presents the volatility of each working process (S1-S6) for each worker (K1-K9).
- (Test-run3) Introducing a preprocess stage. The process throughput is performed synchronously with the reclassification of the process. As shown in Table 9, the theoretical throughput (not including the synchronization idle time) is 400 (min).

TABLE 4. Correspondence between the table labels and the Test-run number

	Table Number	Production process	Working time	Volatility
Test-run1	Table 5	Asynchronous process	627 (min)	0.29
Test-run2	Table 7	Synchronous process	500 (min)	0.06
Test-run3	Table 9	"Synchronization with preprocess" method	470 (min)	0.03

TABLE 5. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181
Deviation		27	39	54	30	29	36

TABLE 6. Volatility of Table 5

K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

Table 10 presents the volatility of each working process (S1-S6) for each worker (K1-K9). On the basis of these results, the idle time must be set to 100 (min). Moreover, the theoretical target throughput ($\mathrm{T}_\mathrm{s}^{'}$) can be obtained using the "Synchronization with preprocess" method. This goal is as follows:

$$\begin{aligned} T_s &\sim 20 \times 6(\text{First cycle}) + 17 \times 6(\text{Second cycle}) \\ &\quad + 20 \times 6(\text{Third cycle}) + 20(\text{Previous process}) + 8(\text{Idolt-time}) \\ &\sim 370 \text{ (min)} \end{aligned} \tag{20}$$

The full synchronous throughput in one stage (20 min) is

$$T_s^{'} = 3 \times 120 + 40 = 400 \text{ (min)} \tag{21}$$

Using the "Synchronization with preprocess" method, the throughput is reduced by approximately 10%. Therefore, we showed that our proposed "Synchronization with preprocess" method is realistic and can be applied in flow production systems. Below, we represent for a description of the "Synchronization with preprocess".

In Table 9, the working times of the workers K4, K7 show shorter than others. However, the working time shows around target time.

Next, we manufactured one piece of equipment in three cycles. To maintain a throughput of six units/day, the production throughput must be as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25 \text{ (min)} \tag{22}$$

where the throughput of the preprocess is set to 20 (min). In Equation (22), the value 28 represents the throughput of the preprocess plus the idle time for synchronization. Similarly, the number of processes is 8 and the total number of processes is 9 (8 plus the preprocess). The value of 60 is obtained as 20 (min) $\times$ 3 (cycles).

In Table 4, Test-run3 indicates a best value for the throughput in the three types of theoretical working time. Test-run2 is ideal production method. However, because it is difficult for talented worker, Test-run3 is a realistic method.

The results are as follows. Here, the trend coefficient, which is the actual number of pieces of equipment/the target number of equipment, represents a factor that indicates the degree of the number of pieces of manufacturing equipment.

Test-run1: 4.4 (pieces of equipment)/6 (pieces of equipment) = 0.73
Test-run2: 5.5 (pieces of equipment)/6 (pieces of equipment) = 0.92
Test-run3: 5.7 (pieces of equipment)/6 (pieces of equipment) = 0.95
Volatility data represent the average value of each Test-run.

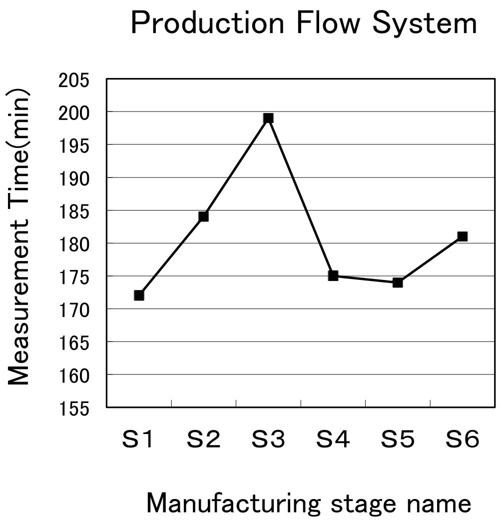

FIGURE 22. Total work time for each stage (S1-S6) in Table 5

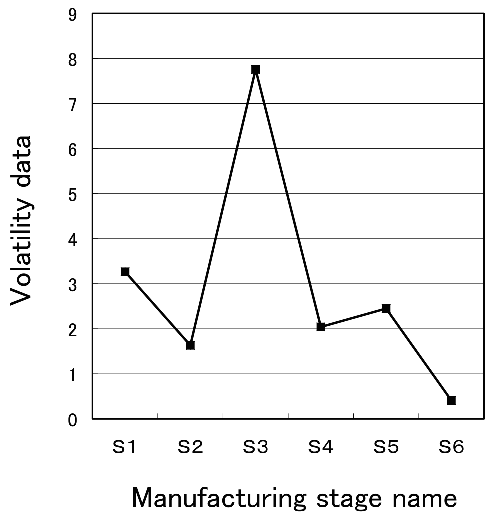

FIGURE 23. Volatility data for each stage (S1-S6) in Table 5

TABLE 7. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180
Deviation		12	16	2	3	2	0

TABLE 8. Volatility of Table 7

K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

TABLE 9. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	20	20	20
K2	20	18	18	18	20	20	20
K3	20	21	21	21	20	20	20
K4	20	13	11	11	20	20	20
K5	20	16	16	17	20	20	20
K6	20	18	18	18	20	20	20
K7	20	14	14	13	20	20	20
K8	20	22	22	20	20	20	20
K9	20	25	25	25	20	20	20
Total	180	165	164	161	180	180	180
Deviation		– 15	–16	–19	0	0	0

TABLE 10. Volatility of Table 9

K1	0.67	0.33	0.67	0	0	0
K2	0.67	0.67	0.67	0	0	0
K3	0.33	0.33	0.33	0	0	0
K4	2.3	3	3	0	0	0
K5	1.3	1.3	1	0	0	0
K6	0.67	0.67	0.67	0	0	0
K7	2	2	2.3	0	0	0
K8	0.67	0.67	0	0	0	0
K9	1.67	1.67	1.67	0	0	0

Appendix B. Analysis of the Test-run Results (ASEP).

TABLE 11. Total manufacturing time at each stage for each worker

	K1	K2	K3	K4	K5	Total
S1	25	25	26	25	25	126
S2	26	25	25	24	24	124
S3	24	25	25	25	25	124
S4	25	26	25	24	24	124
S5	26	25	26	26	25	128
S6	25	25	25	25	25	125
S7	25	24	26	24	26	125
S8	25	26	26	25	25	127
S9	26	25	25	25	25	126
S10	25	25	25	24	25	124
S11	25	25	25	25	25	125
S12	25	26	26	25	26	128
Total	302	302	305	297	300	1506
AVG	25.17	25.17	25.42	24.75	25	125.5

TABLE 12. Volatility of Table 11

	K1	K2	K3	K4	K5	Total
S1	0.028	0.028	0.34	0.063	0	0.25
S2	0.69	0.028	0.17	0.56	1	2.25
S3	1.36	0.028	0.17	0.063	0	2.25
S4	0.028	0.69	0.17	0.56	1	2.25
S5	0.69	0.028	0.34	1.56	0	6.25
S6	0.028	0.028	0.17	0.063	0	0.25
S7	0.028	1.36	0.34	0.56	1	0.25
S8	0.028	0.69	0.34	0.063	0	2.25
S9	0.69	0.028	0.17	0.063	0	0.25
S10	0.028	0.028	0.17	0.56	0	2.25
S11	0.028	0.028	0.174	0.063	0	0.25
S12	0.028	0.69	0.34	0.063	1	6.25
AVG	25.17	25.17	25.42	24.75	25	125.5
σ	0.16	0.16	0.14	0.17	0.17	0.42

TABLE 13. Total manufacturing time at each stage for each worker

	K1	K2	K3	K4	K5	Total
S1	25	25	26	25	25	126
S2	26	25	25	24	24	124
S3	25	25	25	25	25	125
S4	25	26	25	24	24	124
S5	26	25	26	26	25	128
S6	25	25	25	25	25	125
S7	25	24	26	24	26	125
S8	25	26	26	25	25	127
S9	26	25	25	25	25	126
S10	25	25	25	24	25	124
S11	25	25	25	25	25	125
S12	25	26	26	25	25	127
Total	303	302	305	297	299	1506
AVG	25.3	25.2	25.4	24.8	24.9	125.5

TABLE 14. Volatility of Table 13

	K1	K2	K3	K4	K5	Total
S1	0.063	0.028	0.34	0.063	0.007	0.25
S2	0.56	0.028	0.17	0.56	0.84	2.25
S3	0.063	0.028	0.17	0.063	0.007	0.25
S4	0.063	0.69	0.17	0.56	0.84	2.25
S5	0.56	0.028	0.34	1.56	0.007	6.25
S6	0.063	0.028	0.17	0.063	0.007	0.25
S7	0.063	1.36	0.34	0.56	1.18	0.25
S8	0.063	0.69	0.34	0.063	0.007	2.25
S9	0.56	0.028	0.17	0.063	0.007	0.25
S10	0.063	0.028	0.17	0.56	0.007	2.25
S11	0.063	0.028	0.17	0.063	0.007	0.25
S12	0.063	0.69	0.34	0.063	0.007	2.25
AVG	25.3	25.2	25.4	24.8	24.9	125.5
STD	0.13	0.16	0.14	0.17	0.17	0.14

TABLE 15. Total manufacturing time at each stage for each worker

	K1	K2	K3	K4	K5	Total
S1	26	26	26	26	26	130
S2	26	26	25	26	26	129
S3	25	26	25	25	25	126
S4	25	26	25	26	26	128
S5	26	26	26	25	25	128
S6	25	25	25	25	25	125
S7	25	25	26	26	26	128
S8	25	26	26	25	25	127
S9	26	25	25	25	25	126
S10	25	25	25	26	25	126
S11	25	25	25	25	25	125
S12	25	26	26	25	25	127
Total	304	307	305	305	304	1525
AVG	25.3	25.6	25.4	24.4	25.3	127.1

TABLE 16. Volatility of Table 15

	K1	K2	K3	K4	K5	Total
S1	0.45	0.17	0.34	0.34	0.45	8.5
S2	0.45	0.17	0.17	0.34	0.45	3.7
S3	0.11	0.17	0.17	0.17	0.11	1.2
S4	0.11	0.17	0.17	0.34	0.45	0.8
S5	0.45	0.17	0.34	0.17	0.11	0.8
S6	0.11	0.34	0.17	0.17	0.11	4.3
S7	0.11	0.34	0.34	0.34	0.45	0.8
S8	0.11	0.17	0.34	0.17	0.11	0.007
S9	0.45	0.34	0.17	0.17	0.11	1.2
S10	0.11	0.34	0.17	0.34	0.11	1.2
S11	0.11	0.34	0.17	0.17	0.11	4.3
S12	0.11	0.17	0.34	0.17	0.11	0.007
AVG	25.3	25.2	25.4	24.8	24.9	125.5
STD	0.14	0.14	0.14	0.14	0.14	0.4

TABLE 17. Average value of standard deviation of flows to K1-K5 (ASEP)

	Test-run 1	Test-run 2	Test-run 3
S1	0.14	0.15	2.05*
S2	0.94	0.88	1.05*
S3	0.78	0.12	0.38
S4	0.94	0.92	0.42
S5	1.77	1.75	0.42
S6	0.11	0.12	1.05*
S7	0.71	0.75	0.48
S8	0.67	0.68	0.18
S9	0.24	0.22	0.48
S10	0.61	0.62	0.45
S11	0.11	0.12	1.05*
S12	1.67	0.68	0.18
σ_k	0.25	0.22	0.24